Notes: Intrahousehold Models

Jorge M. Agüero
jorge.aguero@ucr.edu

Spring, 2009

Outline of today's lecture

1. Introduction
2. Collective models
3. Testing
4. Efficiency

Possible models

- Unitary model is silent about how decisions are reached.
- *Collective models* is an attempt to open Pandora's box.
- Options:
  - Non-cooperative models: Rotten Kid.
  - Nash bargaining.
  - Collective models.

Collective models

- Proposed by Chiappori and several co-authors.
- They assume Pareto optimality.
- Model (person 1 and 2):

$$\max_{c^1,c^2,Q} u^1(c^1,c^2,Q) \tag{1}$$

$$\text{s.t.}[\mu] \qquad u^2(c^1,c^2,Q) \geq \bar{u}^2(p,P,y^1,y^2,\Theta)$$

$$p(c^1+c^2)+PQ = y^1+y^2 = Y$$

- $\bar{u}^2$ is 2's reservation utility, Θ are *distribution factors*.

Proposition (MWG 16.E.2)

Let (c^1, c^2, Q) be the solution of (1). If the utility possibility set is convex , there exists *Pareto weights* $(\mu_1, \mu_2) \geq 0$ with at least one strictly bigger than zero, such that (1) is equivalent to

$$\max_{c^1, c^2, Q} \mu_1 u^1(c^1, c^2, Q) + \mu_2 u^2(c^1, c^2, Q) \tag{2}$$

$$\text{s.t.} \qquad p(c^1 + c^2) + PQ = y^1 + y^2 = Y$$

where $\mu_1 = \mu_1(p, P, y^1, y^2, \Theta)$
Given (2) we can now consider a two-step approach

Two-step approach

- Consider a model with labor supply (ℓ^i) and no public goods

$$\max_{c^1,c^2,\ell^1,\ell^2} u^1(c^1,c^2,\ell^1,\ell^2) \tag{3}$$

$$\text{s.t.}[\mu] \qquad u^2(c^1,c^2,\ell^1,\ell^2) \geq \bar{u}^2(p,w_1,w_1,y^1,y^2,\Theta)$$

$$p(c^1+c^2)+w_1\ell^1+w_2\ell^2=(w_1+w_2)T+Y$$

- First stage: household bargain in order to allocate resources among members.
- Second stage: given the transfers, each decides optimally (but separately).

Second stage

- After the bargaining, member i received ρ_i of Y and then maximizes his/her utility

$$V^i(p, w_i, \rho_i) = \max_{c^i, \ell^i} \quad u^i(c^i, \ell^i)$$

$$\text{s.t.} \quad pc^i + w_i \ell^i = w_i T + \rho_i$$

First stage

- Family decides how to transfer resources.

$$\begin{aligned} \max_{\rho_1,\rho_2} \quad & V^1(p, w_1, \rho_2) \\ \text{s.t.} \quad & V^2(p, w_1, \rho_2) \geq \bar{u}^2(p, w_1, w_1, y^1, y^2, \Theta) \\ & \rho_1 + \rho_2 = Y \end{aligned}$$

Outcomes

- A series of demand functions and labor supply equations.
- $c = c^1 + c^2 = g(p, w_1, w_2, \rho_1, \rho_2)$
- $\ell^i = \ldots$, for $i = 1, 2$.
- ...and transfers
- $\rho^2 = Y - \rho^1(p, w_1, w_1, y^1, y^2, \Theta)$

Testing the collective model: I

- Browning and Chiappori (ECNT, 1998)
- In the collective model, the Slutsky matrix is

$$S = \Sigma + uv' \qquad SR1 \text{ restriction} \tag{4}$$

- Σ is symmetric negative matrix and uv' is a rank 1 matrix (the sharing rule, ρ)

Testing the collective model: II

- They use a flexible demand function.
- Estimate system of equations.
- Test for support of $SR1$.
- $H_1 : S = \Sigma$
- B&C found evidence in favor of $SR1$

Identification of the sharing rule

1. Variation in non-labor income or distribution factors
   - Chiappori and Lacroix (JPE, 2002) using labor supply.
   - BBCL (JPE) use assignable goods (clothing)
   - Can we find *better* assignable goods?
2. Price variation: wages

Test of Pareto efficiency: I

- Consider a household model with commodities (j) and two household members $i = 1, 2$.

$$c^j = c_1^j + c_2^j = g_1^j(\boldsymbol{p}, w_1, w_2, Y, \theta_1) + g_2^j(\boldsymbol{p}, w_1, w_2, Y, \theta_2) \qquad j = 1, . \tag{5}$$

$$1 - \ell_i = h_i(\boldsymbol{p}, w_1, w_2, \theta_i) \qquad i = 1, 2 \tag{6}$$

Test of Pareto efficiency: II

- Compute the income effect $(\partial c^j/\partial y_i)$ for person $i = 1, 2$ holding Y constant.
- Note that $\rho_2 = Y - \rho_1$

$$\begin{aligned} \frac{\partial c^j}{\partial y_1} &= \\ &= \frac{\partial g_1^j}{\partial \theta_1}\frac{\partial \theta_1}{\partial y_1} - \frac{\partial g_2^j}{\partial \theta_1}\frac{\partial \theta_1}{\partial y_1} \\ &= \frac{\partial \theta_1}{\partial y_1}\left[\frac{\partial g_1^j}{\partial \theta_1} - \frac{\partial g_2^j}{\partial \theta_1}\right] \end{aligned} \tag{7}$$

- A similar expression can be computed for person 2.

Test of Pareto efficiency: III

- Let's define η_j as the marginal rate of substitution between member's non-labor income.

$$\eta_j = \frac{\partial c^j / \partial y_1}{\partial c^j / \partial y_2} = \frac{\partial \theta_1 / \partial y_1}{\partial \theta_1 / \partial y_2}. \tag{8}$$

- Identify the observable and the unobservable elements of this equation.

Test of Pareto efficiency: IV

$$\eta_1 = \frac{\partial c^1/\partial y_1}{\partial c^1/\partial y_2} = \frac{\partial c^j/\partial y_1}{\partial c^j/\partial y_2} = \eta_j \qquad \forall j = 2, \dots, J \tag{9}$$

- This is testable.

Test of Pareto efficiency: V

- Non-labor income could be endogenous.
- In that case you want to use distribution factors

$$\eta_j = \frac{\partial c^j / \partial \zeta_1}{\partial c^j / \partial \zeta_2} = \frac{\partial \theta_1 / \partial \zeta_1}{\partial \theta_1 / \partial \zeta_2} \qquad \forall j \tag{10}$$

Implementation I

- Two commodities (x)

$$x_i^1 = \alpha_1 + \beta_1 s_i^1 + \beta_2 s_i^2 + \delta_1 z_i + \varepsilon_i^1 \tag{11}$$
$$x_i^2 = \alpha_2 + \beta_3 s_i^1 + \beta_4 s_i^2 + \delta_2 z_i + \varepsilon_i^2$$

Implementation II

- Null hypothesis

$$
\begin{aligned}
H_0 &: \quad \frac{\beta_1}{\beta_2} = \frac{\beta_3}{\beta_4} \\
H_1 &: \quad \frac{\beta_1}{\beta_2} \neq \frac{\beta_3}{\beta_4}
\end{aligned} \tag{12}
$$

- Let $g_R(\beta) = \beta_1/\beta_2 - \beta_3/\beta_4$ the ratio-type expression
- and $g_M(\beta) = \beta_1\beta_4 - \beta_3\beta_2$ the multiplicative version.