Endogenous TFP, Labor Market Policies and Loss of Skills

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Workshop to celebrate Ricardo Lagos’s research
UC Irvine
Motivation

- Large differences in TFP among OECD countries
  - What determines TFP?

- Search frictions in labor market
  - Average productivity of formed matches
    - affect TFP

- Unemployed workers suffer large productivity losses
  - If more, longer unemployment spells
    - economy less productive

Question:
TFP with search frictions & skill loss during unemp?
Effect of labor market policies on TFP and u?
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Question:

- TFP with search frictions & skill loss during unemp?
- Effect of labor market policies on TFP and $u$?
Why skill loss?

- 1960-1995
  - high UI in EU vs US
  - \( u \uparrow \) in EU, \( u \sim \text{constant} \) in US
  - TFP \( \uparrow \) in EU relative to US
Why skill loss?

- 1960-1995
  - high UI in EU vs US
  - $u \uparrow$ in EU, $u \sim$ constant in US
  - TFP $\uparrow$ in EU relative to US

- Theories emphasizing average productivity & role of policy (UI) successful, if UI $\uparrow$
  $\Rightarrow$ better/productive matches formed
  $\Rightarrow$ TFP $\uparrow$
Why skill loss?

- 1960-1995
  - high UI in EU vs US
  - $u \uparrow$ in EU, $u \sim$ constant in US
  - TFP $\uparrow$ in EU relative to US

- Theories emphasizing average productivity & role of policy (UI) successful, if UI $\uparrow$
  $\Rightarrow$ better/productive matches formed
  $\Rightarrow$ TFP $\uparrow$

- However, 1995-onwards
  - UI = or $\uparrow$ in EU vs US
  - $u \sim$ constant or $\uparrow$ in EU, $\sim$ constant in US
  - TFP $\downarrow$ in EU relative to US
This paper

- Model of TFP à la Lagos (2006)
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- Aggregate production, TFP determined by
  - Productivity of matches formed/active
    → reservation productivity
  - Aggregate distribution of skills
    → job finding rate, reservation productivity
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    → job finding rate, reservation productivity
- Job finding rate, reservation productivity
  → “sufficient” statistics for TFP
**Main Results**

- Two opposing channels
  - Average productivity
    - Reservation productivity $\uparrow$
      $\Rightarrow$ raises TFP (average matches more productive)
  - Skill channel
    - Reservation productivity and job finding rate
      $\Rightarrow$ compositional effect on skill distribution
      $\Rightarrow$ affect TFP

- Effects of UI
  - Small on skill distribution, large on reserv value $\Rightarrow$ TFP $\uparrow$
  - Large on distribution, small on reserv value $\Rightarrow$ TFP $\downarrow$

- Additional mechanism joint behavior of UI, $u$, TFP
  - Quantitatively, skill loss affect impact of UI? (wip...)
Related literature

■ TFP and labor market frictions

■ Search and skill loss

■ Development accounting, no search
Related literature

- Job displacement

- Loss of skills, further evidence
  - motherhood and earnings: Mincer (1979 JPE), Mincer & Ofek (1982 JHE)
  - test scores: Estin Gustavsson (2008)
  - breaks in production: health services, David & Brachet (2011 AEJ Micro), Hockenberry (2014 JHE); data entry, Globerson et al (1989 IIE); mechanical assembly, Bailey (1989 MS); car radio production, Shafer (2001 MS)

- Time continuous, discount rate $r$

- Agents, risk averse
  - Workers
  - Firms

- Firms post vacancies to find workers

- Unemployed workers search for jobs
SEARCH & MATCHING FRICCTIONS

- Matching function $m(u, v)$
  - Unemployed $u$, vacancies $v$
  - Market tightness $\theta = v/u$

- Finding rates
  - Workers: $\theta q(\theta) = \frac{m(u,v)}{u}$
  - Firms: $q(\theta) = \frac{m(u,v)}{v}$

- Exogenous separation at rate $s$

- Workers leave labor force/die at rate $\mu$
  - Ensures stationarity of endogenous skill distribution
Production function, firm level

- Lagos (2006), Houtthaker (1955)
  - fixed proportion technologies/Leontieff

- Match production technology: $f(x, n, k) = x \min(n, k)$
  - $x$: match quality, $x \sim G(\cdot)$
  - $n$: hours
  - $k$: capital

- Poisson rate $\lambda$
  - $\rightarrow$ shock to match quality, new draw from $G(\cdot)$

- $k$ capture scale of operation
  - assumption, all projects same scale
Skill Loss

- Workers suffer skill loss during unemployment

- Two skill levels: \( H, L \)
  - Workers born \( H \)
  - If \( H \) unemployed
    \[ \rightarrow \text{become} \; L \; \text{at rate} \; \sigma \]
  - Empirical evidence
    \[ \rightarrow \text{skill loss very persistent, does not wash away} \]
  - Tractable, closed form endogenous TFP
    (but can be generalized)

- Low skill \( L \) output
  - \( \delta f(x, n, k) \), with \( 0 < \delta < 1 \)
Endogenous skill distribution

- Skill distribution
  \[ \to \text{TFP through agg human capital} \]

- Unemployed: \( u_H, u_L \)

- Employed: \( e_H, e_L \)

- Skill distribution
  - Unemployed: \( \Delta_u = u_H/u \)
  - Employed: \( \Delta_e = e_H/e \)
Bellman equations: unemployed workers

- Non-market time
  - $H$ unemp workers: $b$
  - $L$ unemp workers: $b\delta$

- Bellman unemployed

\[
(r + \mu)U_H = b + \theta q(\theta) \int \max\{W_H(z) - U_H, 0\} dG(z) \\
+ \sigma(U_L - U_H)
\]

\[
(r + \mu)U_L = b\delta + \theta q(\theta) \int \max\{W_L(z) - U_L, 0\} dG(z)
\]
Bellman equations: employed workers

- Wages $w_i(x)$, $i = H, L$

- Bellman employed, $i = H, L$

\[(r + \mu)W_i(x) = w_i(x) + \lambda \int \max\{W_i(z) - U_H, 0\} dG(z)\]

\[\quad - (\lambda + s)(W_i(x) - U_i)\]
Bellman equations: firms

- Profits $\pi_i(x), i = H, L$

- Bellman filled job, $i = H, L$

\[(r + \mu)J_i(x) = \pi_i(x) + \lambda \int \max\{J_i(z) - V, 0\} dG(z)\]
\[- (\lambda + s)(J_i(x) - V),\]

- Bellman vacancy

\[rV = -ck + q(\theta)[\Delta_u \int \max\{J_H(z) - V, 0\} dG(z) + (1 - \Delta_u) \int \max\{J_L(z) - V, 0\} dG(z)]\]

- Assume

  - free entry for vacancies $\Rightarrow V = 0$
  - capital must be pre-installed, rental cost $c$
Firms profits, $H$ workers

- Similar to Lagos (2006)

- Revenue: $f(x, n, k)$

- Costs:
  - Rental cost of capital: $ck$
  - Variable cost: $\phi n$
  - “Fixed” cost: $C(x, \phi)k = \max\{\phi - x, 0\}$
    \[ \Rightarrow \text{non-increasing in } x \text{ is enough, Lagos (2006)} \]

\[ \Rightarrow \pi_H(x) = f(x, n, k) - w_H(x) - ck - \phi n - C(x, \phi)k \]
Firms profits, \( L \) workers

- Revenue: \( \delta f(x, n, k) \)

- Tractability, costs proportional too (not essential)
  - Rental cost of capital: \( \delta ck \)
  - Variable cost: \( \delta \phi n \)
  - “Fixed” cost: \( \delta C(x, \phi)k \)

\[
\Rightarrow \pi_L(x) = \delta f(x, n, k) - w_L(x) - c\delta k - \phi \delta n - \delta C(x, \phi)k
\]

- If assume same costs
  - \( \rightarrow \) larger effect of skill loss on TFP (share \( L \uparrow \))
**Equilibrium**

- **Optimal hours**

  \[ n(x) = \begin{cases} 
  k, & \text{if } \phi < x \\
  0, & \text{if } \phi \geq x 
  \end{cases} \]

- **Note: choice of hours**

  ≠ job destruction decision (hoarding possible)

- **Profits**

  \[ \pi^H(x) = (x - c - \phi)k - w^H(x). \]
  \[ \pi^L(x) = (x - c - \phi)\delta k - w^L(x). \]
**Surplus**

- Surplus: \( S^i(x) = J^i(x) + W^i(x) - U^i - V, \ i = L, H \)
  - Increasing in \( x \)
  - \( \exists! \ R_i \) such that \( S^i(R_i) = 0, \ i \in \{H, L\} \)

- Assume NB over wages,
  \( \beta \) worker bargaining strength

\[
w_i(x) = \arg \max_{w_i(x)} [W_i(x) - U_i]^\beta [J_i(x) - V]^{1-\beta}.
\]

- Wages

\[
w_H(x) = \beta(x - c - \phi)k + (1 - \beta)(r + \mu)U_H, \ \forall x \geq R_H
\]
\[
w_L(x) = \beta(x - c - \phi)\delta k + (1 - \beta)(r + \mu)U_L, \ \forall x \geq R_L
\]


**Reservation values**

**Proposition**

The reservation productivity is larger for workers with low human capital, i.e. $R_H \leq R_L$

- Useful to derive aggregate TFP.

- Intuition: match with $H \rightarrow$ larger surplus
  
  $\Rightarrow$ firm and worker willing to form less productive matches
  
  $\Rightarrow$ better matches required with $L$ worker for $> 0$ surplus

- $\delta = 1 \Rightarrow R_L = R_H$
Equilibrium \( \{\theta, R_L, R_H, \Delta_u, \Delta_e\} \) satisfies

- **Job Creation Condition (JC):**
  \[ \Rightarrow \text{Free Entry } V = 0 \]

- **Job Destruction Conditions (JDH, JDL):**
  \[ \Rightarrow S^H(R_H) = 0, \ S^L(R_L) = 0 \]

- **Distributions** \( \Delta_u, \Delta_e \) stationary

- Set \( \delta = 1 \) (no skill loss)
  \[ \Rightarrow \text{Lagos (2006)} \]
Job Creation Condition

\[
\frac{c}{q(\theta)} = (1 - \beta) \left[ \Delta_u \int_{R_H} (z - R_H) dG(z) + (1 - \Delta_u) \delta \int_{R_L} (z - R_L) dG(z) \right] \frac{r + \mu + \lambda + s}{\Delta u}
\]

- Intuition
  - Post vacancies until
    - expected vacancy cost = expected value of filled job
**Job Destruction Conditions**

\[
(R_H - \phi - c)k + \lambda \int_{R_H} \frac{z - R_H}{r + \mu + \lambda + s} kdG(z) - (r + \mu)U_H = 0
\]

\[
(R_L - \phi - c)\delta k + \lambda \int_{R_L} \frac{z - R_L}{r + \mu + \lambda + s} kdG(z) - (r + \mu)U_L = 0
\]

where

\[
(r + \mu)U_L = b\delta + \beta f(\theta) \int_{R_L} \frac{z - R_L}{r + \mu + s + \lambda} \delta kdG(z)
\]

\[
U_H = \beta(\theta) \left( \frac{r + \mu}{r + \mu + \sigma} \right) \int_{R_H} \frac{z - R_H}{r + \mu + s + \lambda} kdG(z)
\]

\[
+ \frac{\sigma}{r + \mu + \sigma} \beta f(\theta) \int_{R_L} \frac{z - R_L}{r + \mu + s + \lambda} \delta kdG(z) + \left( \frac{r + \mu + \delta \sigma}{r + \mu + \sigma} \right) b
\]
From flow equation + stationarity

\[ \Delta u = \frac{\mu [f(\theta)(1 - G(R_L)) + s + \lambda G(R_L) + \mu]}{\mu [f(\theta)(1 - G(R_L)) + s + \lambda G(R_L) + \mu] + \sigma (s + \lambda G(R_L) + \mu)} \]

\[ \Delta e = \frac{1}{1 + \frac{e_L}{e_H}} \]

where

\[ \frac{e_L}{e_H} = \frac{1 - G(R_L)}{1 - G(R_H)} \cdot \frac{\sigma}{\mu} \cdot \frac{s + \lambda G(R_H) + \mu}{f(\theta)(1 - G(R_L)) + s + \lambda G(R_L) + \mu} \]
**Existence and Uniqueness**

**Proposition**

Assume \( \eta < \bar{\eta} \) and \( \theta < \bar{\theta} \). Then the equilibrium exists and is unique.

**Intuition**

- Implicit theorem
  
  \[ \Rightarrow \text{can express } R_L = R_L(\theta) \text{ using JD for } L \text{ workers} \]

- Reduce equilibrium to one JD, one JC

- First condition ensures JC downward sloping
  
  If \( \eta \) “too large”, u distribution improves too much when \( \theta \uparrow \)

- Second condition similar to Lagos (2006), ensures crossing
Aggregation: Preview

- Aggregate, TFP depends on
  - match quality
  - human capital distribution

- $R_L, R_H, \theta$ “sufficient” statistics
  - uniquely determine TFP

- Lagos (2006) $\rightarrow$ TFP determined by match quality

- With skill loss $\rightarrow$ TFP depends on match quality and human capital
Distribution of match quality

- CDF observed match quality: $\tilde{G}^H(\cdot), \tilde{G}^L(\cdot)$

- Flow equation, $i \in \{H, L\}, \forall x \geq R_i$

$$
\frac{d[\tilde{G}^i(x)e^i]}{dt} = \lambda e^H [1 - \tilde{G}^i(x)][G(x) - G(R^i)]
+ f(\theta)u^i[G(x) - G(R^i)] - \lambda e^i\tilde{G}^i(x)[1 - G(x)]
- \lambda e^i\tilde{G}^i(x)G(R^i) - (s + \mu)e^i\tilde{G}^i(x)
$$

- Steady-state, $d[\tilde{G}^i(x)e^i]/dt = 0$

$$
\tilde{G}^i(x) = \frac{G(x) - G(R^i)}{1 - G(R^i)} , \text{ for } i \in \{H, L\}.
$$

(1)
AGGREGATION

- Follow Lagos (2006) method

- Aggregate output, capital, hours/labor: $Y, K, N$
  - Relationship b/w aggregate variables?

- Aggregate capital: $K$

  \[ K = [1 - (1 - \theta)u]k \]

  ⇒ Effective capital $K_e = \frac{1-u}{1-(1-\theta)u}K$
**Aggregating output**

- Firm with worker $i \in \{H, L\}$ produces if
  - $x \geq R_i$ (job active/created)
  - $x \geq \phi$ (>0 hours)

- Define: $\mu_i = \max\{R_i, \phi\}, i \in \{H, L\}$

- Aggregate output

$$Y = (1 - \Delta_e)(1 - u) \int_{\mu_L} \delta f(x, n(x), k) d\tilde{G}_L(x)$$

$$+ \Delta_e (1 - u) \int_{\mu_H} f(x, n(x), k) d\tilde{G}_H(x).$$
Similarly, aggregate hours $N$

\[
N = (1 - u)\Delta_e \int_{\mu_H} n(x) d\tilde{G}^H(x)
+ (1 - u)(1 - \Delta_e) \int_{\mu_L} n(x) d\tilde{G}^L(x)
\]
Distribution match quality

- Assume $G(.) \sim$ Pareto, $\varepsilon > 0$, $\alpha > 1$

$$G(x) = \begin{cases} 
0, & \text{if } x < \varepsilon \\
1 - \left(\frac{\varepsilon}{x}\right), & \text{if } \varepsilon \leq x 
\end{cases}$$

- Distribution, observed matches

$$\tilde{G}^i(x) = \begin{cases} 
0, & \text{if } x < R_i \\
1 - \left(\frac{R_i}{x}\right), & \text{if } R_i \leq x 
\end{cases}$$
**Proposition**

Let $\gamma \equiv 1/\alpha$. The economy’s aggregate production function $Y = F(K_e, N)$ satisfies

$$Y = F(K_e, N) = AK_e^\gamma N^{1-\gamma},$$

where $A$ is the economy’s TFP, with

$$A = \begin{cases} 
    A_l = \frac{\Delta e R_H^\gamma + (1 - \Delta e) R_L^\gamma}{\Delta e R_H^\gamma + (1 - \Delta e) R_L^\gamma} \cdot \frac{1}{1-\gamma} \cdot \frac{1}{1-\gamma} & , \text{if } R_H < R_L \leq \phi \\
    A_m = \frac{\Delta e R_H^\gamma + (1 - \Delta e) R_L^\gamma \delta \phi^{-1}}{\Delta e R_H^\gamma + (1 - \Delta e) \delta R_L^\gamma} \cdot \frac{1}{1-\gamma} \cdot \frac{1}{1-\gamma} & , \text{if } R_H < \phi < R_L \\
    A_h = \left[\Delta e R_H + (1 - \Delta e) \delta R_L\right] \cdot \frac{1}{1-\gamma} & , \text{if } \phi \leq R_H < R_L 
\end{cases}$$
Discussion

- Lagos (2006)
  - $R$ sufficient statistic for TFP

- Skill loss
  - $R$ and $\theta$ (job finding rate) sufficient statistics for TFP
Discussion

- Lagos (2006)

\[ A = \frac{R}{1 - \gamma} \]

- Skill loss

\[
A = \begin{cases} 
A_l = \frac{\Delta e R_H^{\frac{1}{\gamma}} + (1 - \Delta e) R_L^{\frac{1}{\gamma}} \delta}{\left[ \Delta e R_H^{\frac{1}{\gamma}} + (1 - \Delta e) R_L^{\frac{1}{\gamma}} \right]^{1 - \gamma}} \cdot \frac{1}{1 - \gamma}, & \text{if } R_H < R_L \leq \phi \\
A_m = \frac{\Delta e R_H^{\frac{1}{\gamma}} + (1 - \Delta e) R_L \delta \phi^{\frac{1}{\gamma} - 1}}{\left[ \Delta e R_H^{\frac{1}{\gamma}} + (1 - \Delta e) \phi^{\frac{1}{\gamma}} \right]^{1 - \gamma}} \cdot \frac{1}{1 - \gamma}, & \text{if } R_H < \phi < R_L \\
A_h = [\Delta e R_H + (1 - \Delta e) \delta R_L] \cdot \frac{1}{1 - \gamma}, & \text{if } \phi \leq R_H < R_L 
\end{cases}
\]

- \( \delta = 1 \) (no skill loss) \( \Rightarrow \) Lagos (2006)
Skill channel

- No skill loss

  - If $R \uparrow \Rightarrow TFP \uparrow$
  - Two economies with same $R$

    $\Rightarrow$ same TFP, even if job finding rates very $\neq$
Skill channel

- No skill loss
  - If $R \uparrow \Rightarrow TFP \uparrow$
  - Two economies with same $R$
    $\Rightarrow$ same TFP, even if job finding rates very $\neq$

- With skill loss, further channel
  - TFP depends on skill distribution $\Delta^e$
    & match quality (i.e. $R_i$)
  - If skill distribution improves $\Delta^e \uparrow$
    $\Rightarrow$ TFP $A \uparrow$ ($\frac{\partial A}{\partial \Delta^e} > 0$)
**Skill channel**

- **No skill loss**
  - If $R \uparrow \Rightarrow TFP \uparrow$
  - Two economies with same $R$
    \[ \Rightarrow \text{same TFP, even if job finding rates very } \neq \]

- **With skill loss, further channel**
  - TFP depends on skill distribution $\Delta^e$
    & match quality (i.e. $R_i$)
  - If skill distribution improves $\Delta^e \uparrow$
    \[ \Rightarrow \text{TFP } A \uparrow (\frac{\partial A}{\partial \Delta^e} > 0) \]

- **Distribution $\Delta^e$ determined by both**
  - Job finding rate $f(\theta)$
  - Reservation values $R_H, R_L$ (\(\frac{\partial \Delta^e}{\partial R_L} > 0\), \(\frac{\partial \Delta^e}{\partial R_H} < 0\))
Labor market policies

- Advantage w/ method in Lagos (2006)
  - $R$ “sufficient” statistic
    → uniquely determines TFP
  - Effect of labor mkt policies?
    Need effect on $R$ alone $\Rightarrow$ TFP
  - If policy $\uparrow R$ $\Rightarrow$ TFP $\uparrow$

- Same with skill loss, except
  → sufficient statistics: $R$ and $f(\theta)$ (skill channel)

- If policy $\uparrow R$ but $\downarrow$ distribution $\Delta_e$ (e.g. if job finding $\downarrow$)
  $\Rightarrow$ overall effect depends on relative size
Labor market policies

- Policies
  - UI ($R_i \uparrow$, $\theta \downarrow$)
  - Hiring subsidy ($R_i \uparrow$, $\theta \uparrow$)
  - Employment subsidy ($R_i \downarrow$, $\theta \uparrow$)
  - Firing tax ($R_i \downarrow$, $\theta \downarrow$)

- In particular, UI may lower TFP
  - Non-linear/monotonic effect of UI on TFP
Skill channel and policy

- EU-US unemployment and TFP behavior

- Labor policy (UI) may raise TFP if
  - raises reservation productivity
  - small effect on job finding rate, skill distribution

- If policy starts affecting skill distribution
  ⇒ may reduce TFP and raise unemployment
CONCLUSION

- Model of endogenous TFP à la Lagos (2006)

- TFP depends
  - Average productivity formed/active matches
  - Skill distribution

- Sufficient statistics
  - reservation productivities
  - job finding rate

- Skill channel ⇒ policy can lower TFP
  
  *even* if average match quality ↑

- Next step: Quantitative exercise
  - Quantitative effect of mkt policy w/ skill loss?