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A "Chinese Eratosthenes" Reconsidered: Chinese and Greek Calculations and Categories[1]

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In the third century BC, Eratosthenes of Cyrene (276-196) attempted to calculate the circumference of the earth using gnomon measurements and the properties of similar triangles. His calculation is widely taken as one of the great achievements of Greek science. In "A Chinese Eratosthenes of the Flat Earth: A Study of a Fragment of Cosmology in Huai Nan tzu 淮南子," Christopher Cullen remarks that a comparison of Greek and Chinese calculations provides a good example of the characteristics of success and failure in science. Eratosthenes had two hypotheses of considerable predictive power, despite the fact that he would have found some difficulty in justifying them: "(a) the earth is spherical; (b) the sun is for practical purposes at an infinite distance so that its rays reach the earth sensibly parallel. As it will appear, the Chinese author believed neither of these things."[2]

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Several second century (BC) Chinese texts use gnomon shadow lengths to calculate terrestrial and celestial distances, including the distance from the earth to the sun. They assumed that the earth was flat and the sun at a measurably finite distance. Their calculations were less accurate than Eratosthenes', but no less mathematically well reasoned. These apparently similar calculations invite consideration from another viewpoint, namely the role of quantification in the origins of science.

I Introductory Considerations

The development of physical methods and mathematical techniques for "accurate" measurement and the manipulation of the data of such measurement is widely accepted as one of the bases of the European "scientific revolution." The cachet of measurement and calculation has extended from the physical sciences to far more interpretive contexts; contemporary examples include psychological testing and computer simulations of human behavior. These developments are typically contrasted with a "premodern" science based on categorical thinking rather than extrapolation from accurate measurement, for example, in this statement by Joseph Needham: "In so far as numerical figures entered into them [premodern sciences], numbers were manipulated in forms of 'numerology' or number-mysticism constructed a priori, not employed as the stuff of quantitative measurements compared a posteriori [as in modern science]."[3] Here we have a clean divide between "quantities" and "categories," the stuff of numerology and "number-mysticism." Both terms need further consideration.

Categories, Quantities, Metrology and Metrosophy

The terms "category" and "categorical thinking" have been applied to a wide range of conceptual schemes based on the classification by qualities. The pioneering work of Lucien Levy-Bruhl and Claude Lévi-Strauss first called attention to the complexity of the classificatory schemata used by primitive societies.[4] These terms have been applied to a range of humoural theories, Greek, Chinese and otherwise, including the correlative cosmologies of Han China, as well as to

[2] Cullen 1976: 107, reprinted as Appendix A in Major 1993. See also Lloyd 1996: 60. As will be apparent throughout, the present study is particularly indebted to Christopher Cullen's studies of Chinese mathematics and astronomy and his pioneering work on the *Zhoubi suan jing*.

[3] Needham 1979: 15.

[4] Levy-Bruhl 1923 and 1966; Lévi-Straus 1969.

the classificatory schemes of Chinese histories and encyclopedias.[5] The latter have been widely studied within the context of Chinese cosmology, but less on in their relations to calculation and the quantitative measurement.

"Quantification" refers in general to the expression of properties in numerical terms, including, but not limited to, estimation, measurement and calculation (most measurements incorporate some degree of estimation). It is distinct from both precision (exactitude) and accuracy (correctness), which is applicable to qualities as well as quantities.[6] It is similar to metrology, as distinct from metrosophy.[7] Metrology pertains to the arts of calculation using numbers, weights and measures; metrosophy refers to speculation using numbers, especially in cosmology.[8]

Even within a clearly metrological context, it is necessary to distinguish between modern and traditional concepts of precision, accuracy and error, in part because modern scientific concepts of precision have no equivalent in premodern periods. In the ancient world, very particular problems present themselves. For example, precision in early Greek astronomy was impeded by a number of social factors, including the lack of standardized weights and measures between city-states and the private, rather than institutional, character of Greek astronomical measurements.[9] The notion of precision has a long and complex history in China. Clear metrological interest appears in medical and cosmological contexts in the second and first centuries BC. Examples include the "Treatise on [Mathematical] Harmonics and Astronomy" (*Lüli zhi* 律曆志) of the *Hanshu* (discussed below),[10] and discussions of anatometrics in the *Hanshu* and *Huang Di neijing*

5 Correlative cosmologies included detailed analogies between the body, state and cosmos, and appeared in such works as the *Huang Di neijing* 黃帝內經, the *Chunqiu fanlu* 春秋繁露 and later canonical Confucian works such as the *Baihu tong* 白虎通. See Graham 1986: 91-92, Graham 1989, and Henderson 1984: 2-22. For the classificatory schemes of some Chinese histories and encyclopedias, see Bodde 1981.

6 *Van Nostrand's Scientific Encyclopedia* (Considine and Considine, 1989: 2309) defines precision as "the degree of exactness with which a quantity is stated" or "the degree of discrimination or amount of detail," and notes that a result may have more precision than it has accuracy. Within the context of instruments and scientific measuring systems, it defines accuracy as "the conformity of an indicated value to an accepted standard value, or true value." Thus the "exactness" of precision refers to the refinement with which an operation is performed, a measurement stated, or a number represented. Accuracy refers to correctness, understood negatively as freedom from mistake or error and positively as conformity to a standard, model, or to truth, including the degree of conformity of a measure to a standard or true value.

7 I take these terms from Vogel 1994, to which the following discussion is indebted.

8 Or, as one author defines it, "number speculation within cosmological philosophemes" (Figala 1988: 3 as quoted in Vogel 1994: 135). For further discussion of metrosophy, see Fleckenstein 1975.

9 Lloyd 1994: 157.

10 *Hanshu* 21A: 955-972.

lingshu.[11] In late Imperial China, precision becomes an explicit issue in critiques of traditional astronomy and cosmology.[12]

Both metrology and metrosophy are centrally concerned with the use of number and the development of number systems, but in very different ways. In metrology numbers are used for counting, measuring and defining, in metrosophy, for the symbolic, magical and correlative values ascribed to them.[13] Despite these differences, these two approaches to number cannot but have overlapped, sometimes with considerable tension. An example is provided by Chinese geometrical cosmography, described as: "The division of space, celestial and terrestrial, civic and agrarian, and ceremonial and secular, into regular rectilinear units."[14] Geometrical cosmography is both metrosophical and metrological; it involves both "qualitative" correlation and "quantitative" measurement.[15]

In cases of overlap and tension between metrosophical and metrological systems, quantities can become categories. Consider chapter 21A of the "Treatise on

11 E.g., the account of dissection in *Hanshu* 99B: 4145-4146, as well as several treatises in the *Huang Di neijing*: "Cardinal Waters" (*Jing shui* 經水, *Lingshu* 12), "Dimensions of the Bones" (*Gu du* 骨度, *Lingshu* 14), "Dimensions of the Pulsating Vessels" (*Mo du* 脈度, *Lingshu* 17), "Intestines and Stomach" (*Chang wei* 腸胃, *Lingshu* 31) and "A Normal Person Abstains from Cereals" (*Ping ren jue gu* 平人絕穀, *Lingshu* 32), all as discussed in Yamada 1991.

12 For example, the Song Neo-Confucian thinker Zhu Xi 朱熹 (1130-1200), in a reported conversation with the polymath Cai Yuanding 蔡元定 (1135-1198) about the inaccuracy of calendrical systems, remarked that "if it were studied with enough precision to yield a definitive method of computation, there would be no further discrepancies ... the astronomical techniques of the Ancients were imprecise (*shukuo* 疏闊, lit. 'loose') but there were few discrepancies. The more precise (*mi* 密, lit. 'tight') the systems of today are, the more discrepancies appear!" (*Zhuzi yulei* 57: 14a-17a, trans. Sivin 1986, 163). In the foregoing interpretation, increases of precision led to greater expectations of accuracy. An alternative reading is to retain the literal meanings of "tight" and "loose." Read thus, the looser systems of the sages of antiquity showed fewer discrepancies than the overly tight systems of Zhu Xi's day, with the implication that the looser systems were preferable because of their greater overall consistency (Vogel 1996, especially 80-82). Concepts of precision also played an important part in sixteenth- and seventeenth-century Chinese critiques of traditional cosmology. One of Xu Guangqi's 徐光啓 (1562-1633) priorities in adopting the astronomy of Tycho Brahe (1546-1601) was to introduce more accurate astronomical knowledge based on precise observation (Hashimoto 1988, especially 1-6, 49-52 and 227-228). Fang Yizhi 方以智 (1611-1671) and Wang Fuzhi 王夫之 (1619-1692) held that discrepancies and irregularities were an inherent part of the cosmos and are therefore not predictable. The astronomical version of this view was that indeterminacy was inherent in the fabric of the cosmos, and a corresponding imprecision in human knowledge of the world, regardless of care or precision in observation and calculation (Henderson 1984: 246).

13 See Vogel 1994, especially 135-136.

14 Henderson 1984: 59.

15 Henderson 1984: 59-87 and 119-136.

[Mathematical] Harmonics and Astronomy" (*Lüli zhi*) of the *Hanshu*.[16] The first part of the chapter introduces a system that integrates metrology and metrosophy, the latter including numerology, musical pitch, celestial phenomena, astronomy, cosmology, historiography, politics and ethics. It describes harmonics (*lü*) as consisting of five categories: complete numbers (*beishu* 備數), harmonized sounds (*hesheng* 合聲), reliable length measures (*shendu* 審度), good capacity measures (*jialiang* 嘉量), and weights and balances (*quan heng* 權衡).[17] These categories also describe a hierarchy of *kinds* of quantities: number, pitch, length, volume and weight, with units of measure for each, forming an integrated metrological system based on the length of the *huangzhong* pitch-pipe. This system in turn was placed in a wider network of corresponding metrological and metrosophical phenomena, with correlations to *yin* 陰 and *yang* 陽, heaven and earth (*tiandi* 天地), the four seasons (*sishi* 四時), the five phases (*wuxing* 五行), the ten-thousand things (*wanwu* 萬物), the five constant virtues (*wuchang* 五常) and so forth.[18] Texts such as this show the limitations on our ability to distinguish between a priori "numerology" and a posteriori "quantitative measurement" in premodern texts.

Quantification is not necessarily more accurate than qualitative description. For example, inaccurate measurement may be evidence of an attempt at quantification or mathematical reasoning about nature. In each case we must ask where the numbers came from, including what verification procedures were used to ascertain them and how they were used. Simplistic oppositions between quantification and categorical thinking impede our understanding of complex interactions between cosmology, mathematics, observation and analysis in ancient science. The Chinese and Greek gnomon shadow measurements allow us to problematize this divide; we can use them to explore two distinct contexts for measurement, calculation and quantification in the ancient world. The details and contexts of these calculations illustrate a balance between "quantities" and "categories." Their Greek and Chinese authors had discovered that they could combine mathematical knowledge and direct measurements to calculate distances that could not be measured directly. Apparent similarities in mathematics and measurement methods belie important differences in assumptions, models, interpretation of data, conclusions and context, including the nature of the "similarity" between large and small and the use of conventional values rather than actual measurements.

16 *Hanshu* 21A: 955-972.

17 *Hanshu* 21A: 956, trans. Vogel 1994: 137. All five categories are ascribed to the *huangzhong* pitch pipe, traditionally attributed to the Yellow Emperor (*Hanshu* 21A: 959; *Guoyu* 3: 42; Needham 1962: 199.

18 A full treatment of *Hanshu* 21 is beyond the scope of the present study. For further discussion, see Vogel 1994.

Quantification and the Origins of Science

Quantification in the ancient world also bears on claims for and against the uniquely Western origins of science. Of the historians of science who credit the Greeks with fundamental insights not duplicated elsewhere, some emphasize a distinct, and qualitative, "mode of thought; such claims do not turn on measurement or the collection and manipulation of accurate data."[19] The work of Joseph Needham directly challenged accounts of ancient science that moved from unique Greek insights to claims that ancient science was uniquely Greek.[20] Needham's entire project rejected the claim that the origins of science were uniquely Western, but nonetheless asserted that only *modern* science was genuinely universal:

> The sciences of the medieval world were tied closely to their ethnic environments, and it was very difficult, if not impossible, for the people of those different cultures to find any common basis of discourse.[21]

According to Needham, universal modern (rather than distinctively Western) science emerged from the unique "scientific revolution" of the West, which prominently included quantification:

> It is surely quite clear by now that the history of science and technology of the Old World must be thought of as a whole. But when this is done, a great paradox presents itself. Why did modern science, the mathematization of hypotheses about nature, with all its implications for contemporary technology, take its meteoric rise only in the West at the time of Galileo?[22]

Nathan Sivin subsequently problematized Needham's position, and argued, as historians of science now typically do, that the "scientific revolution" of seventeenth-century Europe was not a unique event in world history. Sivin's article

19 For example, A.C. Crombie, writing as recently as 1994 (1994: 1213), credits the origins of science to ancient Greek philosophers, mathematicians and physicians who developed the "two fundamental conceptions of universal natural causality matched by formal proof." For the view that Greek science was a new creation that was inseparable from Greek philosophy, see Kahn 1991.

20 The first volume of *Science and Civilisation in China* appeared in 1959. Six years later, Joseph Needham's "Poverties and Triumphs of the Chinese Scientific Tradition" appeared in Alistair Crombie's 1963 anthology *Scientific Change: Historical Studies in the Intellectual, Social and Technical Conditions for Scientific Discovery and Technical Invention, from Antiquity to the Present* (Needham 1963). It is more frequently cited as reprinted in *The Grand Titration: Science and Society in East and West* (Needham 1979).

21 Needham 1963: 118.

22 Needham 1963: 117.

demonstrates a different scientific revolution in Song-dynasty China and argues that the difference between the two "revolutions" was one of degree rather than kind.[23] In a parenthetical remark, he points out a potential "scientific revolution that did not take place" in Hellenistic Greece:

> I remember how puzzled a classicist colleague once was at why Archimedes didn't set off a scientific revolution. My colleague was convinced that Archimedes had the mathematical tools to invent dynamics; why did he stop short, eighteen hundred years before Galileo? Why didn't someone form a committee and fund an enormous research project so that Archimedes' dynamics and his talent for invention could have saved the Hellenistic world from its enemies? Unfortunately, neither new ideas without human organization to carry them out, nor new associations rearranging clichés, can change the world.[24]

The most important of two ostensible shifts between ancient and modern science is the development of methods and mathematical techniques for accurate measurement of physical phenomena and the manipulation of the resulting data, Needham's "mathematization of hypotheses about nature."[25] (The other is heliocentrism, discussed below.) Following from this, the usual answer to the question, Why the Scientific Revolution Did Not Take Place in Greece — Or Didn't It? — which Sivin argues is a false question based on bad assumptions — is the claim that Greek science and all science before Galileo, was qualitative and lacked exact quantitative expression or data. This claim has been taken by some, most notably Alexandre Koyré, to diminish any claims by Greek science to be science at all.[26] As Koyré put it:

> ... no one had the idea of counting, of weighing and of measuring. Or, more exactly, no one ever sought to get beyond the practical use of number, weight, measure in the imprecision of everyday life.[27]

Geoffrey Lloyd takes Koyré's claim head-on by examining its merits in several areas of Greek science: dynamics, element theory, geophysics, astronomy, harmonics, optics, the use of standardized weights and the applications of counting. He demonstrates that, while the first two fit Koyré's claim, the others do not. Optics, harmonics, astronomy and geophysics provided important counterevidence that the ancient Greeks sought exactness in the formulation of rigorous theories and in the collection of precise data. He argues that cases must be taken individually, that "the ancients' performance in different contexts and at different periods varies, and each field and period must be judged on its own merits."[28]

Lloyd's study of "Measurement and Mystification" also demonstrates the limits of overvaluing quantification for its own sake. Premature or poorly grounded mathematization or quantification in some Greek medical theories impeded their speculations about nature.[29] Other appeals to mathematics and precision were rhetorical (e.g. symbolic numbers) or confused precision with accuracy. Other cases of "exact measurement" used instruments of very limited accuracy. Nor were the limits of appeals to precision lost upon the Greeks themselves; ἀκρίβεια, precise measurement, is already a target of Aristophanes, who depicts Socrates trying to measure the length of the jump of a flea.[30]

II Greek Geodesy and Celestial Measurement

During the third century BC several Greek mathematicians and astronomers developed hypotheses and calculations regarding the motion and sizes and distance of the sun, moon and earth. Because of the direct link between them, I examine the arguments and calculations of Aristarchus, Archimedes and Eratosthenes. What were they attempting to accomplish in their celestial measurements, calculations and mathematical hypotheses? Were these attempts at accurate measurement or geometric proofs using numbers of only vague relation to actual measurements?

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23 Sivin 1982: 65. A complete treatment of the complex arguments of this paper is beyond the scope of the present study. For example, Sivin contrasts the marginalized position of premodern Western astronomers and other scientists in European society with the highly official character of Chinese astronomy. For an incisive treatment of the history and significance of Sivin's work on these and other problems see Chu's (2001) double review of Sivin 1995b and 1995c.

24 Sivin 1982: 61-62. Elsewhere, he makes a sociological distinction between what others have called modern "universal" biomedicine and the diverse practices of traditional medical systems, which have more in common with each other than with the modern medicine that is challenging and replacing them: "modern medicine, on the other hand, has certain basic characteristics that can be seen anywhere that it is practiced, despite great variation in its social organization and the way it is understood by laymen. ... It uses surgery, asepsis, anesthesia and powerful drugs to overcome life-threatening traumas, infections and biochemical dysfunctions that no traditional system can control." Cf. Sivin 1987: 6.

25 Koyré 1957 and Kuhn 1957.

26 For statements of this claim, see Koyré 1948: 806-823 (rpt. 1961: 311ff.), Koyré 1968: 89ff., and Kuhn 1961: 161-193 (rpt. 1977).

27 Koyré 1961: 318, as quoted in Lloyd 1987: 216.

28 Lloyd 1987: 278.

29 For example, overly numerical theories of periodicity in embryology and Galen's attempts to grade degrees of hot, cold, wet and dry (Lloyd 1987: 280).

30 Aristophanes, *Clouds*, 143ff. Cf. Lloyd 1987: 280n218. Unless otherwise noted, Greek texts refer to the editions in the Loeb Classical Library series.

Aristarchus of Samos

Aristarchus of Samos (310-230 BC) is known to have been a pupil of Strato of Lampsacus, a natural philosopher who succeeded Theophrastus as head of the Peripatetic school in 288 (or 287) BC and held that post for eighteen years.[31]

He is best known as the first Greek exponent of a heliocentric theory. Our major source for it is the works of his younger contemporary Archimedes of Syracuse (287-212 BC), who recorded it, but, like his contemporaries, did not believe it. (Copernicus did and quoted Aristarchus in his own formulation.[32]) Aristarchus' contemporaries rejected the theory in part because it conflicted with the commonsense view that the earth did not move.[33] According to Plutarch, Cleanthes (d. 232 BC), the head of the Stoic school at Athens, urged that Aristarchus be indicted on charges of impiety for "putting the Hearth of the Universe in motion."[34]

The one surviving work of Aristarchus is *On the Sizes and Distances of the Sun and Moon*. In it, Aristarchus uses six (unexplained) hypotheses to prove eighteen propositions. The hypotheses are: (1) that the moon receives light from the sun; (2) that "the earth holds the relation of point and center to the sphere of the moon," in other words, that the moon revolves around the earth; (3) that "when the moon appears halved, the observer's eye lies on the plane of the great circle dividing its bright and dark halves," in other words, that the centers of the sun, earth and moon form a right triangle with its right angle at the center of the moon; (4) that "when the moon appears halved, its [angular] distance from the sun is one-thirtieth quadrant short of a quadrant (87°);" (5) that the earth's shadow has a breadth of two moons; and (6) that the moon subtends one-fifteenth of a sign of the zodiac.[35]

These hypotheses vary in the extent to which they could have been, and were, actually observed. The first three and fifth seem to be the results of actual observations that do not depend on high-precision measuring devices. The two hypotheses that required precise measurement were (4) and (6), and it is difficult to ascertain whether either was actually observed. The instrumentation available to Aristarchus would not have yielded the needed precision to observe (4), and even if the value of 1/30 quadrant short of a quadrant was not determined instrumentally, there still remains the question of whether it was observed.[36] Although (6) could have been observed, it is not clear that it was.[37]

Vitruvius credits Aristarchus with the invention of an improved sand-dial, the σκάφη. Instead of a plane, its surface was a concave hemisphere with a vertical pointer in its center and lines marked on its surface.[38] The pointer's shadow made it possible to measure the direction and height of the sun; however, such a device could not have been used at night to measure the direction and height of the moon.

In the propositions, Aristarchus uses assumptions and the properties of similar triangles to "demonstrate" ratios between the diameters of the earth, sun and moon and their distances from each other. For example:

> Proposition 7. The distance of the sun from the earth is greater than 18 times, but less than 20 times the distance of the moon from the earth.[39]

Where do the numbers come from? The Greek text is a narrative, listing procedures for naming points and drawing lines and circles between them. It is unclear whether the figures it uses are actual measurements, and there is no exact visual representation of quantitative information. (By contrast, translations of this passage typically include an illustrative diagram.) The Greek text has the look and feel of a proof by deduction from the properties of similar triangles. It is not clear whether their lengths and angles are determined by the initial assumptions or by actual measurements.[40]

One explanation of the errors in Aristarchus' calculations is that very small elongations are difficult to estimate precisely. This explanation implies that Aristarchus was attempting to obtain an accurate value. On the contrary, the terms of

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31 Aëtius i.15.5 and Galen, *Histor. Philos.* 3, in *Doxographi Graeci*, ed. H. Diehls 1879: 313 and 601, as quoted in Heath 1913: 299.

32 Cf. Heath 1913: 301-302, and Koyré 1957: 28, 281n2. See Nicolaus Copernicus (1473-1543), *De revolutionibus orbium caelestium* (Nuremberg, 1543), trans. Duncan 1976.

33 For discussion of the reasons for the rejection of Aristarchus' heliocentric hypothesis throughout antiquity, see Lloyd 1973: 57-61.

34 Plutarch, *De facie in orbe lunae*, 6: 922F-923A, as cited in Heath 1913: 304. According to Diogenes Laertius, Cleanthes' works included a tract "Against Aristarchus." By contrast, the Babylonian astronomer Seleucus (c.150 BC), one of the few adherents of heliocentrism in antiquity, maintained it as both a mathematical proposition and a physical fact (Heath 1913: 304-308). Aristarchus views also contradicted Plato (*Timaeus* 39b-d, *Laws* vii 822a).

35 Aristarchus, *De Mag. et Dist. Solis et Lunae*, as translated in Heath 1913: 352-354.

36 As Alan C. Bowen has pointed out (personal communication), the observational status of (4) depends on whether Aristarchus was able to observe both the sun and moon visible above the horizon when the moon was at quadrature. If so, it would have been possible by various means to estimate that the angular distance separating the sun and moon as slightly less than 90°. If not, however, it would be clear that the value was neither determined by measurement nor directly observed.

37 G.E.R. Lloyd has suggested (personal communication) that *Sizes and Distances*, may be a geometric exercise and that Aristarchus arbitrarily picked, rather than measured, the number for hypothesis (6).

38 Vitruvius, *De architectura* (i.x 8 (9).1), Heath 1913: 300.

39 Aristarchus, *De Mag. et Dist. Solis et Lunae*, as cited in Heath 1913, secs. 376.1-380.28.

40 For further discussion of this demonstration, see Lloyd 1973: 56-57.

his exposition suggest that Aristarchus was more interested in a mathematical demonstration than in a practical problem. His "calculations" are problems in deductive reasoning that draw on proportions and ratios, rather than exact measurements of quantities.

Archimedes of Syracuse

One of Koyré's few concessions in his negative judgment of Greek science was in the area of celestial physics. When we turn to the works of Archimedes, we might expect to find a greater concern with the kind of precision Koyré is looking for. Archimedes' broader range of interests included engineering and measurement, as well as mathematics. Whether or not we accept Vitruvius' story of his use of precise measurements to detect the adulteration of a gold crown (by comparing the water it displaced to the displacement of a pure gold weight), his treatise *On Floating Bodies* makes it clear that he understood the workings of specific gravity through the measurements of the volumes and weights of solids.[41]

The Ψαμμίτης or *Sand-Reckoner* of Archimedes uses an astronomical example to propose a number system capable of expressing large numbers.[42] It is also the source for our association of Aristarchus with heliocentrism.[43] The *Sand-Reckoner* was addressed to Gelo[n], the son and co-ruler of King Hieron II of Syracuse.[44] In the first section, Archimedes explains the purpose of the work, to introduce a new system for the expression of large numbers:

41 Lloyd 1987: 249-250.

42 Archimedes, *Arenarius*, in J.I. Heiberg (1880-1881): ii. 218, 7-18, and Mugler 1971: vol. 1, 10-20.

43 According to Archimedes, Aristarchus published (or made available) written statements (or illustrations) of certain hypotheses. One was that the "fixed stars" (ἀπλανέα τῶν ἄστρων) and the sun remain motionless and that the earth revolves around the sun "along the circumference of a circle" (κατὰ κύκλου περιφέρειαν). Aristarchus suggests that the ratio of the earth's orbit to its distance from the fixed stars was the same as the ratio of the (size of the) center of a sphere to its surface, with the sense that the universe is very large, since the center of a sphere has no magnitude. It is tempting but anachronistic to read into this passage modern claims for an infinite universe (such claims rest on modern ideas of division by zero). Aristarchus is more likely to have meant that the universe was very large. The point of the remark that a center has no magnitude might simply have been that the ratio was improperly formed.

44 Gelo (or Gelon) was the son of King Hiero II of Syracuse (r. 265-216 BC) and Philistis. He married Nereis, the daughter of King Pyrrhos. Gelo co-ruled with his father from about 240 BC until the latter's death, at which time Gelo's son Hieronymos inherited the throne.

> There are some, King Gelon, who think that the number of [grains of] the sand is infinite in multitude ... Again there are some who, without regarding it as infinite, yet think that no number has been named which is great enough to exceed its multitude ... But I will try to show you by means of geometrical proofs which you will be able to follow, that, of the numbers named by me ... some exceed ... that of a mass [of sand] equal in magnitude to the universe.[45]

He begins with a common definition of the universe as a sphere whose center is the earth and whose periphery is the center of the sun (with no stated reference to the fixed stars). He contrasts it with Aristarchus' view that the earth revolves around an unmoving sun within the sphere of the fixed stars, an immense distance away. Archimedes' purpose was to calculate the number of grains of sand it would take to fill a large but finite universe. The interest of his example for present purposes is that it is more concerned with numbers and ratios than with measurement. The astronomical data specifies the conditions of *his* problem, the expression of large numbers.

He describes the sizes of the earth, sun and moon in relative terms, as ratios, rather than as observed measurements of actual sizes. He introduces four assumptions to specify the size of the earth, the relative diameters of the sun, earth and moon (sun's diameter no greater than 30 times the moon's), the relation between the diameter of the sun and the circumference of the fixed stars (the universe) and the relative diameters of three spheres: the universe (circumscribed by the fixed stars), the orbit of the sun (around the earth) and the (spherical) earth itself. Finally comes the size of a grain of sand; no more than 10,000 grains of sand would fit into a sphere whose diameter was 1/40 a finger-width:[46]

> 1. the perimeter of the earth is 3 million [τ μυριάδων, lit. 300 myriad] stades and not more ...[47]

This figure was offered for the sake of the argument; Archimedes probably knew it to be false because he acknowledges that some had argued for a figure of only 300,000 stades. The second assumption follows the views of the astronomers of his time:

> 2. the diameter of the earth is greater than the diameter of the moon, and the diameter of the sun is greater than the di-

45 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 134-135; C. Heath 1897 (rpt. 1912): 221, and Lloyd 1973: 41-42.

46 Archimedes, *Aren. 1-2*, Mugler 1971: vol. 2, 136-145; Translation slightly modified from Heath 1897/1912: 222-227.

47 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 136; Heath 1897: 222.

> ameter of the earth.[48]
> 3. the diameter of the sun is about 30 times the diameter of the moon, and not greater.[49]

Archimedes states that this assumption departs from the views of Eudoxus, Archimedes' own father Pheidas, and Aristarchus, who argued for a figure between 18 and 20. Why does Archimedes use larger numbers than his predecessors? One possible reason is improved accuracy of measurement. Another is to establish an irreproachably conservative upper bound for his calculation. (His choice of a problem had already created a context for the expression of the largest numbers possible.) Finally,

> 4. The diameter of the sun is greater than the side of the chiliagon inscribed in the greatest circle in the (sphere of the) universe.[50]

Archimedes' explanation of this section introduces an actual experimental improvement on the measurement of Aristarchus, who found that the sun appeared to be 1/720th part of the circle of the zodiac. Archimedes uses experimental methods to arrive at a higher and lower limit for the angle subtended by the sun. He invented a sighting tube device to measure the angle subtended by the disk of the sun, beginning with Aristarchus' conventional figures. His measurements used magnitudes of error in the form of upper and lower limits, based on actual measurement. He shows that the angle subtended by the diameter of the sun was less than 1/104th part, but greater than 1/200th part, of a right angle.[51] Finally, he adds a fifth assumption in the second section:

> 5. Suppose a quantity of sand taken not greater than a poppy-seed, and suppose that it contains not more than 10,000 grains. Next, suppose the diameter of the poppy-seed to be not less than 1/40 of a finger-breadth.[52]

In the third section of the *Sand-Reckoner*, Archimedes proposes a number system capable of expressing large numbers. Greek arithmetical calculations at that time used an alphabetic notation of 27 signs to represent numbers, in addition to the names of the numbers 10 (δέκα), 100 (ἑκατόν), 1000 (χίλιοι) and 10,000 (μύριοι).[53]

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48 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 136; Heath 1897: 223.

49 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 136; Heath 1897: 223.

50 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 137; Heath 1897: 223.

51 Archimedes, *Aren.* 1, Mugler 1971: vol. 2, 138; Heath 1897: 224.

52 Archimedes, *Aren.* 2, Mugler 1971· vol. 2, 145; Heath 1897: 227.

53 The letters α through θ represented the numbers 1 through 9, with 6 represented not by Zeta ζ but by the Phoenician letter Stigma ϛ. The letters Iota ι through Pi π represented the numbers 10 through 80, with 90 represented by the Phoenician letter Koppa ϙ. The

Thus the number 3 million would be written as 300 myriad or τ μυριάδων, where the letter Tau stood for the number 300. This was a decimal system without decimal or other positional notation, and it did not provide for the naming of numbers beyond a hundred million (10^8), a myriad myriads (μυρίας μυριάδας). The Greeks used a decimal system for naming integers; they also used the sexagesimal notation of Babylonian astronomical texts for describing fractions.[54] Thus Greek astronomers, including Archimedes, used Babylonian-style sexagesimal notation for fractions and used the ordinary alphabetic notation for integers, degrees and hours.[55] They also used Egyptian fractions to represent quantities less than one unit.[56]

Archimedes defines the numbers expressed in the existing system as numbers of the first *order*, allowing for the expression of a myriad myriad orders of numbers, which in turn become the basis for *periods* of numbers.[57] In the fourth sec-

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last eight letters of the Greek alphabet, Rho ρ through Omega ω, represented the first eight numbers of the hundred series (100-800), with the Phoenician letter Sampi ϡ signifying the number 900. The thousands were represented by the letters for the numbers 1 through 9, with a stroke subscripted to the left of the number (e.g. ͵α for 1000). A macron written over the letter indicated that it represented a number. See Thomas 1951, vol. 1: 43, and Cajori 1928: 21-29.

54 In sexagesimal notation, 60 units of one kind are written as 1 unit of the next order higher, for example, in the following multiplication table of numbers: 4/40, 5/50, 6/1, 7/1,10, 8/1,20, 9/1,30, 10/1,40, 11/1,50, 12/2, 13/2,10. See Neugebauer 1957: 12-17, especially 16.

55 Cajori 1928: 26-29. For sexagesimal fractions in numerical calculations, see Ptolemy, *Syntaxis* i. 10, ed. Heiberg (1880-1881): i. 31. 7-32. 9, as quoted in Thomas 1951: vol. 2, 412-413. For a comparative perspective, see Lloyd 1994.

56 Cf. Neugebauer 1957: 21, 50, and 72-74.

57 Archimedes, *Aren.* 3, Mugler 1971: vol. 2, 145-147; Heath 1897: 227-229. He defines the numbers from 1 to a myriad myriads (100,000,000 or 10^8 in modern notation), the numbers expressible through existing names, as numbers of the first order (πρῶτοι ἀριθμοί). He then posits a myriad myriads, the last number of the first order, as the units of numbers of the second order (δευτέρων ἀριθμῶν μονάδες), from 100,000,000 (10^8) to $100,000,000^2$ (10^{16}). The last number of the second order (10^{16}) becomes the unit of numbers of the third order (τρίτων ἀριθμῶν μονάδες), which includes numbers from $100,000,000^2$ (10^{16}) to $100,000,000^3$ (10^{24}). He continues through numbers of the fourth (10^{24} to 10^{32}) and fifth orders (10^{32} to 10^{40}) to the myriad-myriadth order of numbers, ending with a myriad myriads taken a myriad-myriad times (μυριακισμυριοστῶν ἀριθμῶν μυρίας μυρίαδας) or $100,000,000^{100,000,000}$ ($_{10}8*10^8$). He then defines the numbers from 1 to $100,000,000^{100,000,000}$ as the numbers of the first period (ἀριθμοὶ πρώτας περίοδου) of a myriad-myriad of orders. He takes the last number of the first period as the unit of numbers of the first order of the second period (μονὰς δευτέρας περίοδου πρώτων ἀριθμῶν) and moving upwards by octads (series from 10^1 to 10^8), he defines the remaining orders of the second period and additional periods up to the 10^{8th} period, a myriad myriad units of numbers of the myriad myriadth order of the myriad myriadth period (τὰς μυριακισμυριοστῶν ἀριθμῶν μυρίας μυριάδας) This number would require 80,000 million millions of zeros to express.

tion, Archimedes applies the system described in the third section to the problem and assumptions introduced in the first two sections. He concludes that the number of grains of sand necessary to fill the universe would be fewer than a thousand myriads of numbers of the eighth order (,α μυριάδες τῶν ὀγδόων ἀριθμῶν), or 10^{63} in modern notation.[58]

Whereas Aristarchus may have been satisfied with a conventional figure, Archimedes introduced an observational method through his invention of a sighting tube to measure the angle subtended by the disk of the sun, as well as the use of upper and lower limits.[59] Nonetheless, the argument of the *Sand-Reckoner* suggests that Archimedes too may not have been primarily interested in geophysical measurement, astronomical observation, or the creation of precision instruments.

Eratosthenes of Cyrene

Eratosthenes of Cyrene was known as a student of the philosopher Ariston of Chios, of the grammarian Lysanias of Cyrene and of the poet Callimachus.[60] His range of interests earned him the nicknames Pentathlos and Beta (for being second-place in everything). Ptolemy Euergetes invited him to Alexandria as tutor to his son Philopator and later made him Librarian at Alexandria. The one book attributed to him, Πλατονικός, seems to have addressed mathematics in relation to Platonic philosophy.

Much of our knowledge of Eratosthenes comes through Archimedes. Of the twelve known complete works of Archimedes, five are undedicated,[61] and five are addressed to the astronomer Dosithios of Pelusium (c. 230 BC).[62] Of the remaining two, the first is the *Sand-Reckoner*, discussed above, addressed to Archimedes' patron. The second is the *Method* or Εφοδος (classed as) a lost work until 1906. It is addressed to Eratosthenes. The language of the dedication makes clear the central concerns of Archimedes' investigations. It is addressed to Eratos-

58 Archimedes, *Aren*. 4, Mugler 1971: vol. 2, 149-156; Heath 1897: 229-232.

59 Bowen (personal communication) notes that we do not know exactly how Archimedes understood these upper and lower bounds: Whether he meant them to be simply extreme values of a range of possible values, or numerical values bracketing the real value. In either event, it would be anachronistic to introduce here the modern notion of magnitudes of error.

60 *Suidas* s.v. Eratosthenes, quoted in Thomas 1951: vol. 2, 261. For discussion of Eratosthenes and Greek mathematics, see Heath 1921 and Heath 1932.

61 *On the Equilibrium of Planes* I and II; *Floating Bodies* I and II and *Measurement of a Circle*.

62 *Quadrature of the Parabola*; *On the Sphere and Cylinder* I and II; *On Spirals*; and *On Conoids and Spheroids*. Dosithios maintained Archimedes' connection with Alexandrian astronomy.

thenes as an eminent philosopher who "gives due honor to mathematical inquiries" (μαθήμασιν θεωρίαν):

> I have thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, furnished with which, you will be able to make a beginning in the investigation by mechanics of some of the problems of mathematics.[63]

Archimedes recommended the use of the mechanical method for discovery and the geometric method for demonstration. Eratosthenes used the latter to calculate the size of the earth by means of the shadow cast by a gnomon at noon of the summer solstice at two points on the same meridian. An upright gnomon at Syene (near contemporary Aswan in Egypt) cast no shadow; a similar gnomon at Alexandria cast a shadow of a fiftieth of a circle (7.2°). Given a distance between the two points of 5000 stades, Eratosthenes used the geometry of similar triangles to calculate the circumference of the earth as 250,000 stades. The description comes from a contemporary account by Cleomedes, "On the Circular Movement of the Heavenly Bodies" (c. 200 BC).[64] It probably derives from an account of Eratosthenes by the Stoic Posidonius (135-51 BC), the teacher of Cicero.[65] Cleomedes notes that Eratosthenes' method depends on a geometric argument and describes it as follows:

> Let us suppose, in this case also, first that Syene and Alexandria lie under the same meridian circle; secondly, that the distance between the two cities is 5000 stades; and thirdly, that the rays sent down from different parts of the sun upon different parts of the earth are parallel; for the geometers proceed on this assumption. Fourthly, let us assume that, as is proved by the geometers, straight lines falling on parallel straight lines make the alternate angles equal, and fifthly that the arcs subtended by equal angles are similar, that is, have the same proportion and the same ratio to their proper circles — this also being proved by the geometers.
>
> ... But the arcs are similar since they are subtended by equal angles. Whatever ratio, therefore, the arc in the bowl of the

63 Archimedes, *Meth.*; Praef., Archim. ed. Heiberg (1880-1881): ii, 426. 3-430, 22; cf. Thomas 1951: vol. 2, 220-223.

64 This dating is from Bowen and Todd 2001. Bowen has suggested (personal communication) that Cleomedes may have structured or recast Eratosthenes' argument to draw out its function as an *ephodos*, a procedure for gaining knowledge about matters not directly accessible through observation and measurement (of which there are many in Cleomedes' treatise.) As a Stoic, Cleomedes was clearly concerned with *ephodoi* and his reportage of Eratosthenes' argument may reflect that interest.

65 Cf. Lloyd 1987: 231ff. and Lloyd 1973: 49-50.

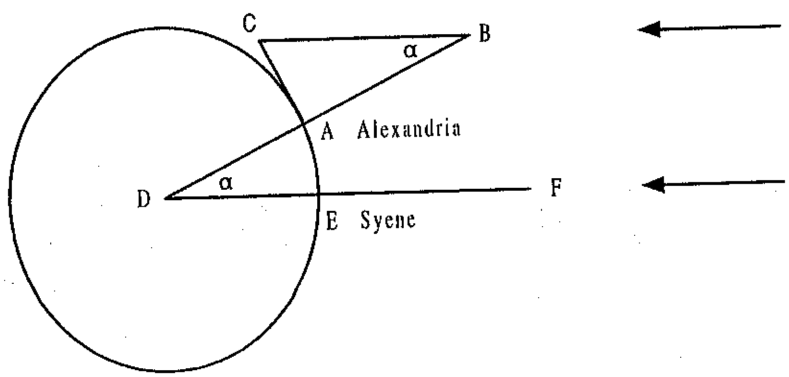
> sundial has to its proper circle, the arc reaching from Syene to Alexandria has the same ratio. But the arc in the bowl is found to be the fiftieth part of its proper circle. Therefore the distance from Syene to Alexandria must necessarily be a fiftieth part of the great circle of the earth. And this distance is 5000 stades. Therefore the whole great circle is 250,000 stades.[66]

Thus, in Figure 1 (below) AB is the Alexandria gnomon (sundial), with a right angle at A and a shadow length of AC; EF is the Syene gnomon, with a right angle at E and no shadow (because of summer solstice); CB and EF are parallel; D is the center of the earth; the angle at B is 7.2° (one fiftieth part of a circle); the angle at D is 7.2° (by the properties of similar triangles); AE is 5000 stades (a conventional figure, discussed below). Thus one right triangle was formed by the gnomon at Alexandria (AB) and its shadow (AC), the other by the 5000 stades between Syene and Alexandria (EA) and the radius of the earth (ED). The angle (α) at the center of the earth (D) was the same as that of the angle at the tip of the Alexandria gnomon (B). The key ratio is:

$$1 : 50 = 5000 : CE$$

where CE = the circumference of the Earth.

Figure 1: The Calculation of Eratosthenes



66 Cleomedes, *De motu circulari corporum caelestium* i. 10. 52, ed. Ziegler in Thomas 1951: vol. 2, 267-269.

Several specifically quantitative assumptions complicate our interpretation of this account.[67] One is that Syene and Alexandria are on the same meridian; actually, Syene is 3° east of Alexandria. A second is the claim that Syene was 5000 stades from Alexandria. Cleomedes does not say how the latter figure was obtained. On the one hand, there is no evidence to decide whether the 5000 stades was a measured quantity; on the other, there is no evidence of any standardization of the stade.[68] The degree of accuracy of Eratosthenes' measurement depends primarily on the length of the stade. The most accurate result comes from the value for the stade of 157.2 m. (deduced from Pliny); the value of 166.7 m. probably used by Eratosthenes produces a less accurate result.[69]

These Greek scientists had influential positions and wealthy patrons. Aristarchus was a pupil of the head of the Peripatetic school. Archimedes was a friend and kinsman of Hiero of Syracuse.[70] Eratosthenes enjoyed the patronage of the Ptolemies.[71] Both Hiero and the Ptolemies had an ongoing practical interest in the measurement of terrestrial distances, but that knowledge was not significant to the legitimation of their authority.[72] Archimedes' considerable reputation in his own lifetime was due, neither to his astronomy nor his mathematics, but to his invention of war engines at the explicit behest of King Hiero of Syracuse.[73]

67 Cf. Lloyd 1973: 232n61-64, Fischer 1975: 152-167, Goldstein 1984: 411-416, and Goldstein and Bowen 1983: 330-340.

68 There has been several efforts to determine the length of the stade, given conflicting remarks by later authors. Some scholars argue that the 5000 stades was not a measurement, but a round number based on the number of days it took to march or sail from one point to the other, time values already traditional by Eratosthenes' time (Cf. Goldstein 1984: 411-412). Other scholarship has pointed to the existence of more accurate astronomy in the Hellenistic world, for example, Diller 1949 and Drabkin 1942-1943.

69 For the various values scholars have given for the stade, see Gulbekian 1987.

70 Plutarch, *Marcellus* xiv.7, Thomas 1951: vol. 2, 22-23.

71 Suidas, s.v. Eratosthenes, Thomas 1951: vol. 2, 260-261.

72 This is not to say that maps and geography were not of interest to Greek rulers, ever since Herodotus' (5.49) account of how Aristagoras of Miletus tried to use a map to persuade Cleomenes of Sparta to attack Asia Minor on behalf of the Ionians. Greek cartography began with Eratosthenes, who applied a coordinate system to the surface of the earth and reached its height with Ptolemy (fl. 120-170 AD) For a comparative treatment of the origins of Greek and Chinese geography and cartography, see Needham 1959: 526-527.

73 According to Plutarch (Marcellus 14), "These machines [Archimedes] had designed and contrived, not as matters of any importance, but as mere amusements in geometry; in compliance with King Hiero's desire and request, some little time before."

III Chinese Terrestrial and Celestial Measurements

The preceding discussion suggests that Greek mathematicians and astronomers may have been more interested in mathematical demonstration than the "counting weighing and measuring" of such concern to Koyré.[74] Now let us turn to Chinese measurements of the size of the earth and its distance from the sun. What techniques were used to measure terrestrial and celestial distances? What was being measured, and why?

Warring States and Han texts from the third and second centuries BC provide examples of several kinds of terrestrial and celestial measurement and calculation. An initial difficulty is that these measurements and calculations are not attributed to specific individuals and appear in a range of texts, including the *Guanzi* 管子, the *Classic of Mountains and Seas* 山海經 (*Shanhai jing*), the *Springs and Autumns of Master Lü* 呂氏春秋 (*Lü Shi chunqiu*) and the *Huainanzi* 淮南子.[75] A very different method of calculation of the dimensions of the earth applies ratios and the properties of similar triangles (more on this distinction later) to measurements of the length of the shadows cast by upright posts called gnomons, taken under carefully controlled circumstances. Gnomon measurements are used to calculate both the size of the earth and the distance from the earth to the sun in both the *Huainanzi* and in a first-century BC mathematical work, the *Gnomon of the Zhou* [dynasty] or *Zhoubi suanjing* 周髀算經 (henceforward the *Zhoubi*), which also includes a calculation of the diameter of the sun.[76]

Geodesy by Direct Measurement

The first five books of the *Shanhai jing* describe the mountains of the five cardinal regions of the earth. This section ends with a statement attributed to the sage king Yu that the world contains 5370 named mountains, over 64,056 *li* of inhabited land,[77] and

74 In putting the issue thus I do not mean to suggest any inherent opposition between demonstration and measurement. At issue is the extent to which one may have taken precedence over the other.

75 The *Guanzi* dates from the fifth to first centuries BC; the *Lü Shi chunqiu* from the third and the *Huainanzi* to the second. For dating of these texts, see Loewe (ed.) 1993. For controversies surrounding dating of the *Shanhai jing*, see Fracasso 1983: 665-667.

76 For this translation and for dating of this text, see Needham 1959, vol. 3: 19-20 and Cullen's (1996) translation of the *Zhoubi suanjing* 周髀算經. Needham translates the title as *Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*. For problems concerning the title and its translation, see Cullen 1996: xi and 163-172.

77 One *li* 里 was equal to 576 meters and 180 *zhang* 丈. One *zhang* equalled two "steps" *bu* 步 or ten "feet" *chi* 尺. One foot equalled ten "inches" *cun* 寸 or one hundred

> 天地之東西二萬八千里，北南二萬六千里，
> 出水之山者八千里，受水者八千里。
>
> [The extension of] heaven and earth is 28,000 *li* from east to west and 26,000 *li* from north to south. The mountains from which rivers flow cover 8000 *li* and into which rivers flow cover 8000 *li*.[78]

This passage gives no indication of how these measurements were taken, but as the passage continues, it becomes clear that the context for these distances is economic. These areas of land are divided into sources of grain and sources of metal:

> Those that produce copper number 467, while those that produce iron number 3690. This is the way Heaven and Earth have divided up the land for planting grain and provided sources for weapons and money. Capable rulers will have more than enough, while those who are stupid will suffer shortages.[79]

The same passage appears in the *Guanzi*, in a passage where Duke Huan [of Qi] asks about "methods involving the earth." Guanzi replies with the discourse ascribed to Yu in the *Shanhai jing*. The next discourse in the *Guanzi* elaborates the successful methods of the sage kings, which depended on economic factors, but also on the measurement, and demarcation, of territory:

> Huang Di questioned Bogao saying "I wish to mold the world into one family. Is there a way to do this?" Bogao responded by saying "I suggest that you cut down the sedge grass and erect boundary markers (*shu* 樹)."[80]

fen 分.

78 *Shanhai jing jiaozhu* 山海經校注, 5: 179-180. Cf. Cheng et al. 1985: 144; Fracasso 1983: 691; and Rickett 1985: 422. These chapters have been taken as a separate work, the *Classic of Mountains* (*Shan jing* 山經) or the *Classic of the Mountains of the Five Treasuries* (*Wuzang shanjing* 五臧山經).

79 *Shanhai jing*, 5: 180; cf. Cheng et al. 1985: 144; Fracasso 1983: 691; and Rickett 1985: 422.

80 *Guanzi* 管子 (*Sibu beiyao* ed.), 23: 1b; Rickett 1985: 423. There is debate about whether this term refers to markers for mineral deposits or to planting grain. See Rickett 1985: 423n10.

The *Lü Shi chunqiu* supplements this information by elaborating

> ... what lays within the Four Ends measures 597,000 *li* from east to west and 597,000 *li* also from south to north.[81]

Both the *Shanhai jing* and *Huainanzi* contain accounts of how the sage king Yu measured these distances. According to the *Huainanzi*:

> 帝乃使太章步自東極至于西極，二億三萬三千五百里七十五步。使豎亥步自北極至于南極，二億三萬三千五百里七十五步。
>
> The Emperor [Yu] then ordered Taizhang to walk from the eastern extremity [of the world] to the western extremity: two hundred thousand (*yi*), three myriad (*wan*), three thousand, five hundred [233,500] *li* and 75 steps; after this he ordered Shu Hai to walk from the northern extremity to the southern extremity: two hundred thousand, three myriad, three thousand, five hundred [233,500] *li* and 75 steps.[82]

The *Shanhai jing* provides a more complete account, with a different set of measurements, a more detailed method for obtaining them and varying accounts of who ordered them:

> 帝命豎亥步自東極至于西極，五億十選九千八百步。豎亥右手把算，左手指青丘北。一曰禹命豎亥。一曰五億十萬九千八百步。
>
> The Emperor ordered Shu Hai to step from the eastern extremity [of the world] to the western extremity: five hundred thousand (*yi*), ten ten-thousands (*xuan*), nine thousand, eight hundred [519,800] steps. Shu Hai held counting rods in his right hand and his left hand pointed north of Qing Qiu. Some say Yu ordered Shu Hai. Some say five hundred thousand (*yi*), ten myriad (*wan*), nine thousand, eight hundred [519,800] steps.[83]

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81 *Lü Shi chunqiu* 呂氏春秋 (*Sibu beiyao* ed.), XII: 3a. Cf. Wilhelm 1928: 159 and Fracasso 1983: 691-692.

82 *Huainanzi* 淮南子 (*Zhuzi jicheng* ed.), 4: 56; cf. Fracasso 1983: 692. The *bu* 步 or step was a double stride, conventionally reckoned at about 2 meters. At 180 *zhang* 丈 per *li* and 2 *bu* per *zhang*, there were 360 *bu* per *li*.

83 *Shanhai jing*, 9: 258; cf. Cheng et al. 1985: 172. This is based on the value of *yi* as 10^5 or 100,000. It has also been taken as 10^8 or 100,000,000 (Needham 1959: 87). *Wan* 萬 "myriad" and *xuan* 選 "ten-thousand" denote the number 10,000. For *wan* as the place-value term for 10^4, see Needham 1959: 83 and 87.

It may be useful to think of these accounts as of proto-measurement, but the term requires some clarification. Consider the following passage from Hesiod's *Theogony*, which describes the distances between Heaven, Earth and Tartarus in the following terms:

> ... a brazen anvil falling down from heaven nine nights and days would reach the earth upon the tenth: and again, a brazen anvil falling from earth nine nights and days.[84]

The *Huainanzi* and *Shanhai jing* measurements improve upon this in several respects. They restrict themselves to humanly accessible locations (unlike Heaven or Tartarus); the "steps" of the *Huainanzi* and *Shanhai jing* purport to be actual measurements that could have been made by actual persons. This is not to say that they were real measurements, or that these "measurements" had any observational basis. A "step" was hardly a consistent unit of measure (until it became standardized during the Qin dynasty), and other Qin and Han texts contain examples of imaginery data that sound like direct measurements, but probably are not. Nonetheless, the use of a calculating device (the counting rods) and the importance assigned to the accurate measurement and demarcation of territory attest to the early importance accorded to geographical measurement. Techniques for accurate measurement of the area and directional orientation of land were clearly central aspects of these quasi-mythical accounts of the establishment of royal power.

Geodesy by Calculation Based on Gnomon Measurements

Accounts of techniques for calculating distances based on measurement of a shadow cast by a gnomon and on the properties of right triangles appear in the third book of the *Huainanzi* and in the *Zhoubi*, which includes discussions on the properties of right triangles and the use of gnomons. Shorter variations on these techniques occur in the *Mozi* 墨子, the *Rites of Zhou* or *Zhouli* 周禮 and the *Nine Chapters on the Mathematical Art* or *Jiuzhang suanshu* 九章算術.

They all assumed some kind of round heaven over a square (and flat) earth.[85] In the *gaitian* 蓋天 theory, which informs the *Zhoubi*, heaven covered earth like the canopy of a chariot; that is, a square within the circle of an umbrella-like hemisphere, defined by an axis mundi at its center. Earth and heaven were parallel planes, in some versions flat, in others shallow arcs or hemispheres. The pur-

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84 Hesiod, *Theogony*, 720.

85 For a discussion of the antiquity of these beliefs, see Allan 1991. For the Han context, see Major 1993: 32-43.

pose of *gaitian*-oriented gnomon measurements of the *Zhoubi* was to measure the distance between these two planes (or arcs), the "height of heaven."[86]

Several flaws in the *gaitian* theory, including its inability to account for sunrise and sunset, led to its rejection in favor of the *huntian* 渾天 theory, in which heaven was a sphere surrounding the square flat earth.[87] Although the *Huainanzi* probably predates the *gaitian*-oriented *Zhoubi*, it cannot be classed clearly under either theory. Cullen suggests that it predates either theory and may be the earliest known Chinese attempt at a quantitative empirical cosmology.[88] If so, this would present a serious challenge to Koyré's claim.

Huainanzi 3 ends with a section on the use of gnomons to calculate terrestrial and celestial distances. It is probably a later addition and not part of the original *Huainanzi* text. It gives directions for a series of gnomon techniques, beginning with determination of the directions of sunrise and sunset, and thence of the cardinal directions.[89] These preliminaries are followed by methods "to know the breadth and length of east, west, north and south," effectively a more refined technology for the measurements originally ascribed in the *Shanhai jing* to the orders of the legendary sage king Yu. The following discussion seems to assume that the sun's journey begins at the point in heaven parallel and at the eastern extremity of the earth, allowing the construction of a triangle formed by an observer, the rising (or setting) sun and the easternmost (or westernmost) parallel extremity of the earth:

> 欲知東西南北廣袤之數者立四表以爲方一里距。先春分若秋分十餘日從距北表參望日始出及旦以候相應。相應則此與日直也。輒以南表參望之以入前表數爲法。除舉廣除立表袤以知從此東西之數也。
>
> (d) If you wish to know the figures for the breadth and length of east, west, north and south set up four gnomons to make a right-angled figure one *li* square. More than ten days before the spring or autumn equinox sight along the northern gnomons of the square on the sun from its first appearance to its rise above the horizon. Wait for [the day when] they coincide. When they coincide they are in line with the sun. Each time take a sight on it [the sun] with the southern gnomons, and take the amount by which it is within the forward gnomons as the divisor. Divide the whole width and divide the length [between] the standing gnomons in order to know the measurements east and west from here.[90]

A more modest terrestrial calculation in the *Jiuzhang suanshu* helps us clarify the ambiguities of the last sentence. In that example, a square of four gnomons is used to calculate the distance between an observer and a tree (something that could be measured directly, terrain allowing). The left gnomons are aligned with the tree, it is sighted from the rear right gnomon (*cong hou you biao* 從後右表), and appears within the front right gnomon (as in Figure 2, below).[91] In addition to providing an answer (333 *chi* and 3.3 *cun*), the text also describes the method:

> 術曰令一丈自乘爲實，以三寸爲法，實如法而一。
>
> Square the one *zhang* [= 10 *chi* = 100 *cun*] and make it the dividend [*shi*]. Using the three *cun* as divisor [*fa*], divide the dividend by the divisor.[92]

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86 The competing *huntian* theory (which replaced the *gaitian* theory by the second century AD) also exerted an important influence on the *Zhoubi*. See Cullen 1996: 35-39 and Henderson 1984.

87 The *gaitian* theory was unable to account for the interval between the sun's setting in the west and rising in the east. According to the "Discourse on the Vault of Heaven" or *Qiongtian lun* 穹天論 of Yu Xi 虞喜 (c. 265 AD) "the sun turns round the pole [of heaven], disappearing at the west and returning from the east, but neither emerges from nor goes below [lit. enters] the earth." (Needham 1959: 211.) Wang Chong 王充, a proponent of the *gaitian* theory, attempts to explain the apparent rising and setting of the sun as an optical illusion for any particular observer, caused by the sun moving closer and further away as heaven rotates with the sun attached to its underside. See *Lun heng* 論衡 (*Sibu congkan* ed.), 11: 8b-10a; Cullen 1996: 61.

88 Cullen 1976: 108-109; Major 1993: 270-271.

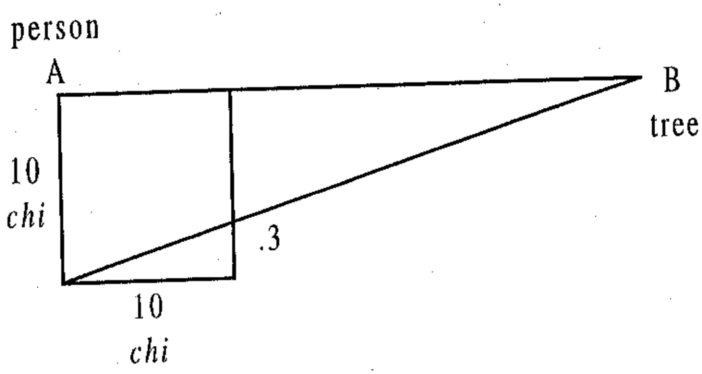
89 Variations on these passages appear in the *Mozi* (*Mozi yinde* 墨子引得, Shanghai: Guji, 1982), 35: 6-7, the "Artificer's Record" (*Kaogong ji* 考工記) of the *Zhouli* 周禮 (*Sibu congkan* ed.), 12: 15b, and the *Zhoubi suan jing* 周髀算經 (ed. Qian Baocong 錢寶琮 in *Suanjing shishu* 算經十書, Beijing: Kexue, 1963): 56. Cf. Cullen 1976, sections a-c. Both the original article and the version reprinted in Major (1993) use the same section divisions.

90 *Huainanzi*, 3: 53-54, translation slightly modified from Cullen 1976: 116. My discussion follows Cullen's sections.

91 The square is 1 *zhang* (10 *chi*) in length. The tree appears 3 *cun* within the forward right gnomon. The distance is calculated as 333 *chi*, 3 1/2 *cun*.

92 *Jiuzhang suanshu* 九章算術 (ed. Qian Baocong 錢寶琮 in *Suanjing shishu*, Beijing: Kexue, 1963), 9: 257; Cullen 1976: 117.

Figure 2: Measuring the Distant Tree according to the *Huainanzi* and *Jiuzhang suanshu*



$$\frac{.3}{10} = \frac{10}{AB}$$

AB = 333 *zhang* = 3.3 *cun*

This exposition clarifies the technique for measuring the breadth of the cardinal directions. It has three components. The first is to set up the gnomons in the correct alignment. This means both orienting the square correctly along a north-south and east-west axis and taking the measurement when the sun is correctly aligned with it. An observer at the northwest sights along the northern two gnomons to ascertain the time that they are in line with the sun (see Figure 3, below). The second was to measure the distance of the sun within the "forward gnomon" (the eastern) at sunrise. An instruction to "take a sight on it with the southern gnomons" presumably meant that one observer stood at the southwest gnomon and a second proceeded due north from the southeast gnomon until he was directly between the southwest observer and the sun. Once this distance was measured it was possible to compute the distance to the eastern limit from the northwest gnomon. The passage continues:

假使視日出，入前表中一寸。足寸得一里也。一里積萬八千寸得從此東萬八千里。視日方入，入前表半寸，則半寸得一里。半寸而除

一里積寸得三萬六千里除，則從此西里數也。并之東西里數也，則極徑也。

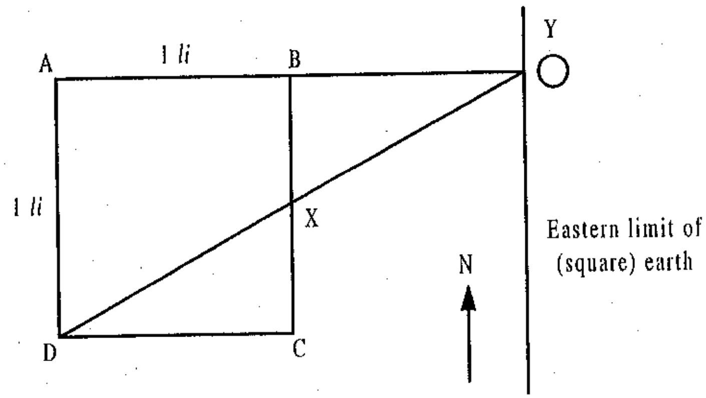
(e) Suppose that the rising sun enters one inch within the forward gnomons. This implies that for an inch one gets one *li*. One *li* contains 18,000 inches, so one gets 18,000 *li* eastwards from here to the sun. [Suppose] one observes the sun just as it sets, and it sets half an inch within the forward gnomon: then for half an inch one gets one *li*. Dividing the number of inches in a *li* by half an inch, one gets 36,000 *li*. Divide, and then [you have] the number of *li* westwards from here. Add them, [and you have] the number of *li* east and west, which is the diameter of the extreme limits.[93]

In summary, four gnomons A B C and D are set up so in a square so that the lines AD and BC point straight north-south and AB and CD straight east-west. An observer at A sights along B to the sun to ascertain the time that the northern two are in line with the sun (Y). The procedure is:

1. Measure CX (distance of the sun within the eastern gnomons). Presumably, one observer stood at D and a second proceeded north on the line CB until he was directly between the observer at D and the sun. This allows use of the properties of the similar right triangles DAY and DCX to compute AY, the distance to the eastern limit.
2. CD and AD are each 1 *li* (by setup).
3. $\frac{CX}{CD} = \frac{AD}{AY}$
4. $\frac{CX}{CD} = \frac{1\ \text{inch}}{1\ li} = \frac{1}{18{,}000}$ (1 *li* = 18,000 inches)
5. AY = 18,000 *li*
6. A similar measurement for the distance to the west is taken at sunset. Presumably one observer stood at C, the second proceeded north on the line DA until he was directly between the observer at C and the sun.
7. Given a measurement of 1/2 inch within the forward gnomon, the ratio is 1/2 inch to 1 *li*. The same calculation yields 36,000 *li*.
8. The sum of the two yields the number of *li* east and west, the diameter of the extreme limits.

93 *Huainanzi*, 3: 54, translation slightly modified from Cullen 1976: 119. (Figure 3 is also based on Cullen's exposition.)

Figure 3: Calculation of the Eastern Limit according to the *Huainanzi*



This computation of the dimensions of the earth as 54,000 *li* east-west is larger by an order of magnitude than the *Shanhai jing* measurement of 519,800 steps or 1443 *li*.[94]

This calculation, like the hypotheses of Aristarchus, is flawed by an incorrect *a priori* and a disinterest in actual measurement. The introductory "suppose that" (*jiashi*) suggests the use of hypothetical data, as do the implausible measurement of half-inch and inch distances from a distance of one *li*.[95] Even more important is the unjustified equivalence between one inch and one *li* measuring east (*shi cun de yi li ye* 是寸得一里也) and between a half inch and one *li* measuring west. As Cullen points out, the actual distance between the earth and sun of 93 million miles or 300 million *li* would put the sun within the forward gnomons at a distance of six hundred thousandths of an inch (0.00006 inches).[96]

94 This calculation is based on an understanding of the number *yi* where 1 *yi* = 100 thousand. Alternatively, if 1 *yi* = 100 million (500,019,800 steps), the number is 1,388,944 *li*.

95 Cullen 1976: 119n12.

96 Cullen 1976: 119.

From Measurement to Calculation: Celestial Distances

The *Huainanzi* and the *Zhoubi* also describe the use of gnomons to calculate the height of the sun, or the height of heaven, the last gnomon calculation described in *Huainanzi* 3:

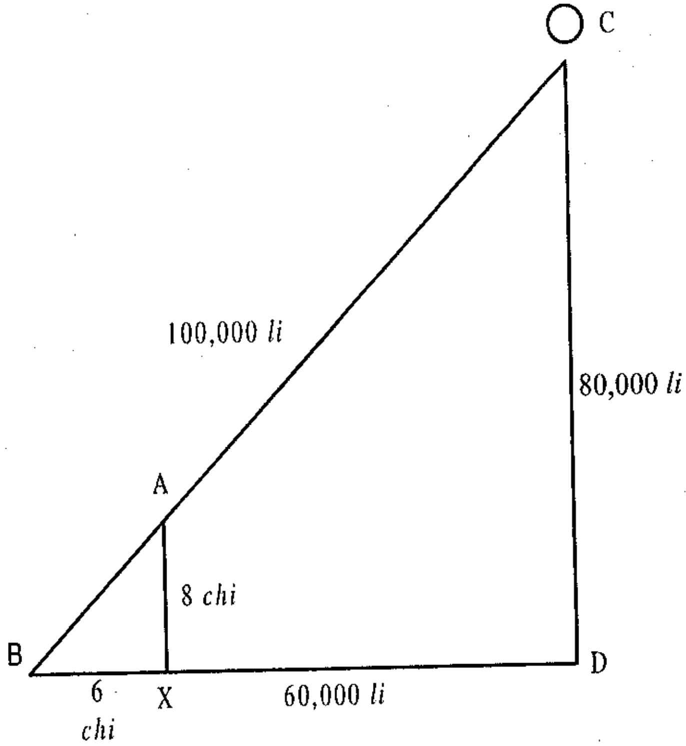
> 欲知天之高樹表高一丈正南北相去千里。同日度其陰。北表二尺，南表尺九寸。是南千里陰短寸南二萬里則無景。是直日下也。陰二尺而得高一丈者，是南一而高五也。則置從此至日下，里數因而五之爲十萬里，則天高也。若使景與表等即高與遠也。
>
> (k) To find the height of heaven, set up gnomons 1 *zhang* [ten feet] high and 1000 *li* apart due north-south. Measure their shadows [at noon] on the same day. The north gnomon [shadow] is two feet, and the south gnomon [shadow] is one foot nine inches. Thus a thousand *li* due south shorten the shadow by one inch and twenty thousand *li* due south there is no shadow at all. This is directly below the sun. A two-foot shadow corresponds to a height of 1 *zhang* [10 feet] so for each unit southwards one rises five units. Therefore if one takes the number of *li* from this position south to the subsolar point and multiplies by five, making 100,000 *li*, this is the height of heaven. Supposing the shadow is equal to the gnomon, then the height is equal to the distance.[97]

The *Zhoubi* calculation starts with an eight foot (*chi*) gnomon at noon of the summer solstice whose shadow measures one *chi* six *cun*. In the *Zhoubi* and elsewhere, the components of a right triangle are described as the hook (*gou* 句 or 勾), thigh (*gu* 股) and bowstring (*xian* 弦); these correspond to what, in modern parlance, we would call the base, altitude and hypotenuse of a right triangle. Later I discuss several issues that arise from this terminology.

The 6 *chi* hook (BX) and 8 *chi* thigh (AX) of the gnomon and its shadow (triangle AXB) correspond to the 60,000 *li* hook (BD) and 80,000 *li* thigh (CD) of the triangle formed by the sighting point (B), the sun (C) and the point where the sun casts no shadow (D). Its thigh is the distance from the earth to the sun, as shown in Figure 4, below.

97 *Huainanzi*, 3: 54, modified slightly from Cullen 1976: 123. This technique is also discussed in detail by Needham 1959: 225.

Figure 4: The Height of Heaven according to the *Zhoubi suanjing*



髀者，股也。正晷者，句也。正南千里，句一尺五寸。正北千里，句一尺七寸。日益南，晷益長。

B10. The gnomon is the thigh. The exact shadow is the hook. One thousand *li* due south the hook is one *chi* five *cun*. One thousand *li* due north the hook is one *chi* seven *cun*. As the sun goes further south, the shadow grows [correspondingly] longer.[98]

98 *Zhoubi*, 1: 26; cf. Cullen 1996: #B10. Henceforward references to Cullen's translation are given by his section numbering. These translations at points diverge from, but are all substantially indebted to it, as is Figure 4.

Thus a thousand *li* of terrestrial distance corresponds to a *cun* change in the shadow length. A bamboo sighting tube one *cun* in diameter and eight *chi* long is used to sight the sun just when the gnomon's shadow reaches six *chi* in length and the sun exactly covers the bore of the sighting tube:

由此觀之，率八十寸而得徑一寸。故以句爲首，以髀爲股。從髀至日下六萬里而髀無影。從此以上至日，則八萬里。

B11. From this you can observe that 80 *cun* [of distance] corresponds to (*lü*) one *cun* of diameter. Therefore take the hook as the head [*shou*, starting point] and the gnomon as the thigh. From the *bi* to the subsolar point is 60,000 *li* to where the gnomon has no shadow. From this point directly upward to the sun is 80,000 *li*.[99]

The passage concludes with a summary of this regular correspondence:

法曰：周髀長八尺，句之損益寸千里。

B12. The method dictates: for a gnomon 8 *chi* in length, a *cun* of decrease or increase in the thigh is 1000 *li*.[100]

Although the gnomon measurements of terrestrial and celestial distances use the same techniques, there is an important difference between them. The terrestrial measurements replicate the accounts of direct measurement in the *Huainanzi* and elsewhere. Terrestrial distances can, in principle at least, be measured directly, rather than calculated, and the texts above claim to have done so. The use of gnomon measurements to calculate celestial distances presents a different case as there is no alternative form of direct measurement available.

The *Zhoubi* is explicit about this difference. The text begins with a discourse between the Duke of Zhou 周公 and a certain Shang Gao 商高 in which the Duke asks for instruction. His stated reasons for needing it begin from, and amplify on, the activities of the sages in measuring terrestrial distances:

竊聞乎大夫善數也，請問古者包犧立周天曆度，夫天不可階而升，地不可得尺寸而度，請問數安從出？

A1. I have heard, Sir, that you excel in numbers. May I ask how Bao Xi laid out the successive degrees of the circum-

99 *Zhoubi*, 1: 26; cf. Cullen 1996: #B11.
100 *Zhoubi*, 1: 34; cf. Cullen 1996: #B12.

ference of heaven in ancient times? Heaven cannot be scaled like a staircase, and earth cannot be measured out with a footrule. Where do the numbers come from?[101]

This passage is interesting for several reasons. First, it provides a rationale for the need for calculation, rather than direct measurement. In the case of celestial measurements, it is obvious that the direct measurement of distance is impossible. Applying the same reasoning to terrestrial measurement, however, is a subtler point. Shang Gao's introductory exposition ends:

是故知地者智，知天者聖。智出於句，句出於矩。夫矩之於數，其制裁萬物，唯所爲耳。

A7. Therefore those who have correct knowledge of the Earth are wise but those who understand Heaven are sages. Wisdom comes from the hook. The hook comes from the [carpenter's] square. It is through [relations with] numbers that the [carpenter's] square regulates the myriad things.[102]

Shang Gao now recounts a discourse in the ancient past, where one Rong Fang 榮方 asks a master Chen Zi 陳子 whether his *dao* comprehends an understanding of (among other things): The height and size of the sun (*zhi ri zhi gao da* 知日之高大), its nearest and farthest distances [from earth], and the dimensions of the world [from north to south] (*tiandi zhi guangmao* 天地之廣袤).[103] This introduction also clearly positions the calculations in the *Zhoubi* within a discourse of sagacity. Further, it claims a higher level of knowledge for those who have gone beyond terrestrial measurement, which can be done directly, to celestial, which must be done by calculation.[104]

101 *Zhoubi*, 1: 13; cf. Cullen 1996: #A1.

102 *Zhoubi*, 1: 23; cf. Cullen 1996: #A7.

103 *Zhoubi*, 1: 23-24; cf. Cullen 1996: #B1. The commentary takes *tiandi* as "the world" ("the length and breadth of east, west, north and south") rather than "heaven and earth."

104 An alternative interpretation is that those who know heaven have higher knowledge simply because heaven is more noble than earth. In this view, neither heaven nor earth can be measured directly, since one is too high and the other too vast. As a result, any numerical description must come from indirect measurements and calculations. I am grateful to an anonymous reader for EASTM for pointing out this possibility.

Accumulating Trysquares and the Pythagorean Theorem

The first section of the *Zhoubi* contains a demonstration of a method it describes as *ji ju* 積矩 or "accumulating trysquares."[105] (Cullen uses the term "trysquare" as a translation for *ju* 矩, the carpenter's square.)

故折矩以爲勾廣三，股修四，徑隅五。既方其外，半之一矩。環而共盤，得成三，四，五。兩矩共長二十有五。是爲積矩。

A3. Therefore fold a trysquare so as to make the hook three in breadth, the thigh four in extension, and the diameter five in length. Make a square around its outside, and halve it to one trysquare. Placing them round together in a ring, one can form three, four and five. The two trysquares have a combined length of twenty-five. This is called the accumulation of trysquares.[106]

It ends by linking its method to the rule of the sage king Yu:

故禹之所以治天下者，此數之所生也。

Thus we see that what made it possible for Yu to set the realm in order was what numbers engender.[107]

This passage explains how to calculate the distance between the end points of a right-angled figure whose dimensions are three and four. The components of the figure can be drawn easily and its dimensions measured directly. Cheng-Yih Chen's explanation of this passage in contemporary terms describes it as a dissection method demonstration of the Pythagorean theorem.[108] Some of the difficulties of that explanation are discussed below.

A second *Zhoubi* passage uses the "accumulating trysquares" method to calculate a distance that cannot be measured, that is, the distance from the earth to the sun:

105 Joseph Needham (1959: 22) translates this term as "piling up the rectangles."

106 *Zhoubi*, 1: 14; cf. Cullen 1996: #A3.

107 *Zhoubi*, 1: 14; cf. Cullen 1996: #A3.

108 Chen Cheng-Yih glosses the *gougu* method as the "*gougu* (Pythagorean) theorem" and translates the central section very differently: "circumscribe it by half-rectangles so as to form a square plate. Then the 3-4-5 relation can be established, since the total difference between [the square plate] and the two rectangles is an area of 25. [This method of proof] is called 'piling up the rectangles'" (Chen 1987: 35-36). This translation and analysis anachronistically assumes the Pythagorean theorem. For a more extensive account of the history of interest in right triangles in early China, see Lam Lay-yong 1984: 87-112.

若求邪至日者，以日下爲句，日高爲股。
句，股各自乘，并而開方。

> B11. If we require the oblique [*xie*] distance [from our position] to the sun, take [the distance to] the subsolar point as the hook, and take the height of the sun as the thigh [base and altitude]. For the hook and thigh multiply each by itself [*ge zicheng*], combine [*bing*], and then extract the root [*kai fang*].[109]

In other words: square both base and altitude, add them and take the square root, which gives the oblique distance to the sun.

Expositions of these passages by Needham, Chen and others seem to make the tacit assumption that their authors used and understood the properties of similar triangles and the Pythagorean theorem. Indeed, such a view dates back to the initial transmission of Euclidean mathematics to China, and Matteo Ricci's collaborator Xu Guangqi's 徐光啓 attempts to compare traditional Chinese and Euclidean methods of triangular measurement, incuding a study of the properties of right triangles titled *Gougu yi* 勾股義 (literally, *Principles of Hook and Thigh*).[110]

Recent scholarship has raised other interpretations and possibilities. Were the correspondences described in the passages the corresponding sides of similar triangles, or were their authors simply using analogical reasoning and positing "correspondences" between the lengths of gnomon shadows and celestial distances? Did these texts even employ a notion of a triangle as a type of plane figure?

In his recent translation of the *Zhoubi*, Cullen shifts from others' and his own prior use of the term "similar triangles" to describe the reasoning of the Chinese gnomon texts and argues that the *Zhoubi* does not *calculate* the dimensions of the similar triangles, but relies on ratios among similar *categories*. In this view, the Chinese authors used the measurements of gnomon shadow lengths, not as part of mathematical calculations, but rather as instances of the corresponding "categories" of *chi* and *li*. In *Zhoubi* B11, six *chi* correspond to eight *chi* as 60,000 *li* correspond to 80,000 *li*. Thus the distance to the sun is not measured but inferred from a rule of 1 *chi* to 1000 *li*. Cullen suggests that this method of "calculation" is more verbal than computational, more of a Saussurean syntagm than mathematical thinking.[111] This interpretation would seem to explain this passage as a

109 *Zhoubi*, 1: 26; cf. Cullen 1996: #B11.

110 Hashimoto 1988: 12-13. The more general comparison appears in a text titled the *Celiang yitong* 測量異同 or *Similarity and Difference in [Chinese and Western Methods of] Mensuration* of 1617.

111 Cullen 1996: 77-79. Nor is the situation entirely transparent on the Greek side. For discussion of diverse notions of "ratio" in Greek mathematical thinking, see Fowler 1991. For the role of mathematics and cosmography in Plato's view of science, see Mourelatos

demonstration by analogy, rather than a mathematical demonstration according to prevailing Western usage.[112]

Quite aside from the problem of similar triangles, we must also consider the status of any kind of triangle in these texts. The *Zhoubi* terms for the components of a right triangle identified what we would now call the base, altitude and hypotenuse of a right-angled triangle — *gou* or hook, *gu* or thigh, and *xian* or bowstring, as well as the combination of hook and thigh to make the right angle of the trysquare (*ju* 矩). *Zhoubi* A3 uses the term diameter, *jing* 徑, normally the diameter of a circle, for what the commentary identifies as *xian*, the hypotenuse.

Zhao Shuang 趙爽 (third century AD), the first commentator on the *Zhoubi*, adds explanatory essays and diagrams to each of these passages.[113] The first one, between B11 and B12, discusses the relations between the base, altitude and hypotenuse of right triangles. It includes an illustrative diagram, titled the "Bowstring Diagram" 弦圖 (*xian tu*) in which a right triangle is inscribed within a square on a grid (Figure 5). The square's lower left side forms the hypotenuse of 5; it is labeled: "bowstring [hypotenuse] 5" (*xian wu* 弦五). The other two sides of the unnamed figure are labelled similarly "hook [base] 3" (*gou san* 句三) and "thigh [altitude] 4" (*gu si* 股四).[114] Whilst the passage uses terms for the circle (*yuan* 圓) and square (*fang* 方), neither the *Zhoubi* nor Zhao Shuang's commentary uses a term for a whole triangle, right or otherwise (*sanjiao* 三角 or *sanjiao xing* 三角形).

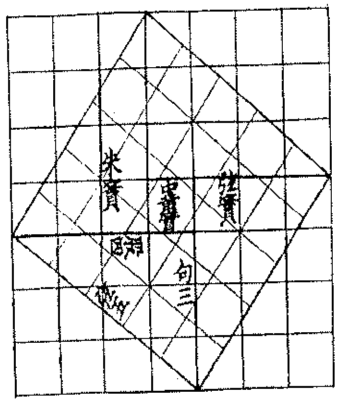
1991: 23-25.

112 In recent years, a considerable body of scholarship has addressed the question of whether and in what manner Chinese mathematics contained notions of demonstration and proof, and the nature and degree of philosophical reflection in Chinese mathematical works. For an assessment of some of the broader problems of comparative mathematics, see Cullen 1995. The question of demonstration and proof is further explored in a series of studies by Karine Chemla (1991, 1992, 1997 and 1999).

113 Most of his commentary is straightforward exposition of terminology. The major exception is four extended essays and four diagrams. The first two of the essays and diagrams appear following the two passages that deal with accumulating trysquares. Cf. Gillon 1977 and Cullen 1996: 69-71.

114 *Zhoubi*, 1: 18; Cullen 1996: 205.

Figure Five: The Bowstring Diagram as added by Zhao Shuang to the *Zhoubi suanjing*



As Cullen remarks about the *Zhoubi*:

> The problem is that at no point does the text compare one triangle with another, and indeed it contains no noun corresponding to our 'triangle' at all. Plane figures bounded by three straight lines just do not figure as a unit of discourse.[115]

This observation applies not only to the gnomon measurements in the *Huainanzi*, but also to the propositions on geometry in the Mohist Canon. Mohist geometry starts with definitions of the circle and square, and uses both, but does not refer to a triangle.[116] And there is curiously little discussion in the secondary literature of when the concept of a triangle first entered Chinese mathematical (or other) discourse.[117] Whether or not the *Zhoubi* authors had an abstract concept of "triangle," it is clear that they knew how to use them. The emphasis in the *Zhoubi* passage on the importance of "accumulating trysquares" and the evidence of Zhao Shuang's commentary suggest that the Chinese authors understood the use of the Pythagorean theorem.

The *Jiuzhang suanshu* or *Nine Chapters on the Mathematical Art* also gives many examples of calculations using various kinds of solid figures with triangular

115 Cullen 1996: 77-78.

116 *Mozi*, 40: 1-10. These passages are coherently reconstructed in Angus C. Graham's *Later Mohist Logic, Ethics and Science* (Graham 1978: 301-316). Nor does the term appear in the *Huainanzi* gnomon measurement passages, or elsewhere in the *Huainanzi*.

117 Cf. Needham 1959: 25-31 and passim; Graham 1978; Chen 1987; and Martzloff 1987 (trans. Wilson, 1997): 132-133 and passim.

faces, even though it never uses the word "triangle" (*sanjiao* 三 角).[118] Its first chapter, "Surveying Fields" (*Fang tian* 方 田), contains 38 examples of calculations of the areas of fields of different shapes, including rectangular shapes and trapezoids, circles, irregular fields and "tablet shaped fields," *gui tian* 圭 田, in the shape of an isosceles triangle."[119] Its last chapter, "Hook and Thigh" (*Gougu* 句 股), uses 24 examples to elaborate the properties of right triangles, but these are described in terms of hook, thigh and bowstring.[120] None of these examples use a specific term for triangle, though it does use explicit terms for several triangular solids. *Bienao* 鼈 臑 is a right-triangular base pyramid with one lateral edge perpendicular to the base; *qiandu* 塹 堵 is a right prism with a right triangular base; and *yangma* 陽 馬 is a rectangular-base pyramid with one lateral edge perpendicular to the base.[121]

These considerations present suggestive evidence that early Chinese mathematics, and possibly Chinese culture more broadly, did not use an explicit concept of a triangle. The circle and the square are the two plane figures that appear consistently in the *Zhoubi*, the Mohist canon, and in Chinese cosmology, which seemed to have no need for a triangle. An explicit concept of triangle is not necessary to demonstrate the Pythagorean theorem; contemporary mathematicians frequently do it by means of graph coordinates.[122] At first glance, its absence might appear to weaken the apparent analogy between the Chinese and Greek gnomon measurements. However, references to either triangles or similar triangles are also absent from Cleomedes' account of Eratosthenes' method, which refers, not to the side of a right triangle, but rather to the "arc in the bowl of the sundial" (the segment AC in Figure 1, above). In other words, according to Cleomedes at least, Eratosthenes relies, not on the properties of similar triangles, but on the properties of similar sectors of a circle.[123]

118 The authorship and early history of the *Jiuzhang suanshu* are unclear. It seems to have been current in the early years of the Later Han and was considered a Han book by its third century commentator Liu Hui 劉 徽, probably containing mathematical knowledge dating back to the Warring States.

119 *Jiuzhang suanshu*, 1: 93-94 and 99-111. For *gui tian* see *Jiuzhang suanshu*, 1: 101-102, examples 25-26.

120 *Jiuzhang suanshu*, 9: 241-258. The *Sea Island Mathematical Manual* or *Haidao suanjing* 海 島 算 經, also ascribed to Liu Hui, contains further descriptions of the use of triangulation in the measurement of distant (terrestrial) objects. See Needham 1959: 30-40.

121 *Jiuzhang suanshu*, 5. The text uses these terms in problems 14 (*qiandu*), 15 (*yangma*) and 16 (*bienao*). Liu Hui introduces them in his commentary in problems 10, 14, 15, 17 and 18 (*qiandu*), 10, 12, 15, 16, 17 and 18 (*yangma*), and 15 and 17 (*bienao*). See also Wagner 1979 and Lloyd 1996: 152-153n22. A commentary by the Song polymath Shen Gua 沈 括 (1031-1095) in the *Hanyu da cidian* 漢 語 大 詞 典 (vol. 11, 1069) notes the *Jiuzhang suanshu* as the *locus classicus* for the term.

122 John C. Baez, personal communication.

123 I am grateful to an anonymous reader for EASTM for calling attention to this

Metaphors of Quantification and Precision

Both the Chinese and Greek material seem to show some disregard for exact measurement, but there is a striking and perhaps significant difference between the Greek and Chinese scientific contexts. Metaphors of quantification and precision appear across the spectrum of Chinese Warring States philosophical and political thought so repeatedly that they may function as what Sarah Allan and others have described as "root metaphors." (Allan uses the work of A. C. Graham to argue that philosophers' conceptual schemes rely on "pre-logical" patterns of names and categories.)[124] Before the term *gui ju* 規 矩, "compass and square," lost most of its literal meaning and came to mean "morally well regulated," metaphors of the accuracy of craftsmen's measuring tools expressed a wider range of notions that included moral and technological advancement, innovation, human relations, skill and natural world. [125]

Mozi 墨 子 compares the mandate of heaven to the wheelwright's compass and the carpenter's square. The metaphor is one of accurate fit.[126] Mencius 孟 子 makes an analogy between the "perfection of squares and circles" of the compass and square and the sage's "perfection of human relationships.[127] Xunzi 荀 子 analogizes the certainty they provide (as to squareness and roundness) to the *junzi's* 君 子 use of the rites, and distinguishes its standards from the arguments of the like of Hui Shi 惠 施 and Deng Xi 鄧 析.[128] Zhuangzi 莊 子 equates "real skill" with throwing them away, but still speaks in terms of them.[129] The *Shang Jun shu* 商 君 書 argues against turning away from the clear standards established by the former kings in favor of private assessments.[130] Han Fei 韓 非 treats them as correctives to wrongdoing. Models and distinctions, calibrations and measurements were established because: "Once calibrations and measure-

point. Cleomedes' description merely assumes that (1) straight lines falling on parallel straight lines make the alternate angles equal; and (2) the arcs subtended by equal angles have the same proportion and the same ratio to their proper circles. The latter assumption, rather than the properties of similar right triangles (CAB and EAD in Figure 1), is the basis for the ratios of 1, 50, 5000 and the circumference of the earth.

124 Allan 1997: 13-14; Graham 1992; Lakoff and Johnson 1980: 3, 18, 22.

125 Cf. Bodde 1981: 134f.

126 *Mozi*, 26: 41. For translation, see Mei 1929: 140. Cf. *Mozi* 墨子, 4: 2-3, 27: 63-67 and 28: 44-45 (Mei 1929: 13, 149 and 156) and Graham 1978: Canons, 316.

127 *Mengzi*, 4A: 2, in *Lunyu yinde. Mengzi yinde* 論語引得，孟子引得 (Shanghai: Guji, 1986). Cf. 4A: 1, 6A: 20 and 7B: 5.

128 Cf. *Xunzi yinde* 荀子引得 (Shanghai: Guji, 1986), 1: 1-2, 11: 43 and 19: 32-34. Cf. Knoblock 1988, 1990 and 1994: vol. 1, 135, vol. 2, 55, and vol. 3, 61, respectively.

129 *Zhuangzi yinde* 莊子引得 (Shanghai: Guji, 1982), 8: 13, 9: 5 and 10: 26. For translation, see Graham 1986b: 201 and 204, and 209.

130 *Shang Jun shu* 商君書 (*Zhuzi jicheng* ed.) 14: 24-25. Cf. Duyvendak 1928: 262. Cf. *Shang Jun shu* 4: 10 and 24: 39, cf. Duyvendak 1928: 205 and 318.

ments have been made true, there is no chance for a Bo Yi to slip from what is right, or for a Robber Zhi to do what is wrong."[131]

This tendency continues into the Han. Dong Zhongshu 董 仲 舒 makes the analogy between the compass and square, which are necessary to draw circles and squares correctly, and the *dao* of the former kings, which is a "compass and square" for the world.[132] The *Huainanzi* makes an analogy between the "six measures" (*liu du* 六 度), the six precision measurement instruments, and heaven, earth and the four seasons: Heaven and the plumb-line or inked cord (*sheng* 繩), Earth the water level (*zhun* 準), spring the compass (*gui* 規), summer the balance (*heng* 衡), autumn the carpenter's square (*ju* 矩) and winter the weights (*quan* 權).[133] A similar analogy in the "Five Phases Produce Each Other" (*wuxing xiangsheng* 五 行 相 生) section of the *Chunqiu fanlu* 春 秋 繁 露 compares them to the five phases: (1) wood, spring, the east, the compass, birth; (2) fire, summer, the south, the carpenter's square, growth; (3) earth, summer-end, the center, the plumb-line, maturation; (4) metal, autumn, the west, the weights, destruction; and (5) water, winter, the north, the balance, storage.[134]

A more extended example appears in the memorial of the Han physician Chunyu Yi 淳 于 意, who uses a similar analogy in his account of his system of medical diagnosis.[135] He invokes a tacit metaphor of accuracy as a mark of the activities of the sage kings and ascribes the invention of vessel theory and accurate prognostic techniques to the sages of antiquity.

> By establishing compass and square, suspending weights and balances, applying the inked cord, and harmonizing *yin* and *yang* [the ancient sages] distinguished the vessels of the body and named each, in mutual resonance with heaven and earth, and blended together in the body. As a consequence, thereafter people made distinctions among the Hundred Ailments by distinguishing between them [the pulses]. Those who have this technique of prognostication are able to distinguish between them; those who do not consider them the same.[136]

131 *Han Feizi jishi* 韓 非 子 集 釋 (ed. Chen Qiyou 陳 奇 猷, Beijing: Zhonghua shuju, 1958) 26: 492. Cf. Liao 1939: vol. 1, 267. Cf. *Hanfeizi*, 6: 88 and 27: 498 (Liao 1939: vol. 1, 45 and 270).

132 *Chunqiu fanlu* 春秋繁露, attributed to Dong Zhongshu, in *Chunqiu fanlu jinzhu jinyi* 春秋繁露今註今譯 (Taibei: Shangwu, 1984), 1: 11.

133 *Huainanzi* (*Zhuzi jicheng* ed.), 5: 86-87; cf. Vogel 1994: 139. Needham (1962: 15-17) translates *heng* as steelyard and *quan* as balance.

134 *Chunqiu fanlu*, 13: 334-340.

135 Cf. Raphals 1998b: 7-28 and Sivin 1995: 177-204.

136 *Shiji* 史 記 (Beijing: Zhonghua, 1959), 105: 2813; cf. Bridgman 1955: 45. My translation is indebted to Elisabeth Hsu.

Sima Qian structures this biography around a defining incident in Chunyu's life: Charges brought against him to the Han throne, his reprieve through the memorial of his daughter Ti Ying 緹縈 and his subsequent memorial on the merits of his medical practices and prognostic ability.[137] In the memorial he claimed the ability to predict accurately which illnesses were fatal and which curable. No explicit notion of precision is specified in this passage, which reads:

> 知人死生，决嫌疑，定可治。
>
> He understood who would live or die, was decisive about dubious cases, and certain about what could be cured.[138]

He acknowledges that his prognosis is not perfect.[139] Nonetheless, the force of his rhetoric is to claim that, in the treatment and diagnosis of disease, he is able "to use pulse diagnosis to distinguish between life and death with infallible results."[140] This claim justifies his withholding treatment in certain cases and represents pulse diagnosis as a technique that separates him from potentially competing physicians, based on qualitative claims for accuracy.

Nor was Chinese interest in quantification restricted to the use of metaphors of precision and accuracy as a philosophical ideal. To mention a few oft-repeated examples: dating and recording of celestial and human events, the standardization of measurements, the decimalization of weights and measures, and the early presence of a wide range of instrumentation including sliding calipers, rain gauges, astrolabes and water clocks.[141] To these we can add an early interest in and use of sociological quantitative methods, most obviously, the population census and collection of other demographic data, starting in the Qin. Other human "quantifications" include the precise gradations of punishments in legal codes and the ranking of moral qualities. A striking example of the latter is the tabulations of *Gujin renbiao* 古今人表 or "Table of Ancient and Modern Persons" that comprises the twentieth chapter of the *Han History* 漢書, which ranks 1955 persons according to nine grades of intellectual and moral worth.[142] As Christoph

137 This incident is dated to 167 BC, during the reign of Han Wen Di (r. 180-157 BC). The biography consists of arguments by Ti Ying, by Chunyu Yi and by Sima Qian. Ti Ying's argument against mutilating punishments persuades Xiao Wen, not only to release her father, but to change the law. They are repeated verbatim at *Lienü zhuan* 列女傳 (*Sibu beiyao* ed.) 6.16 and are a very interesting example of argumentation by women. See Raphals 1998 and 1998b.

138 *Shiji*, 105: 2794.

139 *Shiji*, 105: 2817.

140 診病决生死有驗精良. *Shiji*, 105: 2796.

141 Cf. Bodde 1981: 138-141.

142 This ninefold classification of 1955 individuals from legendary times to the Qin dynasty was begun by Ban Gu 班固 (32-92 AD) and completed by his sister Ban Zhao 班昭 (d. circa 125 AD).

Harbsmeier points out, this table is also important evidence of the early sophistication of the Chinese concept of a class (*lei* 類), a notion of fundamental importance to the history of science.[143]

IV Quantification in Ancient Science Reconsidered

The foregoing discussion suggests several interrelated issues that are important for a nuanced understanding of measurement, calculation and quantification, both in Chinese and Greek science and in the broader context of claims for the Scientific Revolution. We need to understand the effects of *a priori* assumptions and the roles of competing theories and perspectives if we are to understand the microhistory of either context, let alone for any comparison. Taken together, the Greek and Chinese evidence indicates that talk of an antinomy between quantities and categories is very problematic and that the whole notion of quantification needs to be nuanced and clarified.

In Koyré's account, the "inaccuracies" of both the Chinese and Greek calculations are simply evidence for lack of quantitative measurement. Misguided focus on "accuracy" obscures important questions of us whether or what observations or measurements were actually attempted because it overlooks important distinctions in kinds and sources of "inaccuracy." First, false assumptions can introduce inaccuracies into calculations based on relatively accurate observational data and otherwise sound mathematical reasoning. The Greek and Chinese gnomon calculations attest to the importance of *a priori* assumptions in affecting or determining results. The Chinese assumption of a flat earth made an accurate determination impossible, and the "inaccuracy" is not a result of poor measurement or quantification. By contrast, the Greek assumption that the sun was, for practical purposes, infinitely distant from the earth, was not the major source of error in Eratosthenes' calculation. The accuracy of his measurement depends primarily on the length of the stade. Second, inaccuracies can be evidence for genuine attempts at accurate measurement, as distinct from purely qualitative methods of analysis such as humoural theories. A third kind of inaccuracy results from genuinely inaccurate measurement, or no measurement at all. Both the Chinese and Greek authors apply similar mathematical techniques to data that is, to varying degrees, conventional or idealized. Yet Aristarchus' use of conventional or idealized numbers in ratios is a very different kind of quantification from Archimedes' attempts at actual observational data.

It may be more useful to view both the Chinese and Greek calculations as elements in ongoing cosmographical speculations over such questions as the shape of the earth, the shape of the cosmos, the movements of the sun and earth, and how or whether to measure the distance between them. Competing Greek

143 Harbsmeier 1998: 218. For further discussion of this issue, see Raphals 2000.

cosmographies differed over whether the earth revolved around the sun or the sun revolved around the earth, and the magnitude of the distance between the two. They agreed, however, that the two were far apart and that the earth was a sphere. The Chinese cosmographies that informed the *Zhoubi* and *Huainanzi* calculations all posited some kind of round heaven over a square (and flat!) earth. They differed over the details of the round heaven, and whether the earth was square, but not whether it was flat.[144] This Chinese cosmological consensus on a flat earth disallowed consideration of the earth's curvature. (Nonetheless, the accuracy of the Chinese measurements is comparable to those of Aristarchus.)

The Greek and Chinese gnomon calculations also reflect the differing status of apodeixis in Chinese and Greek philosophy and science. The Chinese authors were concerned with the universality of their demonstrations, but their formulation did not proceed from axioms.[145] By contrast, Archimedes' description of *his* method is explicit. It is useful for initial investigation, but, more importantly, for the demonstrative proof that follows it:

> For certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions to supply the proof than it is to find it without any previous knowledge.[146]

However useful the mechanical method might be for investigation, and for other practical purposes, in Archimedes' view it was no substitute for the "demonstration" of formal proof. (The texts discussed earlier do not use it.)

The foregoing discussion has questioned the apparent similarity of Chinese and Greek understandings of triangles and the properties of similar triangles. There is suggestive evidence that the early Chinese mathematicians may not have had an explicit concept of a triangular plane figure, even though they clearly identified the parts of triangles and knew how to use them in practical contexts.

By contrast, triangles were central to Greek mathematical thinking, beginning with the cosmology of the *Timaeus*. Plato argues that the four ultimate constituents of matter are solid bodies, that solid bodies are bounded by plane surfaces, that all rectilinear planes are composed of triangles, that all triangles can be broken down into two kinds of right triangle. "All triangles originate from two triangles" (τὰ δὲ τρίγωνα πάντα ἐκ δυοῖν ἄρχεται τριγώνοιν) — the use of the dual is striking — the isosceles and the scalene.[147] He goes on to identify the four ultimate constituents of matter with four regular solids constructed out of triangles: the tetrahedron (fire), cube (earth), octahedron (air) and icosahedron (water).[148]

It is also noteworthy that the Chinese and Greek calculations seem to refer to *different* properties of similar triangles. Eratosthenes' calculation is based on their identical *angles;* the Chinese calculations (or correspondences) are based on an understanding of the proportionality of their line lengths. Angles are not discussed, nor even named.

The problem of the status of triangles in China and Greece illustrates the limitations of treating categories and quantities as mutually exclusive modes of explanation and forcing the evidence into one or the other mold. The categorical extreme assigns them to qualitative and non-measurable categories, ignoring the possibility that they may be actual measurements. Another problem is that "applying categories" does not require or explain the development of the gnomon measuring technique. If these were simply "categories," why create the triangle or "trysquare" of the gnomon and its shadow, rather than simply asserting them, as in other Han correlative cosmological texts? The quantitative extreme treats numbers indiscriminately as observational data, without regard to where they come from or how they are verified. Most discussions of Chinese gnomon measurements tend toward the quantitative extreme. They assume the existence of the abstract concepts of a triangle, similar triangles, and the Pythagorean theorem, and further that the numerical values were actual measurements.[149] This approach masks differences in how numerical measurements involving triangles were used. There is a tautological element to this antithesis between quantities and categories. Defining the Scientific Revolution as a shift from categories to quantities introduces a polarity that the microhistories do not bear out.

V Conclusions

Generalizations break down both within and between the two cultures, but overall, the Greek and Chinese mathematicians surveyed here seem to have been more interested in ratios and proportions than in measurement. The Greek mathematicians show a consistent preference for epideictic demonstration and the properties of numbers and ratios; they differ considerably in their interest in observation and measurement. Claims for an early Chinese empiricism also seem to break down upon close reading of the texts. This Greek picture, however, is

144 Cullen 1976: 108.

145 Recent publications by Geoffrey Lloyd have described this difference at length, especially Lloyd 1996 and Lloyd 1998.

146 Archimedes, *Meth.*; Praef., Archim., ed. Heiberg (1880-1881), ii, 426. 3-430, 22, trans. Heath 1912: 13. Cf. Mugler 1971: vol. 3, 83-84 and Lloyd 1998: 355.

147 Plato, Timaeus, 53d.

148 Plato, *Timaeus*, 53e-55c.

149 Cf. Needham 1959, Cullen 1976 and Chen 1987. By contrast, Dmitri Panchenko has argued that the extreme inaccuracy of the Chinese 1 inch per 1000 *li* ratio suggests that it was a garbled translation of some other ratio from some other culture (personal communication). See also Panchenko 1993: 387-414.

skewed by its focus on foundational theories of mathematics and astronomy. The high-precision skills of sixth and fifth century Greek architects and engineers and the temples and structures they created suggest a different picture of highly quantitative applied, if not theoretical, geometry.

A striking and perhaps significant difference between the Greek and Chinese scientific and philosophical contexts is the extent to which metaphors of craftsmen's measuring tools appear as metaphors across the spectrum of Chinese Warring States philosophical and political thought. These metaphors, in particular the "compass and square" expressed a range of notions of moral and technological excellence, and were put to the service of a wide range of arguments. These analogies are so pervasive as to suggest that accurate measurement and precision functioned as root metaphors in early China. These metaphors contain an interesting tension between precision and accuracy. In their later "Confucianized" usage of good order and moral rectitude, they clearly refer to accuracy. Yet it seems reasonable to suggest that the focus of their initial usage was on precision, since precision and exactitude of measurement, rather than accuracy, was what the actual practices of real craftsmen required.

The foregoing examples show important differences in techniques, goals, conceptual foundations and contexts for Greek and Chinese gnomon measurement calculations. They show the limitations of both unnuanced comparison and of a history of science constructed on contrasts between distinct world views of ancient and modern science.

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The Yabuuti Paradigm in the History of Chinese Science

The Ninth International Conference of the History of Science in China, City University of Hong Kong, 12 October 2001, Plenary Speech

Nakayama Shigeru

[Nakayama Shigeru retired in 2000 to become Professor Emeritus, but sti teaches graduate students. During his research career, his interests have shifte gradually from the history of traditional East Asian sciences to more contempo raneous science policy studies. From time to time, however, he returns to ol subjects, especially mathematical astronomy. His most recent publication i English is A Social History of Science and Technology in Contemporary Japar *Volume 1, Melbourne: Trans Pacific Press, 2001.]*

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It seems that the works of the late Professor Yabuuti Kiyosi 藪内清 have nc been fully translated into either Chinese or Western languages. Parts of his majo works are available in Chinese, thanks to Du Shiran 杜石然. In the West, hi work has been discussed in several articles, but most of them are quite sketch overviews. His analysis of Western influence on Chinese astronomy has bee translated by Benno van Dalen, while his small popular account entitled *Chines Mathematics* was recently translated into French, which has led readers in tha country to consider him a historian of Chinese mathematics.

When I visited Joseph Needham for the first time in 1957, Volume 3 of *Sci ence and Civilisation in China* (Cambridge: Cambridge University Press, 1954 hereafter abbreviated SCC) was in galley proofs. I noticed a grave omissior Needham had overlooked the significance of Chinese calendrical science, sayin "although there is a very large literature, still growing almost daily, on the Chi nese calendar, its interest is, we suggest, much more archaeological and historica than scientific," and "the whole history of calendar-making is that of successiv attempts to reconcile the irreconcilable, and the numberless systems of interca lated months, and the like, are thus of minor scientific interest." (SCC, 3: 390).

I told him that calendrical science was central to Chinese exact science, an that Yabuuti was spending his whole life exploring it by thoroughly studying th