Dual Channel Distribution:
The Case for Cost Information Asymmetry

Long Gao
School of Business Administration, The University of California Riverside, Riverside, CA 92521,
long.gao@ucr.edu

Liang Guo
CUHK Business School, Chinese University of Hong Kong, Shatin, Hong Kong, China,
liangguo@baf.cuhk.edu.hk

adem.orsdemir* School of Business Administration, The University of California Riverside, Riverside, CA 92521,
adem.orsdemir@ucr.edu

Dual channel distribution benefits upstream manufacturers but may irritate downstream retailers. The channel conflict only seems to aggravate when retailers are put at information disadvantage. We show this need not be the case. (i) We demonstrate upstream private information can improve channel efficiency and consumer surplus. The main mechanism is the offsetting interplay of signaling distortion and double marginalization: with private selling cost, the manufacturer may signal her cost by cutting the wholesale price; the price cut encourages the retailer to buy more, thereby reducing double marginalization and improving channel efficiency. (ii) We qualify the received wisdom. The general insight that cost information asymmetry reduces efficiency does not work in dual-channel settings. We show incorporating cost information asymmetry can change dual-channel equilibrium substantially—it can turn the retailer and channel from the victims of manufacturer encroachment to its beneficiaries. Also, we rationalize why the retailer can benefit from his information disadvantage, and when he can gain from the manufacturer’s selling cost improvement, despite retail competition. (iii) We demonstrate our results are robust for other prevailing arrangements, e.g., two-part tariffs, price competition, imperfect substitution, and simultaneous moves. Our results suggest a more nuanced view of manufacturer encroachment: as private cost information can ease channel conflict and improve consumer surplus, previous studies may have overestimated the harm of encroachment. By highlighting the critical role of cost information asymmetry, this study sharpens our understanding of dual-channel theory and practice.

Key words: dual channel, strategic uncertainty, information asymmetry, signaling

1. Introduction

In the age of e-commerce, manufacturers are increasingly relying on dual channel for distribution. They not only sell through traditional retail channels, but also sell directly to consumers, encroaching on their retailers’ territory. Such dual-channel distribution is a double-edged sword: although it can increase sales, it can also irritate retailers. How should manufacturers leverage dual channels to improve profits?

* Corresponding author. Author names are in alphabetical order.
Two factors complicate the problem. The first is channel conflict. In a dual-channel arrangement, the manufacturer and retailer are independent firms with divergent interests (Zhang et al. 2010). The arrangement empowers the manufacturer with the dual-selling option: it can broaden customer reach, deepen market penetration, and strengthen sales revenue (Alba et al. 1997, Chiang et al. 2003). But dual selling can also irritate the retailer, erode his market share, and depress his sales. Consequently, resentment arises and channel conflict worsens (Geyskens et al. 2002). Indeed, retailers such as Home Depot make no secret of their resentment (Brooker 1999): “We recognize that a vendor has the right to sell through whatever distribution channels it desires. However, we too have the right to be selective in regard to the vendors we select, and we trust that you can understand that a company may be hesitant to do business with its competitors.”

The second complication is information asymmetry. In a dual-channel relationship, the manufacturer often enjoys the information advantage of her selling cost, due to her superior knowledge on workforce, facilities, and selling ability (Corbett et al. 2004, Zhang 2010). For example, the direct selling cost has a complicated structure, including managing the online websites and the operating cost of both personnel and facilities. These costs are usually proprietary information; some are tightly-guarded trade secrets, inaccessible to the retailer (www.virtual-sales.com). The ever-changing nature of e-commerce further complicates the cost forecast. Unlike physical store selling, e-commerce operations are less visible, run at a fast “clock-speed” (Fine 2010), with constantly changing technologies and competitive landscape (Hazzard 2017). These dynamics induce additional uncertainty, making it impossible for others to forecast the cost precisely. The best one can hope for is a rough estimation of the cost distribution.

For the retailer, that cost information is crucial for a wide range of decisions. To plan sales, the retailer must know the intensity of retail market competition. Without a good understanding of manufacturer’s selling ability (cost), he cannot compete effectively. In practice, retailers infer the cost distribution from a variety of sources, such as industry reports and price comparison. For example, Home Depot offers in-store customers price-matching guarantees to monitor its online competitors (Wu et al. 2018). Such active price comparison allows Home Depot to gauge its manufacturers’ selling abilities, so that it can plan and respond properly in timing and magnitude.

The cost information asymmetry can induce strategic uncertainty, exacerbating channel conflict. Behind the veil of information asymmetry, the manufacturer may manipulate: although she can share the information

---

1 We focus on the situations where no mechanism is available for credible information disclosure or sharing. Such situations arise when detailed public information disclosure is infeasible, and may risk antitrust litigation (McAfee 2009). For example, the exchange of customer-specific pricing information is widely regarded to facilitate oligopolistic collusion; interfirm exchange of cost information has long been treated with suspicion by antitrust authorities. In these situations, neither party wants to risk antitrust litigation, and the retailer has to rely on public information to estimate the competitor’s direct selling cost.

2 We thank the senior editor for this line of reasoning. Indeed, Home Depot’s price match policy allows its store managers to discount up to $1000 without calling the regional director (www.consumerreports.org). These practices have an equalizing effect, producing similar price changes in frequency and size for traditional and online channels. For example, Cavallo (2017) finds that 72% of time the traditional and online channels offer identical prices.

3 By strategic uncertainty, we mean the uncertainty concerning the purposeful behaviour of players in an interactive decision situation (Brandenburger 1993).
to improve retailer efficiency, she can also abuse it to extract higher surplus. Without precise cost information, the retailer must recognize such strategic uncertainty, form his own belief, and predict the distribution of possible rival actions. Given the possibility of intense retail competition, he may rationally cut back orders, suppress sales, and hurt the channel. To ease channel conflict and assure the retailer, the manufacturer may have to make excessive concession, which may further damage channel efficiency. For example, Goodyear and Michelin promise their dealers a share of the online sales (Ulrich 2015, MTD 2016), while Nike allows their retailers to make 25% discount year around (Patos 2017). In general, information asymmetry can profoundly change the channel relationship. In such a situation, how should the manufacturer choose the selling strategy, taking into account its impact on retailer’s belief and response?

In this paper, we seek to understand how cost information asymmetry affects dual-channel performance. We model the dual-channel relationship as a game of incomplete information. The manufacturer (she) can sell through both the retailer channel and her direct channel. She is less efficient in selling, but enjoys an information advantage: she knows her true selling cost (high or low), whereas the retailer (he) knows only the distribution. At the outset, the manufacturer offers a wholesale-price contract to the retailer, who then decides his order quantity. Afterwards, the manufacturer decides her direct selling quantity and engages in quantity competition in the retail market. Both parties are strategic and profit maximizing.

We make three contributions to the channel literature. First, we identify how cost information asymmetry affects dual-channel performance. We show it entails new interactions. (i) The manufacturer and retailer engage in competition, cooperation, and signaling, simultaneously. They are at once channel partners and retail competitors. As such, the manufacturer sets wholesale price for multiple purposes, e.g., to extract wholesale revenue from the retailer, and to coordinate dual-channel sales for her own benefit. (ii) Two firms also engage in a signaling game. Facing information disadvantage, the retailer wants to infer manufacturer’s direct selling cost from the wholesale price. That information can guide him to better predict the upcoming retail competition and to calibrate his order properly. Anticipating this, however, the manufacturer may manipulate the price—she always wants the retailer to believe her cost is high. As such, cost information asymmetry can change channel dynamics in a substantial way.

We find cost information asymmetry can create an offsetting mechanism, improving efficiency. The mechanism works as follows. The low cost manufacturer has an incentive to inflate her cost; knowing this, the high cost manufacturer will distort her wholesale price downward. This price cut serves makes the wholesale price too low for the low cost manufacturer to follow, thereby credibly signaling the true cost. The lower wholesale price mitigates double marginalization and improves efficiency. Hence, cost information asymmetry can have a bright side.

This finding qualifies the conventional wisdom. In the supply chain literature, a general insight is that, cost information asymmetry invites manipulation, creates distortion, and reduces efficiency (Corbett et al. 2004, Zhang 2010, Hu and Qi 2018). We show this insight may not work in dual channel settings. We demonstrate
that cost information asymmetry can improve efficiency, due to the interplay of two offsetting forces—double marginalization and signaling distortion. Therefore, cost information asymmetry per se does not reduce efficiency; its impact depends on how it interacts with other forces. The efficiency improvement is substantial, when the proportion of the high cost type is large (so the price cut is likely), and when the cost gap between the two types is moderate (so the signaling incentive is high).

Second, we derive two implications for dual-channel management. (i) The retailer can benefit from the rival’s cost improvement. This is driven by the non-monotonic relationship between retailer payoff and manufacturer selling cost. Intuitively, improving manufacturer’s cost should hurt the retailer by shifting her sales away to the direct channel. But the cost improvement can also benefit the retailer by strengthening signaling incentives and intensifying price distortion. When the cost gap between two types is moderate, the retailer can actually benefit from rival’s cost improvement. (ii) The retailer can benefit from his information disadvantage. Relative to the full information he can gain from downward price distortion. This result suggests cost information asymmetry can help ease channel conflict, and manufacturers need not make excessive concessions—the downside of dual selling may have been overestimated (Blair and Lafontaine 2005).

Third, we demonstrate that our results are robust for other prevailing arrangements, e.g., price competition, imperfect substitution, and simultaneous moves. We find that even under two-part tariffs, cost information asymmetry can still benefit the retailer. This is somewhat puzzling. It is well-known that in the single channel, the manufacturer can use two-part tariffs to eliminate double marginalization and extract all channel surplus. This is still true for a dual channel under full information. But it is no longer the case under cost information asymmetry: the encroachment threat depresses the retailer’s order quantity, pushes up the wholesale price, thereby causing double marginalization. So, she can no longer extract the full retailer surplus under asymmetric information. When the market size is small, the manufacturer must downward distort both the fixed and wholesale price to signal her cost: while wholesale price distortion reduces double marginalization, it is the fixed fee distortion that leaves the retailer with a positive surplus.

1.1. Related Literature

Our work contributes to the channel and signaling literature. A central theme of the channel literature is how to reduce channel conflict and improve efficiency. Two culprits of inefficiency are double marginalization and information asymmetry (Klibanoff and Morduch 1995). The literature has examined various aspects of dual-channel distribution; e.g., regulation and public policy (Blair and Lafontaine 2005), sales effort (Vinhas and Anderson 2005), personal pricing (Liu and Zhang 2006), price discrimination (Li et al. 2015), service competition (Chen et al. 2008), product return (Ofek et al. 2011), quality differentiation (Ha et al. 2015), and brand externality (Kalnins 2016).

A central question is whether dual selling can improve channel efficiency and benefit downstream retailers. Some studies find that dual selling intensifies retail competition, discourages downstream selling effort,

Most of these studies rely on two key assumptions. The first is full information. This assumption simplifies the analysis by avoiding incentive compatibility constraints. But it also implies that all parties are willing and able to communicate and act upon all the information fully, costlessly, and instantaneously. In such a frictionless world, there is no place for the strategic uncertainty. But real firms rarely operate in a frictionless world—they must act with limited information. Once information asymmetry is considered, however, the strategic uncertainty sets in, the new interactions emerge, and the second inefficiency arises.

There are extensive studies on cost information asymmetry; see, e.g., Ha (2001), Corbett and De Groote (2000), Corbett and DeCroix (2001), Corbett et al. (2004), Zhang et al. (2010), Bolandifar et al. (2017), Hu and Qi (2018). The general insight is that, private cost information reduces efficiency: in standard screening, the efficiency loss comes from information rents paid for truth-telling; in standard signaling, the efficiency loss comes from the wasteful effort exerted for separation. We show this insight may not work for dual channels. In our model, private cost information can produce downward price distortion, reduce double marginalization, and hence improve efficiency. In the marketing literature, Jiang et al. (2016) find the manufacturer’s demand signaling can benefit the retailer. Our finding complements theirs. The main differences are: they consider a single channel with private demand information, while we focus on dual channels with private cost information.

The second key assumption of the literature is the wholesale price contract—a root cause of double marginalization (Kolay and Shaffer 2013). In the single channel, it is well known that the manufacturer can use two-part tariff to eliminate double marginalization and extract all retailer surplus (Moorthy 1987). We find this is no longer the case in the dual channel: under two-part tariffs, double marginalization still persists, and the retailer can still gain from information disadvantage.

---

4 For example, Corbett and De Groote (2000) state “Global efficiency is reduced by information asymmetry…”. Ha (2001) finds “the joint profit and the supplier’s profit are lowered due to information asymmetry (as both the optimal order quantity and the optimal price are distorted).”

5 In the dual-channel literature, Li et al. (2014) show that, private demand information can cause downward quantity distortion, amplify double marginalization, and reduce efficiency. We complement their study by identifying a different mechanism that can improve efficiency.

6 The intuition is as follows. To maximize his own profit, the manufacturer needs to increase either the size or his share of the pie (the channel surplus). Doing so imposes conflicting pressures on the wholesale price, which alone cannot achieve both goals.

7 Coordination instruments include quantity discounts (Jeuland and Shugan 1983), two-part tariff (Moorthy 1987), franchise agreements (Desai and Srinivasan 1995), product returns (Padmanabhan and Png 1997), bargaining power (Iyer and Villas-Boas 2003), retail price floor/ceiling (Iyer 1998); see Cachon (2003) for a review of early static models. In essence, these distinct instruments are variants of the same nonlinear pricing scheme (Hermalin 2009): they can achieve the first best under full information.
Our work builds on the signaling literature. Besides pricing, the literature has also studied many other signaling devices; e.g., advertising (Bagwell and Ramey 1988), scarcity (Stock and Balachander 2005, Yu et al. 2014), slotting allowance (Desai 2000) and warranties (Balachander 2001). Two main insights are: (i) the uninformed party is usually penalized for his lack of information (Ackerloff 1970); (ii) costly signaling usually reduces efficiency. In contrast, we show that the uninformed party can gain from information disadvantage, and that costly signaling can improve efficiency.

### Table 1 Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>prior on type-$\ell$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>posterior belief of the retailer</td>
</tr>
<tr>
<td>$a$</td>
<td>market size</td>
</tr>
<tr>
<td>$b$</td>
<td>price sensitivity</td>
</tr>
<tr>
<td>$c_i$</td>
<td>type-$i$ manufacturer’s selling cost</td>
</tr>
<tr>
<td>$c_0$</td>
<td>manufacturer’s average selling cost, $c_0 = \mu_0 c_\ell + (1 - \mu_0) c_h$</td>
</tr>
<tr>
<td>$q_R$</td>
<td>retailer’s order quantity</td>
</tr>
<tr>
<td>$q_M$</td>
<td>manufacturer’s order quantity</td>
</tr>
<tr>
<td>$Q$</td>
<td>total product quantity sold</td>
</tr>
<tr>
<td>$w_i$</td>
<td>type-$i$’s wholesale price decision in separating equilibrium</td>
</tr>
<tr>
<td>$w^*$</td>
<td>manufacturer’s wholesale price decision in pooling equilibrium</td>
</tr>
<tr>
<td>$\sigma^r$</td>
<td>equilibrium outcome of regime $r \in {N, SI, AI}$</td>
</tr>
<tr>
<td>$\Pi_M$</td>
<td>manufacturer’s profit under asymmetric information</td>
</tr>
<tr>
<td>$\Pi_R$</td>
<td>retailer’s profit under asymmetric information</td>
</tr>
<tr>
<td>$\Pi_{SC}$</td>
<td>supply chain’s profit under asymmetric information</td>
</tr>
<tr>
<td>$\Gamma^{SI}$</td>
<td>dual-channel regime with symmetric information</td>
</tr>
<tr>
<td>$\Gamma^{AI}$</td>
<td>dual-channel regime with Asymmetric Information</td>
</tr>
<tr>
<td>$\Gamma^N$</td>
<td>single-channel regime with no encroachment</td>
</tr>
<tr>
<td>$\tilde{\Gamma}^N$</td>
<td>single-channel regime under two-part tariff; similar definitions for $\tilde{\Gamma}^{SI}$ and $\tilde{\Gamma}^{AI}$</td>
</tr>
<tr>
<td>$\equiv$</td>
<td>equal by definition</td>
</tr>
</tbody>
</table>

### 2. Model

Consider a dual-channel distribution. The manufacturer has two channels to sell a single product to consumers. The first is the retailer channel, through which she sells quantity $q_R$ at wholesale price $w$; the second is the direct channel, through which she sells quantity $q_M$ directly to consumers. The market clearance price is $P = a - bQ$, where $Q \equiv q_R + q_M$ is the total quantity, $a$ the market size, and $b$ the price sensitivity. Both parties are risk neutral and profit maximizing. To focus on the main effects, we normalize the manufacturer’s production cost and the retailer’s selling cost to zero—a standard treatment in the literature (Arya et al. 2007).

The manufacturer is less efficient than the retailer in retailing, and she incurs (additional) direct selling cost $c_i > 0$. The cost $c_i$ can be either low or high, with type $i \in \{\ell, h\}$, $c_\ell < c_h$, and prior $\mu_0 \equiv \Pr\{c_i = c_\ell\}$. The cost $c_i$ itself is the manufacturer’s private information, but its prior $\mu_0$ is common knowledge. Ex-ante, the

---

There is a stream of literature that examines firms’ information sharing decisions under different supply-chain structures. Exemplary studies include Ha and Tong (2008), Ha et al. (2011), Shamir and Shin (2015) for competing channels, Anand and Goyal (2009), Kong et al. (2013) on upstream information leakage, and Huang et al. (2018) under dual channel. In contrast, we do not focus on information sharing.
retailer knows only prior $\mu_0$; upon observing price $w_i$, he updates $\mu_0$ to the posterior belief $\mu$. Table 1 defines all the notation.

This is a game $\Gamma^{AI}$ of Asymmetric information. It plays out as follows (see Figure 1). First, the manufacturer privately observes her cost $c_i$. She then offers a wholesale price $w_i$ to the retailer, who orders quantity $q_R$. Afterwards, the manufacturer decides her direct-channel quantity $q_M$. Finally, the market clears, the manufacturer gets profit $\Pi_M = w \cdot q_R + (P - c_i) \cdot q_M$, the retailer gets $\Pi_R = (P - w) \cdot q_R$, and the supply chain gets $\Pi_{SC} = \Pi_M + \Pi_R$. Importantly, this game has exogenous dual-channel structure but endogenous selling strategy. The manufacturer has three selling options: retailer selling only, direct selling only, and dual selling. As we shall show, either retailer selling only or dual selling strategy can be optimal, depending on selling cost $c_i$. However, the direct-selling-only strategy is never optimal, regardless of information structure.

We now discuss the main assumptions. (i) Our base model studies Cournot quantity competition. Depending on business specifics, the competition can take place on other dimensions, e.g., price (Chiang et al. 2003), sales effort (Tsay and Agrawal 2004), service (Chen et al. 2008). We examine price competition in §6.3. (ii) We assume only the manufacturer has private selling cost. This is a first order approximation to common situations where the manufacturers’ direct selling cost are much harder for others to estimate: in terms of selling, a manufacturer has idiosyncratic ability, while her retailer the industry average ability. When both parties have private selling abilities, new interactions arise and more complex contracts are necessary. For example, the manufacturer may design a menu of contracts for two purposes: signaling her own cost and screening the retailer’s private cost. This is an interesting direction for future research. (iii) Given the goal of this paper, we abstract away from other well-studied issues, e.g., production cost, product differentiation, market segmentation, demand enhancing activities, encroachment decision and channel-structure choice. These issues are well-studied and important in their own right, but they are not our focus (see §1.1).

Our model features information asymmetry, wholesale price, and quantity competition. Except information asymmetry, the setup is canonical in the dual-channel literature (Arya et al. 2007). It captures the essential

---

9 For the sake of realism, we can certainly incorporate many other factors, but this approach would complicate the model and obfuscate the main effects of upstream information asymmetry—our main focus. Instead, we follow the literature (Arya et al. 2007) and build a parsimonious model that captures the main features of our problem.
dynamics in a parsimonious way, facilitates a direct comparison with the existing insights, and allows us to articulate the main mechanism in the simplest fashion. The problem is a complex game with compounding effects. To build intuition, first we sketch full-information benchmarks (§3); then we examine our main case, the dual-selling equilibrium of regime $\Gamma^{AI}$ (§4); finally we consider other arrangements for robustness check (§6).\footnote{For example, the manufacturer may contract the retailer with a two-part tariff (§6.2), two parties may compete on price (§6.3), and retail quantity or price decisions may be unobservable to each other (Online Appendix).} We shall show, although certain details differ, the main mechanism remains the same.

## 3. Full Information Benchmarks

We first establish two benchmark regimes under full information. The first is the single-channel regime $\Gamma^N$ with no direct-selling option, in which the manufacturer can only sell through the retailer channel ($q_{M} = 0$). The second is the dual-channel regime $\Gamma^{SI}$ with symmetric information, in which the manufacturer can use both channels. Their performance difference characterizes the effects of dual selling under full information.

\textbf{Lemma 1.} (a) In the single-channel regime $\Gamma^N$, the equilibrium outcome is as follows:

$$w^N = \frac{a}{2}, \quad q^N_R = \frac{a}{4b}, \quad q^N_M = 0, \quad \Pi^N_M = \frac{a^2}{8b}, \quad \Pi^N_R = \frac{a^2}{16b}. \quad (\sigma^N)$$

(b) In the dual-channel regime $\Gamma^{SI}$, the equilibrium outcome is as follows:

\begin{align*}
\begin{cases}
\text{if } w^SI_I = \frac{2 - c_i}{6}, & q^SI_R = \frac{2c_i}{3b}, & q^SI_M = 0, & \Pi^SI_M = \frac{3a^2 - 6ac_i + 7c_i^2}{12b}, & \Pi^SI_R = \frac{2c_i^2}{9b}, & \forall c_i \in (0, \frac{3a}{5}]. \quad (\sigma^SI_1) \\
\text{if } w^SI_I = \frac{3a - c_i}{4}, & q^SI_R = \frac{a - 3b}{b}, & q^SI_M = 0, & \Pi^SI_M = \frac{3c_i^2 - 2a(3b - c_i)}{2b}, & \Pi^SI_R = \frac{(a - c_i)^2}{2b}, & \forall c_i \in (\frac{3a}{5}, \frac{5a}{6}); \quad (\sigma^SI_2) \\
\text{if } w^SI_I = \frac{a}{2}, & q^SI_R = \frac{a}{4b}, & q^SI_M = 0, & \Pi^SI_M = \frac{a^2}{8b}, & \Pi^SI_R = \frac{a^2}{16b}, & \forall c_i \in (\frac{5a}{6}, \infty). \quad (\sigma^SI_3)
\end{cases}
\end{align*}

Lemma 1 is standard in the literature (Arya et al. 2007). Part (a) reveals how double marginalization harms the single-channel regime $\Gamma^N$. Relative to the first-best FB, wholesale pricing depresses output $q^N_R = \frac{a}{4b} > \frac{a}{2b} = q^N_B$, reduces channel efficiency $\Pi^N_M = \frac{a^2}{16b} < \frac{a^2}{4b} = \Pi^F_B$, and hurts consumer surplus.

Part (b) characterizes how dual channel changes equilibrium. In regime $\Gamma^{SI}$, the very existence of the dual channel empowers the manufacturer with the direct-selling option, threatening the retailer with encroachment. The manufacturer now has three possible strategies: retailer only ($q_{R_i} > 0, q_{M_i} = 0$), direct selling only ($q_{R_i} = 0, q_{M_i} > 0$), and dual selling ($q_{R_i} > 0, q_{M_i} > 0$). The optimal strategy depends on the selling cost $c_i$. (i) When the selling cost is low ($c_i \leq \frac{3a}{5}$), the manufacturer exercises the encroachment option, and dual selling is optimal. (ii) When the selling cost is moderate ($\frac{3a}{5} < c_i \leq \frac{5a}{6}$), retailer selling only is optimal. Although the manufacturer does not exercise the encroachment option ($q^{SI}_M = 0$), encroachment is still a credible threat that changes retailer’s order. (iii) When the selling cost is high ($c_i > \frac{5a}{6}$), retailer only is again optimal. Interestingly, because the high cost makes the encroachment threat incredible, all parties ignore direct selling ($\frac{\partial}{\partial c_i} q^{SI}_R = 0$), acting as if they were in the single-channel regime $\Gamma^N$. (iv) Importantly, the direct-selling-only strategy is never optimal. This is because the retailer has superior cost efficiency ($c_i > 0$). For any quantity
sold through direct channel, he can sell at less cost for higher revenue. As such, the manufacturer will never use direct selling alone; in part (b), \( c_i > 0 \) implies \( q_{RI} > 0 \).\(^{11}\)

Lemma 1 reveals that dual selling is optimal for \( c_i < \frac{3a}{5} \). Relative to \( \Gamma^N \), dual selling can either benefit or hurt channel members. In particular, (i) dual selling always benefits the manufacturer, because she gains from additional sales and reduces double marginalization. (ii) Dual selling benefits the retailer when \( \frac{3a}{4\sqrt{2}} < c_i < \frac{3a}{5} \), because he benefits from wholesale price cut \( (w_{SI} \downarrow c_i) \) and sales increase \( (q_{RI} > 0) \). (iii) Dual selling benefits the supply chain, when \( 0 < c_i < \frac{3a}{6+\sqrt{7}} \) or \( \frac{3a}{6-\sqrt{7}} < c_i < \frac{3a}{5} \), because dual selling increases sales and limits double marginalization.\(^\text{12}\)

The key insight is that, dual selling can benefit the retailer and improve channel efficiency. The underlying mechanism relies on the interplay of dual selling and double marginalization. Indeed, dual selling can motivate the manufacturer to cut wholesale price: although the price cut reduces her share of retail surplus, it can also boost wholesale revenue and benefit the manufacturer. Thus, when her selling cost is moderate, the manufacturer may cut the wholesale price, which can benefit the retailer, reduce double marginalization, and improve channel efficiency.

3.1. Remarks

These are classical results (Tsay and Agrawal 2004, Cattani et al. 2006, Arya et al. 2007). They rest on the full-information assumption. The assumption paints a rather simplistic picture of how firms behave—they are omniscient with infinite rationality. Each party has the ability to read the rival’s mind, observe his in-house operations, predict his future actions, and calibrate countermeasures, all in perfect precision. Yet the same superrational firm is willing to open books, reveal internal workings, and share trade secrets with the rival, despite the risk of being exploited later.

This simplistic picture is hard to square with business reality. In practice, channel remember has limited visibility into rivals’ internal working (Ha 2001, Corbett and De Groote 2000, Zhang et al. 2010, Li 2018). They may have reasonably good forecast (distribution) about rivals’ costs and actions, but not the perfect clarity. Such strategic uncertainty can cloud calculation and change the relationship in a fundamental way (Brandenburger 1993, Stiglitz 2002). In the dual-channel relationship, for example, the retailer lacks the precise knowledge of the manufacturer’s direct selling ability. So he can no longer predict directing selling quantity with perfect precision. Instead, he must form the prior \( \mu_0 \) of rival’s selling ability \( c_i \), update the belief \( \mu \) based on newly observed action \( w_i \), and adjust his order accordingly. Theoretically, the full-information

---

\(^{11}\) In regime \( \Gamma^{SI} \), the manufacturer should use the dual-selling strategy only when direct selling is cheap \( (c_i \leq \frac{3a}{5}) \); otherwise, she should use the retailer-only strategy.

\(^{12}\) When \( c_i \in \left( \frac{3a}{2}, \frac{5a}{6} \right) \), the manufacturer uses the direct selling channel only as a threat (he does not sell through it). In this case, relative to regime \( \Gamma^N \), the manufacturer prefers \( \Gamma^{SI} \) if \( \frac{3a}{4} < c_i \leq \frac{5a}{6} \), the retailer prefers \( \Gamma^{SI} \) if \( \frac{3a}{5} < c_i \leq \frac{4\sqrt{2}a}{5} \), and the supply chain prefers \( \Gamma^{SI} \) if \( \frac{3a}{6} < c_i < \frac{3a}{5} \). The manufacturer uses direct selling threat to charge a higher wholesale price than \( \Gamma^N \) when \( c_i \) is large. But, she uses it to sell a higher quantity by reducing the wholesale price when \( c_i \) is small. The reduction in wholesale price mitigates double marginalization, and thus the retailer and supply chain fair when \( c_i \) is sufficiently small.
framework $\Gamma^\text{SI}$ has no place for beliefs and their updating. As such, it is inadequate for studying the interplay of information asymmetry, strategic behavior, and dual-channel performance, which is at the heart of our problem. To make credible predictions, we must model information asymmetry, studying more realistic firm behaviors.

4. Dual-Selling Equilibrium under Information Asymmetry

When cost information is private, the signaling game $\Gamma^\text{AI}$ arises. We focus on the case where dual selling is optimal for both types. This requires the dual-selling condition (C1):

$$a > a \equiv \frac{1}{3} (4c_h + c_\ell) + \frac{1}{3} \sqrt{2c_h c_\ell + c_h^2 - 3c_\ell^2}. \quad (C1)$$

It implies that manufacturer prefers dual-channel over single-channel distribution ($\Pi_M^\text{AI} > \Pi_M^\text{SI}$, $\Pi_M^\text{AI} > \Pi_M^\text{SI}$). Under (C1), the two parties compete on quantity in the end market. To decide his quantity $q_R$, the retailer must estimate the competitor’s cost information $c_i$. Yet that information is private, and the retailer can only infer it from the wholesale price $w_i$ the manufacturer offers. Knowing this, type-$\ell$ manufacturer may have an incentive to mimic type-$h$’s price, because of two advantages she enjoys: (i) the information advantage over the retailer (knowing $c_i$ vs. $\mu_0$), and (ii) the selling cost advantage over type-$h$ (since $c_\ell \leq c_h$). Indeed, if type-$\ell$ can mislead the retailer to believe she is of high cost, she may benefit from tricking the retailer to order more. This outcome may hurt type-$h$; so she may attempt to signal her cost, by distorting her price to the extent unprofitable for type-$\ell$ to mimic.

To formalize this intuition, we work backward, starting with the best quantity responses $q_R$ and $q_{M^i}$. At the final stage, given price $w$ and quantity $q_R$, the manufacturer solves the problem

$$\max_{q_{M^i} \geq 0} q_R \cdot w + \left[ \frac{a - b(q_R + q_{M^i})}{2} - c_i \right] \cdot q_{M^i}, \quad i \in \{\ell, h\}. \quad (1)$$

Thus

$$q_{M^i}(q_R) = \left( \frac{a - c_i - bq_R}{2b} \right)^+. \quad (2)$$

Anticipating this response, the retailer solves the following problem, given price $w$ and belief $\mu$:

$$\max_{q_R \geq 0} \mu \left[ a - b(q_R + q_{M^i}(q_R)) - w \right] \cdot q_R + (1 - \mu) \left[ a - b(q_R + q_{M^i}(q_R)) - w \right] \cdot q_R - w.$$ 

Thus

$$q_{R}(w, \mu) = \left( \frac{a + c_0 - 2w}{2b} \right)^+, \quad (3)$$

where $c_0 = \mu c_\ell + (1 - \mu)c_h$. By Eqs. (2) and (3), we can write type-$i$’s payoff as

$$\Pi_M(i, \mu, w) = \frac{(a - 2c_i - c'_0 + 2w)^2}{16b} + \frac{w \cdot (a + c'_0 - 2w)}{2b}. \quad (4)$$

It follows that $\Pi_M(\ell, 0, w_\text{SI}^\ell) > \Pi_M(\ell, 1, w_\text{SI}^\ell)$, and $\Pi_M(h, 1, w_\text{SI}^\ell) < \Pi_M(h, 0, w_\text{SI}^h)$, which implies:
LEMMA 2. Under dual selling, only type-ℓ manufacturer has the incentive to mimic the other type.

Intuitively, type-ℓ has the incentive to inflate the cost, because in the retail market, a higher direct selling cost implies lower direct sales and less intense competition. If she can convince the retailer that she is of high cost, the retailer would order more $q_R$, thereby benefiting type-ℓ ($\Pi_M(\ell, 0, w_{SI}^h) > \Pi_M(\ell, 1, w_{SI}^h)$). However, because of the cost disadvantage ($c_h > c_\ell$), mimicking low cost type is never optimal for type-$h$ ($\Pi_M(h, 1, w_{SI}^\ell) < \Pi_M(h, 0, w_{SI}^\ell)$). See Figure 2 for an illustration.

For game $\Gamma AI$, the proper solution concept is perfect Bayesian equilibrium (PBE). We first consider a separating PBE $\sigma^* \equiv \{\mu, q_R^*(\cdot), q_M^*(\cdot), w_\ell^*, w_h^*\}$, where $\mu$ is the posterior probability, $q_R^*$ and $q_M^*$ the equilibrium quantities, and $w_\ell^*$ and $w_h^*$ the equilibrium wholesale prices. By definition, two types must set distinct wholesale prices ($w_\ell^* \neq w_h^*$), so that the retailer can fully infer the cost $c_i$.

To characterize $\sigma^*$, we must specify the posterior belief $\mu$. On the equilibrium path, $\mu$ is well-defined by Bayes’ rule. Off the equilibrium path, however, belief $\mu$ can take arbitrary form, as long as it supports the equilibrium. Thus, we can set $\mu$ off the equilibrium path to make deviation as costly as possible—a common approach in games with continuous strategy set (Chu 1992). Specifically, we assume when the retailer is off the equilibrium path, he believes the manufacturer to be type-ℓ, with posterior belief $\mu = 1$.

Next we characterize the separating prices $w_\ell^*$ and $w_h^*$. This relies on two incentive compatibility (IC) constraints. The first is to ensure that type-$h$ has no incentive to deviate from her equilibrium price $w_h^*$:

$$\Pi_M(h, 0, w_h^*) \geq \Pi_M(h, 1, w), \quad \forall w \neq w_h^*.$$  \hspace{1cm} (IC$_h$)

---

13 The PBE concept requires that each party acts optimally given their posterior beliefs, that the posterior beliefs are consistent with others’ best responses, and that posterior beliefs are updated with Bayes’ rule whenever possible.

14 This specification ensures the worst outcome for a deviating type-$h$—being regarded as type-ℓ.
The second IC constraint is to ensure neither type has incentive to mimic one another. By Lemma 2, only type-$\ell$ has such incentive. So it suffices to require:

$$\Pi_M(\ell, 1, w_{\ell}^*) \geq \Pi_M(\ell, 0, w_{h}^*), \quad \forall w_{h}^* \neq w_{\ell}^*.$$  \hspace{1cm} (IC$_{\ell}$)

That is, type-$\ell$ must prefer setting $w_{\ell}^*$ for revelation, rather than setting $w_{h}^*$ for imitation. As a result, the two constraints narrow down the set of equilibria:

**Lemma 3.** Under dual selling, a separating PBE $\sigma^*$ satisfies:

(a) Type-$\ell$ manufacturer sets the wholesale price at the first best level, $w_{\ell}^* = w_{SI\ell}$.

(b) Type-$h$ manufacturer sets price $w_{h}^* \in \mathcal{H} = [w, \overline{w}]$, with $w < \overline{w} < w_{h}^{SI} < w_{SI\ell}$, where

$$w = (3a - c_{h})/6 - \sqrt{(c_{h} - c_{\ell})(3c_{h} + c_{\ell})}/3, \quad \overline{w} = (3a + c_{h} - 2c_{\ell})/6 - \sqrt{(c_{h} - c_{\ell})(c_{h} + 3c_{\ell})}/3.$$

Figure 2 illustrates Lemma 3. It reveals how the manufacturer should set price. Part (a) shows that, despite information asymmetry, type-$\ell$ still prices efficiently, in the first best sense. This is because, in the separating equilibrium $\sigma^*$, type-$h$ has made all her prices $w \in \mathcal{H}$ too costly for type-$\ell$ to mimic. Unable to manipulate, type-$\ell$ is better off to price at the first best level.

Part (b) reveals that type-$h$ manufacturer downward distorts her price, below the first best ($w_{h}^* < w_{h}^{SI}$). This is driven by the endogenous sorting condition—the relative marginal benefit of the retailer selling is higher for type-$h$ than that for type-$\ell$. As a result, if type-$h$ reduces her price and type-$\ell$ follows, then both need to sell more through the retailer channel. But this is more costly for type-$\ell$ to bear (as her direct channel is more efficient, $c_{\ell} < c_{h}$). Consequently, if type-$h$ sets her price sufficiently low, type-$\ell$ would stop mimicking. That is why type-$h$ should downward distort price.

One may ask, why does not type-$h$ upward distort price? Because that would reduce the imitation barrier, invite type-$\ell$ to mimic, and thus defeat the very purpose of distortion—to signal her type. Unlike canonical signaling games, however, our sorting condition arises endogenously, depending on sales allocation of each manufacturer and their interactions.\(^{15}\)

When the two IC constraints are binding, we pin down the boundary prices $w$ and $\overline{w}$ of set $\mathcal{H}$:

$$\Pi_M(h, 0, w) = \max_w \Pi_M(h, 1, w), \quad \Pi_M(\ell, 0, \overline{w}) = \Pi_M(\ell, 1, w_{SI\ell}).$$

Still, the set $\mathcal{H}$ admits too many prices. To pinpoint price $w_{h}^*$, we apply the intuitive criterion (Cho and Kreps 1987):

**Proposition 1.** Under dual selling, the separating PBE $\sigma^*$ is given by Table 2, with $w_{h}^* = \overline{w} < w_{h}^{SI}$.

\(^{15}\) The marginal benefit of selling through the retailer channel is not fixed: it depends on how the manufacturer allocates quantities for each channel, and how the two types interact with each other and the retailer.
Table 2 Equilibrium $\sigma^*$ under (C1)

<table>
<thead>
<tr>
<th>equilibrium</th>
<th>type-$\ell$</th>
<th>type-$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^*$</td>
<td>$w_{SI}^\ell$</td>
<td>$\bar{w}$</td>
</tr>
<tr>
<td>$q^*_R$</td>
<td>$\frac{2q}{3b}$</td>
<td>$\frac{a+c_\ell-2\pi}{2b}$</td>
</tr>
<tr>
<td>$q^*_M$</td>
<td>$\frac{3a-5c}{6b}$</td>
<td>$\frac{a-3c+2\pi}{4b}$</td>
</tr>
<tr>
<td>$\Pi^*_R$</td>
<td>$\frac{2c^2}{9b}$</td>
<td>$\frac{(a+c_\ell-2\pi)^2}{8b}$</td>
</tr>
<tr>
<td>$\Pi^*_M$</td>
<td>$\frac{3a^2-6ac_\ell+7c^2}{12b}$</td>
<td>$\frac{(a-3c+2\pi)^2+8\pi(a+c_\ell-2\pi)}{16b}$</td>
</tr>
</tbody>
</table>

The intuition as to why $w^*_h = \bar{w}$ is the only separating price is as follows. First, under the intuitive criterion, the retailer believes that any manufacturer with price $w \in H$ is type-$h$ (with posterior belief $\mu = 0$), and type-$\ell$ would not set price $w^*_\ell \in [w, \bar{w}] \subset H$.\(^{16}\) Second, among all prices in set $H$, the price $\bar{w}$ gives type-$h$ the highest payoff (by the concavity of payoff $\Pi_M$; see Figure 2); all other prices in $H$ cannot survive the intuitive criterion. Hence, she should downward distort price, from $w_{SI}^h$ to $w^*_h = \bar{w}$.

The downward price distortion produces an *offsetting mechanism*. The wholesale price cut can encourage the retailer to order more, improve retail efficiency, thereby reducing double marginalization. It turns out, this offsetting mechanism has a profound impact on channel performance, a key property we explore later.

Under the intuitive criterion, type-$h$ manufacturer always prefers separating to pooling. (i) In a pooling equilibrium, type-$h$ must charge the same price as type-$\ell$. So the retailer cannot infer the manufacturer’s cost, and he has to order based on average cost $c_0 \equiv \mu_0 c_\ell + (1-\mu_0)c_h$, for which $c_0 < c_h$. Consequently, type-$h$ gets less retailer orders than in regime $\Gamma^{SI}$, and she relies more on her inefficient direct channel. Here type-$h$ suffers the mismatch cost of quantity (based on $c_0$, not $c_h$). (ii) In a separating equilibrium, however, type-$h$ can lower her price $w^*_h < w^*_\ell$ to signal her cost $c_h$, thereby benefiting from more sales through the efficient retailer channel. Here type-$h$ suffers the signaling cost of price distortion. (iii) In regime $\Gamma^{AI}$, the signaling cost is always smaller than the mismatch cost. Hence type-$h$ always prefers separation:

**Proposition 2.** Under dual selling, no pooling equilibrium survives the intuitive criterion.

5. The Role of Private Cost Information

In our model, private cost information hurts the manufacturer. This follows immediately from the standard signaling logic (Salanie 2005): to signal her type, the manufacturer must deviate from profit-maximizing wholesale price $w_{SI}^h$—the first best. Formally,

**Proposition 3.** Under dual selling (C1), the manufacturer suffers from cost information asymmetry:

$$\Pi_{M_i}^ SI \geq \Pi_{M_i}^ AI, \quad E\Pi_{M_i}^ SI > E\Pi_{M_i}^ AI.$$\(^{16}\) In fact, the IC constraint ensures that $\bar{w}$ is the lowest price that type-$\ell$ can tolerate to imitate: $\Pi_M(\ell, 1, w_{SI}^\ell) = \Pi_M(\ell, 0, \bar{w})$. Since any price smaller than $\bar{w}$ would make mimicking unprofitable, we assume that at $\bar{w}$, type-$\ell$ reveals her type by setting $w_{SI}^\ell$.\(^{16}\)
In this section, we examine how private cost information affects other parties: (i) How does it affect channel performance? (ii) Can it turn the victims of manufacturer encroachment to its beneficiaries? (iii) How does it affect retailer behavior?

5.1. How Does Cost Information Asymmetry Affect Channel Performance?

This question evokes two contradicting arguments. The conventional view argues that cost information asymmetry reduces efficiency (Ha 2001, Corbett et al. 2004). In standard screening, the efficiency loss comes from the information rents paid for truthtelling; in canonical signaling, the efficiency loss comes from the wasteful effort exerted for separation. In both cases, distortion is a device to prevent imitation, a necessary price paid for ensuring truthtelling and separation. As such, information asymmetry induces strategic maneuvers, creates distortions, and reduces efficiency—it is a defect to correct. In our model, the same logic seems to apply. Indeed, type-ℓ manufacturer has the incentive to inflate the cost and imitate type-h; this threat compels type-h to distort wholesale price for separation. As such, the price distortion seems to be a “wasteful” action, and cost information asymmetry should hurt channel performance.

On the other hand, one may argue, cost information asymmetry can produce the offsetting mechanism. It can induce strategic uncertainty and change the nature of the channel relationship: in regime $\Gamma_{AI}$, type-h attempts to imitate while type-ℓ cuts wholesale price for signaling. Such “wasteful” price distortion can encourage the retailer to buy more, thereby increasing retail channel sales and reducing double marginalization. Because of this offsetting mechanism, cost information asymmetry may benefit channel performance.

Is cost information asymmetry beneficial? To address the question, we compare the dual-channel performance under private and full information ($\Gamma_{AI}$ vs. $\Gamma_{SI}$). Except the privacy of selling cost $c$, the two regimes are identical. If cost information asymmetry is beneficial, the retailer should expect higher payoff in regime $\Gamma_{AI}$; so should the supply chain and the consumers. Formally, for $k \in \{R, C, SC\}$, we define payoff gap $\Delta_k \equiv \Pi_{AI}^k - \Pi_{SI}^k$. Hence, $\Delta_R \Pi_R$ measures the effect of cost Information asymmetry on retailer payoff in dual channels. We find:

**Proposition 4.** Under dual selling, cost information asymmetry benefits the retailer, the consumers, and the supply chain:

$$\Delta_R \Pi_R > 0, \quad \Delta_C \Pi_C > 0, \quad \Delta_{SC} \Pi_{SC} > 0.$$  \hspace{1cm} (5)

The proposition reveals two insights. First, in a dual-channel relationship, the retailer’s lack of information can actually improve his fortune. When he does not know the manufacturer’s type, he will be able to infer it from the wholesale price, and being uninformed invites price cut. This boosts the retailer’s order quantity, pushes up his sales, and improves his expected payoff. This dynamics also improves consumer surplus, because of higher sales quantity and lower retail price.

Second, cost information asymmetry can improve channel efficiency. In our model, the manufacturer is the informed party, and type-h manufacturer uses downward price distortion to signal her cost. In response,
the retailer increases his order quantity, thereby mitigating double marginalization and improving the retail channel efficiency. In essence, the improvement is the windfall benefit from signaling distortion—the price cut taken by type-$h$ for separation. The magnitude of the improvement depends on the proportion of type-$h$ (who take the distortion action). In figure 3, the more the signaling type (smaller $\mu_0$), the bigger the improvement. When few take signaling action ($\mu_0 \rightarrow 1$), the improvement vanishes ($\sigma^{AI} \rightarrow \sigma^{SI}$).

![Figure 3 Profits under symmetric and asymmetric information](image)

Our result qualifies the conventional view. In the supply chain literature, a general insight is that cost information asymmetry reduces efficiency. For example, Corbett and De Groote (2000) find: “global efficiency is reduced by information asymmetry.” Ha (2001) concludes: “the joint profit and the supplier’s profit are lowered due to information asymmetry (as both the optimal order quantity and the optimal price are distorted).” We show this insight hinges on the single-channel structure, and it does not extend to the dual-channel settings. When dual channel is in place, cost information asymmetry can actually improve efficiency.

5.2. Can Cost Information Asymmetry Turn the Victims of Encroachment to Its Beneficiaries?

In the dual-channel literature, a key result is when manufacturer encroachment can hurt the retailer and supply chain. However, this result relies on full information and simplistic firm behavior. As we have shown, once cost information asymmetry is considered, firms will entertain new interactions and channel efficiency can improve. The new mechanism is the offsetting interplay of signaling distortion and double marginalization. As such, one may wonder, are there conditions under which encroachment hurt the retailer and supply chain under symmetric information, but benefit them under asymmetric information?

To have a precise answer, we now quantify when cost information asymmetry can turn the victims of encroachment to its beneficiaries. We proceed in two steps. First, we define $\Delta \Pi_k^r \equiv E[\Pi_k^r] - E[\Pi_k^n]$, for $r \in \{SI, AI\}$, $k \in \{R, SC\}$. Hence, the payoff gap $\Delta \Pi_k^n$ measures the effect of encroachment on Retailer payoff, while $\Delta \Pi_k^{AI}$ measures the joint effects of encroachment and information asymmetry. Second, for the retailer, we partition the entire parameter space-$(\mu_0, a)$ into three sets of scenarios:\footnote{Define $A_k^4 \equiv \{ (\mu_0, a) : \Delta \Pi_k^{SI} > 0, \Delta \Pi_k^{AI} < 0 \}$. Since cost information asymmetry always benefits the retailer, we have $A_k^4 = \emptyset$.}

- $A_k^1 \equiv \{ (\mu_0, a) : \Delta \Pi_k^{SI} < 0, \Delta \Pi_k^{AI} > 0 \}$
- $A_k^2 \equiv \{ (\mu_0, a) : \Delta \Pi_k^{SI} > 0, \Delta \Pi_k^{AI} > 0 \}$
- $A_k^3 \equiv \{ (\mu_0, a) : \Delta \Pi_k^{SI} < 0, \Delta \Pi_k^{AI} < 0 \}$
We do the same partition for the supply chain. Intuitively, set $A^1_R$ specifies all the scenarios wherein the retailer suffers from encroachment under full information, but he benefits from it under asymmetric information; set $A^1_{SC}$ specifies such scenarios for the supply chain. We find:

**PROPOSITION 5.** On $A^1_R \cap A^1_{SC} \neq \emptyset$, encroachment benefits both the retailer and supply chain, only if the cost information is private.

Proposition 5 characterizes when the reversion occurs. It implies that, ignoring cost information asymmetry, previous studies may have overestimated the harm of encroachment in these scenarios. See Fig. 4 for an illustration.

![Figure 4](image)

**Figure 4** Comparison of retailer and supply chain expected profits: $b = 1$, $c_h = 0.375$, $c_\ell = 0.3$.

### 5.3. How Does Private Direct Selling Cost Information Affect the Retailer?

In regime $\Gamma^{si}$, the retailer has to compete with the manufacturer for consumers. A key question for the retailer is, how competitor’s selling cost $c_i$ affects price $w_i^*$ and payoff $\Pi^*_R$? We address the question in two propositions.

**PROPOSITION 6.**

(a) Under dual selling, the wholesale price $w_h^*$ decreases in $c_h$.

(b) Under dual selling, the wholesale price $w_h^*$ decreases in $c_\ell$ iff cost ratio $c_h/c_\ell > 3/2$.

Figure 5 illustrates the proposition. Part (a) reveals: (i) higher direct selling cost $c_h$ reduces wholesale price $w_h^*$, and (ii) it intensifies distortion ($w_{SI}^* - w_h^*$). Intuitively, raising cost $c_h$ has two effects. The first is the *allocation effect*: type-$h$ must make the best use of both channels; as her direct channel becomes less efficient, she allocates more sales to the retailer channel, by reducing her price (see Proposition 1). The second is the *signaling effect*: the larger the cost $c_h$, the greater the manufacturer heterogeneity (holding $c_\ell$
Gao et al.: Dual-Channel Distribution

\[ c_\ell = 0.3 \]

\[ c_h = 0.375 \]

Figure 5  The impact of cost \( c \) on wholesale price \( w^*_h \): \( a = 1, b = 1 \).

constant), the stronger the mimicking incentive, the larger the price distortion. In Figure 5.(a), the dashed curve \( w^*_h \) is driven by the allocation effect alone, while the solid curve \( w^*_h \) is driven by both effects. Further, the increasing distortion \( (w^*_h - w^*_h) \) is driven by the need to neutralize the intensified imitation incentive.

Part (b) depicts how the type-\( \ell \)'s cost affects type-\( h \)'s price. First, in full information regime \( \Gamma^\text{SI} \), the signaling effect is absent. As such, type-\( h \) always prices efficiently, immune to type-\( \ell \)'s cost; see dashed constant curve. Second, in regime \( \Gamma^\text{AI} \), price distortion \( (w^*_h - w^*_h) \) is the largest for moderate cost ratio \( c_h/c_\ell \).

This result is driven by the interplay of the allocation and signaling effects, in that the imitation incentive is the weakest when cost \( c_\ell \) approaches 0 or \( c_h \). (i) Indeed, when cost \( c_\ell \) approaches 0, the allocation effect dominates. As such, type-\( h \) strongly prefers her own allocation strategy, using mainly the direct channel. So she has little incentive to mimic type-\( h \), who mainly uses the retailer channel. (ii) When cost \( c_\ell \) approaches \( c_h \), two types use a similar allocation strategy; again, there is little incentive to mimic. Only when cost \( c_\ell \) is moderate, the mimicking incentive is the strongest, which must be neutralized by intense price distortion. Thus, the interplay of allocation and signaling produces the U-shaped curve \( w^*_h \).

Next, we study how reducing competitor’s selling ability \( (c_i \uparrow) \) affects retailer payoff \( \Pi^*_R \).

**Proposition 7.** (a) Under dual selling, the retailer benefits from increasing cost \( c_h \).

(b) Under dual selling, the retailer suffers from increasing cost \( c_\ell \), iff \( \frac{2}{3} c_h < c_\ell < c_h \) and \( \mu_0 < \mu_3 \); he benefits otherwise.

Figure 6 illustrates the proposition. Part (a) is intuitive. Ex ante, the retailer buys from type-\( \ell \) at price \( w^*_\ell \) with probability \( \mu_0 \), and from type-\( h \) at price \( w^*_h \) with probability \( (1 - \mu_0) \). Raising cost \( c_h \) reduces price \( w^*_h \), but does not change price \( w^*_\ell \). Hence, on average, the retailer benefits from raising cost \( c_h \) of type-\( h \).

What is puzzling is part (b). In classical quantity competition, one would prefer a weaker rival (Tirole 1988). In regime \( \Gamma^\text{AI} \), two parties compete on quantity for consumers. Yet part (b) suggests that the retailer may prefer a stronger rival, with lower selling cost \( c_\ell \) (in the range where \( \frac{d\Pi^*_R}{dc_\ell} < 0 \)). This seems counterintuitive. The
The key to understand the result is the interplay between the allocation and signaling effects. When cost $c_\ell > \frac{2}{3}c_h$, improving cost $c_\ell$ has two countervailing effects on the retailer. (i) The first is the negative, allocation effect from type-$\ell$ herself: reducing direct cost $c_\ell$ allows her to increase direct sales, reduce wholesale $q^*_R$, thereby hurting the retailer. (ii) The second is the positive, signaling effect from type-$h$: reducing cost $c_\ell$ heightens type-$h$’s need for signaling, intensifies distortion ($w^h_R - w^*_h$), and reduces price $w^*_h$, thereby benefiting the retailer. When the proportion of type-$\ell$ is sufficiently small ($\mu_0 < \mu_3$), the positive signaling effect dominates. Hence, the retailer can prefer a stronger rival, and benefit from the manufacturer’s cost improvement.

6. Extensions

Our base model focuses on dual selling, linear wholesale pricing, quantity competition, and sequential move. In practice, alternative arrangements may prevail. A key question is, will our results still hold? In what follows, we address this question for cases where dual-selling is suboptimal (§6.1), two-part tariff is in place (§6.2) and the competition is on price (§6.3). The simultaneous-move games have similar insights; we detail them in Online Appendix.

6.1. When Dual Selling is Suboptimal

We now study the game $\Gamma^{AI}$ under condition (C2):

$$\max\{4c_h - 3c_\ell, 6c_\ell/5\} < a < a.$$  \hspace{1cm} \text{(C2)}

Condition (C2) serves three purposes. First, it implies that dual selling is suboptimal for some manufacturers ($a < a$). Second, it rules out the uninteresting case where cost information asymmetry is irrelevant ($6c_\ell/5 < a$).18 Third, it ensures tractability ($4c_h - 3c_\ell < a$). We find:

**PROPOSITION 8.** Under condition (C2), the unique separating PBE $\sigma^{AI}$ is characterized by Table 3.

---

18 When $6c_\ell/5 > a$, both manufacturers in $\Gamma^{SI}$ adopt the retailer-only strategy (cf. Lemma 1). Therefore, they have no incentive to imitate each other in $\Gamma^{AI}$. As such, the equilibrium of $\Gamma^{SI}$ and $\Gamma^{AI}$ coincide, and cost information asymmetry is irrelevant.
The proposition reveals that, under (C2), type-ℓ engages in dual selling while type-h shuts down the direct channel, using retailer only strategy. There are three cases. (i) In case $\sigma_{AI}^1$, dual selling cost is low. Both types would engage in dual selling if they were in full-information regime $\Gamma_{SI}$. But in asymmetric-information regime $\Gamma_{AI}$, only type-ℓ engages in dual selling. He attempts to imitate type-h, who cuts wholesale price and benefits everyone. The driving force is the offsetting mechanism between signaling distortion and double marginalization. (ii) In case $\sigma_{AI}^2$, direct selling cost is moderate. Type-ℓ attempts to imitate; in response, type-h cuts price and benefits the retailer. Despite the price cut, the supply chain output remains the same ($q_{AI}^R = q_{SI}^R$), and the price cut merely reallocates more channel surplus from type-h to the retailer. Hence, the offsetting mechanism still operates. (iii) In case $\sigma_{AI}^3$, direct selling cost is so high that no manufacturer would use dual selling under full information $\Gamma_{SI}$. The offsetting mechanism stops working, and cost information asymmetry hurts everyone. In this case, they prefer regime $\Gamma_{SI}$ over $\Gamma_{AI}$.

We are unable to characterize the equilibrium only for the scenario $6c_f/5 \leq a \leq 4c_f - 3c_\ell$. The technical challenge here is the proliferation of subcases and the multitude of mimicking incentives. This is an well-known challenge in the signaling literature: multiple mimicking incentives may deny the existence of equilibrium (Gertner et al. 1988, Farrell and Gibbons 1989, Zhao et al. 2019).

In summary, when dual selling is optimal in $\Gamma_{SI}$ but suboptimal in $\Gamma_{AI}$, the offsetting mechanism can still work its magic: type-ℓ will attempt to mimic while type-h will cut wholesale for signaling; the price cut can mitigate double marginalization, thereby benefiting the retailer, the supply chain, and the consumers (i.e., in $\sigma_{AI}^1$).

### 6.2. The Impact of Two-Part Tariffs

So far the results hinge on the interplay of signaling and double marginalization. It is well-known that two-part tariff can eliminate double marginalization in the conventional retailer channel (i.e., regime $\Gamma^N$). What happens if a two-part tariff is employed in a dual channel?

In this section, we study the regimes in which the retailer channel is governed by a two-part tariff $(\tilde{T}, \tilde{w})$ (instead of wholesale price only). Here $\tilde{T}$ is the fixed fee, and $\tilde{w}$ is unit wholesale price.\(^{20}\) We denote the

\(^{19}\) In our context, for $a < q$, the profit functions are no longer concave in $w$; they are piecewise and change characteristics at multiple thresholds. As such, the problem $\Gamma_{AI}$ becomes prohibitively complex, and the separating equilibrium may not exist. Despite this complexity, we are able to characterize the equilibrium as long as (C2) holds and an equilibrium exists.

\(^{20}\) To focus on the encroachment cases ($q_{MI} > 0$), we assume $a > 3c_\ell$. 

<table>
<thead>
<tr>
<th>Case</th>
<th>Type-ℓ</th>
<th>Type-h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{AI}^1$</td>
<td>$w_\ell^1$</td>
<td>$q_{M_\ell}^1$</td>
</tr>
<tr>
<td>$\sigma_{AI}^2$</td>
<td>$w_\ell^2$</td>
<td>$\frac{3a-5c_\ell}{60}$</td>
</tr>
<tr>
<td>$\sigma_{AI}^3$</td>
<td>$w_\ell^3$</td>
<td>$\frac{a-3c_\ell+2c_\ell}{4b}$</td>
</tr>
</tbody>
</table>

Note: the column $\sigma_{SI}$ indicates the corresponding equilibrium in regime $\Gamma_{SI}$. 


relevant variables by tilde, e.g., $\tilde{\Gamma}^N$. The next lemma characterizes full information benchmarks.

**Lemma 4.** (a) In the single-channel game $\tilde{\Gamma}^N$, the equilibrium outcome is

$$\tilde{T}^N = \frac{a^2}{4b}, \quad \tilde{w}^N = 0, \quad \tilde{q}_R^N = \frac{a}{2b}, \quad \tilde{\Pi}_R^N = \frac{a^2}{4b}, \quad \tilde{\Pi}_M^N = 0.$$

(b) In the symmetric information game $\tilde{\Gamma}^SI$ with dual selling, the equilibrium outcome is

$$\tilde{T}_i^SI = \frac{2c_i^2}{b}, \quad \tilde{w}_i^SI = \frac{a}{2} - \frac{3c_i}{2}, \quad \tilde{q}_{Ri}^SI = \frac{2c_i}{b}, \quad \tilde{q}_{M_i}^SI = \frac{a - 3c_i}{2b}, \quad \tilde{\Pi}_{M_i}^SI = \frac{a^2 - 2ac_i + 5c_i^2}{4b}, \quad \tilde{\Pi}_{R_i}^SI = 0.$$

Lemma 4 reveals that two-part tariff cannot eliminate double marginalization from dual channels. Specifically, part (a) is the standard single-channel result: the manufacturer can use two-part tariff to extract all channel surplus (with the fixed fee), sell at marginal production cost ($\tilde{w}^N = 0$), and thereby eliminate double marginalization. Part (b) says, in the dual-channel $\tilde{\Gamma}^SI$, the manufacturer can still use two-part tariff to extract all retailer surplus ($\tilde{\Pi}_{R_i}^SI = 0$), but he can no longer eliminate double marginalization. The latter is driven by the dual-channel structure: the very existence of a direct channel gives the manufacturer the option to encroach. This threatens the retailer with ex post retail competition, which discourages him from ordering the first type. Anticipating this, the manufacturer will price above the marginal cost ($\tilde{w}_i^SI > 0$), reduce wholesale quantity, thereby causing double marginalization. Ironically, the manufacturer suffers from her own direct channel investment: had she had only the retailer channel, she would have done better ($\tilde{\Pi}_{M_i}^SI < \tilde{\Pi}_M^N$).

We now consider the asymmetric information regime $\tilde{\Gamma}^AI$. We find:

**Lemma 5.** In regime $\tilde{\Gamma}^AI$, only manufacturer type-$\ell$ has the incentive to mimic the other type.

This may force type-$h$ to distort the contract ($\tilde{T}, \tilde{w}$) for signaling her higher cost:

**Proposition 9.** In regime $\tilde{\Gamma}^AI$, there exists a unique separating PBE $\tilde{\sigma}^*$ (in Table 4) that survives the intuitive criterion. Relative to the first best in $\tilde{\Gamma}^SI$, it has the following properties:

(a) Type-$\ell$ manufacturer offers the same contract ($\tilde{T}_i^\ell, \tilde{w}_i^\ell$) as in full information regime $\tilde{\Gamma}^SI$. 

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Equilibrium outcomes under two-part tariff</th>
</tr>
</thead>
<tbody>
<tr>
<td>equilibrium</td>
<td>type-$\ell$</td>
</tr>
<tr>
<td>$\tilde{T}_i^\ell, \tilde{w}_i^\ell$</td>
<td>$\tilde{T}_i^{SI}, \tilde{w}_i^{SI}$</td>
</tr>
<tr>
<td>$a \leq \tilde{a}$</td>
<td>$(\frac{3a^2 + 2ac_i - 4ac_i - 4c_i - 2c_i}{8b}, 0)$</td>
</tr>
<tr>
<td>$\tilde{q}_R^\ell$</td>
<td>$\frac{2c_i}{b}$</td>
</tr>
<tr>
<td>$\tilde{q}_M^\ell$</td>
<td>$\frac{a - 2c_i}{2b}$</td>
</tr>
<tr>
<td>$\tilde{\Pi}_R^\ell$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tilde{\Pi}_M^\ell$</td>
<td>$\frac{a^2 - 2ac_i + 5c_i^2}{4b}$</td>
</tr>
</tbody>
</table>
(b) If market size $a > \tilde{a}$, $\tilde{a} \equiv 2\sqrt{(3c_\ell + c_h)(c_h - c_\ell) + c_h + 2c_\ell}$, then type-$h$ manufacturer sets lower wholesale price and higher fixed fee, the retailer receives zero payoff, and the supply chain produces lower surplus:

$$\tilde{w}_h^* < \tilde{w}_h^{SI}, \quad \tilde{T}_h^* > \tilde{T}_h^{SI}, \quad E\tilde{\Pi}_R^* = E\tilde{\Pi}_R^{SI} = 0, \quad E\tilde{\Pi}_{SC}^* < E\tilde{\Pi}_{SC}^{SI}.$$  

(c) If market size $a \leq \tilde{a}$, then type-$h$ manufacturer prices at the marginal cost, the retailer receives higher payoff, and the supply chain produces lower surplus:

$$\tilde{w}_h^* = 0 < \tilde{w}_h^{SI}, \quad E\tilde{\Pi}_R^* > E\tilde{\Pi}_R^{SI} = 0, \quad E\tilde{\Pi}_{SC}^* < E\tilde{\Pi}_{SC}^{SI}.$$  

Moreover, $\tilde{T}_h^* < \tilde{T}_h^{SI}$ iff $a < \frac{1}{3} (2c_\ell - c_h) + \frac{2}{3} \sqrt{2c_\ell c_h + 25c_h^2 - 11c_\ell^2}$.

(d) The consumers are better off in regime $\tilde{\Gamma}^{AI}$ than in $\tilde{\Gamma}^{SI}$.

Proposition 9 shows that, even under two-part tariff, our main mechanism—the countervailing interplay of signaling distortion and double marginalization—still operates. Specifically, type-$\ell$ manufacturer needs no distortion, behaving as under full information $\tilde{\Gamma}^{SI}$. To signal high cost, however, type-$h$ must distort the contract in two ways. First, for large market size, type-$h$ only needs to distort the wholesale price $\tilde{w}_h^*$, while setting the fixed fee efficiently—in the same manner as in $\tilde{\Gamma}^{SI}$ (for given price $\tilde{w}_h^*$). So the change in the fixed fee only reflects the change in the wholesale price. As a result, she can still use the fixed fee to extract all retailer surplus.

Second, for small market size, however, type-$h$ must distort both terms. Intuitively, a small market size makes the retailer sales more important, thereby intensifying the imitation incentive. To prevent mimicking, type-$h$ needs to distort not only price $\tilde{w}_h^*$ (to the marginal cost), but also the fixed fee—in the downward distortion manner relative to $\tilde{\Gamma}^{SI}$ (for given price $\tilde{w}_h^*$). Both distortions benefit the retailer. As a result, the manufacturer can no longer extract all the surplus, and the retailer gets positive payoff ($E\tilde{\Pi}_R^* > E\tilde{\Pi}_R^{SI} = 0$)—the windfall benefit of the signaling distortion. Moreover, signaling distortion can still increase the value of encroachment ($E\tilde{\Pi}_R^* > E\tilde{\Pi}_R^{SI} = E\tilde{\Pi}_R^{AI}$); i.e., encroachment can still strictly benefit the retailer, but again only under asymmetric information.

### 6.3. Sequential Price Competition

To further assess the robustness of the main mechanism, we now study price competition with imperfect product substitution. This scenario arises when the two parties compete on retail price, and consumers have heterogenous shopping preferences. For example, some consumers value human interactions in brick-and-mortar stores, while others favor the convenience of online shopping.

The game plays out in three stages: (i) the manufacturer sets wholesale price $w$; (ii) the retailer decides retail price $P_R$; (iii) the manufacturer decides her direct price $P_M$. The inverse demand curve for firm $i$ is $P_i = \alpha - q_i - \beta q_j$, where $i, j \in \{R, M\}$, $i \neq j$. The parameter $\beta \in (0, 1)$ measures the degree of product
substitution; e.g., $\beta = 0$ means independent products, while $\beta = 1$ means perfect substitutes. The demand in the end market is

$$q_i = \frac{\alpha}{1 + \beta} - \frac{P_i}{1 - \beta^2} + \frac{\beta P_j}{1 - \beta^2}, \quad i, j \in \{M, R\}, i \neq j.$$  \hfill (7)

The full information benchmarks $\Gamma^N_p$ and $\Gamma^SI_p$ are well-studied (Arya et al. 2007):

**LEMMA 6.** (a) In single-channel game $\Gamma^N_p$, the equilibrium outcome is same as in the quantity game $\Gamma^N$ (Eq. ($\sigma^N$)):

- $w^N = \frac{\alpha}{2}$,
- $P^N_R = \frac{3\alpha}{4}$,
- $\Pi^N_M = \frac{\alpha^2}{8}$,
- $\Pi^N_R = \frac{\alpha^2}{16}$.

(b) In the dual-channel game $\Gamma^SI_p$ with encroachment, the equilibrium outcome is

$$w^SI_i = \frac{\alpha}{2} - \frac{\beta^2 (\alpha (1 - \beta) + \beta c_i)}{2 (\beta^4 - 5 \beta^2 + 8)}, \quad P^SI_R = \frac{\alpha (2 - \beta - \beta^2) + \beta c_i + 2 w^SI_i}{2 (2 - \beta^2)}, \quad P^SI_M = \frac{\alpha (\beta - 1)((\beta - 2) \beta - 4) + (4 - \beta^2) c_i + 2 \beta (3 - \beta^2) w^SI_i}{4 (2 - \beta^2)}.$$  \hfill (8)

The lemma confirms that, under price competition, the retailer can still gain from encroachment, because of the wholesale price cut, i.e., $w^SI_i < w^N$. The size of the price cut, however, is smaller than in quantity competition, i.e., $w^SI_i$ is higher than that in quantity competition $\Gamma^SI$ (§3). This is because retail prices are **strategic complements**. As such, the manufacturer should set a higher wholesale price $w^SI_i$ (relative to that in $\Gamma^SI$), induce a higher retailer price $P^SI_R$ (cf Eq. (8)), and thus benefit more.

We now study price competition $\Gamma^SI_p$ under asymmetric information. We find:

**LEMMA 7.** In the price game $\Gamma^SI_p$, only type-$h$ manufacturer has the incentive to mimic the other type.

---

**Figure 7**  Manufacturer profits: $\alpha = 4.6, \beta = 0.85, c_e = 0.3, c_b = 0.55
This is because in the retail market, a lower direct selling cost $c_i$ entails lower direct price $P_{M_i}$, which intensifies price competition and reduces retailer price $P_R$. If type-$h$ can convince the retailer that she is of low cost, the retailer would cut price $P_R$ and order more $q_R$, thereby benefiting type-$h$ ($\Pi_M(h, 1, w_{SI}^h) > \Pi_M(h, 0, w_{SI}^h)$). However, having the cost advantage ($c_\ell < c_h$), type-$\ell$ will never imitate high-cost type ($\Pi_M(\ell, 0, w_{SI}^\ell) < \Pi_M(\ell, 1, w_{SI}^\ell)$). See Figure 7. The imitation threat from type-$h$, however, may compel type-$\ell$ to signal her cost by cutting the wholesale price:

**Proposition 10.** In price game $\Gamma^A_{IP}$, the unique separating PBE $\sigma^*_i$ under intuitive criterion satisfies:
(a) Type-$h$ sets the wholesale price at the first best level, $w^*_h = w_{SI}^h$.
(b) Type-$\ell$ sets her wholesale price below the first best: $w^*_\ell = \widebar{w}_p < w_{SI}^\ell$.
(c) The consumers, retailer and supply chain are better off when the manufacturer’s direct selling cost information is private.

Proposition 10 demonstrates that our offsetting mechanism still works its magic. Specifically, (a) type-$h$ will price efficiently; (b) but type-$\ell$ will cut the wholesale price for signaling,\(^21\) which encourages the retailer to order more, and thereby reduces double marginalization. Although this regime $\Gamma'^A_p$ differs substantially from the base model $\Gamma^A$ (e.g., price vs. quantity competition, type-$\ell$ vs. type-$h$ price cut), the offsetting mechanism remains the same. As a result, the retailer, the supply chain, and the consumers can still benefit from upstream information asymmetry.

7. Conclusion

Managing dual-channel distribution is inherently challenging, especially when the manufacturer holds superior cost information. This paper shows how incorporating cost information asymmetry changes channel performance in substantial ways. We find cost information asymmetry can help ease channel conflict, improve efficiency, and enhance consumer surplus.

We characterize the role of cost information asymmetry. (i) Cost information asymmetry induces strategic uncertainty, entails new interactions, and creates the offsetting mechanism. The mechanism relies on signaling distortion to offset double marginalization: to signal her cost, the manufacturer may cut price and commit to less aggression in retail competition; this concession encourages the retailer to buy more, thereby reducing double marginalization and improving efficiency. The improvement is substantial, when the proportion of high cost type is large, and the cost gap is moderate. (ii) Cost information asymmetry can turn the victims of encroachment to its beneficiaries. It helps rationalize two puzzles: the retailer can benefit from his information disadvantage, and he can gain from the rival’s cost improvement. Ignoring cost information asymmetry, however, previous studies may have overestimated the harm of encroachment, and some manufacturers may

---

\(^21\)The rationale for type-$\ell$ to cut the wholesale price is as follows. The price cut can intensify price competition and increase sales through direct selling. If type-$h$ were to mimic, then both need to sell more through the direct channel. But this is more costly for type-$h$ to bear ($c_h > c_\ell$).
be too eager to make excessive concessions. (iii) A key question is what happens if alternative arrangements prevail. We find when the two parties engage in price competition or move simultaneously, the offsetting mechanism can still operate and cost information asymmetry can still benefit. Even under two-part tariff, double marginalization still persists and the retailer can still benefit from cost information asymmetry.

This study sharpens our understand of dual-channel management. Methodologically, the results are of normative nature. The question of real impact of cost information asymmetry, however, is a positive one. To have a definitive answer, one needs comprehensive empirical studies of business specifics. For such empirical inquiry, we provide a modeling framework and testable implications. We hope this work can spark more research in this exciting area.

References


Hermalin, Benjamin. 2009. Lecture notes for economics.


Appendix

Proof of Lemma 1:
Part (a) is trivial so we omit the proof. The solutions to part (b) can be found in Arya et al. (2007). We detail the proof for completeness.

We first find the manufacturer best response given retailer’s quantity decision. For simplicity, we drop superscripts and subscripts when it is clear from the context. Because the manufacturer’s profit function is concave we can easily show that her best response is

\[ q_M(q_R) = \left( \frac{a - b q_R - c_t}{2b} \right) = \begin{cases} \frac{a - b q_R - c_t}{2b} : q_R < \frac{c_t}{b}, \\ 0 : q_R \geq \frac{c_t}{b}. \end{cases} \]  

(9)

Therefore,

\[ \Pi_M(q_R) = \begin{cases} \pi_{M_a} : q_R < \frac{c_t}{b}, \\ \pi_{M_b} : q_R \geq \frac{c_t}{b}. \end{cases} \]  

(10)

where \( \pi_{M_a} \equiv (a - b(q_R + \frac{a-b-q_R}{b}) - w)q_R \) and \( \pi_{M_b} \equiv (a - b(q_R) - w)q_R \). These functions are concave. By checking the derivative of these functions at \( q_R = \frac{c_t}{b} \), we split the problem into three cases \( a < 2c_t \), \( 2c_t \leq a < 3c_t \), and \( 3c_t \geq a \). We now solve for the first as this is the most involved one, and the other cases can be derived from this case. When \( a < 2c_t \), using (10) we can write the retailer’s best response is as follows.

\[ q_R(w) = \begin{cases} \frac{a-c_t^2}{2a} : w \geq \frac{a+c_t}{2}, \\ \frac{b}{c_t} - w < \frac{a+c_t}{2}, \\ \frac{a}{2b} : 2c_t - a \leq w < \frac{a+c_t}{2}, \\ w < 2c_t - a. \end{cases} \]  

(11)

Now, we solve for the manufacturer’s wholesale price decision. Using (11), we can write the manufacturer’s profit as

\[ \Pi_M = \begin{cases} \frac{3 a^2 - 6 a c_t + 7 c_t^2}{12 b} : w \geq \frac{a+c_t}{2}, \\ \frac{a c_t^2}{2(3a+2b(2c_t-a))} + (3a+12c_t-12c_t^2) w : \frac{b}{c_t} - w < \frac{a+c_t}{2}, \\ \frac{a c_t^2}{2b} : 2c_t - a \leq w < \frac{a+c_t}{2}, \\ w < 2c_t - a. \end{cases} \]  

(12)

By checking the derivatives of this function at the threshold \( w \)s where it changes characteristics, we can find the optimal wholesale price \( w \). Then using equations (9) and (11), we can find the equilibrium quantities and profits as stated in the lemma. The cases for \( a \geq 2c_t \) can be found similarly and can be combined with \( a < 2c_t \) to obtain the result in the proposition.

Proof of Lemma 2:
From Proposition 1, the symmetric equilibrium wholesale prices are \( w^2_t = a/2 - c_t/6 \) and \( w^0_t = a/2 - c_t/6 \). Plugging these into the manufacturer’s profit in Equation (4), we obtain

\[ \Pi_M(\ell, 1, w^2_t) = \frac{3a^2 - 6ac_t + 7c_t^2}{12b}; \quad \Pi_M(\ell, 1, w^0_t) = \frac{3(a-c_t)^2 + 4c_t c_t}{12b}; \]

\[ \Pi_M(h, 0, w^2_t) = \frac{3a^2 - 6ac_t + 7c_t^2}{12b}; \quad \Pi_M(h, 1, w^0_t) = \frac{3(a-c_t)^2 + 4c_t c_t}{12b}. \]

Comparison of these profits yields the result.

Proof of Lemma 3:
It is easy to see that whenever it is not profitable for manufacturer type- \( \ell \) to mimic manufacturer type- \( h \), her optimal action would be to play the symmetric equilibrium wholesale price, which proves part (i).

For part (ii) define

\[ \hat{\Delta}(w) \equiv \Pi_M(\ell, 0, w) - \Pi_M(\ell, 1, w^2_t) = \frac{-9a^2 - 6ac_t + 12ac_t + 3c_t^2 - 16c_t^2}{48b} + \frac{w(36a + 12c_t - 24c_t^2)}{48b} - \frac{3w^2}{4b}. \]

Thus \( \hat{\Delta} \) determines the additional profit manufacturer type- \( \ell \) makes by mimicking type- \( h \). Therefore, for \( 1c_t \) constraint to be satisfied, we need \( \hat{\Delta}(w) \leq 0 \). \( \hat{\Delta} \) is a concave function of \( w \) with two roots \( \Pi \) and \( \frac{1}{3}(3a+2\sqrt{(3a-c_t)(c_t+3c_t^2)+c_t-2c_t}) \equiv \Pi' \), where \( \Pi < \Pi' \). Therefore, \( 1c_t \) holds if and only if \( w^* \leq \Pi \) or \( w^* \geq \Pi' \).

Similarly, from \( 1c_h \) we need \( \Pi_M(h, 0, w) \geq \max \{ \Pi_M(h, 1, w), \Pi_M(h, 1, w) \} \) is a concave function of \( w \). Therefore, from the first order condition, the optimal \( w \) that maximizes the right hand side of the inequality is \( w = \frac{1}{3}(3a+c_t-2c_t) \). Plugging this into the profit function on the right hand side of the inequality, we find \( \frac{w^2}{4b} \geq 0 \).
This requires that either \( w \geq w_0 \) or \( w \leq \frac{1}{3} (3a + 2\sqrt{(c_c - c_t)(3c + c_t)} - c_t) \equiv w_* \). It can be easily verified that \( w < w_0 < w_* \). Therefore, IC and IC are satisfied simultaneously if and only if \( w_* \in [w_0, w] \).

Finally, we show that \( w < w_* \). First, note that it is straightforward to see that \( w_* < w_0 \). Second,

\[
\Pi(h, 0, w) = \frac{(a + c + 2w)^2}{16b} + \frac{w(a + c - 2w)}{2b},
\]

\[
d\Pi(h, 0, w) = \frac{-3}{2} < 0.
\]

with a maximum at \( a - c - 6b > w \). As a result, \( w_* = w < w_0 \) does not survive the intuitive criterion and only \( w_* = w \) fulfills the intuitive criterion. \( \square \)

**Proof of Proposition 1:**

Suppose that manufacturer type-\( h \)'s equilibrium wholesale price \( w_*^h \) is such that \( w_*^h < w \). Consider the off-the-equilibrium deviation \( w^* = w_0 \). We know that \( \Pi(h, 0, w^* - w_0) < \Pi(h, 0, w_0) \) due to the IC constraint. Therefore, if the retailer observes \( w^* - w_0 \), it must rationally believe that the manufacturer is type-\( h \) with probability 1. However, in that case, \( \Pi(h, 0, w_0^h) < \Pi(h, 0, w_0 - w^* - w_0) \), type-\( h \) would have an incentive to deviate from \( w_*^h \) to \( w - w_0 \). It is indeed the case that \( \Pi(h, 0, w_0^h) < \Pi(h, 0, w_0 - w^* - w_0) \). To see why, note that

\[
\Pi(h, 0, w) = \frac{(a - 3c + 2w)^2}{16b} + \frac{w(a + c - 2w)}{2b},
\]

\[
d\Pi(h, 0, w) = \frac{3}{2} > 0.
\]

with a maximum at \( a - c - 6b > w \). As a result, \( w_* = w < w_0 \) does not survive the intuitive criterion and only \( w_* = w \) fulfills the intuitive criterion. \( \square \)

**Proof of Proposition 2:**

Consider a pooling equilibrium \( \sigma^* = (\mu, \sigma_q^*, \sigma_0^*, \sigma_0^*, w^*) \), where \( \mu \) is the posterior belief, and \( (\sigma_q^*, \sigma_0^*, w^*) \) the equilibrium decisions. In this case, the manufacturer’s best response is \( \sigma_q^*(\varphi, \mu) = \frac{w}{w_0} \). Due to pooling, the retailer cannot infer manufacturer type. So he hold constant belief \( \mu = \mu_0 \), and decides his best best response \( \sigma_q^*(w, \mu) = \frac{w}{w_0} \), based on the average cost \( c_0 = \mu_0 c_t + (1 - \mu_0) c_s \).

We first prove some preliminary results (lemmas 8, 9 and 10) that assist us in proving the proposition. Lemma 8 constrains the set of wholesale prices a pooling equilibrium may exist (although, we show that these wholesale prices cannot survive the intuitive criterion afterwards), i.e., IC and IC constraints are satisfied. Lemma 9 is used to identify a probable profitable deviation from the pooling equilibrium for type-\( h \) manufacturer. Lemma 10 identifies the best belief of retailer that would maximize manufacturer type-\( h \)'s profit. After proving these lemmas, we show that a pooling equilibrium cannot be sustained under intuitive criterion.

Suppose that a pooling equilibrium \( \sigma_q^*(\mu, \sigma_q^*, \sigma_0^*, \sigma_0^*, w^*) \) exists.

**Lemma 8.** An off-the-equilibrium deviation \( \Pi(h, 0, w^* - w_0) < \Pi(h, 0, w_0) \) must satisfy the following properties:

1. \( \Pi(h, 0, w^* - w_0) \leq \Pi(h, 0, w_0) \).
2. \( \Pi(h, 0, w^* - w_0) \leq \Pi(h, 0, w_0) \).
3. \( \Pi(h, 0, w^* - w_0) \leq \Pi(h, 0, w_0) \).

**Proof of Lemma 8:** An off-the-equilibrium deviation must satisfy the following IC constraints:

\[
\Pi(h, \mu, w^* - w_0) \geq \Pi(h, 1, w_0^h),
\]

\[
\Pi(h, \mu, w^* - w_0) \geq \Pi(h, 1, w_0^h),
\]

where \( w_0^h = \arg \max \Pi(h, 1, w_0) = \Pi(h, \mu, w_0) \). Otherwaise, one of the types may have an incentive to deviate. These conditions are equivalent to \( \Pi(h, \mu, w^* - w_0) \leq \Pi(h, \mu, w_0) \) and \( \Pi(h, 1, w^* - w_0) \leq \Pi(h, 1, w_0) \). Otherwise, one of the types may have an incentive to deviate. These conditions are equivalent to \( \Pi(h, \mu, w^* - w_0) \leq \Pi(h, \mu, w_0) \) and \( \Pi(h, 1, w^* - w_0) \leq \Pi(h, 1, w_0) \).

**Lemma 9.** Suppose that the manufacturer encroaches. Define \( w_1 \) as \( \Pi(h, 0, w_1) = \Pi(h, \mu, w_0) \). Then, \( w_1 = \frac{1}{2} (2a + (\mu - 6)c_1 - (1 - \mu)c_s) \). There exists a \( w^* \) satisfying \( w_1 < w^* < w_0^* \) such that \( \Pi(h, 0, w^*) = \Pi(h, \mu, w^*) \).

**Proof of Lemma 9:**

(A1) It can be shown that \( \Pi(h, 0, w_0^h) > \Pi(h, \mu, w_0) \) if and only if \( w > w_1 \). Furthermore, it can be seen that \( w_1 < w^* \).

(A2) \( \partial \Pi(h, 0, w, w_1) / \partial w = 2c_1 / 3b > 0 \), \( \partial \Pi(h, 0, w, w_1) / \partial w = (5 \mu_0 c_t + c_s) / 3b > 0 \).

(A3) It can be shown that \( \Pi(h, \mu, w^* - w_0) > \Pi(h, \mu, w_0) \). Moreover, \( \Pi(h, \mu, w^* - w_0) = \Pi(h, \mu, w^*) \) for any \( w^* \) in \( [w_0, w^*] \). Note that by the virtue of Lemma 8, \( w^* \in [w_0, w^*] \). \( \square \)

**Lemma 10.** Suppose that the manufacturer encroaches. \( \Pi(h, 0, w) > \Pi(h, 1, w) \) if and only if \( w > w_0 \), where \( w_0 \equiv (2a + 5c_t - 6c_s) / 4 \). In addition, \( w_1 > w_0 \).
Therefore, the lemma states that the best belief of the retailer for type-$\ell$ is $\mu = 0$ if and only if $w > w_{12}$.

**Proof of Lemma 10:**

By comparing $\Pi_{d}(\ell,0,w)$ and $\Pi_{d}(\ell,1,w)$, the lemma can be shown easily. Hence, the proof is omitted.

We now show that a pooling equilibrium cannot be sustained under the intuitive criterion. A pooling equilibrium survives the intuitive criterion if there does not exist a wholesale price $w$ such that either (15) or (16) holds.

\[
\begin{align*}
(a) & \quad \Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, \mu_{a}(\ell), w) \quad \text{and} \quad (b) \quad \Pi_{d}(\ell, \mu, w^*) < \Pi_{d}(\ell, 1, w), \quad (15) \\
(a) & \quad \Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, \mu_{a}(\ell), w) \quad \text{and} \quad (b) \quad \Pi_{d}(\ell, \mu, w^*) < \Pi_{d}(\ell, 0, w), \quad (16)
\end{align*}
\]

where $\mu_{a}(i)$ is the retailer’s best belief that will maximize type-$i$ manufacturer’s profit. Otherwise, there exists a wholesale price $w$ such that one of the type would want to deviate to, and which would break the equilibrium. Essentially, these conditions search for a $w$ to deviate such that for one of the types this deviation is dominated by the equilibrium even under the best belief of the retailer (Conditions 15a and 16a). If such $w$ exists, the retailer would rationally believe that $w$ can only be played by the other type. Given this belief of the retailer, if the other type is better off by deviating to this $w$, the equilibrium does not survive the intuitive criterion (Conditions 15b and 16b). If such $w$ does not exist, the equilibrium survives the intuitive criterion.

Note that the inequality (15b) cannot hold because of the requirement of the pooling equilibrium per $\Pi_{d}$ constraint given in inequality (14) (see the proof of Lemma 8). Therefore, we only need to check whether a wholesale price exists that satisfies the inequalities (16a) and (16b). Because we focus on the region where both type of manufacturers encroach, we need to find deviations in which deviating manufacturers continue to output a positive quantity through direct channel.

From Lemma 9, there exists a $w'$ satisfying $w_{i} < w' < w^*$ such that $\Pi_{d}(\ell, \mu, w^*) = \Pi_{d}(\ell, h_{i}, w')$. Suppose that $w' > (3c_{1} - a)/2$ so that both manufacturer types encroach for $w \geq w'$. Also suppose that $w' < w_{2d}$. In this case, $\Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, 1, w')$. This is because $\Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, \mu_{a}(\ell), w)$. By comparing $\Pi_{d}(\ell, 0, w') > \Pi_{d}(\ell, 1, w')$, and $\partial \Pi_{d}(\ell, 1, w_{i})/\partial \mu = (5c_{1} + 11c_{2})/8b > 0$. Therefore, if $\partial \Pi_{d}(\ell, h_{i}, w')/\partial \mu > 0$ (or $\Pi_{d}(\ell, h_{i}, w')/\partial \mu < 0$), type-$h$ manufacturer can benefit from deviating to $w' + \epsilon (w' - \epsilon)$, where $\epsilon$ is an arbitrarily small number. This breaks the intuitive criterion.

On the other hand, if $w' \geq w_{3d}$. Then,

\[
\Pi_{d}(\ell, \mu, w^*) - \Pi_{d}(\ell, \mu, w^*) = \frac{(c_{1} - c_{2})a - \mu c_{3} + 2c_{1} + \mu c_{2} - 2\mu w')}{4b},
\]

\[
\Pi_{d}(\ell, 0, w') - \Pi_{d}(\ell, 0, w') = \frac{(c_{1} - c_{2})a + 2c_{1} + \epsilon - 2\mu w')}{4b}.
\]

Therefore, $\Pi_{d}(\ell, \mu, w^*) - \Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, 0, w')$ and $\Pi_{d}(\ell, \mu, w^*) > \Pi_{d}(\ell, 0, w')$. Therefore, if $\partial \Pi_{d}(\ell, 0, w')/\partial \mu > 0$ (or $\Pi_{d}(\ell, 0, w')/\partial \mu < 0$), type-$h$ manufacturer can benefit from deviating to $w' + \epsilon (w' - \epsilon)$, where $\epsilon$ is an arbitrarily small number. This breaks the intuitive criterion.

Now suppose that $w' \leq (3c_{1} - a)/2$. This implies $w_{i} < (3c_{1} - a)/2$, and because $w_{i}$ and $(3c_{1} - a)/2$ cannot be negative at the same time, we have $(3c_{1} - a)/2 > 0$. We show that type-$h$’s deviation to $(3c_{1} - a)/2$ breaks the intuitive criterion. Note that when the wholesale price is $(3c_{1} - a)/2$, type-$h$ weakly encroaches when $\mu = 0$. On the other hand, from Eqs. (2) and (3), type-$\ell$ manufacturer always encroaches, i.e., $q_{d}^{*}(q_{d}^{*}(3c_{1} - a/2, \mu, \ell)) > 0$ for all $\mu \in [0,1]$. In addition, $\Pi_{d}(\ell, 1, (3c_{1} - a)/2) < \Pi_{d}(\ell, \mu, w^*) < \Pi_{d}(\ell, \mu, \mu)$ and $\Pi_{d}(\ell, 0, (3c_{1} - a)/2) < \Pi_{d}(\ell, \mu, w^*) < \Pi_{d}(\ell, \mu, \mu)$. Combined with the fact that $\Pi_{d}(\ell, \mu, w)$ is a concave function, this implies that the intuitive type-$\ell$ cannot benefit from deviating from $w^*$ to $(3c_{1} - a)/2$ since $w^* < w^* < \mu$ even when the best belief is chosen.

We first show that $\Pi_{d}(h_{i}, 0, (3c_{1} - a)/2) \geq \Pi_{d}(h_{i}, 0, w')$. Suppose not, from the concavity of $\Pi_{d}(h_{i}, 0, w_{i})$, it must be that $\partial \Pi_{d}(h_{i}, 0, (3c_{1} - a)/2)/\partial \mu \leq 0$. But then, $\Pi_{d}(h_{i}, 0, (3c_{1} - a)/2) > \Pi_{d}(h_{i}, 0, w^*)$ and $\Pi_{d}(h_{i}, \mu, w^*)$ since $(3c_{1} - a)/2 < w^*$ and $w^* > w_{3d}$. This contradicts with the assumption that $\Pi_{d}(h_{i}, 0, (3c_{1} - a)/2) < \Pi_{d}(h_{i}, 0, w^*)$ since $\Pi_{d}(h_{i}, 0, w^*) = \Pi_{d}(h_{i}, \mu, w^*)$. Therefore, in this case, the manufacturer type-$h$ can benefit from deviating to $(3c_{1} - c_{2})/2$, while the type-$h$ cannot. This breaks the intuitive criterion.

**Proof of Proposition 3:**

Assume condition (C1) holds. First, we show $\Pi_{d}^{1} \geq \Pi_{d}^{2}$. From Lemma 3 and Proposition 1, we know that when the manufacturer is type-$\ell$, the equilibrium of $\Gamma^{w}$ is same as the equilibrium of $\Gamma^{1}$. But, when the manufacturer is type-$h$, she deviates from her first-best strategy in $\Gamma^{w}$, obtaining a lower profit. Therefore, the manufacturer’s expected profit decreases due to information asymmetry.

**Proof of Proposition 4:**

From Proposition 1, in symmetric information the ex-ante total supply chain profit is

\[
\mathbb{E}[\Pi_{d}^{1} + \Pi_{d}^{2}] = \mu_{0} \left\{ \frac{9c_{1}^{2} - 18ac_{1} + 29c_{2}^{2}}{36b} + (1 - \mu_{0}) \frac{9c_{1}^{2} - 18ac_{1} + 29c_{2}^{2}}{36b} \right\}.
\]

In separating equilibrium, the ex-ante total supply chain profit is

\[
\mathbb{E}[\Pi_{d}^{1} + \Pi_{d}^{2}] = \mu_{0}(\Pi_{d}(\ell, 1, w_{1}^{*}) + \Pi_{d}(\ell, 1, w_{2}^{*})) + (1 - \mu_{0})(\Pi_{d}(h, 0, w_{1}^{*}) + \Pi_{d}(h, 0, w_{2}^{*})),
\]

where $w_{i}^{*}$ is the equilibrium wholesale price for type $i$. When manufacturer is type-$\ell$, the supply chain profit is same for the symmetric information and separating equilibrium cases. Therefore, we only need to compare the profits when the manufacturer is type-$h$, i.e., we only need to show that $\Pi_{d}(h, 0, \mu) + \Pi_{d}(h, 0, \mu) > \Pi_{d}(h, 0, w_{1}^{*}) + \Pi_{d}(h, 0, w_{2}^{*})$. This inequality is equivalent to

\[
-c_{1} \left\{ \sqrt{2c_{1}c_{2} - 3c_{3}^{2} + c_{4}^{2} - 4c_{5}} \right\} + 5c_{4} \left\{ \sqrt{2c_{1}c_{2} - 3c_{3}^{2} + c_{4}^{2} - c_{5}} \right\} + c_{5} > 0.
\]

\footnote{Note that $w_{1}$ and $(3c_{1} - a)/2$ cannot be negative at the same time. Therefore, $w' > 0$, and it is a proper deviation.}
The above holds if and only if 
\[(5c_h - c_i)\sqrt{3(3c_h + c_i)(5c_h - c_i)} > (5c_h + c_i)(c_h - c_i) \iff 2(2c_h^2 + c_i^2 + 25c_i^2 + c_i^2) > 0.\]
Thus, the supply chain’s profit is higher in separating equilibrium, i.e., \( \Delta_2 \Pi_{SC} > 0 \).

For the retailer’s profit, we again only need to compare the profits when the manufacturer is type-\( h \), i.e., we need to check
\[
\Pi_{h}(h, 0, \pi) - \Pi_{h}(h, 0, \pi^*) = \frac{(2c_h^2 + c_i^2 + c_i^2 + c_i^2)^2}{18b} - \frac{2c_h^2}{9b} > 0.
\]

The above holds if and only if
\[
\frac{(2c_h^2 + c_i^2 + c_i^2 + c_i^2)^2}{18b} - \frac{4c_h^2}{9b} > 0 \iff (c_h - c_i)(c_h + 3c_i) + c_h + c_i > 2c_h
\]
\[(c_h + 3c_i) > c_h - c_i.\]

The last inequality shows that the retailer’s profit in separating equilibrium is higher, i.e., \( \Delta_2 \Pi_{h} > 0 \).

These prove that both the retailer and the supply chain are better off in the separating equilibrium compared to the symmetric information case. Finally, we show the consumers are better under asymmetric information. First, the expected consumer surplus under symmetric information is
\[
\mathbb{E}[\Pi_{i}^S] = \mu_b \int_0^{\pi} \left( \frac{q_{i}^{SI}}{2} \right)^2 + (1 - \mu) b \left( \frac{q_{h}^{SI} + q_{i}^{SI}}{2} \right)^2 = \mu \left( \frac{3a - c_i}{} \right)^2 + (1 - \mu) \left( \frac{3a - c_h}{} \right)^2.
\]

Second, expected surplus under asymmetric information is
\[
\mathbb{E}[\Pi_{i}^A] = \mu_b \int_0^{\pi} \left( \frac{q_{i}^{SI}}{2} \right)^2 + (1 - \mu) b \left( \frac{q_{h}^{SI} + q_{i}^{SI}}{2} \right)^2 = \mu \left( \frac{3a - c_i}{} \right)^2 + (1 - \mu) \left( \frac{3a - c_h}{} \right)^2.
\]

It follows immediately that \( \mathbb{E}[\Pi_{i}^A] > \mathbb{E}[\Pi_{i}^S] \). Hence the consumers are better off under asymmetric information, i.e., \( \Delta_2 \Pi_{i} > 0 \). \qed

**Proof of Proposition 5:**
We find the necessary and sufficient conditions first for \( \Pi_{SC}^L \) and then for \( \Pi_{SC}^R \). To do so, we first prove some preliminary results (lemmas 11, 12 and 13). For simplicity, we use \( \Pi_{i}^S(i) \) ( \( \Pi_{i}^V(i) \) ) and \( \Pi_{i}^S(i) \) ( \( \Pi_{i}^V(i) \) ) to denote equilibrium supply chain (retailer) profits when the manufacturer is type-\( i \) under symmetric and asymmetric information, respectively.

**Lemma 11:** Define \( p_i \equiv \Pi_{i}^S(h) - \frac{1}{\mu_b} \). Then \( p_i \) is a convex function of \( d \) with two real roots \( s_1 \) and \( s_2 \), where \( s_1 < s_2 \).

**Proof of Lemma 11:**
\( d^2 p_i/dd^2 \equiv 1/b^2. \) Therefore, it is a convex function of \( a \). In addition \( \lim_{d \to 2} d^i p_i > 0, \lim_{d \to 0} p_i > 0, p_i(1) < 0. \) \( \alpha^* \) is the minimizer of \( p_i \), and \( \alpha^* = 4c_h \). Therefore, there must exist two roots, \( s_1 \) and \( s_2 \) such that \( s_1 < s_2 \). \qed

**Lemma 12:** Define \( p_1 \equiv \lim_{d \to 1} d^i p_i \). Then \( p_1 \) is a third order polynomial of \( d \) with three real roots \( r_1, r_2, \) and \( r_3. \) Furthermore, \( r_1 < c_h < r_2 < c_i < r_3 < c_i \), where \( c_i \equiv \frac{\sqrt{4 - \sqrt{3}}}{2} \approx 0.387965 \) and \( c_i \equiv \frac{\sqrt{4 + \sqrt{3}}}{2} \approx 0.736616 \). \( p_1 > 0 \) if and only if \( r_1 < c_i < r_2 < r_3 < r_1 \) or \( r_1 < c_i < r_2 \).

**Proof of Lemma 12:**
\( d^2 p_1/dd^2 \equiv 35414.5. \) Therefore, \( p_1 \) is a third order polynomial of \( a \). \( \lim_{d \to 2} d^i p_1 = -661c_h < 0, \lim_{d \to 0} p_1 = 113.042c_h > 0, \lim_{d \to 0} p_2 = -137.242c_h < 0, \lim_{d \to 0} p_3 = 256c_h > 0. \) Therefore, \( p_1 \) intersects \( x \)-axis three times. Therefore, \( p_1 \) has three roots satisfying \( r_1 < c_h < r_2 < c_i < r_3 < c_i \). \( p_1 > 0 \) if and only if \( r_1 < c_i < r_2 \) or \( r_1 < c_i < r_2 \) or \( r_1 < c_i < r_2 \) or \( r_1 < c_i < r_2 \). \qed

**Lemma 13:** Define \( f(i) \equiv \Pi_{i}^V(i) - \frac{1}{\mu_b} \). Then \( f(i) \) is a convex function of \( a \) with two real roots \( c_h \) and \( c_i \), where \( c_i \equiv \frac{1}{2} \left( 6 + \sqrt{7} \right) \) and \( c_i \equiv \frac{1}{2} \left( 6 - \sqrt{7} \right) \). Furthermore, the followings hold: 1) \( c_h > c_i > c_i \) if and only if \( c_h \approx 0.387965 \). 2) \( c_h > c_i > c_i \) if and only if \( c_h > 0.834204 \).\( c_h > c_i \) if and only if \( c_i > 0.316381 \).

**Proof of Lemma 13:**
This can be shown similar to Lemma 11. Hence, the proof is omitted. \( \blacksquare \)

We now characterize \( \Pi_{SC} \). For \( \Pi_{SC} \), we need to find the parameter region in which \( \mathbb{E}[\Pi_{i}^S] > 3a^2/16b \) and \( \mathbb{E}[\Pi_{i}^V] < 3a^2/16b \). Where \( \Pi_{SC}^L \) and \( \Pi_{SC}^R \) denote supply chain profits under asymmetric and symmetric information cases, respectively. Note that \( 3a^2/16b \) is the supply chain’s profit without encroachment (see Proposition 1). Define \( \mu_b \) as the prior probability of type-\( h \) such that \( \mathbb{E}[\Pi_{i}^V(\Pi_{i}^S(i)) = 3a^2/16b \) whenever it exists, and \( \mu_b \) is the prior probability of type-\( h \) such that \( \mathbb{E}[\Pi_{i}^S(i)] = 3a^2/16b \) whenever it exists. Given that \( \Pi_{i}^S(i) = \Pi_{i}^V(i) \) and \( \Pi_{i}^S(h) > \Pi_{i}^V(h) \), we have \( \mathbb{E}[\Pi_{i}^S(i)] > 3a^2/16b \) and \( \mathbb{E}[\Pi_{i}^V(i)] < 3a^2/16b \) if and only if one of the following cases holds (i.e. region \( A_{i} \) consists of the following cases):

1. \( \Pi_{i}^V(i) < 3a^2/16b \), \( \Pi_{i}^V(h) < 3a^2/16b \), \( \Pi_{i}^S(h) > 3a^2/16b \) and \( \mu_b < \mu_b \).
2. \( \Pi_1^a(f) < 3a/16b \) and \( \Pi_2^a(h) > 3b/16a \) and \( \mu_1 < \mu_2 < \mu_2^a \). 
3. \( \Pi_1^a(f) > 3a/16b \) and \( \Pi_2^a(h) < 3b/16a \) and \( \Pi_2^a(h) < 3b/16a \) and \( \mu_2 < \mu_1 < \mu_2^a \) or \( \Pi_2^a(h) > 3b/16a \) and \( \mu_2 < \mu_1 \).

Case 1. We now find the necessary and sufficient conditions for Case 1. We need to identify the conditions such that \( p_h > 0 \). Note that for Case 1, we have \( f(h) \leq 0 \) and \( f(t) < 0 \). From Lemma 13, these inequalities hold if and only if \( c_1c_3 < a < c_2c_2 \). We need to find how \( p_h \) behaves in this range. In particular, \( \lim_{a \to c_2c_2} p_h > 0 \) and \( \lim_{a \to c_2c_2} dp_h/da < 0 \). Therefore, from Lemma 11, \( c_1c_3 < s_1 \). From Lemma 12, \( \lim_{a \to c_1c_3} p_h > 0 \) when \( c_1c_3 < c_1 < c_2 < c_2^r \). In addition, \( \lim_{a \to c_1c_3} p_h < 0 \) when \( c_1 < c_2 < c_2^r \). Therefore, when \( c_1 < c_2 < c_2^r \), we have \( s_1 < c_1c_3 < s_2 \). On the other hand, when \( c_1c_3 < c_1 < c_2 \) and \( r_1 < c_1c_3 < s_2 \) or \( c_1c_3 < a < c_2c_2 \), either \( c_1c_3 < s_1 \) or \( c_1c_3 < s_2 \) hold. By checking the sign of \( \lim_{a \to c_2c_2} dp_h/da \), we can verify that when \( c_1c_3 < c_1 < c_2 \), we have \( c_1c_3 < s_1 \), and when \( r_1 < c_1c_3 < c_2 \), we have \( c_1c_3 < s_2 \). Therefore, the conditions in Case 1 are satisfied when the following hold:

\[
(c_1c_3 < c_1 < c_2 \text{ AND } c_1c_3 < a < c_2c_2) \text{ OR } (r_1 < c_1 < c_2 \text{ AND } c_1c_3 < a < s_1) \\
\text{ OR } (s_1 < c_1c_3 < s_2 \text{ AND } c_1c_3 < a < s_2) \text{ AND } (\mu_1 < \mu_0 < \mu_2) .
\] (C1)

Case 2. We need to identify the conditions such that \( p_h > 0 \). Note that for Case 2, we have \( f(h) > 0 \) and \( f(t) < 0 \). From Lemma 13, these conditions hold if and only if \( c_1c_3 < a < c_2c_2 \) and \( c_1c_3 < c_1c_3 \) or \( c_1c_3 < a < c_1c_3 \) and \( c_1c_3 > c_1c_3 \). Simplifying these conditions using Lemma 13 and combining with the requirement that \( \mu_1 < \mu_0 < \mu_2 \), we have

\[
((c_1c_3 < a < s_1 \text{ OR } s_1 < c_1c_3 < c_2c_2) \text{ AND } \mu_0 < \mu_2) \text{ OR } (s_1 < a \leq s_2 \text{ AND } \mu_1 < \mu_0 < \mu_2).
\] (C2)

Case 3. We need to identify the conditions such that \( p_h > 0 \). Note that for Case 3, we have \( f(h) < 0 \) and \( f(t) > 0 \). From Lemma 13, these conditions hold if and only if \( I_1c_1c_3 < a < c_2c_2 \) and \( c_1c_3 < c_1c_3 \) or \( I_1c_1c_3 < a < c_1c_3 \) and \( c_1c_3 > c_1c_3 \). We look at these cases one by one.

For Case 1, \( \lim_{a \to c_2c_2} p_h > 0 \) and \( \lim_{a \to c_2c_2} dp_h/da < 0 \). Therefore, from Lemma 11, \( c_1c_3 < s_1 \). Using the same arguments, we can show that \( c_1c_3 > s_2 \). Therefore, the conditions for Case 1 are satisfied if and only if

\[
((c_1c_3 < a < s_1 \text{ OR } s_1 < c_1c_3 < c_2c_2) \text{ AND } \mu_0 < \mu_2) \text{ OR } (s_1 < a \leq s_2 \text{ AND } \mu_1 < \mu_0 < \mu_2).
\] (C3a)

For Case 2, using similar arguments to Case 1 along with Lemma 11, it can be verified that \( c_1c_3 > s_2 \). We also need to know the sign of \( \lim_{a \to c_2c_2} p_h \), which is given in Lemma 12. Finally, using similar arguments to Case 1, we can show that the conditions for Case 2 are satisfied if and only if

\[
((c_1c_3 < c_1c_3 < s_2 \text{ AND } \mu_0 < \mu_2) \text{ OR } (s_1 < a \leq s_2 \text{ AND } \mu_1 < \mu_0 < \mu_2).
\] (C3b)

Finally, \( A_{1b}^a \) is defined by \( C_1 \cup C_2 \cup C_{3a} \cup C_{3b} \).

For \( A_{2b}^a \), we need to find the parameter region in which \( E[\Pi_1^a] > a'/16b \) and \( E[\Pi_2^a] < a'/16a \). Define \( h \equiv \Pi_1^a(i)-a'/16b \) and \( \Pi_a \equiv \Pi_2^a(i)-a'/16b \). Also define \( \Pi_a \) as the prior probability of type-\( E \) such that \( E[\Pi_a] = a'/16b \) whenever it exists, and \( \Pi_a \) as the prior probability of type-\( E \) such that \( E[\Pi_a] = a'/16a \) whenever it exists.

From Lemma 3 and Proposition 1, we know that \( \Pi_1^a(f) = \Pi_1^a(h) \) and \( \Pi_1^a(h) > \Pi_1^a(h) > \Pi_1^a(h) \). Therefore, \( A_{2b}^a \) consists of the following cases:

1. \( t_1 < 0, t_2 \leq 0, \eta_2 > 0 \text{ and } \eta_1 < \eta_2 \).
2. \( t_1 < 0, t_2 > 0, \eta_2 > 0 \text{ and } \eta_1 < \eta_2 \).

Case 1. We need to identify the region where \( \eta_2 > 0 \). From the convexity of \( t_1 \)’s it can be shown that \( t_1 < 0 \) and \( t_2 \leq 0 \) if and only if \( a \geq 4\sqrt{3}/3c_2 \text{ and } a > a \) when \( a \geq 4\sqrt{3}/3c_2 \). In addition, it can be shown that in this \( a \) region \( d^2\eta_2/da^2 < 0 \), \( d\eta_2/da < 0 \), \( \lim_{a \to -\infty} \eta_2 > 0 \text{ and } \lim_{a \to +\infty} \eta_2 < 0 \). Therefore, there exists \( s_1 < a \) such that \( \eta_2 > 0 \) if and only if \( a < s_1 \). Therefore, Case 1 holds if and only if

\[
\text{max}\{a, 4\sqrt{3}/3c_2\} < a < s_1 \text{ AND } (\mu_0 < \mu_2) .
\] (C4)

Case 2. It can be shown that \( t_1 < 0 \) and \( t_2 > 0 \) and if only if \( a < 4\sqrt{3}/3c_2 \). In this region \( d^2\eta_2/da^2 < 0 \), \( d\eta_2/da < 0 \), \( \lim_{a \to -\infty} \eta_2 > 0 \text{ and } \lim_{a \to +\infty} \eta_2 < 0 \). In addition, it can be shown that \( a < 4\sqrt{3}/3c_2 \) if and only if \( c_1 < 0.594298c_2 \text{ or } c_1 \geq 0.734129c_1 \). Therefore, Case 2 holds if and only if

\[
(c_1 < 0.594298c_2 \text{ OR } c_1 \geq 0.734129c_2) \text{ AND } \text{max}\{a, 4\sqrt{3}/3c_2\} < a < s_1 \text{ AND } (\mu_0 < \mu_2) .
\] (C5)

Finally, \( A_{2b}^a \) is defined by \( C_{1b} \cup C_{2b} \).

To see \( A_{1b}^a \cap A_{2b}^a \neq 0 \), note, for example, for \( a = 0.85, b = 1, c_1 = 0.3, c_2 = 0.375 \text{ and } \mu_0 = 0.2 \). \( E[\Pi^a_1] = 0.147146, E[\Pi^a_2] = 0.13275, 3a'/16b = 0.135469 \text{ and } 3a'/16b = 0.029 \text{ and } a'/16b = 0.045163. \)
Proof of Proposition 6:
Note that
\[
\frac{d\pi}{dc} = \frac{1}{6} \left( 1 - \frac{2(c_h + c_l)}{\sqrt{c_h - c_l}} \right) < 0
\]
\[
\implies (c_h - c_l)(c_h + 3c_l) > 4(c_h + c_l)^2 < 0 \implies -6c_h c_l - 3c_l^2 - 7c_l^2 < 0.
\]
It is straightforward to see that the last inequality always holds. Finally, \(\frac{d\pi}{dc} = \frac{1}{6} \left( \frac{6c_h - 2c_l}{\sqrt{(c_h - c_l)^2}} - 2 \right) < 0\) if \(c_h \leq 3c_l\), this equality always hold. Thus, suppose that \(c_h > 3c_l\), then the above inequality can be rewritten as \(4c_l (3c_l - 2c_h) < 0 \implies c_h < c_l/2\). Hence, it is proved.

Proof of Proposition 7:
We first show that \(\frac{dE[\Pi_{c_h}]_{dc}}{dc} > 0\). To see this, note that \(\frac{dE[\Pi_{c_h}]_{dc}}{dc} = -(1 - \mu_0)\pi'(c_h)\left(\frac{a + c_h - 2\pi(c_h)}{2b}\right)\), where \(\pi'\) is the derivative of \(\pi\) (with respect to \(c_h\) in this case). We know from Proposition 6 that \(\pi'(c_h) < 0\). Therefore, \(\frac{dE[\Pi_{c_h}]_{dc}}{dc} > 0\).

We now prove the result for \(c_l\).
\[
\frac{dE[\Pi_{c_l}]_{dc}}{dc} = \mu_0 \left(1 - \mu_0\right)\pi'(c_l)\left(\frac{a + c_l - 2\pi(c_l)}{2b}\right),
\]
(19)
The first term of the expression (19) is always positive. From Proposition 6, we know that \(\pi'(c_h) < 0\) if and only if \(c_h > c_l/2\). Therefore, when \(c_l \leq 2c_h/3\), the second term of the expression (19) is non-negative. Hence, when \(c_l \leq 2c_h/3\), the expression (19) is always positive. On the other hand, when \(c_l > 2c_h/3\), there must exist a prior probability \(\mu_0\) such that the second term dominates the first term, where \(\mu_0\) is defined by \(\frac{dE[\Pi_{c_h}]_{dc}}{dc} = 0\). Hence, the result follows.

Proof of Proposition 8:
The outline of the proof is as follows. We first check which manufacturer has a mimicking incentive. Then for the regions there is a mimicking incentive we find the equilibrium of the signaling game. We drop superscripts and subscripts whenever it is clear from the context.

Using a similar approach to the proof of Lemma 1, we can write profit functions of manufacturer as function of the retailer’s beliefs and wholesale prices as follows:
\[
\Pi_m(H, 1, w) = \begin{cases} \frac{(a + c_h)^3}{2b} & \text{if } a + c_h < w \\ \frac{a}{2b} (a + c_h - 2c_l)^2 - \frac{a}{2b} c_l^2 \left(\frac{a + c_h - 2c_l}{2b}\right)^2 & \text{if } a + c_h \geq w > \frac{1}{2}(a + 2c_h + c_l) \\ \frac{a}{2b} (a + c_h - 2c_l)^2 - \frac{a}{2b} c_l^2 \left(\frac{a + c_h - 2c_l}{2b}\right)^2 & \text{if } a + c_h \geq w > \frac{1}{2}(a + c_l) \end{cases}
\]
(20)
\[
\Pi_m(L, 0, w) = \begin{cases} \frac{(a + c_l)^3}{2b} & \text{if } a + c_l \leq w \\ \frac{a}{2b} (a + c_l - 2c_h)^2 - \frac{a}{2b} c_h^2 \left(\frac{a + c_l - 2c_h}{2b}\right)^2 & \text{if } a + c_l \geq w > \frac{1}{2}(a + 3c_h) \\ \frac{a}{2b} (a + c_l - 2c_h)^2 - \frac{a}{2b} c_h^2 \left(\frac{a + c_l - 2c_h}{2b}\right)^2 & \text{if } a + c_l \geq w > \frac{1}{2}(a + c_h) \end{cases}
\]
(21)

Also, \(\Pi_m(H, 0, w)\) and \(\Pi_m(L, 1, w)\) are as given in equation (12). First assume \(a < 2c_l\) and \(\frac{a}{2b} < 2c_l\) (The other cases can be shown similarly).

We solve \(\Pi_m(H, 0, w_i^{(ij)}) < \Pi_m(H, 1, w_i^{(ij)})\) and \(\Pi_m(L, 1, w_i^{(ij)}) < \Pi_m(L, 0, w_i^{(ij)})\) for regions \(a \leq \frac{8c_l}{3}, \frac{4c_l}{3} < a \leq \frac{5c_l}{3}, \frac{4c_l}{3} < a \leq \frac{8c_l}{3}, \frac{4c_l}{3} < a \leq \frac{8c_l}{3}, \frac{4c_l}{3} < a < 2c_l\). We only look at these regions because \(a > 4c_l - 3c_l\). Without this assumption there would be many cases to analyze and the analyses would not be tractable. Using these inequalities, we can show the following

i) When \(a \leq \frac{2c_l}{3}\), they do not mimic each other.

ii) type-l mimic type-h if and only if

\[
\left( \frac{6c_h}{5} < a \leq \frac{6c_h}{5} \right) \cup \left( \frac{c_h}{13} < \frac{15c_l}{13} \cap max \left( \frac{6c_h}{5}, 4c_l - 3c_l \right) < a < \frac{3}{4} \left( c_h + c_l \right) \right)
\]
(22)

iii) type-h mimic type-l if and only if

\[
\left( \frac{c_h}{6} < \frac{7c_l}{6} \cap max \left( 4c_l - 3c_l, \frac{1}{6} \left( 5c_l + 7c_l \right) \right) < a \leq \frac{5c_l}{3} \right) \cup \left( \frac{6c_h}{5} < a < 2c_l \right)
\]
(23)

We, however, need to divide these regions into smaller regions and analyze these case by case because the equilibrium decisions may be different from one case to another within the regions defined above. This is apparent from equations (12), (21) and (20). In particular, we need to analyze the intersection of (22) and (23) with \(a \leq \frac{2c_l}{3}, \frac{2c_l}{3} < a \leq \frac{4c_l}{3}, \frac{2c_l}{3} < a \leq \frac{4c_l}{3}, \frac{2c_l}{3} < a < 2c_l\) one by one.
We now analyze $5c_t < a \leq \frac{5c_t}{2}$. This is one of the most involved cases and the other cases follow a similar reasoning. In this case, we know that type-$t\ell$ mimics type-$h$. Similar to the proof of Lemma 3, we need to find the wholesale prices that satisfy $IC_1$ and $IC_2$, constraint, and also the intuitive criterion. What makes the analyses difficult is that these wholesale prices may be in different regions of the profit functions. From the $IC_2$ constraint,

$$\Pi_t(L, 0, w) \leq \Pi_t(L, 1, w_{t\ell}^\ast),$$

where $w_{t\ell}^\ast = a/2 - c_t/6$ in this region. To solve the inequality, we first need to characterize how $\Pi_t(L, 0, w)$ behave. From (21), there are 4 thresholds that this function changes characteristics. Figure 8 demonstrates these 4 thresholds. Define $D_1 = \lim_{a \rightarrow 0} \Pi_t(L, 0, 1)$, $D_2 = \lim_{a \rightarrow 1} \Pi_t(L, 0, 1)$, $D_3 = \lim_{a \rightarrow \infty} \Pi_t(L, 0, 1)$, and $D_4 = \lim_{a \rightarrow -\infty} \Pi_t(L, 0, 1)$.

![Figure 8](image)

**Figure 8** Type-$t\ell$'s profit when she mimics type-$h$, i.e., $\Pi_M(L, 0, w)$, and $5c_t < a \leq \frac{5c_t}{2}$, $a = 1$, $b = 1$, $c_b = 0.375$, $c_t = 0.3$.

By checking the derivatives of the profit at these thresholds from left and right, we can show that there exist wholesale prices such that $w_{t\ell}^\ast$ behaves in our parameter region. According to equation (20), this is a piecewise function changing characteristics in 4 different thresholds. Figure 9 demonstrates these thresholds. By checking the derivatives of the profit at these thresholds from left and right, we can show that there exist wholesale prices such that $w_{t\ell}^\ast$ behaves in our parameter region. According to equation (20), this is a piecewise function changing characteristics in 4 different thresholds. Figure 9 demonstrates these thresholds.

![Figure 9](image)

**Figure 9** Type-$h$'s profit when she mimics type-$t\ell$, i.e., $\Pi_M(H, 1, w)$, and $5c_t < a \leq \frac{5c_t}{2}$, $a = 1$, $b = 1$, $c_b = 0.375$, $c_t = 0.3$.
that it has a unique maximizer and it is either \( w^*_h(L) \in (\frac{3c_h}{5}, \frac{3c_h}{5} + 2) \) or \( w^*_h(L) \in (\frac{3c_h}{5} + 2, \frac{3c_h}{5} + 3) \). \( w^*_h(L) \) is the optimal if and only if
\[
\frac{5c_h}{6} < c_l < \min \left( \frac{4c_h - 3c_l}{3}, \frac{5c_l}{3} \right) < a < \frac{1}{3} (4c_h + c_l) \Rightarrow A_{h1},
\]
and \( w^*_h(L) \) is the optimal if and only if
\[
\frac{5c_h}{6} < c_l < \frac{1}{3} (4c_h + c_l) < a \leq \frac{5c_h}{3} \Rightarrow A_{h2}.
\]
Note that these two regions are disjoint.

We now check whether \( \Pi_u(H, 0, w^*_h) > \Pi_u(H, 1, w^*_h) \) in \( A_{h1} \), and \( \Pi_u(H, 0, w^*_h) > \Pi_u(H, 1, w^*_h) \) in \( A_{h2} \). Otherwise, the equilibrium will not exist because type-\( h \) will have an incentive to deviate. We can show that the second inequality always holds. Hence, in \( A_{h2} \), equilibrium always exists. We can also show that the first inequality holds if and only if
\[
c_h < 6c_l < \frac{2\sqrt{-2c_h c_l + c_h^2 + c_l^2}}{\sqrt{3}} + \frac{5c_l}{3} < a < \frac{1}{3} (4c_h + c_l) \Rightarrow A_{h1}.
\]
Note that \( A_{h1} \subset A_{h2} \).

Finally, when \( A_{h1} \cup A_{h1} \), the equilibrium wholesale price is \( w^*_h = w^*_h \), where \( w^*_h \in (2c_h - a, \frac{3c_h}{5} + 2) \). Therefore, \( w^*_h = w^*_h, q^*_h = \frac{4c_h}{5} \), and \( q^*_h = 21c_h \). All other cases, i.e., \( \frac{3c_h}{5} < a < \frac{2c_h}{5}, \frac{c_h}{5} < a \leq \frac{3c_h}{5} \), and the cases for \( a \geq 2c_h \) and \( 5c_h/b > 2c_h \) can be shown similarly (We omit these cases for brevity). Then, we can obtain the following regions:
\[
s^*_n = \max \left( 4c_h - 3c_l, \frac{6c_l}{5} \right) < a < a_1,
\]
\[
s^*_n = \left( \frac{7c_h}{9} < c_l < \frac{4c_h}{5} \cap \frac{c_h - 3c_l}{3} < a < \frac{5c_h}{3} \right) \cup \left( \frac{4c_h}{5} < c_l < \frac{33c_h}{3 \sqrt{3} + 39} \cap \left( \frac{4c_h - 3c_l}{3} < a < \frac{1}{3} (4c_h + c_l) \cup \frac{1}{3} (4c_h + c_l) < a < \frac{5c_h}{3} \right) \right) \cup \left( \frac{33c_h}{3 \sqrt{3} + 39} < c_l < \frac{5c_h}{6} \cap \left( \frac{1}{3} \left( \frac{2 \sqrt{3}c_h - 2 \sqrt{5}c_h + 5c_l}{3} \right) < a < \frac{1}{3} (4c_h + c_l) \cup \frac{1}{3} (4c_h + c_l) < a < \frac{5c_h}{3} \right) \right) \cup \left( \frac{5c_h}{6} < c_l < \frac{1}{3} \left( \frac{2 \sqrt{3}c_h - 2 \sqrt{5}c_h + 5c_l}{3} \right) < a < \frac{1}{3} (4c_h + c_l) \cup \frac{1}{3} (4c_h + c_l) < a < \frac{5c_h}{3} \right) \right),
\]
\[
s^*_n = \left( \frac{21c_h}{2 \sqrt{2} + 27} \cap \frac{3c_h - 3c_l}{3} < a < \frac{1}{3} (4c_h + c_l) \right) \cup \left( \frac{7c_h}{9} < c_l < \frac{5c_h}{6} \cap \frac{5c_h}{3} \leq a < \frac{1}{3} (2c_h c_l + c_h^2 - 3c_l^2) + \frac{1}{3} (4c_h + c_l) \right) \cup \left( \frac{8c_h}{6} < c_l < \frac{5c_h}{3} \leq a < \frac{1}{3} (2c_h c_l + c_h^2 - 3c_l^2) + \frac{1}{3} (4c_h + c_l) \right).
\]
where \( a_1 \) is the unique \( a \) satisfying \( a \in \left( \frac{c_h}{5}, \frac{2c_h}{5} \right) \) and \( \frac{1}{4} \left( 2 \sqrt{9a^2 + 6c_h(c_h - a) + 3c_h(5c_h - 8a) + 4c_h^2 + 3a - c_h} \right) = w^*_h. \)

**Proof of Lemma 4**
The equilibrium can be found easily by backward induction. Hence, the proof is omitted.

**Proof of Lemma 5**
Given contract \((\hat{T}, \hat{\nu})\) and the posterior belief \( \mu \), the equilibrium quantity decisions are same as in Equations (2) and (3) because the fixed fee \( \hat{T} \) does not alter these decisions (i.e., \( \hat{q}_u(\hat{q}_u) = (a - c_1 - bq_u)^2/2b \) and \( \hat{q}_u(\hat{\nu}, \mu) = (a + c_1 - 2\hat{\nu})/2b \)). The manufacturer type-\( i \)'s payoff is
\[
\hat{\Pi}_u(i, \mu, (\hat{T}, \hat{\nu})) = \frac{(a - 2c_i - \hat{c}_i + 2\hat{\nu})^2}{16b} + \frac{\hat{\nu} \cdot (a + c_1 - 2\hat{\nu})}{2b} + \hat{T}_i.
\]
From Equation (24) and Lemma 4, note that
\[
\hat{\Pi}_u(1, (\hat{T}^n_1, \hat{\nu}^n_1)) = \frac{a^2 - 2ac_i + 5\hat{c}_i^2}{4b}, \quad \hat{\Pi}_u(0, (\hat{T}^n_0, \hat{\nu}^n_0)) = \frac{(a - c_1)^2 + 4ac_i c_l}{4b}
\]
\[
\hat{\Pi}_u(2, (\hat{T}^n_1, \hat{\nu}^n_1)) = \frac{a^2 - 2ac_i + 5\hat{c}_i^2}{4b}, \quad \hat{\Pi}_u(1, (\hat{T}^n_1, \hat{\nu}^n_1)) = \frac{(a - c_1)^2 + 4ac_i c_l}{4b}.
\]
Comparison of the above profit functions yields the result.
Proof of Proposition 9

Note that by using Equations (2) and (3), we can write the retailer’s profit given the posterior belief \( \mu \) and \((\bar{T}_e, \bar{\nu}_e)\) and that the manufacturer is type-\( i \) as

\[
\Pi_i(\mu, \bar{T}_e, \bar{\nu}_e) = \frac{(a + \epsilon_{i} - 2\bar{\nu}_e)(a + 2\epsilon_i - c_{i} - 2\bar{\nu}_e)}{8b} - \bar{T}_e. \tag{25}
\]

Using the profit functions given in Equations 24 and (25), we first identify the set of \((\bar{T}_e, \bar{\nu}_e)\)'s where the manufacturer type-\( \ell \) does not mimic type-\( h \). Noting that if type-\( \ell \) cannot mimic type-\( h \) her best strategy is to play the symmetric equilibrium decisions.

\[
\Pi_{\bar{\mu}}(\ell, 1, (\bar{T}_e, \bar{\nu}_e)) \geq \Pi_{\bar{\mu}}(\ell, 0, (\bar{T}_e, \bar{\nu}_e)) \iff \frac{a^2 - 2ac_{\ell} + 5c_{\ell}^2}{4b} \geq \bar{T}_e + \frac{4\bar{\nu}_e(3a + c_{\ell} - 2c_{\ell}) + (-a + c_{\ell} + 2c_{\ell})^2 + 2(a + c_{\ell} - 2\bar{\nu}_e)^2 - 12\bar{\nu}_e^2}{16b} \iff \zeta_i - \zeta_{\ell}(\bar{\nu}_e) \geq \bar{T}_h. \tag{26}
\]

Therefore, from Eq. (25) and Inequality (26), we must have

\[
\min \left\{ \zeta_i - \zeta_{\ell}(\bar{\nu}_e), \frac{(a + c_{\ell} - 2\bar{\nu}_e)^2}{8b} \right\} \geq \bar{T}_h. \tag{27}
\]

In addition to the above, manufacturer type-\( h \) must not have an incentive to deviate from the equilibrium given the off-the-equilibrium belief \( \mu = 1 \). Therefore,

\[
\Pi_{\bar{\mu}}(h, 0, (\bar{T}_e, \bar{\nu}_e)) \geq \max_{\bar{\mu}_h} \Pi_{\bar{\mu}}(h, 1, (\bar{T}_e, \bar{\nu}_e)) \text{ where } \bar{T} = \frac{(a + c_{\ell} - 2\bar{\nu}_e)^2}{8b}
\]

The optimal values in the right side of the inequality are \( \bar{T} = (a + c_{\ell} - 2\bar{\nu}_e)^2/8b \) and \( \bar{\nu}_e = (a - c_{\ell} + 2c_{\ell})/2 \). Therefore, we have

\[
\frac{4\bar{\nu}_e(3a + c_{\ell} - 2c_{\ell}) + (-a + c_{\ell} + 2c_{\ell})^2 + 2(a + c_{\ell} - 2\bar{\nu}_e)^2 - 12\bar{\nu}_e^2}{16b} \geq \frac{a^2 - 2ac_{\ell} + 5c_{\ell}^2}{4b} \iff \zeta_i - \zeta_i(\bar{\nu}_e) \leq \bar{T}_h. \tag{28}
\]

From inequalities (26) and (28), we have

\[
\zeta_i - \zeta_i(\bar{\nu}_e) \leq \bar{T}_h \leq \min \left\{ \zeta_i - \zeta_i(\bar{\nu}_e), \frac{(a + c_{\ell} - 2\bar{\nu}_e)^2}{8b} \right\} \tag{29}
\]

These inequalities define the candidate \((\bar{T}_e, \bar{\nu}_e)\)'s where neither type-\( \ell \) nor type-\( h \) has an incentive to deviate. Note that in order the equilibrium to survive intuitive criterion, \((\bar{T}_e, \bar{\nu}_e)\) must to maximize the type-\( h \) manufacturer’s profit in this region. Therefore,

\[
\begin{align*}
\text{max} & \quad \Pi_{\bar{\mu}}(h, 0, (\bar{T}_e, \bar{\nu}_e)) \\
\text{s.t.} & \quad \zeta_i - \zeta_i(\bar{\nu}_e) \leq \bar{T}_h \leq \min \left\{ \zeta_i - \zeta_i(\bar{\nu}_e), \frac{(a + c_{\ell} - 2\bar{\nu}_e)^2}{8b} \right\}
\end{align*}
\]

We can show that \( \zeta_i - \zeta_i(\bar{\nu}_e) < \frac{(\bar{\nu}_e - 2w_{\mu})^2}{8b} \) if and only if

\[
\begin{align*}
\Phi & \leq 2\sqrt{(c_{\ell} - c_i)(c_{\ell} + 3c_{\ell}) + c_{\ell} + 2c_{\ell} \equiv \bar{\alpha} \text{ AND } \bar{\nu}_e < \frac{1}{2}(a - c_{\ell} - 2c_{\ell})} + \sqrt{(c_{\ell} - c_i)(c_{\ell} + 3c_{\ell}) < \bar{\nu}_e} \\
\Phi & > 2\sqrt{(c_{\ell} - c_i)(c_{\ell} + 3c_{\ell}) + c_{\ell} + 2c_{\ell} \equiv \bar{\alpha} \text{ AND } \bar{\nu}_e \equiv \frac{1}{2}(a - c_{\ell} - 2c_{\ell})} + \sqrt{(c_{\ell} - c_i)(c_{\ell} + 3c_{\ell}) < \bar{\nu}_e} < \bar{\nu}_e < \bar{\nu}_e/2
\end{align*}
\]

We first ignore the lower bound in the constraint of the optimization problem \( P \). After finding the optimal solution, we confirm that the optimal solution satisfies this lower bound. Therefore, \( \bar{T}_h = \min \{ \zeta_i - \zeta_i(\bar{\nu}_e), \frac{(\bar{\nu}_e - 2w_{\mu})^2}{8b} \} \).

Case 1. Suppose that \( \Phi \leq \bar{\alpha} \). In this case, if \( \bar{\nu}_e < \bar{\nu}_{h_{\ell}} \), then \( \bar{T}_h = \zeta_i - \zeta_i(\bar{\nu}_e) \). Otherwise, if \( \bar{\nu}_e \geq \bar{\nu}_{h_{\ell}} \), \( \bar{T}_h = \frac{(\bar{\nu}_e - 2w_{\mu})^2}{8b} \). Therefore, type-\( h \) manufacturer’s profit is a piecewise function that changes characteristics at \( \bar{\nu}_e = \bar{\nu}_{h_{\ell}} \). In addition,

\[
\lim_{\nu_e \to \nu_{h_{\ell}}^+} \frac{\partial \Pi_{\bar{\mu}}(h, 0, (\bar{T}_e, \bar{\nu}_e))}{\partial \nu_e} = \frac{c_{\ell} - c_i}{2b} < 0, \quad \lim_{\nu_e \to \nu_{h_{\ell}}^-} \frac{\partial \Pi_{\bar{\mu}}(h, 0, (\bar{T}_e, \bar{\nu}_e))}{\partial \nu_e} = \frac{a - 3c_{\ell} - 2\nu_{h_{\ell}}}{4b} < 0.
\]

Thus, \( \nu_{h_{\ell}} = 0 \) and \( \bar{T}_h = \zeta_i - \zeta_i(0) \).
Case 2. Suppose that \( a > \bar{a} \). In this case, if \( w_1 < \bar{w}_1 < w_2 \), then \( \tilde{T}_\ell = \zeta_1 - \zeta_1(\bar{w}_1) \). Otherwise, \( \tilde{T}_\ell = \frac{\omega_{a-2\bar{a}}a^2}{w_2} \). Therefore, type-\( h \) manufacturer’s profit is a piecewise function that changes characteristics at \( \bar{w}_1 = w_1 \) and \( \bar{w}_2 = w_2 \). Similar to the previous case, by checking the derivatives of the profit function at \( \bar{w}_1 \) and \( \bar{w}_2 \) we can show that \( \bar{w}_1^+ = w_1 \) and \( \bar{w}_2^+ = w_2 \).

Finally, we need to show that the lower bound is always satisfied in these cases. To see this, note that the lower bound is equal to \( \frac{3a^2 - 2a c_a + 8c_a c_e - c_e^2 + 4c_e^2}{16b} \). Therefore, for Case 1, we need to show that the lower bound evaluated at \( \bar{w}_1 = 0 \) is smaller than \( \zeta_1 - \zeta_1(0) \). In particular,

\[
\frac{3a^2 - 2a c_a + 8c_a c_e - c_e^2 + 4c_e^2}{16b} < \frac{-3a^2 - 2a c_a + 4a c_e + 4c_a c_e + c_e^2 - 16c_e^2}{4b} \iff b(a - 3c_e)(c_e - c_a) < 0.
\]

The last equality holds because \( a > 3c_e \) due to our assumption. For Case 2, we need to show that the lower bound evaluated at \( \bar{w}_2 = w_2 \) is smaller than \( \frac{\omega_{a-3\bar{a}}a^2}{w_2} \). In particular,

\[
\frac{10c_a c_e + 2c_a \sqrt{c_a - c_e} (c_a + 3c_e) + 6c_e \sqrt{c_a - c_e} (c_a + 3c_e) + 3c_e^2 - 5c_e^2}{4b} < \frac{\sqrt{(c_a - c_e) (c_a + 3c_e) + c_e}}{2b} \iff b(c_e - c_a) < 0.
\]

It is straightforward to see that the last equality always holds. The comparison of profits and the decisions for symmetric information and the asymmetric information cases can be easily done. Hence, the proof is omitted.

Finally, we show the consumers are better off under asymmetric information. In symmetric information game, consumer surplus can be found as

\[
E[\Pi_c^S] = \mu b \int_0^{\alpha \mu c_a} \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) dq + (1 - \mu)b \int_0^{\alpha \mu c_a} \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) dq
\]

\[
= \mu b \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) + (1 - \mu)b \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) = \mu \left( a + c_a \right)^2 + (1 - \mu) \left( a + c_a \right)^2
\]

For the asymmetric information case, we have

\[
E[\Pi_c^A] = \mu b \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) + (1 - \mu)b \left( \frac{q_{\ell}^2 + q_{h}^2}{2} \right) = \begin{cases} 
\mu \left( a + c_a \right)^2 + (1 - \mu) \left( a + c_a \right)^2 & \text{if } a \leq \bar{a}, \\
\mu \left( a + c_a \right)^2 + (1 - \mu) \left( \frac{b c_a - 2 c_e^2}{3b} \right)^2 & \text{if } a > \bar{a}.
\end{cases}
\]

It follows immediately that \( E[\Pi_c^A] > E[\Pi_c^S] \).

Proof of Lemma 6:
The proof is similar to Lemma 1. Hence it is omitted.

Proof of Lemma 7:
At the final stage, given price \( w \) and price \( P_e \), the manufacturer solves the problem

\[
\max_{P_{\ell0} \geq 0} \left( \begin{array}{c}
w \cdot q_\ell(P_e, P_{\ell0}) + (P_{\ell0} - c) \cdot q_\ell(P_e, P_{\ell0}), \\
\text{retailer channel} \\
\end{array} \right) \text{subject to} \begin{array}{c}
\text{direct channel} \\
\end{array}
\]

Thus

\[
P_{\ell0}(w, P_e) = \frac{\alpha + c_e + \beta(P_e + w)}{2}.
\]

Anticipating this response, and given price \( w \) and posterior belief \( \mu \), the retailer finds her retail price by solving

\[
\max_{P_e \geq 0} \mu (P_e - w) \cdot q_e(P_e, P_{\ell0}(w, P_e)) + (1 - \mu)(P_e - w) \cdot q_e(P_e, P_{\ell0}(w, P_e)).
\]

Thus

\[
P_e(w, \mu) = \frac{\alpha (2 - \beta - \beta^2) + \beta c_e + 2w}{2 (2 - \beta^2)}
\]

In the first stage, the manufacturer sets the wholesale price \( w \) in anticipation of retail prices \( P_{\ell0} \) and \( P_e \). Similar to the quantity game we focus on the region where \( q_{\ell0} > 0 \). To ensure this, we assume \( \alpha > \alpha_0 \), where \( \alpha_0 = \frac{1}{\beta^2 + \beta (\beta^2 - 2c_e^2)} \).
To ensure that \( q_m > 0 \), we assume \( \alpha > \alpha_0 \). Using equations (31)-(33), we can find

\[
\Pi_{\ell}(h, 0, w_p^0) = \frac{\alpha^2(\beta - 1)\beta(\beta(2\beta - 1) - 9) + 4) + 12 - 2\alpha(\beta - 1)(\beta(\beta(\beta - 1) - 5) + 4) + 8)\beta + (-2\beta^3 + 9\beta^2 - 8) c_I^2}{4(\beta^2 - 1)(\beta^4 - 5\beta^2 + 8)}.
\]

\[
\Pi_{\ell}(h, 1, w_p^1) = \frac{1}{4(\beta^6 - 6\beta^4 + 13\beta^2 - 8)} (-2\alpha(\beta - 1) \left( (\beta(\beta^2 - 5\beta^2 + 2) + 8) c_I - 2(2\beta^2 - 2) c_I + \beta^3 (-c_I^2 + c_I^2) \right) +
\beta^2 c_I (5c_I + 4c_I) - 8c_I^2) + \alpha^2(\beta - 1)(\beta(\beta(\beta - 1) - 9) + 4) + 12),
\]

\[
\Pi_{\ell}(\ell, 1, w_p^1) = \frac{1}{4(\beta^6 - 6\beta^4 + 13\beta^2 - 8)} (-2\alpha(\beta - 1) \left( (\beta(\beta^2 - 5\beta^2 + 2) + 8) c_I - 2(2\beta^2 - 2) c_I + \beta^3 (-c_I^2 + c_I^2) \right) +
\beta^2 c_I (4c_I + 5c_I) - 8c_I^2 + \alpha^2(\beta - 1)(\beta(\beta(\beta - 1) - 9) + 4) + 12).
\]

Comparison of these profit functions yield the result.

**Proof of Proposition 10:**

It is easy to see that whenever it is not profitable for manufacturer type-\( \ell \) to mimic manufacturer type-\( \ell \), her optimal action would be to play the symmetric equilibrium wholesale price, which proves part (i).

For part (ii), define \( \Delta_{\pi}(w) \equiv \Pi_{\ell}(h, 1, w) - \Pi_{\ell}(h, 0, w_p^0) \).

\( \Delta_{\pi} \) is a quadratic concave function of \( w \) and it has two roots \( \pi_{\alpha} \) and \( \pi_{\beta} \), where \( \pi_{\alpha} < \pi_{\beta} \). Thus \( \pi_{\alpha} \) is satisfied if and only if \( w \leq \pi_{\alpha} \) and \( w \geq \pi_{\beta} \). Similar to proof of Lemma 3, we can show that \( \pi_{\alpha} \) is satisfied if and only if \( w \leq w_{\ell}^* \), where the upper and lower bounds are defined by \( \Pi_{\ell}(h, 1, w) - \max, \Pi_{\ell}(\ell, 0, w) = 0 \). In addition, \( w_{\ell}^* < \pi_{\alpha} \). Therefore, a separating equilibrium exists if and only if \( w_{\ell}^* < \pi_{\alpha} \). This holds when \( \beta < \beta_0 \approx 0.564 \).

Note that any \( w \in \left[ w_{\ell}^*, \pi_{\beta} \right] \) can be a separating equilibrium; however, only \( \pi_{\beta} \) satisfies the intuitive criterion because \( w_{\ell}^* < \pi_{\alpha} < w_p^0 \) and type-\( \ell \) manufacturer's profit function is concave in \( w \). This proves part (iii).

By comparing the profits of the supply chain and the retailer under symmetric and asymmetric information we can readily show that asymmetric information improves the both. As for consumer surplus, we use the definition in Pressman (1970). Accordingly, consumer surplus is given by

\[
E \int_0^{q_m} P_u(q_m, 0)dq - P_{\ell} (q_m) + \int_0^{Q_u} P_s(q_m, q)dq - P_{\ell} (q_m).
\]

Note that the consumer surplus under symmetric and asymmetric information is same when the manufacturer is type-\( h \). Thus we only need to compare the consumer surpluses when the manufacturer is type-\( \ell \). Then \( \Pi_{\ell} = \frac{4(\beta^6 - 6\beta^4 + 13\beta^2 - 8)}{4(\beta^6 - 6\beta^4 + 13\beta^2 - 8)}(-2\alpha(\beta - 1) \left( (\beta(\beta^2 - 5\beta^2 + 2) + 8) c_I - 2(2\beta^2 - 2) c_I + \beta^3 (-c_I^2 + c_I^2) \right) +
\beta^2 c_I (5c_I + 4c_I) - 8c_I^2) + \alpha^2(\beta - 1)(\beta(\beta(\beta - 1) - 9) + 4) + 12),
\]

Additionally, \( \Pi_{\ell} = \frac{1}{4(\beta^6 - 6\beta^4 + 13\beta^2 - 8)} (-2\alpha(\beta - 1) \left( (\beta(\beta^2 - 5\beta^2 + 2) + 8) c_I - 2(2\beta^2 - 2) c_I + \beta^3 (-c_I^2 + c_I^2) \right) +
\beta^2 c_I (4c_I + 5c_I) - 8c_I^2 + \alpha^2(\beta - 1)(\beta(\beta(\beta - 1) - 9) + 4) + 12),
\]

Finally, comparison of \( \Pi_{\ell} \) and \( \Pi_{\ell} \) yields the result.
Online Appendix

Simultaneous Quantity Competition

In this section, we consider a model where the manufacturer and retailer simultaneously decide on their sales quantities \( \hat{q}_M \) and \( \hat{q}_R \) after the manufacturer’s wholesale price decision \( \hat{w} \). We first present the benchmark case in which all information is common knowledge.

**Proposition 11.** In the symmetric information game with encroachment (i.e., \( \hat{q}_{M_i}^{SI} > 0 \)) the equilibrium decisions and the profits are

\[
\hat{w}_i^{SI} = \frac{a_i - c_i}{10}, \quad \hat{q}_{M_i}^{SI} = \frac{5a_i - 7c_i}{10b}, \quad \hat{q}_{R_i}^{SI} = \frac{2c_i}{5b}, \quad \hat{\Pi}_{M_i}^{SI} = \frac{5a^2 - 10ac_i + 9c_i^2}{20b}, \quad \hat{\Pi}_{R_i}^{SI} = \frac{4c_i^2}{25b}.
\]

We now consider the case with asymmetric information in which the manufacturer’s direct selling cost is private information. At the final stage, the manufacturer and retailer solve the following problems simultaneously:

\[
\max_{\hat{q}_M \geq 0} \hat{w} \cdot \hat{q}_R + \left( [a - b(\hat{q}_R + \hat{q}_{M_i})] - c_i \right) \cdot \hat{q}_M, \quad i \in \{\ell, h\}, \quad (34)
\]

\[
\max_{\hat{q}_R \geq 0} \mu \left( [a - b(\hat{q}_R + \hat{q}_{M_i})] - \hat{w} \right) \cdot \hat{q}_R + (1 - \mu) \left( [a - b(\hat{q}_R + \hat{q}_{M_i})] - \hat{w} \right) \cdot \hat{q}_R. \quad (35)
\]

These problems lead to the following quantity decisions given the wholesale price \( \hat{w} \)

\[
\hat{q}_{M_i}(\hat{w}, \mu) = \frac{2(a + \hat{w}) - c'_i - 3c_i}{6b}, \quad (36)
\]

\[
\hat{q}_{R_i}(\hat{w}, \mu) = \frac{a + c'_i - 2\hat{w}}{3b}, \quad (37)
\]

where \( c'_i = \mu c_\ell + (1 - \mu)c_h \). Using Equations (36), and (37), we can write the manufacturer type-\( i \)'s equilibrium payoff, given wholesale price \( \hat{w} \) and the retailer’s posterior belief \( \mu \), as

\[
\hat{\Pi}_M(i, \mu, \hat{w}) = \frac{(2a - 3c_i - c'_i + 2\hat{w})^2}{36b} + \frac{\hat{w}(a + c'_i - 2\hat{w})}{3b}. \quad (38)
\]

The next lemma shows that only manufacturer type-\( \ell \) has an incentive to mimic type-\( h \), similar to the case when the manufacturer and the retailer decide their outputs sequentially (see Section 4).

**Lemma 14.** \( \hat{\Pi}_M(\ell, 0, \hat{w}_h^{SI}) > \hat{\Pi}_M(\ell, 1, \hat{w}_h^{SI}) \) and \( \hat{\Pi}_M(h, 1, \hat{w}_\ell^{SI}) < \hat{\Pi}_M(h, 0, \hat{w}_h^{SI}) \). Therefore, only manufacturer type-\( \ell \) has an incentive to mimic the other type.

The lemma shows that only manufacturer type-\( \ell \) mimics the other type to increase her profit. The intuition is the same as in the sequential move game. If manufacturer type-\( \ell \) can convince the retailer that she is the manufacturer with a high direct selling cost, the retailer would order a higher quantity, i.e., higher \( \hat{q}_M \). This would increase manufacturer type-\( \ell \)'s profit.

The next proposition derives the separating equilibrium that survives the intuitive criterion for the simultaneous move game.
Table 5 Equilibrium actions and profits of simultaneous move game

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Type-ℓ</th>
<th>Type-h</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{w}^*)</td>
<td>(\hat{w}_{\ell}^{SI})</td>
<td>(\hat{w}_{h})</td>
</tr>
<tr>
<td>(\hat{q}_R^*)</td>
<td>(\frac{2c}{3b})</td>
<td>(\frac{a+2c-h-2\pi}{3b})</td>
</tr>
<tr>
<td>(\hat{q}_M^*)</td>
<td>(\frac{5a-7c_{\ell}}{10b})</td>
<td>(\frac{5a-2c_{\ell}+h}{3b})</td>
</tr>
<tr>
<td>(\hat{\Pi}_R^*)</td>
<td>(\frac{4c_{\ell}^2}{5b})</td>
<td>(\frac{(a-c_{\ell}+2\pi)^2}{9b})</td>
</tr>
<tr>
<td>(\hat{\Pi}_M^*)</td>
<td>(\frac{5a^2-10ac_{\ell}+9c_{\ell}^2}{20b})</td>
<td>(9b + \frac{\pi(2c_{\ell}+2\pi)}{3b})</td>
</tr>
</tbody>
</table>

Table 5 shows the equilibrium decisions and profits. Manufacturer type-\(h\) reduces her wholesale price from \(\hat{w}_h^{SI}\) to \(\hat{w}_{h}\) in order to signal her type to the retailer. Similar to the sequential move game, this wholesale price reduction may alleviate the double marginalization problem and increase the supply chain and the retailer expected profits. It may, therefore, be better for the supply chain and the retailer when the retailer is oblivious to the manufacturer’s direct selling cost information, as shown in the following proposition.

**Proposition 12.** Suppose that the manufacturer and the retailer simultaneously decide their selling quantities \(q_M\) and \(q_R\). Then, there exists a unique separating equilibrium satisfying the intuitive criterion where the following properties hold:

(i) Type-\(\ell\) manufacturer sets the wholesale price at the first best level, \(\hat{w}_{\ell}^* = \hat{w}_{\ell}^{SI}\).

(ii) Type-\(h\) manufacturer sets her price to \(\hat{w}_{h}^* = \hat{w}_{h}\) where \(\hat{w}_{h} = \frac{(a+c_{\ell}+2\pi)^2}{9b} + \frac{\pi(2c_{\ell}+2\pi)}{3b}\).

In addition, \(\hat{w}_h < \hat{w}_h^{SI}\).

**Proposition 13.** In the simultaneous move game, both the retailer and the supply chain are better off when the manufacturer’s direct selling cost information is private rather than common knowledge.
Figure 10 depicts the proposition. The proposition demonstrates that the retailer and the supply chain always benefit from information asymmetry, similar to the sequential move game. The next proposition shows that information asymmetry also increases the value of encroachment for the retailer and the supply chain.

**Figure 11** Comparison of retailer and supply chain expected profits in simultaneous move game: $b = 1$, $c_h = 0.61$

**Proposition 14.** There exist parameter regions $\hat{A}_{1R}$ and $\hat{A}_{1SC}$ such that $\hat{A}_{1R} \cap \hat{A}_{1SC} \neq \emptyset$, where in $\hat{A}_{1R}$, $E[\hat{\Pi}_{NR}] < E[\Pi_{NR}^N]$ but $E[\hat{\Pi}_R^R] > E[\Pi_R^N]$, and in $\hat{A}_{1SC}$, $E[\hat{\Pi}_{SR}^S + \hat{\Pi}_{SM}^S] < E[\Pi_R^N + \Pi_M^M]$ but $E[\hat{\Pi}_R + \hat{\Pi}_M^S] > E[\Pi_R^N + \Pi_M^M]$.

Figure 11 demonstrates the proposition. In addition to the regions $\hat{A}_{1R}$ and $\hat{A}_{1SC}$ defined in the proposition, we depict in the figure $\hat{A}_{2R}^3$ and $\hat{A}_{2SC}^3$ ($\hat{A}_{3R}^1$ and $\hat{A}_{3SC}^1$) where the retailer and the supply chain expected profits
increase (decrease) simultaneously with encroachment in both the asymmetric and symmetric information cases. Note that the figure is structurally similar to Figure 4. For both the supply chain and the retailer, information asymmetry may comparably increase the value of encroachment when it originates from the manufacturer’s private selling cost information. In sum, the analysis in this section shows that our main results in the sequential move game continue to hold.

Simultaneous Price Competition

We now consider the simultaneous price game \( \hat{\Gamma}^N_p \): the manufacturer moves first and sets wholesale price \( \hat{w} \); then the manufacturer and the retailer set their retail prices \( \hat{P}_M \) and \( \hat{P}_R \) simultaneously. For benchmarks, we first study game \( \hat{\Gamma}^N_p \) and \( \hat{\Gamma}^{SI}_p \).

**Proposition 15.** (a) In single-channel game \( \hat{\Gamma}^N_p \), the equilibrium outcome is same as the sequential game given in Lemma 6.

(b) In the dual-channel game \( \hat{\Gamma}^{SI}_p \), the equilibrium outcome is

\[
\hat{w}^{SI} = \frac{\alpha \left( \beta^3 + 8 \right)}{2(\beta^2 + 8)}, \quad \hat{p}^{SI}_R = \frac{\alpha \left( 2 - \beta^2 - \beta \right) + \beta \left( c_i + \beta \hat{w}^{SI} \right) + 2\hat{w}^{SI}}{4 - \beta^2},
\]
\[
\hat{p}^{SI}_M = \frac{\alpha \left( 2 - \beta^2 - \beta \right) + \beta \hat{w}^{SI} + 2c_i}{4 - \beta^2}.
\]

We now consider the game \( \hat{\Gamma}^{SI}_p \) under asymmetric information. At the final stage, the manufacturer and retailer solve the following problems simultaneously:

\[
\max_{\hat{P}_M \geq 0} \hat{w} \cdot (\alpha - \hat{P}_R + \beta \hat{P}_M) + (\hat{P}_M - c_i) \cdot (\alpha - \hat{P}_M + \beta \hat{P}_R), \quad i \in \{\ell, h\}, \quad (39)
\]
\[
\max_{\hat{P}_R \geq 0} \mu(\hat{P}_R - w) \cdot (\alpha - \hat{P}_R + \beta \hat{P}_M) + (1 - \mu)(\hat{P}_R - w) \cdot (\alpha - \hat{P}_R + \beta \hat{P}_M). \quad (40)
\]

Thus, from (39) and (40), we have, respectively,

\[
\hat{P}_M(\hat{P}_R) = \frac{1}{2} \left( \alpha(1 - \beta) + c_i + \beta(\hat{P}_R + w) \right), \quad (41)
\]
\[
\hat{P}_R(\hat{P}_M, \mu) = \frac{1}{2} \beta(1 - \mu)P_{Mh} + \frac{1}{2} \beta \mu P_{M\ell} + \frac{1}{2}(\alpha(1 - \beta) + w). \quad (42)
\]

Solving for these best responses we can find the equilibrium price decisions. Anticipating these price decisions, the manufacturer sets the wholesale price to maximize her profit. Similar to the sequential price game, we assume market is large enough such that \( q_{Mi} > 0 \). To ensure this we let \( \alpha > \hat{\alpha}_0 \), where \( \hat{\alpha}_0 \) is defined in the proof of Lemma 15. We find:

**Lemma 15.** In game \( \hat{\Gamma}^{SI}_p \), only manufacturer type-\( h \) has an incentive to mimic the other type.

Next, we characterize the separating equilibrium under the intuitive criterion. Such equilibrium exists if \( \beta \) is sufficiently small. The next proposition characterizes the equilibrium outcome.
PROPOSITION 16. In price game $\hat{\Gamma}_{AI}^P$, the unique separating equilibrium the unique separating PBE $\hat{\sigma}^*$ under intuitive criterion satisfies:

(i) Type-$h$ sets the wholesale price at the first best level, $\hat{w}^*_h = \hat{w}^{SI}_h$.

(ii) Type-$\ell$ sets her price to $\hat{w}^*_\ell = \hat{w}_{pr}$. In addition, $\hat{w}_{pr} < \hat{w}^{SI}_\ell$.

(iii) The consumers, retailer, the supply chain are better off when the manufacturer’s direct selling cost information is private.

In summary, although the price game differs from the quantity game in certain details, our main results remain: the upstream private information can improve channel efficiency and consumer surplus; the main mechanism is still the offsetting interplay of signaling distortion and double marginalization.

Proofs of results in Online Appendix

PROOF OF PROPOSITION 11:

Suppose that the manufacturer is type $h$ and this information is common knowledge. Then, from equations (36) and (37), we have

$$\hat{q}_M(h, 0, \hat{w}) = \frac{a - 2c_h + \hat{w}}{3b}, \quad \hat{q}_R(\hat{w}, 0) = \frac{a + c_h - 2\hat{w}}{3b}. \quad (43)$$

Plugging these into the manufacturer’s and retailer’s profits given in (34) and (35), we obtain

$$\hat{\Pi}_S(h, 0, \hat{w}) = \frac{\hat{w}(5a - c_h) + (a - 2c_h)^2 - 5w^2}{9b}, \quad \hat{\Pi}_R(h, 0, \hat{w}) = \frac{(a + c_h - 2\hat{w})^2}{9b}. \quad (44)$$

From the concavity of $\hat{\Pi}_M$ as a function $\hat{w}$, we can find $\hat{w}^{SI}_i = \frac{a}{2} - \frac{c_i}{10}$. The equilibrium for manufacturer type-$\ell$ can be found similarly. Therefore, the symmetric information equilibrium and the profits are given by

$$\hat{w}^{SI}_i = \frac{a}{2} - \frac{c_i}{10}, \quad \hat{q}^{SI}_M = \frac{5 - 7c_i}{10b}, \quad \hat{q}^{SI}_R = \frac{2c_i}{5b}, \quad \hat{\Pi}^{SI}_S = \frac{5a^2 - 10ac_i + 9c_i^2}{20b}, \quad \hat{\Pi}^{SI}_R = \frac{4c_i^2}{25b}. \quad (45)$$

PROOF OF LEMMA 14

Using Equation (38) and the equilibrium wholesale prices in Proposition 11, we can find the manufacturer profits as

$$\hat{\Pi}_M(\ell, 0, \hat{w}^{SI}_h) = \frac{5a^2 - 10ac_\ell + 4c_hc_\ell + 5c_\ell^2}{20b}, \quad \hat{\Pi}_M(h, 1, \hat{w}^{SI}_\ell) = \frac{(5a - c_h)^2 + 4c_hc_\ell}{20b}. \quad (46)$$

Comparing these profits with $\hat{\Pi}^{SI}_{Mh}$ in Proposition 11, we can easily prove that

$$\hat{\Pi}_M(\ell, 0, \hat{w}^{SI}_h) > \hat{\Pi}_M(\ell, 1, \hat{w}^{SI}_\ell).$$

Note: In price game $\Gamma^{AI}_P$, type-$h$ cuts wholesale price and intensifies price competition; in quantity game $\Gamma^{AI}$, type-$\ell$ cuts price and softens quantity competition.
However,

$$\hat{\Pi}_M(h, 1, \hat{w}^\text{SI}_h) < \hat{\Pi}_M(h, 0, \hat{w}^\text{SI}_h).$$

This proves that type-$\ell$ manufacturer has an incentive to mimic. However, type-$h$ manufacturer does not mimic type-$\ell$ manufacturer. ■

**Proof of Proposition 12:**

This can be shown using the similar steps used for the sequential move game in Lemma 3 and 1 in Section 4. Hence, the proof is omitted. ■

**Proof of Proposition 13:**

From Proposition 11, in the symmetric information game the ex-ante total supply chain profit is

$$E[\hat{\Pi}^\text{SI}_M + \hat{\Pi}^\text{SI}_R] = \mu_0 \frac{25a^2 - 50ac_\ell + 61c^2_h}{100b} + (1 - \mu_0) \frac{25a^2 - 50ac_h + 61c^2_\ell}{100b}. \quad (45)$$

Using Proposition 12 the ex-ante total supply chain profit in the separating equilibrium of asymmetric information game can be found similarly. Note that when the manufacturer is type-$\ell$, the supply chain profit is same for the symmetric information and separating equilibrium cases. Therefore, we only need to compare the profits when the manufacturer is type-$h$, i.e., we only need to show that $\hat{\Pi}_R(h, 0, \hat{w}) + \hat{\Pi}_M(h, 0, \hat{w}) > \hat{\Pi}^\ast_R + \hat{\Pi}^\ast_M$. This inequality is equivalent to

$$\frac{d_1 + d_2}{50b} > 0, \quad (46)$$

where $d_1 \equiv -(c_h - c_\ell)(9c_h + c_\ell)$ and $d_2 \equiv \sqrt{(c_h - c_\ell)(c_h + 3c_\ell)(9c_h - c_\ell)}$. Inequality (46) holds if and only if $4c_\ell(c_h - c_\ell)(-9c_\ell c_h + 72c^2_h + c^2_\ell) > 0$. It is straightforward to see that this inequality always holds.

Note that the retailer obtains a lower wholesale price when the manufacturer is type-$h$ in the separating equilibrium compared to symmetric information case. Hence, it can never be worse off in the separating equilibrium compared to symmetric information case. ■

**Proof of Proposition 14**

We find the necessary and sufficient conditions first for $A^1_{\text{SC}}$ and then for $A^1_R$. To do so, we first prove a preliminary result Lemma 16. For simplicity, we use $\hat{\Pi}^\ast_{\text{SC}}(i)$, $\hat{\Pi}^\ast_{\text{R}}(i)$, and $\hat{\Pi}^\ast_{\text{SC}}(i)$, $\hat{\Pi}^\ast_{\text{R}}(i)$ to denote equilibrium supply chain (retailer) profits when the manufacturer is type-$i$ under symmetric and asymmetric information, respectively.

First define

$$\hat{p}^\ast_{\text{SC}}(h) = \hat{\Pi}^\ast_{\text{SC}}(h) - \frac{3a^2}{16b}, \quad \hat{p}^\ast_{\text{SC}}(\ell) = \hat{\Pi}^\ast_{\text{SC}}(\ell) - \frac{3a^2}{16b}, \quad \hat{p}^\ast_h = \hat{\Pi}^\ast_{\text{SC}}(h) - \frac{3a^2}{16b}, \quad (47)$$

**Lemma 16.** (i) $\hat{p}^\ast_{\text{SC}} < 0$ and $\hat{p}^\ast_h < 0$ if and only if one of the followings hold:

1. $c_\ell \in (0.231148c_h, 0.355969c_h) \cup (0.899033c_h, c_h)$ and $a \in (c_5c_h, c_6c_\ell)$. 


2. \( c_\ell \in (0.355969 c_h, 0.899033 c_h) \) and \( a \in (\hat{a}, c_6 c_\ell) \)

(ii) \( \hat{p}_h^\text{SI} \leq 0 \) and \( \hat{p}_h^\text{SI} > 0 \) if and only if one of the followings hold:

1. \( c_\ell \in (0.226734 c_h, 0.231148 c_h) \) and \( a \in (\hat{a}, c_6 c_\ell) \).
2. \( c_\ell \in [0.231148 c_h, 0.355936 c_h) \cup c_\ell \in (0.899033 c_h, 0.972277 c_h) \) and \( a \in (\hat{a}, c_5 c_h) \).
3. \( c_\ell \in (0.972277 c_h, c_h) \) and \( a \in [c_5 c_\ell, c_5 c_h) \).

(iii) \( \hat{p}_h^\text{SI} \geq 0 \) and \( \hat{p}_h^\text{SI} \leq 0 \) if and only if one of the followings hold:

1. \( c_\ell \in (0, 0.231148 c_h) \) and \( a \in [c_5 c_h, c_6 c_h) \).
2. \( c_\ell \in (0.231148 c_h, c_h) \) and \( a \in [c_6 c_\ell, c_6 c_h) \).

(iv) \( \hat{p}_h \) is a convex function of a with roots \( \hat{s}_1 \) and \( \hat{s}_2 \), where \( \hat{s}_1 < \hat{s}_2 \).

Furthermore, \( c_5 \equiv 4 - \frac{2\sqrt{39}}{3} \) and \( c_6 \equiv \frac{2}{3} (10 + \sqrt{39}) \).

**Proof of Lemma 16:**

Parts (i), (ii), (iii) can be show easily by checking the roots of \( \hat{p}_h^\text{SI} \) and \( \hat{p}_h^\text{SI} \). For (iv), note that \( d^2 \hat{p}_h / da^2 = 1/8b > 0 \), \( \lim_{a \to \max(\hat{a}_1 \leq c_6 c_\ell)} \hat{p}_h > 0 \) and \( \lim_{a \to c_6 c_\ell} \hat{p}_h > 0 \) and \( \min_a \hat{p}_h < 0 \). Note that \( \hat{a} \) is defined in the proof of Lemma 14. Furthermore, define the first root of \( \hat{p}_h \) as \( \hat{s}_1 \) and the second root as \( \hat{s}_2 \).

We now find the necessary and sufficient conditions for \( \hat{A}_S^\text{DI} \). We need to find the parameter region in which \( E[\hat{\Pi}_S^\text{DI}] > 3a^2/16b \) and \( E[\hat{\Pi}_S] < 3a^2/16b \). Note that \( 3a^2/16b \) is the supply chain’s profit without encroachment (see Proposition 1). Define \( \hat{\mu}_1 \) as the probability of type-\( \ell \) such that \( E[\hat{\mu}_1, \hat{\Pi}_S^\text{DI}] = 3a^2/16b \) whenever it exists, and \( \hat{\mu}_2 \) as the probability of type-\( \ell \) such that \( E[\hat{\mu}_2, \hat{\Pi}_S^\text{DI}] = 3a^2/16b \) whenever it exists. We know that \( \hat{\Pi}_S^\text{DI}(\ell) = \hat{\Pi}_S^\text{DI}(\ell), \hat{\Pi}_S^\text{DI}(h) > \hat{\Pi}_S(h) \). Therefore, \( \hat{A}_S^\text{DI} \) consists of the following cases:

1. \( \hat{p}_h^\text{SI} < 0, \hat{p}_h^\text{SI} \leq 0, \hat{p}_h > 0 \) and \( \mu_0 < \hat{\mu}_2 \).
2. \( \hat{p}_h^\text{SI} \geq 0, \hat{p}_h^\text{SI} \leq 0 \) and \( \hat{\mu}_1 < \mu_0 < \hat{\mu}_2 \).
3. \( \hat{p}_h^\text{SI} \geq 0, \hat{p}_h^\text{SI} \leq 0 \) and \( \hat{\mu}_1 < \mu_0 < \hat{\mu}_1 \).

**Case 1.** Lemma 16 part (i) provides a region in which this case is feasible. To find the necessary and sufficient conditions, we need to identify when \( \hat{p}_h > 0 \) in this feasible region. Using part (iv) of the same lemma, we can show that the necessary and sufficient conditions for Case 1 are as follows:

\[
\left( 0.355969 c_h \leq c_\ell \leq 0.899033 c_h \text{ AND } \hat{a} < a < \hat{s}_1 \right) \text{ OR } \left( 0.231148 c_h \leq c_\ell \leq 0.355969 c_h \text{ AND } \hat{a} < a < \min(c_6 c_\ell, \hat{s}_1) \right) \text{ OR } \left( 0.899033 c_h \leq c_\ell \leq 0.978527 c_h \text{ AND } c_5 c_h < a < \hat{s}_1 \right) \text{ OR } \left( 0.978527 c_h \leq c_\ell \leq c_h \text{ AND } c_5 c_h < a < \hat{s}_1 \text{ OR } \hat{s}_2 \text{ or } c_6 c_\ell \right) \text{ AND } \left( \mu_0 < \hat{\mu}_2 \right),
\]

**Case 2.** From Lemma 16, Case 2 holds if and only if parameters fall in the region defined by conditions in part (ii) of the lemma. Define this region as \( \hat{C}_2 \).

**Case 3.** Lemma 16 part (iii) provides a region in which this case is feasible. To find the necessary and sufficient conditions, we need to identify when \( \hat{p}_h > 0 \) and \( \hat{p}_h < 0 \) in this feasible region. Using part (iv) of the same lemma, we can show that the necessary and sufficient conditions for Case 3 are as follows:

\[
\left( c_\ell \leq 0.231148 c_h \text{ AND } c_5 c_h < a < \hat{s}_1 \text{ OR } \hat{s}_2 < a < c_6 c_\ell \right)
\]
whenever it exists.

Similar to Proposition 5 it can be easily shown that Case 1 holds if and only if

\[ E \left[ \hat{\Pi}_R \right] = a^2/16b \] and \( E[\hat{\Pi}_S] < a^2/16b \) where \( \hat{\Pi}_R \) and \( \hat{\Pi}_S \) are the retailer’s profits for the symmetric information and asymmetric information cases, respectively. Define \( \tilde{i} \equiv \hat{\Pi}_S(i) - a^2/16b \) and \( \tilde{n} \equiv \hat{\Pi}_S(i) - a^2/16b \). Also define \( \mu_1 \) as the probability of type-\( \hat{\ell} \) such that \( E[\hat{\mu}_1|\hat{\Pi}_S] = a^2/16b \) whenever it exists, and \( \tilde{\mu}_2 \) as the probability of type-\( \hat{\ell} \) such that \( E[\tilde{\mu}_2|\hat{\Pi}_S] = a^2/16b \) whenever it exists.

From Proposition 12, we can see that \( \hat{\Pi}_R^*(h) > \hat{\Pi}_R^*(h) > \hat{\Pi}_R^*(h) \). Therefore, \( \hat{\Pi}_R^1 \) consists of the following cases:

1. \( \hat{i}_h < 0, \hat{i}_h < 0, \hat{i}_h > 0 \) and \( \mu_0 < \tilde{\mu}_2 \).
2. \( \hat{i}_h < 0, \hat{i}_h > 0, \hat{i}_h > 0 \) and \( \hat{\nu}_1 < \mu_0 < \tilde{\mu}_2 \).

Similar to Proposition 5 it can be easily shown that Case 1 holds if and only if

\[
a > \frac{8c_h}{5} \quad \text{AND} \quad \frac{25a^2 - 40ac_h + 64c_h^2}{40a} < c_h \leq \frac{5a}{8} \quad \text{AND} \quad \mu_0 < \tilde{\mu}_2. \tag{\hat{C}1} \]

Also Case 2 holds if and only if

\[
a > \frac{8c_h}{5} \quad \text{AND} \quad \frac{5a}{8} < c_h < \frac{1}{7} \left( 6a - c_h \right) - \frac{\sqrt{5a^2 + 10ac_h - 23c_h^2}}{7\sqrt{5}} \quad \text{AND} \quad \hat{\nu}_1 < \mu_0 < \tilde{\mu}_2. \tag{\hat{C}2} \]

Finally, \( \hat{\Lambda}_S^4 \) is defined by \( \hat{C}1 \cup \hat{C}2 \).

To see \( \hat{\Lambda}_S^4 \cap \hat{\Lambda}_R^4 \neq \emptyset \), note, for example, for \( a = 1, b = 1, c_h = 0.54, c_h = 0.61 \) and \( \mu_0 = 0.2 \). \( E[\hat{\Pi}_S^4] = 0.193698 \), \( E[\hat{\Pi}_R^4] = 0.264287 \), \( a^2/16b = 0.1875, E[\hat{\Pi}_R^4] = 0.0857254, E[\hat{\Pi}_S^4] = 0.05696 \) and \( a^2/16b = 0.0625 \).}

**Proof of Proposition 15**: The proof is straightforward. Hence it is omitted.

**Proof of Lemma 15**: We define \( \hat{\alpha} = \frac{2(\beta^2 - c_h)}{\beta^2 + 10\beta^2 + 16\beta - 8} \). This \( \alpha \) ensures \( q_{M_i} > 0 \). The proof of the lemma is akin to the Lemma 7, hence it is omitted.

**Proof of Proposition 16**: We define

\[
\hat{\omega}_{pr} \equiv \frac{8\alpha + \beta^3 (\alpha - c_h) + 4\beta (c_h - c_e) + \zeta_5}{2(\beta^2 + 8)}, \quad \text{where} \quad \zeta_5 = \sqrt{\frac{\beta (c_h - c_e) (4\alpha \beta - 1 + \beta (c_e - 5c_h))}{\beta^2 - 1}}.
\]
Because type-$h$ manufacturer does not gain from mimicking, her best strategy is to set $\hat{w}_h^* = \hat{w}_h^{SI}$. This proves part (i).

The rest of the proof is similar to the proof of Proposition 10. Hence we only give the sketch of the proof. For Part (ii), we need to identify the set of wholesale prices that satisfy the $\hat{IC}_h$ and $\hat{IC}_\ell$ constraints. Then we find the $\beta$ values that leads to a non-empty set of wholesale prices. It can be shown that set of wholesale prices is non-empty when $\beta \leq 0.58 \equiv \hat{\beta}_0$. Finally, using the intuitive criterion, we can establish that $\hat{w}_\ell = \hat{w}_{pr}$.

By comparing the profits of the supply chain and the retailer under symmetric and asymmetric information we can readily show that asymmetric information improves the both. The comparison of consumer surpluses is similar the proof of Proposition 10. Hence it is omitted. ■