ECON 184

Economic Growth: capital accumulation and innovation

Questions from Cooper and Kevane readings

- How does Cooper describe the economic situation in Africa since 1940?
- What was the economic performance of Africa between WW-II and 2000?
- What episodes can you identify?
- How does Botswana helps us understand the role of initial conditions?
- What are the limitations of case studies compared to regression analysis?

Contents

1 The Solow Model

1.1 Setup

- Goal: what is the growth rate (of per-capita output) in the long run?
- Previous models (e.g. Harrod-Domar) had a rigid production function: more capital alone will not increase production.
- Robert Solow introduced a model where the production function is **neoclassical**.
- It allows for substitution between inputs.

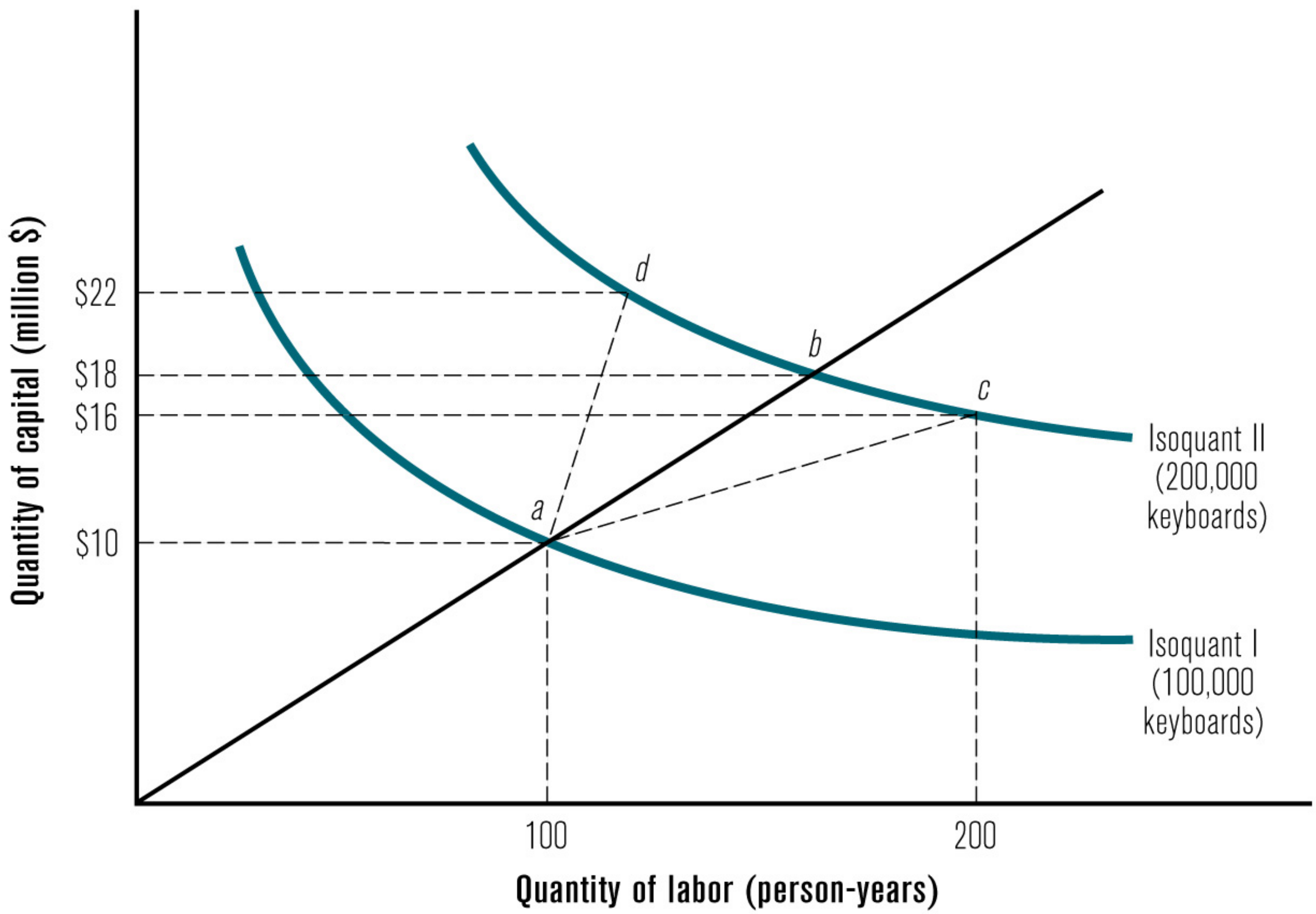

FIGURE 4.2 Neoclassical (variable proportions) Production Function

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Notation

- Production function:

$$Y = F(K, L). \tag{1}$$

- Examples: $F(K, L) = \alpha K + \beta L$ or $F(K, L) = K^{\alpha} L^{\beta}$.
- Aggregate savings:

$$S = sY, \tag{2}$$

where s is the average saving rate.

- If $s = 0.02$, we say that for every dollar the country saves 2 cents (or 2%).
- In a closed economy all savings are converted into investment:

$$S = I. \tag{3}$$

Changes in capital stock

- The change in the stock of capital is given by

$$\Delta K = I - (dK), \tag{4}$$

where d is the depreciation rate.

- Example: $S = 2b$, $K = 30b$, $d = 0.03$ then
$\Delta K = I - dK = S - dK$
$\Delta K = 2 - (0.03 \times 30) = 1.1b$

- Labor supply: $\Delta L = nL$, n=population growth.

- If $n = 0.01$ the population is growing by 1% a year.

- **Key result.** Combining equations (2), (3) and (4) we obtain

$$\Delta K = sY - dK. \tag{5}$$

1.2 Equations

- The production function: $Y = F(K, L)$. K=capital, L=labor, Y=output.
  - Inputs are essential: $F(0, L) = 0 = F(K, 0)$.
  - $F(\cdot)$ has diminishing returns to capital.
  - $F(\cdot)$ has constant returns to scale: $F(\lambda K, \lambda L) = \lambda F(K, L)$.
  - Let $\lambda = \frac{1}{L}$, then in per-capita terms we have:
  - $\frac{Y}{L} = y = \frac{F(\cdot)}{L} = F(\frac{K}{L}, 1) = f(k)$.
  - with $k = \frac{K}{L}$.

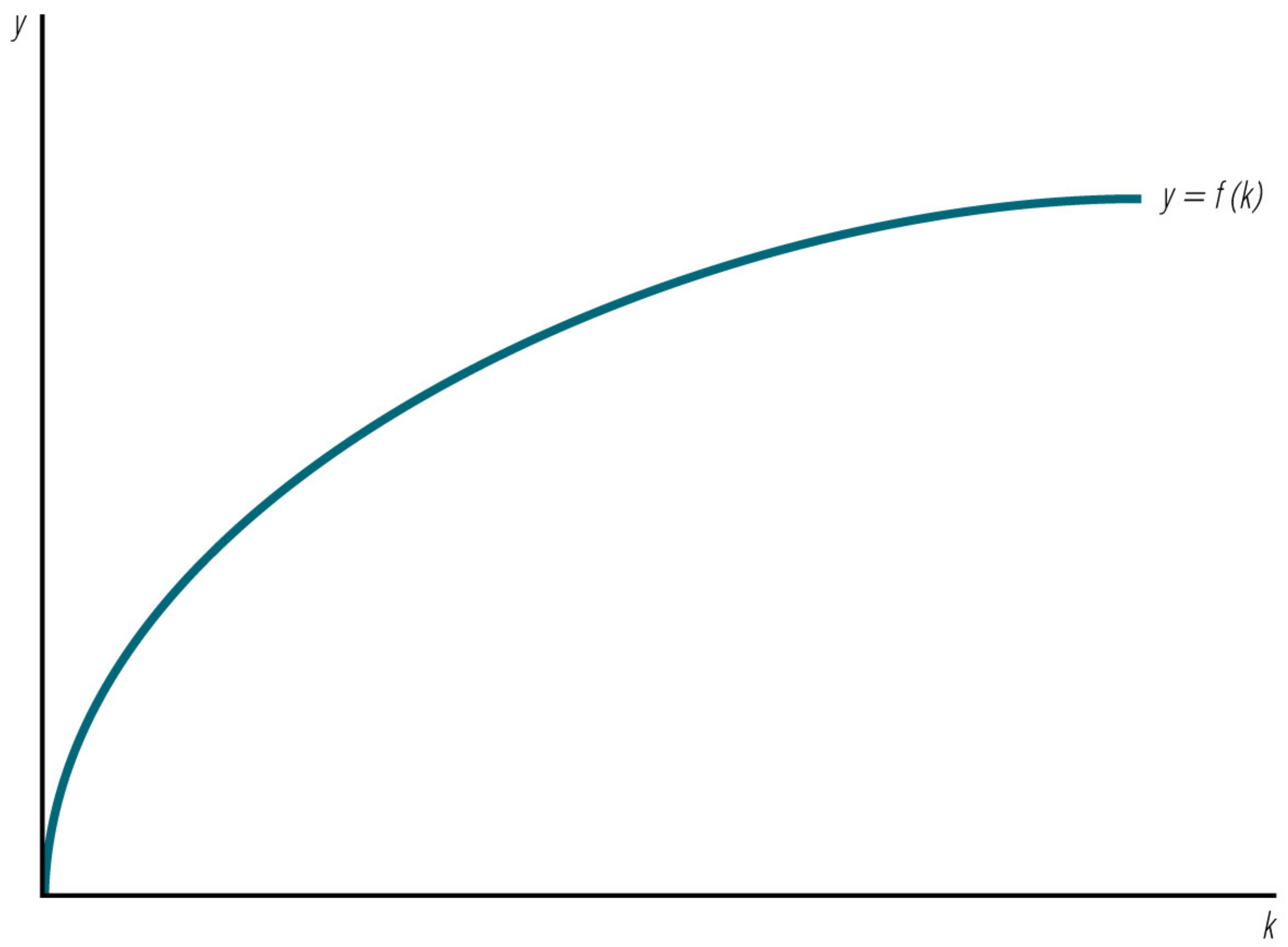

FIGURE 4.3 The Production Function in the Solow Growth Model

Average product of capital decreases with k

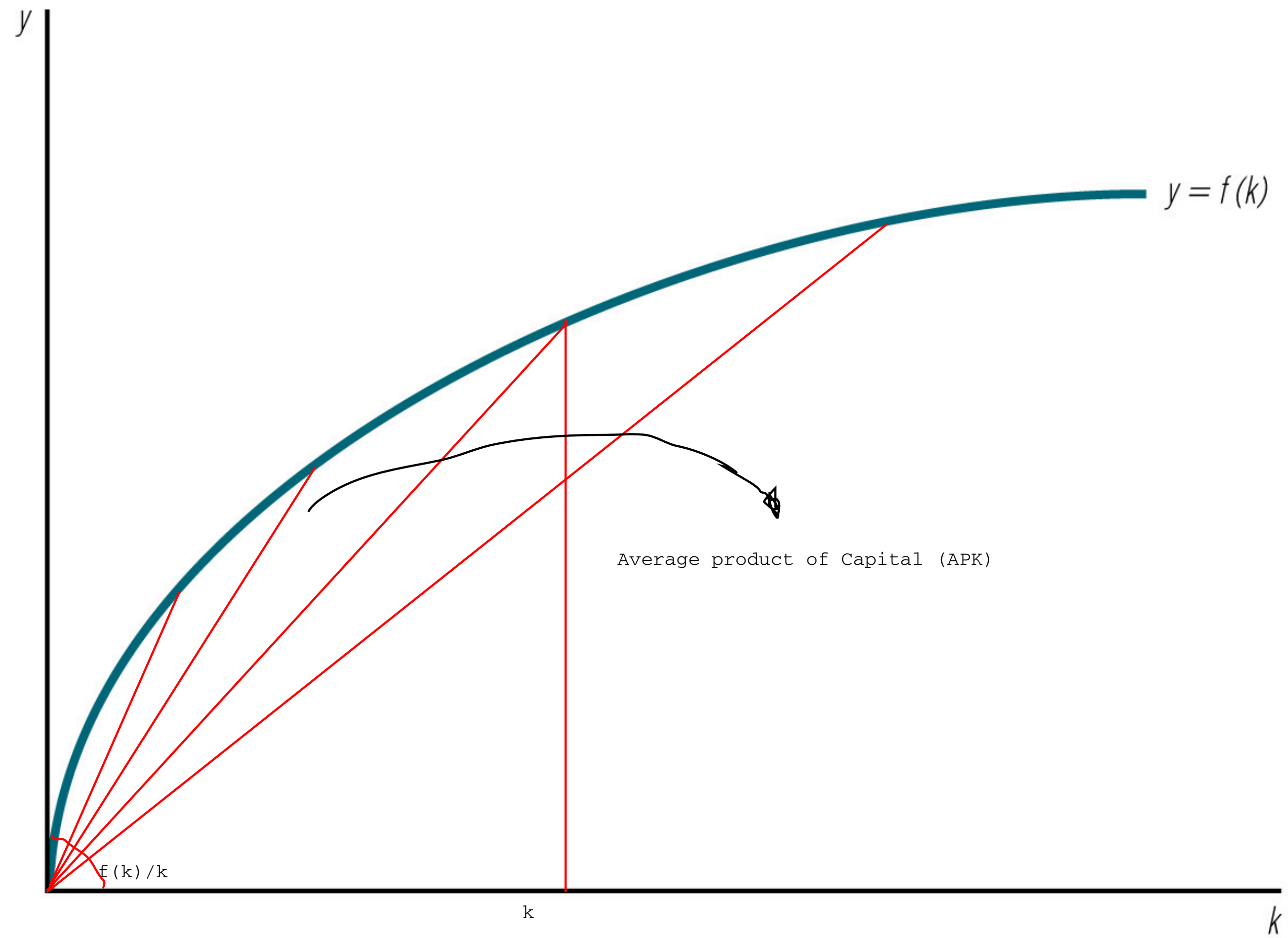

FIGURE 4.3 The Production Function in the Solow Growth Model

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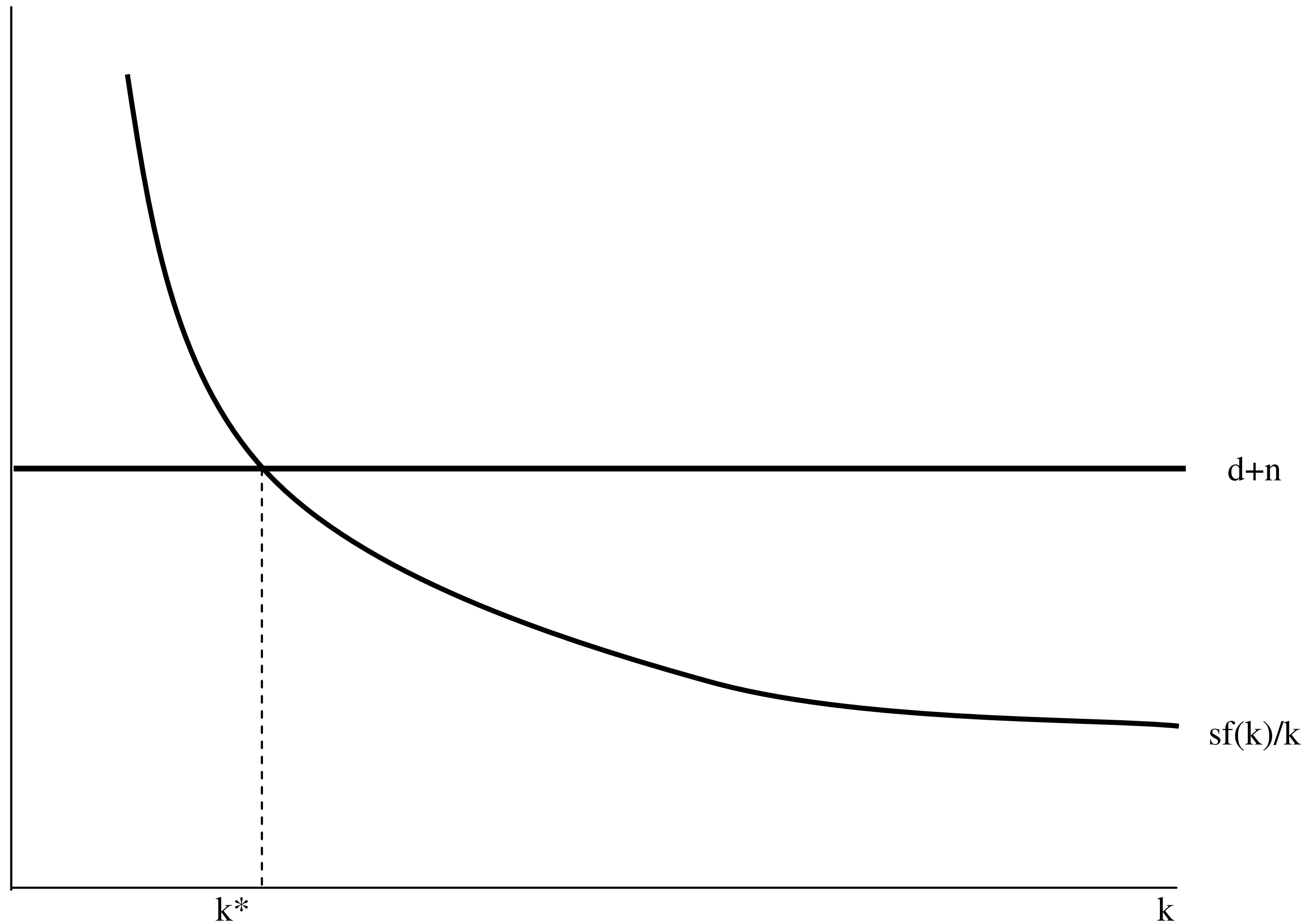
More equations

- Because $y = f(k)$ we just need to find the growth rate of capital per-capita.
- **Goal**: find the value for $\frac{\Delta k}{k}$ in the long run.
- Definition: ΔX measures the changes in X over time.
- Note: $k = \frac{K}{L}$, so $\frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta L}{L}$.
- Assumptions: labor grows at rate $n = \frac{\Delta L}{L}$ and savings are a fraction of income sY.
- Also, changes in capital come from investments (sY) and depreciation (d): $\Delta K = sY - dK$.
- $\Rightarrow \frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta L}{L} = \frac{sY - dK}{K} - n = \frac{\Delta k}{k} = sy - (n + d)$.
- **Key result**: $\frac{\Delta k}{k} = s\frac{f(k)}{k} - (n + d)$.

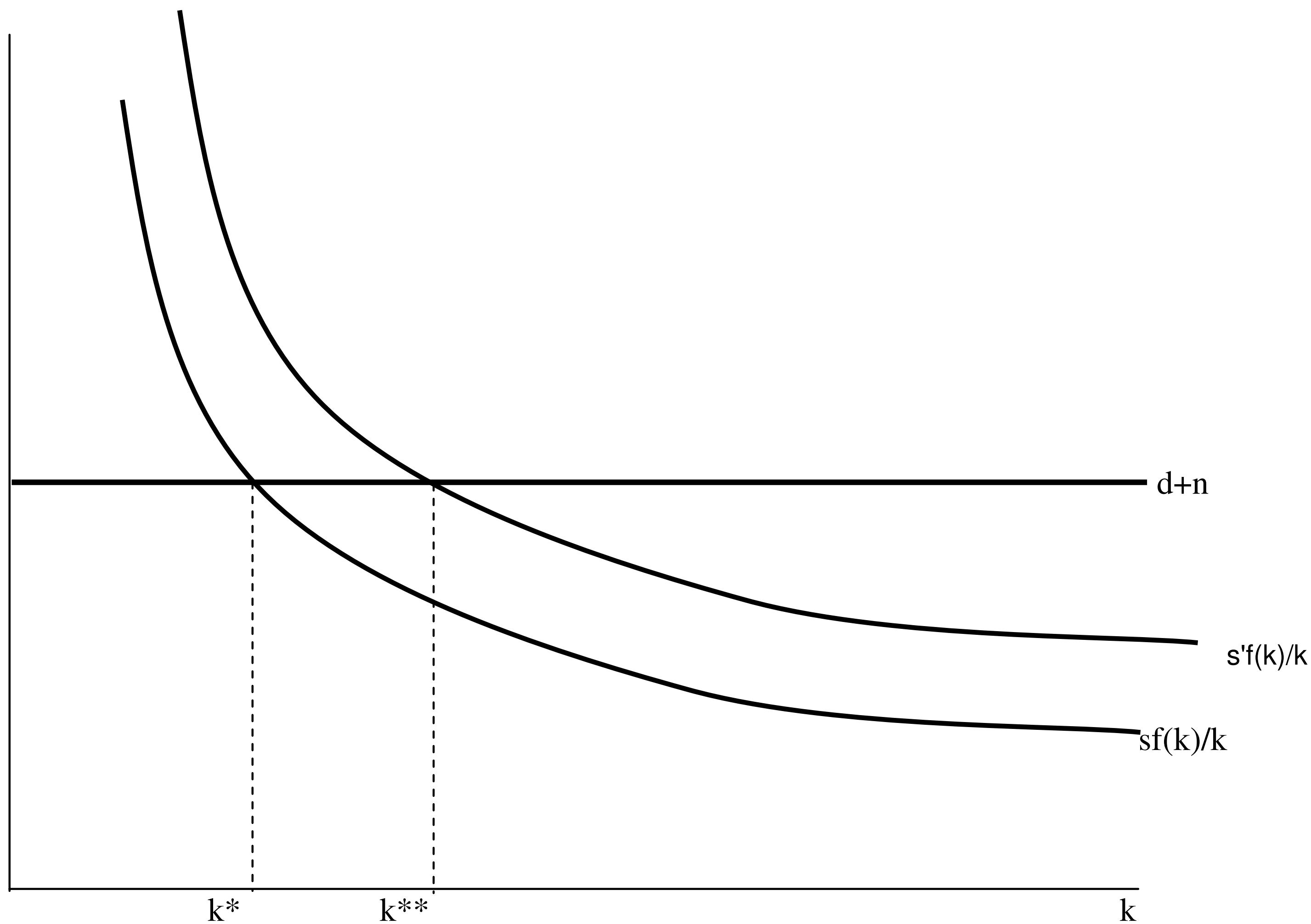
1.3 The long run or steady state



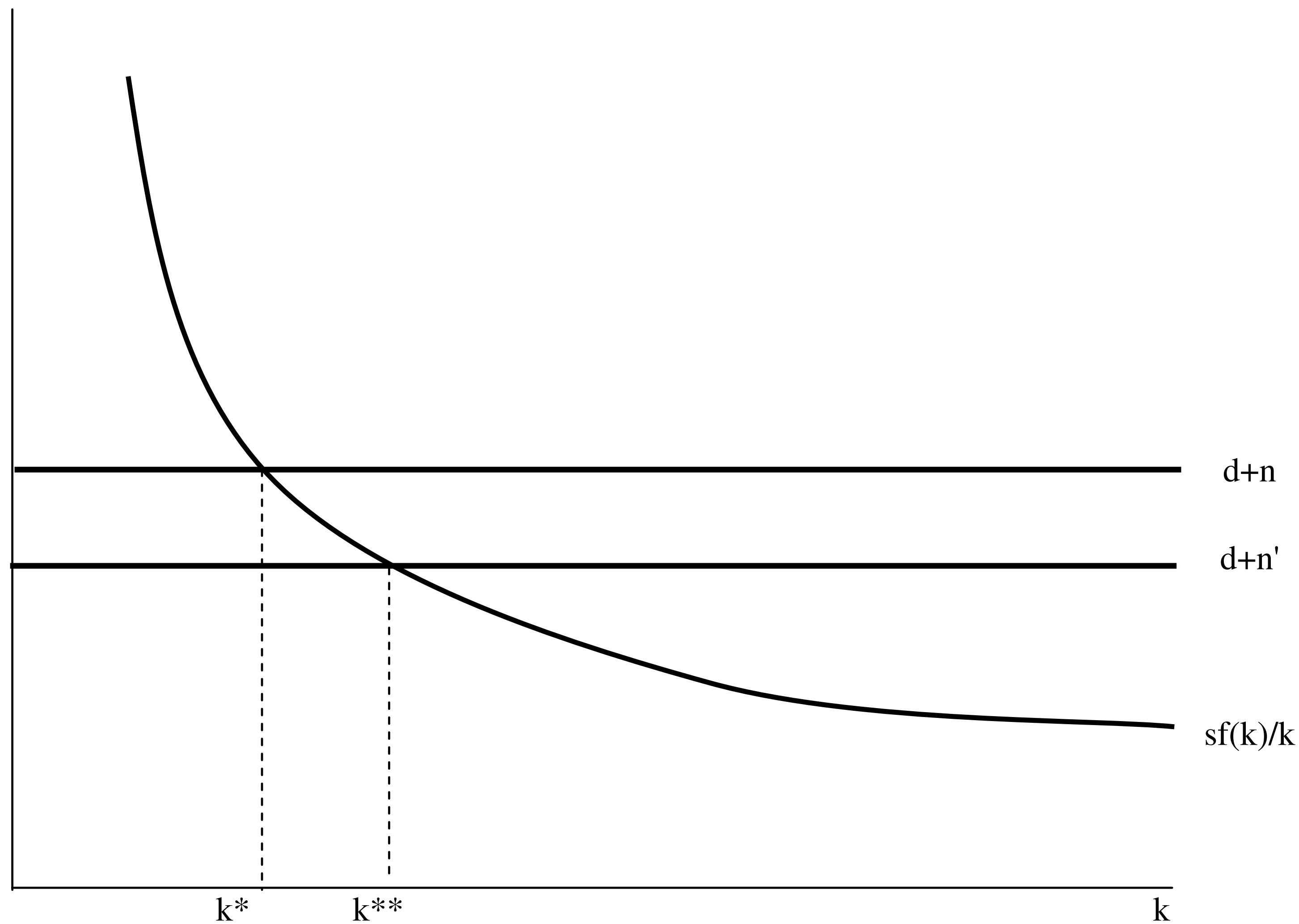
1.4 Policies

- Can we increase per-capita growth?
- Can we increase the long-run level of capital?

Increasing savings rate: from s to s'



Reducing population growth: from n to n'

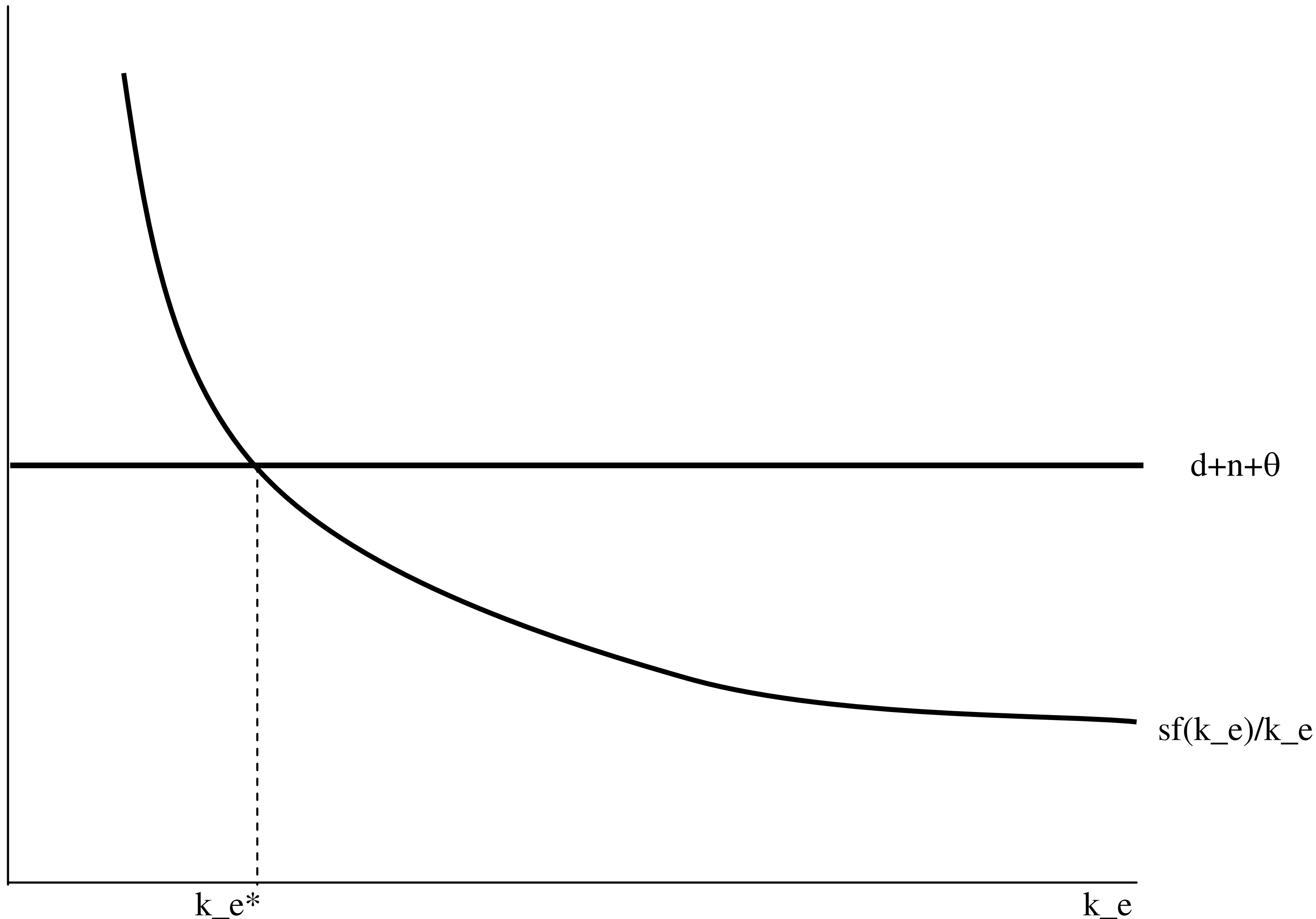


2 The Augmented Solow Model

2.1 Technological change

- **Labor augmenting** technological change: $Y = F(K, T \times L)$.
- Key question: can this economy grow forever in per-capita terms?
- We will express things in terms of **effective units of labor**.
- That is, we divide all terms by $T \times L$ instead of just L.
- We need an expression for $\frac{\Delta k_e}{k_e}$.
- $\frac{\Delta k_e}{k_e} = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta T}{T}$.
- As before, population growth rate is n and let θ be the growth rate of technology ($\frac{\Delta T}{T}$) and they are both constants.
- **Key result**: $\frac{\Delta k_e}{k_e} = s\frac{f(k_e)}{k_e} - (n + d + \theta)$.

Same as before, but with different units: k_e instead of k



2.2 Predictions

- Impact of population growth, savings rate on growth.
- What is the growth rate of output per-capita with and without technological change?
- Without tech. change:
  - In the long run: k^* is fixed.
  - In per-capita terms, there is no growth.
  - Why? Recall that $\frac{\Delta k}{k} = s\frac{f(k)}{k} - (n+d)$.
  - At k^*, $\frac{\Delta k}{k} = 0$.

- With tech. change:
  - In the long run: k_e^* is fixed.
  - In efficiency terms, there is no growth.
  - Same argument as above: $\frac{\Delta k_e}{k_e} = 0$ at k_e^*.
  - What about in per capita terms?
  - $\frac{\Delta k_e}{k_e} = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta T}{T}$
  - That is $\frac{\Delta k_e}{k_e} = 0 = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \theta$
  - then $\frac{\Delta K}{K} - \frac{\Delta L}{L} = \theta = \frac{\Delta k}{k}$
  - In per capita terms, growth is positive and equal to θ.

- Predictions: what is going to happen to output in the long run?
- The convergence debate:
  - Absolute convergence.
  - Convergence among OECD countries: evidence and problems.
  - Conditional convergence: need to account for differences in s, n and θ.

3 Extensions

3.1 Endogenous growth

- So far we assumed decreasing marginal returns.
- Example: $Y = AK^{\alpha}L^{1-\alpha}$.
- The APK is then given by $APK = Ak^{\alpha-1}$ with $\alpha < 1$.
- Now assume that $\alpha = 1$
- What would be the the predictions of the model?

3.2 Variable savings rate

- Before we assumed that saving rate (s) was constant.
- Let's relax this assumption.
- Savings can depend on income (y) and the interest rate (r)

$$s = s(y, r) \tag{6}$$

- What affects income?
- What affects interest rate?

- Income: $y = f(k)$
- Interest rate: $r = f'(k)$. That's the marginal product of capital.
- Using these expressions we get

$$s = s(y, r) = s(f(k), f'(k)) = s(k) \tag{7}$$

- The **Key result** is now

$$\frac{\Delta k}{k} = s(k)\frac{f(k)}{k} - (n + d).$$

- Finally, what are the new predictions of the model?
- What does thin mean for the understanding of African development?

4 What does the data say?

- Problem set 1.