- HW1 (due 10/6/05):
  (from textbook) 1.2.3, 1.2.9, 1.2.11, 1.2.12, 1.2.16
  (extra credit) A fashionable country club has 100 members, 30 of whom are lawyers. Rumor has it that 25 of the club members are liars and that 55 are neither lawyers nor liars. What proportion of the lawyers are liars?

- HW2 (due 10/13/05):
  (from textbook)
  1.3.3,1.3.7,1.3.15,1.4.2,1.4.8,1.4.18,1.4.25,1.4.26,1.4.27
  (extra credit) The game of craps is played by letting the throwers toss two fair die twice until he either wins or loses. The thrower wins on the first toss if he gets a total of 7 or 11; he loses on the first toss if he gets a total of 2,3, or 12. If he get any other total on his first toss, that total is called his "point". He then tosses the dice repeatedly until he obtains a total of 7 or his point. He wins if he gets his point and loses if he gets a total of 7. What is the thrower's probability of winning?

- HW3 (due 10/20/05):
  (from textbook)
  1.5.4(c),1.5.8,1.6.3,1.6.8,1.7.7,1.7.22,1.7.24,1.8.7,1.8.10
  (from Instructor) A continuous random variable X has p.d.f given by $f(x) = cxe^{-x^2}$ for $x > 0$, and zero elsewhere.
  (a) Find c
  (b) Find the CDF functions and sketch the function
  (c)Find E(X)

- Practice Problem for Midterm
  1.Bowl I contains 7 red and 3 white chips and bowl II has 4 red and 6 white chips. Two chips are selected at random and without replacement from I and transferred to II. Three chips are then selected at random and without replacement from II.
  (a) What is the probability that all three are white?
  (b) Given that three white chips are selected from II, what is the conditional probability that two white chips were transferred from I?

2. 1.5.6

3. 1.6.2, 1.6.7

4. 1.7.1, 1.7.6, 1.7.12, 1.7.18, 1.7.20, 1.7.23

5. 1.8.3, 1.8.4, 1.8.6, 1.8.8, 1.8.11, 1.8.14

6. 1.9.25

7. If X is a discrete r.v. with pmf $p(x) = pq^x$, x=0,1,2,3,...,
$0 \leq p \leq 1$, q=1-p.
(a) Find the mgf of X, E(X) and Var(X).
(b) Use Markov's inequality to prove that $P(X \geq \frac{3q}{p}) \leq \frac{1}{3}$
(c) Use Chebyshev's inequality to prove that
$P(|X - \frac{q}{p}| \geq \sqrt{pq}|) \leq \frac{1}{p^3}$

8. If X is a discrete r.v. with pmf $p(x) = \frac{e^{-\theta}\theta^x}{x!}$, x=0,1,2,3,...,find E(X),Var(X) and mgf M(t) of X.

9. 2.1.6, 2.1.8, 2.1.9, 2.1.10, 2.1.13, 2.1.14, 2.1.15, 2.1.16

10. 2.2.1, 2.2.2, 2.2.3, 2.2.4, 2.2.5, 2.2.6, 2.2.7

- HW4 (due 11/10/05):

  1. exercise 2.3.2,2.3.3,2.3.7,2.3.9,2.4.10

2. Consider a sample of size 2 drawn without replacement from an urn containing three balls, numbered 1,2 and 3. Let X be the smaller of the two numbers drawn and Y the larger of the two numbers drawn.

(a) Find the joint pmf of X and Y

(b) Find the marginal pmf $p_X(x)$ for X and $p_Y(y)$ for Y

(c) Find cov(X,Y)

(d) Are X and Y independent?

3. Consider two random variables X and Y having a joint pdf $f(x, y) = 0.5xy$ for $0 < y < x < 2$

(a)draw the joint range of X and Y

(b) Find marginal pdf $f_X(x)$ for X and $f_Y(y)$ for Y, respectively

(c) Are X and Y independent?

4. Suppose that X and Y are jointly continuous r.v.'s, $f(y|x) = 1$ for $x < y < x + 1$, and 0 otherwise, and f(x)=1 for $0 < x < 1$ and 0 otherwise.

(a) draw the joint range of X and Y

(b) Find the joint pdf f(x,y)

(c) Find E(Y), cov(X,Y)

(d) Find $P(X+Y<1)$

(e) Find $f(x|y)$

5. Suppose X and Y have joint pdf $f(x,y) = e^{-y}$ for $0 < x < y$, and 0 otherwise.

(a) find the joint mgf of X and Y

(b) using joint mgf, find ρ, the correlation coefficient of X and Y.

- HW5 (due 11/22/05):
  Exercise 2.5.4, 2.5.8, 2.5.13, 2.6.8, 2.7.5, 2.7.7, 2.7.8, 3.1.19, 3.1.21, 3.2.8, 3.2.13, 3.2.14

- HW6 (due 12/01/05):
  Exercise 3.3.6, 3.3.15, 3.3.24, 3.3.27, 3.4.10, 3.4.14, 3.4.23, 3.4.29, 3.4.31, 3.6.10, 3.6.14, 3.7.11

- **Practice Problem for Final Exam**

1. For each of events, L,M and N, the probability that the event occurs is 1/2, 1/4, 1/8 respectively
(a) if L,M,N are mutually exclusive, find P[at least one event occurs]
(b) if L,M and N are independent, find P[exactly one event occurs]
(c) if $P(L|M) = 1/3$, find $P[M|L]$

2. A parallel system functions whenever at least one of its components works. Consider a parallel system of 3 components and suppose that these components independently work with probabilities 0.7, 0.8, and 0.9.
(a) What is the probability that the system is functioning?
(b) Find the conditional probability that component 1 works given that the system is functioning.

3. A pocket contains five coins, two of which have a head on both sides, while the other three coins are normal. A coin is chosen at random from the pocket and tossed three times
(a) Find the probability of obtaining three heads
(b) If a head turning up all three times, what is the probability that the chosen coin is a normal coin.

4. There are 5 blue balls, 4 red balls in an urn.
a) If you pick 4 balls WITHOUT replacement, what is the probability of drawing (exactly) 2 blue balls? Let X be the number of blue balls in the 4 balls chosen, what distribution that X has?
b) If you draw 4 balls WITH replacement, what is the probability of drawing (exactly) 2 blue balls? Let X be the number of blue balls in the 4 balls chosen, what distribution that X has?

5. A penny and a dime are tossed. Assume that both of the coins are fair. Let X denote the number of heads up. Then the penny is tossed again. Let Y denote the number of heads up on the dime from the first toss and the penny from the third toss.
(a) Find the joint pmf of X and Y and write it in a rectangular array
(b) Find the marginal pmf of X, and marginal pmf of Y
(c) Find the conditional mean and variance of Y, given X=1

6. X and Y are discrete random variables whose joint pmf is given in the table

		X		
		-1	0	1
	-1	1/45	2/45	3/45
Y	0	4/45	5/45	6/45
	1	7/45	8/45	9/45

(a) Find the marginal pmf $p_X(x)$ and $p_Y(y)$ for all possible values of X and Y, respectively
(b) Are the events "X=0" and "Y=1" independent? What about the events X=-1 and Y=-1? Are X and Y independent random variables?
(c) compute $E(X|Y = -1)$.
(d) deduce the correlation coefficient $\rho(X, Y)$

7. Jean is waiting for a ride home from the mall. The time to wait for the next taxi is an exponential random variable with mean 2 minutes. The time to wait for a shuttle bus is an exponential random variable with mean 5 minutes. Taxis and shuttle buses arrive independently, and independently of each other. Let the random variable Z be defined as the time to wait for the next taxi or shuttle bus. And let the random variable W take on the value 1 or 0, depending on whether the next vehicle to arrive is a taxi or shuttle bus, respectively.

a. Derive the CDF for Z

Hints: let X be the waiting time for a taxi, and Y be the waiting time for a shuttle bus. Thus, Z=min(X,Y)

b. Derive the pdf for W

c. Derive $Pr(Z < 10|W = 1)$

d. Are W and Z independent? Why or why not?

8. Let X and Y be two r.v.'s with the joint pdf given by $f(x, y) = 3x$ if $0 < y < x < 1$ and is 0 otherwise.

(a) Find $P(X + Y < 1)$.

(b) let U=-lnX, V=-lnY. Find the joint pdf of U and V using the transformation technique.

(c) Find marginal pdf $f_U(u)$ and $f_V(v)$.

(d) Find the conditional distribution of V given U=u and calculate $P(V \leq 4|U = 1)$.

(e) Find $E(V|U = 1)$

9. Let X,Y,Z have joint pdf
$f(x, y, z) = 2(x + y + z)/3, 0 < x < 1, 0 < y < 1, 0 < z < 1$, zero elsewhere
(a) Find the marginal pdf of X,Y,Z
(b) Compute $P(0 < X < 0.5, 0 < Y < 0.5, 0 < Z < 0.5)$ and $P(0 < X < 0.5) = P(0 < Y < 0.5) = P(0 < Z < 0.5)$
(c) Are X,Y,Z independent?
(d) Calculate $E(X^2YZ + 3XY^4Z^2)$
(e) Find the conditional pdf $f(x, y|z)$ and evaluate $E(X + Y|z)$
(f) Determine the conditional pdf $f(x|y, z)$ and compute $E(X|y, z)$

10. Let X_1 denote a random variable N(0,1) and X_2 denote a random variable $\chi^2(5)$; and let X_1, X_2 be independent.
(a) Find the joint p.d.f. of $Y_1 = X1/\sqrt{X_2/5}$ and $Y_2 = X_2$
(b) Find the marginal p.d.f's $f_{Y_1}(y_1)$, Name this distribution.

11. Suppose that random variable Y_1 and Y_2 follow the normal distribution with $\mu = 0$ and $\sigma = 2$, and are independent.
(a) Find the MGF for Y_1
(b) Using MGF, prove that $E(Y_1) = 0$ and $Var(Y_1) = 4$
(c) Using moment generating function technique, find the distribution function for $\bar{Y} = (Y_1 + Y_2)/2$

12. In the United Kingdom, one person in 250 has the rare blood type AB-. A random sample of 460 blood donors is chosen from the population. Let X represent the number of donors in the sample having the rare blood type AB-.
(a) State the distribution of X.
(b) Give a suitable approximation to this distribution. Find the parameter, λ of this approximate distribution
(c) Using this approximate distribution, calculate the probability that in the sample of 460 donors at least two are AB-.
(d) A hospital urgently requires AB- blood. How large a random

sample of donors must be taken in order that the probability of finding at least one donor of type AB- should be at minimum 0.99?

13. There are two candidates A and B in an election. Voters for A and voters for B arrive at the poll station according to two independent Poisson processes of rates 100,150 voters per hour, respectively.
a) Consider a period of 5 hour, what's the expected number of voters for B to arrive at the poll station? Name the distribution of the number of voters for B to arrive within 5 hours.
c) Let T be the arrival time of the first voter. Find the pdf of T and name the distribution

14. If E(Y)=28 and $E(Y^2) = 793$, use Chebyshev's Inequality determine
(1) an upper bound for $P(|Y - 28| > 6)$;

(2) a lower bound for $P(19 < Y < 37)$

15. The random variables $Y_{11}, ..., Y_{1n_1}$ are independent $N(\mu_1, \sigma^2)$ and $Y_{21}, ..., Y_{2n_2}$ are independent $N(\mu_2, \sigma^2)$. For i=1,2 let

$$\bar{Y}_i = \frac{\sum_{j=1}^{n_i} Y_{ij}}{n_i}$$

$$S_i^2 = \frac{\sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2}{(n_i - 1)}$$

be the means and variances of the two sets of variables.

(1) Write down without proof the distributions of

$\bar{Y}_1, \bar{Y}_2, (n_1 - 1)S_1^2/\sigma^2, (n_2 - 1)S_2^2/\sigma^2$

(2) Using properties of normal and χ^2 distributions, deduce the distributions of $(\bar{Y}_1 - \bar{Y}_2)$ and $(n_1 + n_2 - 2)S^2/\sigma^2$, where

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Carefully stating any independence results used.

(3) Use the above results to explain why the random variable

$$T = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{S\sqrt{(\frac{1}{n_1} + \frac{1}{n_2})}}$$

has the $t_{n_1+n_2-2}$ distribution

16. Textbook Exercise 3.7.4