Homework # 6

P5.41* This is a capacitive load because the reactance is negative.

\[ P = I_{rms}^2 R = (15)^2 100 = 22.5 \text{ kW} \]
\[ Q = I_{rms}^2 X = (15)^2 (-50) = -11.25 \text{ kVAR} \]
\[ \theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}(-0.5) = 26.57^\circ \]

\[ \text{power factor} = \cos(\theta) = 89.44\% \]

P5.44*

\[ I = \frac{1000\sqrt{2} \angle 0^\circ}{100} + \frac{1000\sqrt{2} \angle 0^\circ}{-j265.3} = 14.14 + j5.331 \]
\[ = 15.11\angle 20.66^\circ \]
\[ P = V_{rms} I_{rms} \cos \theta = 10 \text{ kW} \]
\[ Q = V_{rms} I_{rms} \sin \theta = -3.770 \text{ kVAR} \]

\[ \text{Apparent power} = V_{rms} I_{rms} = 10.68 \text{ kVA} \]
\[ \text{Power factor} = \cos(20.66^\circ) = 0.9357 = 93.57\% \text{ leading} \]

P5.47* Load A:

\[ P_A = 10 \text{ kW} \]
\[ \theta_A = \cos^{-1}(0.9) = 25.84^\circ \]
\[ Q_A = P_A \tan \theta_A = 4.843 \text{ kVAR} \]

Load B:

\[ V_{rms} I_{Brms} = 15 \text{ kVA} \]
\[ \theta_B = \cos^{-1}(0.8) = 36.87^\circ \]
\[ Q_B = V_{rms} I_{Brms} \sin(\theta_B) = 9 \text{ kVAR} \]
\[ P_B = V_{rms} I_{Brms} \cos(\theta_B) = 12 \text{ kW} \]

Source:

\[ P_s = P_A + P_B = 22 \text{ kW} \]
\[ Q_s = Q_A + Q_B = 13.84 \text{ kVAR} \]

\[ \text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 26 \text{ kVA} \]

\[ \text{Power factor} = \frac{P_s}{\text{Apparent power}} = 0.8462 = 84.62\% \text{ lagging} \]
P5.49* (a) \[ \cos \theta = 0.25 \]
\[ \theta = 75.52^\circ \]
\[ P = V_{\text{rms}} I_{\text{rms}} \cos(\theta) \]
\[ I_{\text{rms}} = \frac{P}{V_{\text{rms}} \cos(\theta)} = \frac{100 \text{ kW}}{1 \text{kV}(0.25)} = 400 \text{ A} \]
\[ I = 400 \sqrt{2} \angle -75.52^\circ \]

(b) \[ Q_{\text{load}} = V_{\text{rms}} I_{\text{rms}} \sin \theta = 387.3 \text{ kVAR} \]
\[ Q_{\text{total}} = 0 = Q_{\text{load}} + Q_c \]
\[ Q_c = -387.3 \times 10^3 = \frac{(V_{\text{rms}})^2}{X_c} \]
\[ X_c = -2.582 = -\frac{1}{\omega C} \]
\[ C = 1027 \mu \text{F} \]

The capacitor must be rated for at least 387.3 kVAR. With the capacitor in place, we have:
\[ P = 100 \text{ kW} = V_{\text{rms}} I_{\text{rms}} \]
\[ I_{\text{rms}} = 100 \text{ A} \]
\[ I = 100 \angle 0^\circ \]

(c) The line current is smaller by a factor of 4 with the capacitor in place, reducing \( I^2R \) losses in the line by a factor of 16.

P5.52* (a) Zeroing the current source, we have:

Thus, the Thévenin impedance is
\[ Z_t = 100 + j50 = 111.8 \angle 26.57^\circ \Omega \]

Under open circuit conditions, there is zero voltage across the inductance, the current flows through the resistance, and the Thévenin voltage is
\[ V_r = V_{dc} = 200 \angle 0^\circ \]
\[ I_n = \frac{V_r}{Z_l} = 1.789 \angle -26.57^\circ \]

Thus, the Thévenin and Norton equivalent circuits are:

(b) For maximum power transfer, the load impedance is

\[ Z_{load} = 100 - j50 \]

\[ I_{load} = \frac{V_r}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 100 - j50} = 1 \]

\[ P_{load} = R_{load} (I_{rms-load})^2 = 100(1/\sqrt{2})^2 = 50 \text{ W} \]

(c) In the case for which the load must be pure resistance, the load for maximum power transfer is

\[ Z_{load} = |Z_t| = 111.8 \]

\[ I_{load} = \frac{V_r}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 111.8} = 0.9190 \angle -13.28^\circ \]

\[ P_{load} = R_{load} (I_{rms-load})^2 = 47.21 \text{ W} \]

P5.55* For maximum power transfer, the impedance of the load should be the complex conjugate of the Thévenin impedance:

\[ Z_{load} = 10 - j5 \]

\[ Y_{load} = \frac{1}{Z_{load}} = 0.08 + j0.04 \]

\[ Y_{load} = \frac{1}{R_{load}} + j\omega C_{load} = 0.08 + j0.04 \]

Setting real parts equal:

\[ \frac{1}{R_{load}} = 0.08 \]

\[ R_{load} = 12.5 \Omega \]

Setting imaginary parts equal:

\[ \omega C_{load} = 0.04 \]

\[ C_{load} = 106.1 \mu\text{F} \]
P5.58*  
\[ V_L = \sqrt{3} \times V_y = \sqrt{3} \times 440 = 762.1 \text{ V rms} \]
\[ I_L = \frac{V_y}{R} = \frac{440}{30} = 14.67 \text{ A rms} \]
\[ P = 3V_yI_L \cos(\theta) = 3 \times 440 \times 14.67 \times \cos(0) = 19.36 \text{ kW} \]

P5.59*  
\[ Z_y = \frac{1}{1/R + j\omega C} \]
\[ = \frac{1}{1/50 + j377 \times 10^{-4}} \]
\[ = 10.98 - j20.70 \]
\[ = 23.43 \angle -62.05^\circ \]

\[ Z_\Delta = 3Z_y \]
\[ = 70.29 \angle -62.05^\circ \text{ \Omega } \]

P5.63*  
This is a positive sequence source. The phasor diagram is shown in Figure 5.40 in the book. Thus, we have:

\[ V_an = \frac{440 \sqrt{2}}{\sqrt{3}} \angle 0^\circ \]

The impedance of a equivalent wye-connected load is

\[ Z_y = \frac{Z_\Delta}{3} = 1.667 - j0.6667 \text{ \Omega } \]

The equivalent circuit for the a-phase of an equivalent wye-wye circuit is:

![Diagram](image)

Thus, the line current is:
\[ I_{an} = \frac{V_{an}}{0.5 + j0.5 + Z_y} \]
\[ = 116.9\sqrt{2} \angle 4.40^\circ \]
\[ V_{An} = V_{an} - I_{an}(0.5 + j0.5) \]
\[ = 209.8\sqrt{2} \angle -17.40^\circ \]
\[ V_{AB} = 363.4\sqrt{2} \angle 12.60^\circ \]
\[ I_{AB} = \frac{V_{AB}}{Z_\Delta} \]
\[ = 67.49\sqrt{2} \angle 34.40^\circ \]
\[ P_{load} = 3(I_{ABrms})^2 \times 5 = 68.32 \text{ kW} \]
\[ P_{line} = 3(I_{Arms})^2 \times 0.5 = 20.50 \text{ kW} \]