Technical Appendix to “A Note on the Efficiency in the Housing Market with Search Frictions"

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1 Social planner's solution in the model with homogeneous buyers

This section provides the proof for the social planner's allocation in the main text. The Hamiltonian for the social planner's problem is given by

$$H = e^{-rt}\{\varepsilon\tilde{h} - vc^S - ck - v\theta c^B(v\theta) + \lambda_{\tilde{h}}[v\theta m(\theta) - (s+\delta)\tilde{h}] \tag{1}$$

$$+ \lambda_v[c + s\tilde{h} - \delta v - v\theta m(\theta)]\}. \tag{2}$$

The first-order conditions are given by

$$\frac{\partial H}{\partial c} = 0 \Rightarrow \lambda_v = k, \tag{3}$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow (1+\gamma)c^B(b) = (1-\alpha)m(\theta)(\lambda_{\tilde{h}} - \lambda_v), \tag{4}$$

$$\frac{\partial H}{\partial \tilde{h}} = -\dot{\lambda}_{\tilde{h}} + r\lambda_{\tilde{h}} = \varepsilon - (s+\delta)\lambda_{\tilde{h}} + s\lambda_v, \tag{5}$$

$$\frac{\partial H}{\partial v} = -\dot{\lambda}_v + r\lambda_v = -(1+\gamma)\theta c^B(b) - c^S + \lambda_{\tilde{h}}\theta m(\theta) - (\delta + \theta m(\theta))\lambda_v. \tag{6}$$

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Combining the above first-order conditions and observing that at steady state $\dot{\lambda}_{\tilde{h}} = \dot{\lambda}_v = 0$ yields the planner's allocation (13) and (14) in the main text.

To show that the solution to (13) and (14) is indeed the global maximum, re-write the Hamiltonian

$$\hat{H}(v,b,c,h) = \varepsilon\tilde{h} - vc^S - ck - bc^B(b) + \lambda_{\tilde{h}}[M(b,v) - (s+\delta)\tilde{h}] \tag{7}$$

$$+ \lambda_v[c + s\tilde{h} - \delta v - M(b,v)] \tag{8}$$

$$= \varepsilon\tilde{h} - vc^S - ck - bc^B(b) - \lambda_{\tilde{h}}(s+\delta)\tilde{h} + \lambda_v(c + s\tilde{h} - \delta v) \tag{9}$$

$$+ (\lambda_{\tilde{h}} - \lambda_v)M(b,v). \tag{10}$$

Since $bc^B(b)$ is convex, $M(b,v)$ is concave, and for any admissible tuple $\lambda_{\tilde{h}} - \lambda_v = (\varepsilon - (r + \delta)k)/(r + s + \delta)$, it follows that $\hat{H}(v,b,c,h)$ is concave. Thus, the solution is a global maximum.

2 Efficiency with heterogeneity and endogenous participation

In this section we study efficiency in an environment with an alternative entry mechanism. The main departure relative to the framework in section 2 is our assumption that households are heterogenous in how much they value owning a house. The advantage with this mechanism is that we can use data on price dispersion to calibrate the distribution of households' heterogeneity. The main disadvantage, however, is that the framework with heterogeneity generates a counterfactual Beveridge Curve, as we prove below. Despite this shortcoming, the results on efficiency remain remarkably similar to the environment with rising search costs.

Let ε denote the household's individual utility from owning a home, and let x denote the utility from homeownership that is common to all households.[1] Overall, households derive a utility εx from owning a home. Assume that ε follows a known distribution $F(\varepsilon)$. Relative to the model of section 2, buyers now incur a flow of constant costs c^B while searching for a house. Given free entry of buyers, households keep entering the market until the value of becoming a buyer equals zero, the value of their outside option. Intuitively, this free entry condition for buyers pins down

[1] The common component of the utility flow, x, plays no role in our analysis and can, without loss of generality, be normalized to 1. Although beyond the scope of this paper, including it allows one to study the effects of aggregate demand shocks.

the utility of the marginal buyer and determines the endogenous measure of buyers. Given that the value function for buyers is strictly increasing in ε, there exists a unique reservation value ε_R such that only households with $\varepsilon \geq \varepsilon_R$ enter the market.[2]

Let $B(\varepsilon)$ and $H(\varepsilon)$ denote the value functions of a buyer and homeowner with utility ε, and V denote the value function of a vacancy. The value functions satisfy the following Bellman equations

$$rB(\varepsilon) = \max\{0, -c^B + m(\theta)[H(\varepsilon) - B(\varepsilon) - p(\varepsilon)]\}, \tag{11}$$

$$rH(\varepsilon) = \varepsilon x - s[H(\varepsilon) - V - B(\varepsilon)] - \delta[H(\varepsilon) - B(\varepsilon)], \tag{12}$$

$$rV = -c^s + \theta m(\theta) \int_{\varepsilon_R}^{\infty} (p(\varepsilon) - V) \frac{dF(\varepsilon)}{1 - F(\varepsilon_R)} - \delta V. \tag{13}$$

Prices solve the following Nash Bargaining problem

$$p(\varepsilon) = \arg\max_{p(\varepsilon)} \left(S^B(\varepsilon)\right)^{1-\beta} \left(S^S(\varepsilon)\right)^{\beta}, \text{ for all } \varepsilon. \tag{14}$$

Free entry of sellers and buyers imply $V = k$ and $B(\varepsilon_R) = 0$. Intuitively, given a utility of the marginal buyer ε_R, free entry of sellers pins down the market tightness. Given the equilibrium market tightness, free entry of buyers pins down the marginal buyer ε_R and, therefore, the measure of buyers. Solving the model in a similar way as in the baseline model gives the equilibrium[3]

$$\text{(HE):} \qquad \frac{(r+\delta)k + c^S}{\theta m(\theta)} = (1-\beta)\left[\frac{\bar{\varepsilon}x + c^B - (r+\delta)k}{r + s + \delta + \beta m(\theta)}\right] \tag{15}$$

$$\text{(BE):} \qquad \frac{c^B}{m(\theta)} = (1-\beta)\left[\frac{\varepsilon_R x - (r+\delta)k}{r+s+\delta}\right]. \tag{16}$$

[2] Observe that from the assumption of Nash Bargaining, both the buyer and seller agree on the value of ε_R, i.e. $S^B(\varepsilon) \geq 0$ if and only if $S^S(\varepsilon) \geq 0$. We assume that x is large enough, so that all matches yield a positive surplus, akin to search models with endogenous labor market participation.

[3] A main disadvantage of a model with heterogeneity in utility flows and participation is that it generates a downward-sloping Beveridge Curve, contraty to the evidence in Gabrovski and Ortego-Marti (2019, 2021a, 2024, 2022). The reason is the following. The slope of the BE curve depends on two opposite effects. On the one hand, more vacancies imply a lower market tightness, which makes it easier for buyers to find a house and induces entry of buyers. On the other hand, as buyers find homes more quickly the stock of buyers depletes. This is the usual mechanism in search models of housing without buyer entry, and leads to a counterfactual downward-sloping Beveridge Curve. Whether the BE Curve is upward-sloping depends on which effect dominates. It turns out that, given a standard calibration, the second effect dominates and the BE curve is downward-sloping, as in search models of the labor market with labor force participation. See a previous version of this paper for full derivations (Gabrovski and Ortego-Marti, 2021b).

$$\text{(PP):} \qquad p(\varepsilon) = k + \beta\left(\frac{\varepsilon x + c^B + sk + \beta m(\theta)k}{r + s + \delta + \beta m(\theta)} - k\right), \tag{17}$$

We should note that in the pricing equation for the model in the main text, (7), does not include the market tightness, whereas equation (17) does. The reason is that in the homogeneous case the outside option for the buyer in the bargaining game, B, equals 0 due to free entry. In the heterogeneous case, however, this is only true for the marginal buyer. For any buyer with a preference for housing $\varepsilon > \varepsilon_R$ the outside option in the bargaining game $B(\varepsilon)$ is strictly positive and depends on how soon the buyer expects to match with another seller.

2.1 The social planner's allocation

Similar to the benchmark model with homogeneous buyers, the social planner faces two externalities. With heterogeneous buyers, however, the participation externality is driven by compositional effects. The marginal households participating in the market are the households who value housing the least. As a result, the average utility of homeownership declines as more buyers enter the market. This leads to a participation externality because buyers do not internalize how their participation affects the distribution of match surpluses. When the planner internalizes both externalities, the HMP condition is again insufficient to restore the efficient allocation. Intuitively, this condition controls for the congestion externality, but again an additional policy is required to restore entry of buyers to the efficient level.

As before c denotes new construction, $\tilde{h}$ denotes the number of homeowners and N is a large measure of the population (such that there is never a corner solution to the buyer's entry decision). We express buyers as a function of the total measure of homeowners $\tilde{h}$ instead of using the fraction of market participants who are homeowners h, as it simplifies the derivations. The planner maximizes

$$\max_{\tilde{h},v,\theta,\varepsilon_R,c} \int_0^\infty e^{-rt}\left\{\left(\int_{\varepsilon_R}^\infty \varepsilon x \frac{dF(\varepsilon)}{1-F(\varepsilon_R)}\right)\tilde{h} - [N(1-F(\varepsilon_R)) - \tilde{h}]c^B - vc^S - ck\right\}dt \tag{18}$$

subject to

$$\dot{\tilde{h}} = v\theta m(\theta) - (s+\delta)\tilde{h}, \tag{19}$$

$$\dot{v} = e + s\tilde{h} - \delta v - v\theta m(\theta), \tag{20}$$

$$N(1 - F(\varepsilon_R)) - \tilde{h} = \theta v. \tag{21}$$

Solving for the optimal allocation $\{\theta, \varepsilon_R\}$ in steady state yields the two equation system below

$$\frac{(r+\delta)k + c^S}{\theta m(\theta)} = \alpha \left\{ \frac{[(1-h)\bar{\varepsilon} + h\varepsilon_R]x - (r+\delta)k}{r+s+\delta} \right\}, \tag{22}$$

$$\frac{c^B}{m(\theta)} = \left(\frac{1-\alpha}{\alpha}\right) \frac{(r+\delta)k + c^S}{\theta m(\theta)} - \frac{xh(\bar{\varepsilon} - \varepsilon_R)}{m(\theta)}. \tag{23}$$

Comparing the planner's first-order conditions with the corresponding HE and BE conditions in the decentralized economy (15) and (16) shows that the HMP condition does not restore efficiency. The optimal allocation reflects that the planner does not only care about the marginal buyer, she is also concerned about the average composition of buyers. By contrast, in the decentralized equilibrium only the marginal buyer matters for entry.

2.2 Quantifying inefficiency in the housing market

We keep our calibration strategy as close as possible to the strategy in section 3 to make the quantitative predictions of the two models comparable. In particular, the only moments that are different in the calibration are those that pin down the parameters of the utility distribution $F(\varepsilon)$.[4] Kotova and Zhang (2020) report housing price dispersion of 16.84%. The authors further attribute 14.67% of the overall dispersion to buyer heterogeneity. Using these estimates we back out an implied mean-min ratio for prices equal to 1.0884. This yields $\tilde{\alpha} = 1.2698$. It turns out that one can normalize ε_R as it acts as a scaling variable. Accordingly we set $\varepsilon_R = 9.934$ so that the average equilibrium price is 491.2, the average price in thousands of dollars reported in Kotova and Zhang (2020). This normalization, together with the buyer entry condition allows us to back out $\beta = 0.2994$. Lastly, the normalization $N = 10,000$ yields an equilibrium number of buyers $b = 0.8038$.

[4] We also drop the moment which pins down γ in the homogeneous case, as this parameter is absent in the heterogeneous buyers model.

Given the model's calibration we compute the benchmark equilibrium and the planner's socially optimal allocation. The benchmark equilibrium features a vacancy rate of 2.83%. This is very close to the empirically observed vacancy rate for the U.S. of 1.9%. This version of the model also matches relatively well the data on construction: the construction rate in our model is 0.4% whereas the one in the data is 0.27%. Turning our attention to the efficient allocation, the planner finds it optimal to reduce the time-to-sell by almost half. This is achieved by increasing the number of buyers by 13.65% and reducing the number of vacancies by 41.51%. Intuitively, from the planner's perspective the congestion externality induces an over-creation of vacancies in equilibrium. At the same time, the participation externality leads to a sub-optimally low homeownership rate in equilibrium. Therefore, the planner instructs a higher number of households to enter the market. The resulting optimal vacancy rate is 1.64%.

2.3 Entry of buyers and the Beveridge Curve

The BE curve defines a relationship between the measure of buyers and vacancies, and corresponds to the Beveridge Curve in the housing market. In this section we show that a main disadvantage of a model with heterogeneity in utility flows and participation is that it generates a downward-sloping Beveridge Curve. The reason is the following. The slope of the BE curve depends on two opposite effects. On the one hand, more vacancies imply a lower market tightness, which makes it easier for buyers to find a house and induces entry of buyers. On the other hand, as buyers find homes more quickly the stock of buyers depletes. This is the usual mechanism in search models of housing without buyer entry, and leads to a counterfactual downward-sloping Beveridge Curve. Whether the BE Curve is upward-sloping depends on which effect dominates. It turns out that, given a standard calibration, the second effect dominates and the BE curve is downward-sloping, as in search models of the labor market with labor force participation.

To see this more clearly, let h denote the homeownership rate, i.e. the fraction of households participating in the market who own a home. The number of buyers is then given by

$$b = N(1 - F(\varepsilon_R))(1 - h), \tag{24}$$

where N is the large measure of potential buyers and is constant. Increasing vacancies lowers market tightness θ, which lowers the utility of the marginal buyer ε_R, i.e. there is more entry of buyers. This effect leads to a positive relationship between buyers b and vacancies v. At the same time, however, a lower θ lowers the fraction of market participants who are buyers $1-h=(s+\delta)/(m(\theta)+s+\delta)$, since they find homes more quickly. This effect generates a negative relationship between buyers and vacancies. Whether the BE curve describes a positive of negative relationship between buyers and vacancies depends on which effect dominates. Log-differentiating the above expressions and totally differentiating the BE condition gives the following elasticity $\epsilon_{b,v} \equiv (db/dv) \cdot (v/b)$ of buyers with respect to vacancies

$$\epsilon_{b,v} = -\frac{\Delta(\theta, \varepsilon_R)}{1-\Delta(\theta, \varepsilon_R)}, \tag{25}$$

where $\Delta(\theta, \varepsilon_R)$ is given by

$$\Delta(\theta, \varepsilon_R) = \alpha \left[-\frac{f(\varepsilon_R)\varepsilon_R}{1-F(\varepsilon_R)} \frac{\frac{c^B}{m(\theta)}}{\frac{c^B}{m(\theta)} + \frac{(r+\delta)k}{r+s+\delta}} + \frac{m(\theta)}{m(\theta)+s+\delta} \right], \tag{26}$$

and $f(\varepsilon)$ denotes the pdf of the distribution $F(\varepsilon)$. Using the BE condition (16) to substitute $\varepsilon_R \equiv \varepsilon_R(\theta)$ gives an expression that depends only on θ and parameters. Given a standard calibration $\Delta(\theta, \varepsilon_R(\theta)) \in (0,1)$, which implies a downward-sloping BE curve.

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