Tax Evasion and Financial Development under Asymmetric Information in Credit Markets*

Jang-Ting Guo†
University of California, Riverside

Fu-Sheng Hung‡
National Chengchi University

July 24, 2019

Abstract

Recent empirical studies have documented that the incidence of firms’ tax evasion on their sales is negatively correlated with the country’s level of financial development. Our analysis finds that this stylized fact can be theoretically accounted for within a small-open-economy model of optimal tax enforcement under asymmetric information in credit markets. With a more developed financial sector that exhibits smaller agency costs, we analytically show that the government will raise its optimal tax-auditing probability, which in turn leads to more tax compliance. It follows that financial development and tax evasion are inversely related, as observed in the actual data. Our baseline model also yields an empirically-realistic positive correlation between financial development and the ratio of tax revenue over GDP. In an extended setting which allows for size-dependent probabilities of tax detection, we find that consistent with the empirical evidence, large firms comply with taxes whereas small firms evade taxes.

Keywords: Tax Evasion, Financial Development, Asymmetric Information, Credit Rationing.

JEL Classification: D82, H26, H32.

*For helpful comments and suggestions, we thank an anonymous referee, David Lagakos (Co-Editor), Been-Lon Chen, Chih-Fang Lai, Yiting Li, Pei-Ju Liao, Javier Pereira, Cheng Wang, Chien-Chiang Wang, and seminar participants at National Taiwan University, Feng Chia University, the WEAI International Conference, the SAET Annual Conference, and the Joint Conference of CEANA and TEA. Part of this research was conducted while Hung was a Fulbright Scholar at University of California, Riverside, whose hospitality is greatly appreciated. Financial support from the Ministry of Science and Technology of R.O.C. is gratefully acknowledged by Hung. Of course, all remaining errors are our own.

†Corresponding Author. Department of Economics, 3133 Sproul Hall, University of California, Riverside, CA 92521, USA, Phone: 1-951-827-1588, Fax: 1-951-827-5685, E-mail: guojt@ucr.edu.

‡Department of Economics, National Chengchi University, Taipei 116, Taiwan, Phone: 886-2-2938-7369, Fax: 886-2-2939-0944, E-mail: fshung@nccu.edu.tw.
1 Introduction

Recent empirical studies have documented that firms in countries with a more developed financial sector would report a larger fraction of their sales to the tax authority. In particular, using a large sample of survey data across 102 countries, Beck et al. (2014) report that firms in countries with a higher ratio of private credit to GDP or a higher degree of financial outreach (i.e. better information-sharing systems and higher bank branch penetration) will evade taxation on their sales to a lesser extent.\(^1\) Moreover, based on a sample of survey data from 41 countries, Dabla-Norris et al. (2008) find that among firms which regard external financing as a major difficulty, there is a 16% probability whereby these establishments will hide more than 50 percent of their sales. On the contrary, there is only a 7.6% probability of firms hiding such a level of sales when they view obtaining external funds as a minor obstacle. These empirical results together illustrate a discernible negative correlation between the incidence of firms’ tax evasion on their sales revenue and an economy’s financial development that is closely related to the salient feature of asymmetric information in credit markets.\(^2\) As it turns out, this stylized fact has been left mostly unexplored in existing theoretical studies as tax evasion is mainly attributed to the government’s fiscal policy administration through tax rates, the probability of tax detection and its resulting penalty.\(^3\)

Starting from the seminal contribution of Allingham and Sandmo (1972), much effort has been devoted to examining various aspects and effects associated with tax evasion.\(^4\) In these previous studies, tax evasion is postulated as a risky behavior by rational agents maximizing their expected utility from taxable income. A consensus of this literature is that a more aggressive tax-enforcement policy, through an increase in the auditing probability or the magnitude of fines, will raise the marginal cost of tax evasion and hence leads to more tax compliance, as in Andreoni et al. (1998) and Sandmo (2005).\(^5\) On the other hand, pioneered by the work of Stiglitz and Weiss (1981), it has been shown that asymmetric information in credit markets may give rise to adverse selection and/or moral hazard, thus generating credit rationing in equilibrium. This is detrimental to the economy’s aggregate performance as it places consider-

\(^1\)Since King and Levine (1993), the ratio of a country’s private credit over GDP has been used as an empirical indicator of financial development.

\(^2\)See Levine (1997) for a literature review.

\(^3\)Recent studies by Blackburn et al. (2012) and Capasso and Japplli (2013) have incorporated tax evasion into a model with asymmetric information in credit markets. However, the tax rate is exogenously given and there is no role for government policy of tax enforcement in their analyses. Moreover, these authors focus on firms’ tax evasion of the returns from collateral assets, not sales.

\(^4\)See, for example, Andreoni et al. (1998), Alm (1999), Slemrod and Yitzhaki (2002) and Sandmo (2005), among others. Slemrod (2007) provides a literature review on this research topic.

\(^5\)See also survey studies by Spicer and Lundstedt (1976) and Mason and Calvin (1978).
able strain on firms’ investment opportunities. Since financial development is able to influence the degree of asymmetric information in credit markets, our analysis will investigate its theoretical interrelations with firms’ tax evasion on their sales in a simple small-open-economy model under optimal tax enforcement.

In our model economy, each agent/firm is endowed with one unit of time and a risky production project. At the beginning of the period, an agent needs to borrow a certain amount of resources from the financial intermediary, through the optimal financial contract, to obtain state-contingent working capital as an intermediate good for production. As in Bernanke and Gertler (1989), agents are heterogenous because the amount of principal borrowed for each bank loan differs across individual firms. It follows that those establishments who borrow a relatively lower quantity of resources are more efficient agents. After making the loan payment, an agent will combine its retained productive capital with labor hours to produce output and generate sales revenue. The government imposes a tax rate on firms’ reported sales to finance public spending on goods and services that are assumed to be a constant fraction of agents’ total (gross) output. Moreover, both the true level of working capital and the accurate amount of sales are each firm’s private information. As a result, agents may misreport their capital possession to the bank as well as underreport their sales to the fiscal authority. For the remaining inefficient firms who are unable to obtain a bank loan and thus do not produce any output, they are postulated to gain access to an outside option that yields a given level of reservation utility. Following the literature of equilibrium contracts, such as Roy and Serfes (2001), the presence of this alternative opportunity gives rise to a participation constraint on the optimal financial contract between producing agents and the financial intermediary.

Through the standard procedure of backward induction, we first examine agents’ optimal decision problem of tax evasion, and find that the economy may exhibit two tax rates that satisfy the government’s balanced-budget constraint. To maintain consistency with earlier research, our analysis is restricted to the environment in which certis paribus an increase in the probability of tax auditing will raise the degree of tax compliance by agents as well as the expected after-tax rate of returns from undertaking their production projects. Next, the optimal financial contract that stipulates the bank to audit operating establishments under a bad state for capital acquisition is solved. In particular, a competitive bank offers the contract to maximize an agent’s expected utility across two distinct states, taking into account the world interest factor and the fact that the true outcome of firms’ capital acquisition is private

---

6Early examples include Smith and Stutzer (1989), Bencivenga and Smith (1993), and Bose and Cothren (1996, 1997), among others.

7With the exceptions of U.S. and China, all countries in the data samples of Dabla-Norris et al. (2008) and Beck et al. (2014) are small open economies.
information. We show that the financial intermediary will take away all working capital from firms when the bad state takes place, and that less-efficient establishments will be audited more often than their more-efficient counterpart. Under asymmetric information in the credit market, we also analytically derive the equilibrium measure of agents that will obtain working capital and expend labor effort to produce output. This in turn corresponds to the economy’s aggregate amount of lending/borrowing with costly state verification.

Finally, a benevolent government is postulated to maximize the economy’s social welfare that is defined as aggregate expected utilities across all agents. Per this optimization problem, we find that a lower agency or monitoring cost in credit markets will generate an increase in the optimal tax-auditing probability for the government. The intuition for this result is as follows. On the positive side, a reduction in the agency cost is shown to raise firms’ expected after-tax rate of returns from their production and tax evasion decisions; hence, aggregate utilities for producing agents will rise because of their higher consumption spending. On the negative side, a smaller monitoring cost results in more less-efficient firms capable of receiving bank loans, thus the economy’s overall production efficiency will fall. In response to these counteracting effects, we analytically show that the government will raise its optimal probability of tax auditing to maintain the maximum for the economy’s social welfare function. Since a higher tax-auditing probability leads to more tax compliance, our analysis finds that countries with a more developed financial sector are associated with a less degree of tax evasion on firms’ sales. Therefore, this paper provides a theoretical explanation for the observed negative correlation between financial development and tax evasion documented by Beck et al. (2014) and Dabla-Norris et al. (2008).

As it turns out, our baseline model can shed some light on another well-known stylized fact whereby the proportion of a country’s GDP accounted by the government’s tax revenue is relatively lower in developing economies which exhibit a less developed financial sector. For example, Gordon and Li (2009) find that the average taxes-to-GDP ratio in poor countries is about two-thirds of that in rich nations. Similarly, Besley and Persson (2009) report a positive correlation between the taxes-to-GDP ratio and income per capita.⁸ As discussed earlier, a smaller agency cost enables more firms to receive bank loans and produce output, which in turn will raise the total amount of taxes collected. However, a smaller monitoring cost also induces more less-efficient establishments to borrow and produce, hence their contribution to the value-added GDP will be lower compared to that from more-efficient agents. We analytically show that consistent with the empirical evidence, financial development leads to a relatively higher increase in tax revenue than that in aggregate net output, thus raising the taxes-to-GDP ratio.

---

⁸See Besley and Persson (2014) for a review article on this literature.
within our small-open-economy model. This finding can then be regarded as advancing the “will to collect” viewpoint as the government will choose to put more effort into tax collection, through implementing its optimal tax enforcement policy, when the financial system becomes more developed.9

Some recent studies have empirically examined the incidence of tax evasion within developing economies at the firm level; see Bigio and Zilberman (2011), Kleven et al. (2016), Carrillo et al. (2017), Zareh and Peichl (2017), and Bachas et al. (2018), among others. A robust piece of evidence documented by this literature is that large firms comply with taxes, whereas small firms do not. In order to account for this empirical regularity, our baseline model is extended to allow for size-dependent probabilities of tax enforcement. Moreover, the economy will encounter another type of asymmetric information since a large firm may have incentive to mimic a small establishment, and vice versa. In this environment, we consider the following three formulations in turn: (i) when firm size is known to the government; (ii) when firm size is private information and the government imposes additional penalty on pretending large establishments; and (iii) when firm size is private information and the government must satisfy the large firm’s incentive-compatibility constraint. Our analysis finds that within each of these settings, large firms do not evade any taxes while small establishments conceal some of their output from the government. Intuitively, since large firms are efficient agents that are not subject to the concern of asymmetric information in credit markets, the associated socially optimal tax-auditing probability will be set as high as possible, which in turn induces them to truthfully report their sales revenue to the fiscal authority. On the contrary, the socially optimal tax-detection probability on small establishments needs to balance two opposite effects: an increase in the expected after-tax rate of returns from their production and tax evasion decisions versus a decrease in the economy’s overall production efficiency because of more less-efficient agents entering the financial market. As a result, the small firms’ tax compliance rate will be lower than one hundred percent. These results together indicate an empirically-realistic positive correlation between firm size and tax compliance within developing countries.

The remainder of this paper is organized as follows. Section 2 presents our small-open-economy model. Section 3 examines the agent’s decision of tax evasion, and then characterizes the government’s tax policy administration. Section 4 derives the optimal financial contract between firms and banks under asymmetric information in credit markets. Section 5 studies the optimal tax-enforcement policy and its relationship with financial development. Section

9See a series of papers by Besley and Persson (2009, 2010, 2013) that advocate the “ability to collect” viewpoint – developing countries possess weaker “state capacity” to impose income taxation, which in turn leads to a lower taxes-to-GDP ratio.
investigates the interrelations between financial development and the taxes-to-GDP ratio within our baseline model economy. Section 7 explores the connection between firm size and tax compliance in an extended setting that allows for size-dependent probabilities of tax detection. Section 8 concludes.

2 The Economy

Consider a small open economy inhabited by a countably infinite number of agents/firms whose population size is normalized to one. As in Bernanke and Gertler (1989), these heterogenous agents are indexed by \( \phi \) that is uniformly distributed over the interval \([0, 1]\). Each agent is endowed with one unit of time and a risky production opportunity/project. At the beginning of the period, an agent-\( \phi \) needs to borrow the principal of \( \phi \) units of resources from the financial intermediary to obtain state-contingent working capital as an intermediate good for production.\(^{10}\) As a result, low-\( \phi \) establishments will incur relatively smaller borrowing as well as production costs, thus they can be regarded as more efficient agents. Banks are competitive and each has access to perfectly-elastic international supply of loanable funds at an exogenous world gross interest rate factor \( R_w > 1 \). The amount of productive capital acquired by an individual firm (as a funded agent) takes on two possible values: \( \kappa_1 \) and \( \kappa_2 \), where the probability that event \( \kappa_i \) occurs is given by \( \pi_i \in (0, 1) \) with \( \pi_1 + \pi_2 = 1 \). It is assumed that \( \kappa_2 > \kappa_1 > 0 \), hence \( \kappa_1(\kappa_2) \) represents the bad(good) state for capital acquisition. After making the loan payment in state \( i \) (\( = 1, 2 \)), each agent operates a technology that combines its remaining working capital with \( e_i \in (0, 1) \) units of labor hours to produce output and generate sales revenue.\(^{11}\) For the sake of analytical simplicity, we postulate that one unit of capital input will yield one unit of output.

Our model economy exhibits two types of informational imperfection. First, there exists asymmetric information between agents and banks in that the true level of working capital acquired is private information to each firm. Banks can correctly observe the firms’ capital ownership only by employing an auditing technology that absorbs \( \gamma > 0 \) units of working capital per operating establishment. As a result, \( \gamma \) represents the agency cost between the financial intermediary and firms; and a higher value of \( \gamma \) corresponds to a less developed financial sector. It follows that an agent who misreports the bad state of its project, but not being audited by banks, can consume extra output. Second, there exits asymmetric information between producing agents and the government which imposes a tax rate \( \tau \in (0, 1) \)

\(^{10}\)An agent-\( \phi \) who borrows the amount smaller than \( \phi \) will not be able to acquire any working capital.

\(^{11}\)Since our model economy is static with a single time period, firms will not hold inventory; hence, the amount of output produced by each firm is equal to its sales revenue.
on firms’ reported sales revenue. Since the true amount of output produced is each firm’s private information, agents may underreport their sales revenue to tax collectors. Therefore, the government can induce tax compliance by auditing each operational establishment with a probability \( \eta \in (0, 1) \). We also postulate that any under-reporting firm will be audited without error, and that all under-reported sales will be confiscated by the fiscal authority.\(^{12}\)

The sequence of timing for economic activities proceeds as follows. The tax authority announces the tax rate \( \tau \) on firms’ sales revenue and the auditing probability \( \eta \) at the outset. Taking \( \tau \) and \( \eta \) as given, each agent decides whether or not to borrow from the financial intermediary. If an agent chooses to borrow, s/he must sign a financial contract with the bank to obtain working capital as an intermediate productive input. Each firm reports its capital possession to the bank and makes (principal plus interest) payment on the financial contract; and then combines its remaining working capital with hours worked to operate the production technology. After producing output and generating sales revenue, the firm reports its sales to the government and pays taxes as well as penalties when audited. Subsequently, s/he consumes the residual output with a linear utility function that exhibits risk neutrality as follows:

\[
U = c - e, \tag{1}
\]

where \( c \) denotes the net consumption after taxes/penalties and loan payments are made, and \( e \) represents labor disutility from the production of output. When an agent is unable to borrow, s/he will not produce any output but can gain access to an outside option that is postulated to yield a level of reservation utility \( u > 0 \). Notice that the presence of such an alternative opportunity implies that the expected utility for agents who borrow and produce must be at least \( u \) across the two states. This in turn gives rise to a participation constraint on the optimal financial contract (see equation 18 below).

In what follows via the standard process of backward induction, we first analyze the firm’s decision on tax evasion, while taking the government’s tax rate \( \tau \) and tax-auditing probability \( \eta \) as given. This will determine the expected after-tax rate of returns from production. Next, we derive the optimal financial contract between firms and banks that maximizes an agent’s expected utility under a competitive financial system. We then examine the government’s

---

\(^{12}\) In general, the tax authority can induce tax compliance by selecting the probability of tax auditing and the associated penalty rate. However, Schroyen (1997) points out that the penalty rate is usually stated in a country’s law, hence it cannot be directly controlled by the fiscal authority. Moreover, Chen (2003) stipulates that collecting penalties is more efficient than tax auditing since the former is less costly. It follows that it is optimal for the government to set the maximum possible penalty rate, which amounts to confiscating all unreported sales. Given this assumption, our analyses focus on the tax-auditing probability as the only policy instrument for tax compliance.
optimal policy of tax enforcement and its interrelations with the degree of financial development represented by $\gamma$. Finally, we derive the correlation between the taxes-to-GDP ratio and financial development, and extend our model to allow the government selecting distinct probabilities of tax detection for different sizes of firms.

3 Optimal Tax Evasion

Suppose that an agent-$\phi$, after obtaining working capital from the bank and expending its labor hours, produces $y > 0$ units of (before-tax) output/sales. Since the production outcome is private information, the firm may evade taxes by only reporting a $\beta \in (0, 1)$ fraction of its sales to the government. Therefore, the firm derives $\beta(1 - \tau)y$ units of after-tax output, and retains $(1 - \beta)y$ units of sales without being audited by the tax authority. As in Chen (2003), we postulate that the transaction costs to a tax-evading firm is equal to $\frac{(1-\beta)^2}{2}y$. Since the tax authority audits each firm with probability $\eta$ and all unreported output will be confiscated under tax detection, the expected after-tax output/sales $y_{after-tax}$ is given by

$$y_{after-tax} = \beta(1 - \tau)y + (1 - \beta)(1 - \eta)y - \frac{(1 - \beta)^2}{2}y.$$  \hspace{1cm} (2)

Taking $\tau$ and $\eta$ as given, the first-order condition from firms’ maximizing $y_{after-tax}$ leads to the following optimal fraction of sales reported to the fiscal authority:

$$\beta^* = 1 - \tau + \eta,$$ \hspace{1cm} (3)

where $\tau > \eta$ is assumed to ensure that $0 < \beta^* < 1$. Plugging (3) into (2) shows that the expected after-tax rate of returns from the firm’s output production and tax evasion decisions

$$r^* = \beta^*(1 - \tau) + (1 - \beta^*)(1 - \eta) - \frac{(1 - \beta^*)^2}{2},$$ \hspace{1cm} (4)

where $0 < r^* < 1$. Notice that equations (3) and (4) indicate that the firm’s decision of tax evasion is affected by the tax rate $\tau$ and the probability of tax detection $\eta$.

Next, the government is postulated to finance an exogenously-given level of public expenditures $g > 0$ that are set to be a constant proportion $\theta \in (0, 1)$ of the firm’s gross sales $y$. As each establishment reports a fraction $\beta^*$ of its output, the government’s tax revenue is $\beta^*\tau y$. In addition, the unreported sales $(1 - \beta^*)y$ will be confiscated by the tax authority under the auditing probability $\eta$. It follows that the balanced government budget is governed by
\[ \theta = \beta^* \tau + (1 - \beta^*) \eta. \] (5)

**Proposition 1.** For a given probability of tax auditing \( \eta \), there may exist two tax rates (denoted as \( \tau_1^* \) and \( \tau_2^* \)) that satisfy the balanced government budget (5).

Using equations (3) and (5), it is straightforward to obtain Figure 1 which depicts a Laффer curve-type relationship between \( \theta \) and \( \tau \) that lies entirely below the 45-degree line,\(^{13}\) while taking the tax-detection probability \( \eta \) as given. We first note that after substituting \( \beta^* \) from (3) into (5), its vertical axis illustrates that \( \theta \) will be equal to \(-\eta^2 < 0 \) when \( \tau = 0 \). Setting \( \frac{\partial \eta}{\partial \tau} = 0 \) yields that there exists a unique tax rate \( \hat{\tau} = \frac{1}{2} + \eta \in (0, 1) \) which maximizes the government’s revenue (including penalties) as a friction of total output denoted by \( \hat{\theta} \). It follows that an increase in the tax rate \( \tau \) will generate two opposite effects. On the one hand, equation (5) shows that it directly raises the ratio of public spending to aggregate sales. On the other hand, equation (3) shows that it produces less tax compliance \( \frac{\partial \eta}{\partial \tau} < 0 \), which in turn decreases the value of \( \theta \). Figure 1 demonstrates that since the first (second) effect dominates when \( \tau > (\tau) \hat{\tau} \), an inverted-U curve ensues. Moreover, setting \( \theta = 0 \) leads to two intersection points with the horizontal axis: \( \bar{\tau} = \frac{(1+2\eta) - \sqrt{1+4\eta}}{2} \in (0, \hat{\tau}) \) and \( \bar{\tau} = \frac{(1+2\eta) + \sqrt{1+4\eta}}{2} > 1 \). As a result, our model possesses zero (two) interior equilibrium tax rates provided \( \theta > (\theta) \hat{\theta} \). In particular, we find that when \( \theta \in (0, \hat{\theta}) \) as shown in Figure 1, the analytical expressions for \( \tau_1^* \) and \( \tau_2^* \) (as functions of \( \eta \)) are

\[ \tau_1^*(\eta) = \frac{1 + 2\eta - \sqrt{1 - 4(\theta - \eta)}}{2}, \] (6)

and

\[ \tau_2^*(\eta) = \frac{1 + 2\eta + \sqrt{1 - 4(\theta - \eta)}}{2}, \] (7)

where \( 1 - 4(\theta - \eta) > 0 \) and \( 0 < \tau < \tau_1^*(\eta) < \hat{\tau} < \tau_2^*(\eta) < 1 < \bar{\tau} \).

Substituting \( \tau_1^*(\eta) \) and \( \tau_2^*(\eta) \) into (3) derives that the corresponding expressions for firms’ optimal fractions of output/sales reported to the fiscal authority are

\[ \beta_1^*(\eta) = \frac{1 + \sqrt{1 - 4(\theta - \eta)}}{2}, \] (8)

and

\[ \beta_2^*(\eta) = \frac{1 - \sqrt{1 - 4(\theta - \eta)}}{2}, \] (9)

\(^{13}\)Plugging the restriction of \( \tau > \eta \) from (3) into equation (5) yields that \( \theta < \tau \) within our model economy.
Figure 1: Equilibrium Laffer-Curve Relationship between $\theta$ and $\tau$

where $0 < \beta^*_2(\eta) < \beta^*_1(\eta) < 1$. In addition, we note that $\frac{\partial \beta^*_1(\eta)}{\partial \eta} > 0$ and $\frac{\partial \beta^*_2(\eta)}{\partial \eta} < 0$. Based on the pioneer work of Allingham and Sandmo (1972) and numerous subsequent theoretical studies, it is generally accepted that a more aggressive tax-detection policy, such as an increase in the probability of tax auditing $\eta$, is able to mitigate the incidence of tax evasion. On the empirical front, Shefrin and Triest (1992) examine a cross sectional sample of survey data, and find that taxpayers who perceive a higher audit probability will report significantly less understating of their income. However, the result that $\frac{\partial \beta^*_2(\eta)}{\partial \eta} < 0$ contradicts this consensus. To maintain consistency with the existing literature, we will rule out $\beta^*_2(\eta)$ and $\beta^*_2(\eta)$ in the ensuing analyses; and focus on the case with $\tau^*_1(\eta) \equiv \tau^*_1(\eta)$ and $\beta^*(\eta) \equiv \beta^*_1(\eta)$ from now on.

From equation (6) with $\sqrt{1 - 4(\theta - \eta)} \in (0, 1)$, it is straightforward to show that

$$\frac{\partial \tau^*(\eta)}{\partial \eta} = 1 - \frac{1}{\sqrt{1 - 4(\theta - \eta)}} < 0. \quad (10)$$

Since a higher tax-auditing probability induces more tax compliance ($\frac{\partial \beta^*(\eta)}{\partial \eta} > 0$ per equation

---

*14 In general, since data related to the tax auditing probability is not readily available, it is not straightforward to directly test the relationship between the audit probability and tax evasion. Spicer and Thomas (1982) adopt an experimental approach to show that an increase in the tax auditing probability leads to a decrease in the degree of tax evasion. Similarly, Slemrod, Blumenthal, and Christian (2001) analyze a controlled experiment on a group of randomly selected Minnesota taxpayers who were informed in 1995 that the tax returns they were about to file would be “closely examined”. These authors find that low- and middle-income taxpayers, on average, have raised their tax payments compared to those in the previous year.*
8), the government’s balanced budget as in (5) can be maintained with a lower tax rate \( \tau^*(\eta) \). Substituting equations (6) and (8) into (4), we obtain the following expression for the expected after-tax rate of returns as a function of \( \eta \):

\[
\tau^*(\eta) = \frac{(1 - \eta) \left[ 1 - \sqrt{1 - 4(\theta - \eta)} \right]}{2} + \frac{1 + \sqrt{1 - 4(\theta - \eta)}}{4} \left[ 1 - 2\eta + \sqrt{1 - 4(\theta - \eta)} \right] - \frac{1 - \sqrt{1 - 4(\theta - \eta)}^2}{8};
\]

and then find that

\[
\frac{\partial \tau^*(\eta)}{\partial \eta} = \frac{1 - \sqrt{1 - 4(\theta - \eta)}}{2\sqrt{1 - 4(\theta - \eta)}} > 0,
\]

hence an increase in the probability of tax auditing will raise the expected after-tax rate of returns from firms’ production and tax evasion decisions.

4 Optimal Financial Contract

This section derives the optimal financial contract between financial intermediaries and heterogeneous firms. In particular, an agent-\( \phi \) needs to first borrow the principal of \( \phi \) units of resources to obtain state-contingent working capital, which takes on the value of \( \kappa_1(\kappa_2) \) in the bad(good) state, as an intermediate good for production. On the other hand, a competitive bank offers the contract that maximizes agent-\( \phi \)’s expected utility, taking into account the world interest factor \( R_w \), as well as the fact that the true outcome of firms’ capital acquisition is private information. When the bad state (state 1) takes place, the bank audits a firm with probability \( p \in [0, 1] \) and agency/monitoring cost \( \gamma \); and then receives the total payment (principal plus interest) of \( T^a \). We also denote \( T_1 \) and \( T_2 \) as the total payment that an unaudited firm will pay to the bank under the bad and good states, respectively. After making the loan payment, an agent combines its remaining productive capital with \( e_1(e_2) \in (0, 1) \) units of labor hours to produce output and collect sales revenue in the bad(good) state.\(^{15}\) For the sake of analytical simplicity, we postulate that one unit of working capital will generate one unit of output.

With the expected after-tax rate of returns from firms’ production and tax evasion decisions \( \tau^*(\eta) \) à la (11) taken as given, the optimal financial contract is obtained by choosing the bank-auditing probability \( p \) together with the repayment schedule \( \{T^a, T_1, T_2\} \) to maximize agent-\( \phi \)’s expected utility given by

\(^{15}\)We do not need to impose the relationship between \( e_1 \) and \( e_2 \), i.e. whether \( e_1 \leq e_2 \), since it does not affect any of our results below.
\[ \pi_1 \left\{ r^*(\eta) \left[ \kappa_1 - pT^a - (1 - p)T_1 \right] - e_1 \right\} + \pi_2 \left\{ r^*(\eta) (\kappa_2 - T_2) - e_2 \right\}, \quad (13) \]

subject to

\[ \pi_1 [pT^a + (1 - p)T_1 - p\gamma] + \pi_2 T_2 \geq \phi R_w, \quad (14) \]

\[ r^*(\eta) (\kappa_2 - T_2) - e_2 \geq (1 - p) [r^*(\eta)(\kappa_1 - T_1) + r^*(\eta)(\kappa_2 - \kappa_1)] - e_2, \quad (15) \]

\[ r^*(\eta) (\kappa_1 - T^a) - e_1 \geq 0, \quad (16) \]

\[ r^*(\eta) (\kappa_1 - T_1) - e_1 \geq 0, \quad (17) \]

\[ \pi_1 \left\{ r^*(\eta) \left[ \kappa_1 - pT^a - (1 - p)T_1 \right] - e_1 \right\} + \pi_2 \left\{ r^*(\eta)(\kappa_2 - T_2) - e_2 \right\} \geq u, \quad (18) \]

where \([\kappa_1 - pT^a - (1 - p)T_1]\) and \(y_1^{after-tax}\) are the agent’s expected post-loan-payment working capital and after-tax output/consumption in the bad state; whereas \((\kappa_2 - T_2)\) and \(y_2^{after-tax}\) represent the corresponding variables in the good state. Equation (14) states that the expected net revenue (including the agency cost \(\gamma\)) undertaken by a bank is not lower than its total cost of securing funds from the international financial market at the world interest factor \(R_w\). Moreover, equation (15) is the incentive-compatibility (or truth-telling) constraint which prevents establishments under the good state from misreporting the bad state to a financial intermediary.\(^\text{16}\) Equations (16)-(17) require that the realized utility must be nonnegative in the bad state, regardless of whether a firm is audited by the bank or not.\(^\text{17}\) Finally, equation (18) is the participation constraint on a producing agent’s expected utility across the two states (13) to be at least the reservation level \(u\).

**Proposition 2.** For a given probability of tax auditing \(\eta\), the optimal contract that a competitive bank offers to agent-\(\phi\) is characterized by

\[ T^a = T_1 = \kappa_1, \quad (19) \]

\[ T_2 = \kappa_2 - (1 - p) (\kappa_2 - \kappa_1), \quad \text{and} \quad (20) \]

\(^{16}\)Notice that \(r^*(\eta)(\kappa_2 - \kappa_1)\) is the difference of after-tax output/sales between the good and bad states. With probability \(1 - p\), an unaudited agent who is in the good state but misreports the bad state can enjoy \(r^*(\eta)(\kappa_1 - T_1)\) as well as the extra amount of \(r^*(\eta)(\kappa_2 - \kappa_1)\) for consumption.

\(^{17}\)As in Bernanke and Gertler (1989), these two inequalities are the “limited liability” constraints that restrict each firm’s ability to pay the bank if its project’s outcome turns to be bad.
Based on the revelation principle, setting the risk-neutral agent’s utility level to zero in either scenario of the bad state (being audited or not) will eliminate its incentive for misreporting. Hence, equations (16) and (17) will be binding with $T^a = T_1 = \kappa_1 - \frac{e_1}{r^*(\eta)}$. It follows that there are two possible outcomes after the firm makes its loan payment in the bad state: (i) the remaining working capital $[\kappa_1 - pT^a - (1 - p)T_1]$ is positive, thus an agent will expend labor hours to produce output with $y_1^{after-tax} = e_1 > 0$; and (ii) the establishment does not retain any working capital under $T^a = T_1 = \kappa_1$, thus no labor effort is expended and no output will be produced $\left(y_1^{after-tax} = e_1 = 0\right)$. In the Appendix A, we show that the agent’s utility is higher under (ii), hence the financial intermediary will take away all working capital from firms when the bad state takes place. As a consequence, the extra consumption that an unaudited firm can enjoy from misreporting the good state, as in the first term from the right-hand-side of (15), is equal to $(1 - p)\pi^*(\eta)(\kappa_2 - \kappa_1)$. It follows that the binding incentive-compatibility constraint (15) results in $T_2 = \kappa_2 - (1 - p)(\kappa_2 - \kappa_1)$. Substituting these expressions for $T^a$, $T_1$ and $T_2$ into equation (14) with equality (i.e. the zero-profit condition) yields that the competitive bank’s optimal auditing probability $p$ is given by (21). This in turn implies that certis paribus less-efficient agents, who (if funded) borrow a relatively higher amount of resources, will be audited more often by the bank $\left(\frac{\partial p}{\partial \phi} > 0\right)$.

In terms of the range of $\phi$ over which firms will receive loans from financial intermediaries, we first set $p = 0$ and find that

$$\phi = \frac{\kappa_1}{R_w}. \quad (22)$$

For those agents with $\phi < \phi$, their production efficiency is so high that they always report the true outcome of capital acquisition to the bank, and they are able to pay back the loan’s full amount even in the bad state. Therefore, these firms will not be audited by the bank and there is no agency problem associated with them. Next, we set $p = 1$ and find that

$$\bar{\phi} = \frac{\kappa_1 + \pi_2(\kappa_2 - \kappa_1) - \pi_1 \gamma}{R_w}. \quad (23)$$

For those agents with $\phi > \bar{\phi}$, their production efficiency is so low that they are unable to make the loan payment even in the good state.

Moreover, substituting the optimal financial contract (19)-(21) into equation (18) yields that the critical level of $\phi$ which will bind the agent’s participation constraint is given by
\[
\phi^c(\eta) = \frac{\kappa_1 + [\pi_2(\kappa_2 - \kappa_1) - \pi_1 \gamma]}{R_w} \left[1 - \frac{\frac{e_2 + \frac{\gamma}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)}}{r^*(\eta)(\kappa_2 - \kappa_1)}\right],
\]

where \(\frac{e_2 + \frac{\gamma}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \in (0, 1)\). For those inefficient agents with \(\phi > \phi^c(\eta)\), they will choose not to participate in the credit market at the outset, thus no output are produced by these firms.

Since \(0 < \phi < \phi^c(\eta) < \bar{\phi} < 1\), we have shown that agents with \(\phi < \bar{\phi}\) will borrow from the financial intermediary without any informational imperfection; that the economy’s aggregate amount of lending/borrowing with costly monitoring is equal to \(\phi^c(\eta) - \bar{\phi}\); and that agents with \(\phi > \phi^c(\eta)\) will not receive bank loans.

## 5 Financial Development and Optimal Tax Enforcement

In this section, we first consider a benevolent government that chooses the optimal probability of tax auditing \(\eta^*\) to maximize the economy’s social welfare \(SW\), which is defined as the aggregate expected utilities across all agents:

\[
SW = \int_0^{\phi^c(\eta)} \left\{ \pi_1 \left[ r^*(\eta)(\kappa_1 - pT^a) - (1 - p)T_1 - e_1 \right] - u_1 + \pi_2 \left[ r^*(\eta)(\kappa_2 - T_2) - e_2 \right] - u_2 \right\} h(\phi) d\phi + \int_{\phi^c(\eta)}^1 u h(\phi) d\phi,
\]

where the vector \(\{T^a, T_1, T_2, p\}\) is taken from equations (19)-(21) that characterize the optimal financial contract; \(U_1\) and \(U_2\) represent producing agents’ expected utility in the bad and good states, respectively; \(h(\phi) = 1\) is the probability density of an uniform distribution over the interval \([0, 1]\); and \(u\) is the reservation utility from an outside option that can be accessed by agents who do not borrow and produce. Based on the analysis from the previous section, agents/firms in our model can be divided into the following three subgroups:

(a) Agents with \(\phi \in [0, \bar{\phi}]\), where \(\bar{\phi}\) is given by (22) – these firms are sufficiently efficient without default risk, thus the bank will not audit them in the bad state. Substituting (19), (20), (22) and \(p = 0\) into (25) yields that the aggregate welfare for this subgroup is

\[
SW_a = \int_0^{\phi} \pi_2 \left[ r^*(\eta)(\kappa_2 - \kappa_1) - e_2 \right] h(\phi) d\phi.
\]

\(^{18}\)Under the optimal financial contract with \(T^a = T_1 = \kappa_1\), the expected utility (13) is equal to \(\pi_2(1 - p)r^*(\eta)(\kappa_2 - \kappa_1) - e_2\), which must be greater than or equal to the reservation utility \(u\). If \(\frac{e_2 + \frac{\gamma}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} > r^*(\eta)(\kappa_2 - \kappa_1)\), then agents’ expected utility will be less than \(u\) in that \(0 \leq p \leq 1\). As a result, the restriction of \(\frac{e_2 + \frac{\gamma}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \in (0, 1)\) is imposed.

\(^{19}\)Since the bank’s auditing probability \(p \in [0, 1]\), the denominator in equation (21), given by \(\pi_2(\kappa_2 - \kappa_1) - \pi_1 \gamma\), is positive. This, together with \(0 < \frac{e_2 + \frac{\gamma}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} < 1\), implies that \(0 < \phi < \phi^c(\eta) < \bar{\phi} < 1\).
(b) Agents with \( \phi \in [\phi^c(\eta), \phi^c(\eta)] \), where \( \phi^c(\eta) \) is given by (24) – these firms will receive bank loans under asymmetric information in credit markets, subject to the bank-auditing probability \( p \in (0, 1) \) à la (21). It is straightforward to show that the aggregate welfare for this subgroup is

\[
SW_b = \int_{\phi}^{\phi^c(\eta)} \pi_2[(1 - p)r^*(\eta)(\kappa_2 - \kappa_1) - e_2]h(\phi)d\phi.
\]  

(27)

(c) Agents with \( \phi \in [\phi^c(\eta), 1] \) – these inefficient agents do not receive bank loans, hence they will not engage in producing any output. It follows that the aggregate welfare for this subgroup is

\[
SW_c = \int_{\phi^c(\eta)}^{1} \pi h(\phi)d\phi.
\]  

(28)

In the Appendix B, we use (26)-(28) to derive the first-order condition for the government’s optimization problem with respect to its tax-auditing probability \( \frac{\partial SW}{\partial \eta} = 0 \); and then find that the expected after-tax rate of returns from firms’ production and tax evasion decisions will be a constant, denoted as \( M \in (0, 1) \), in equilibrium:

\[
r^* = \frac{1}{(\kappa_2 - \kappa_1)} \sqrt{\frac{[\pi_2(\kappa_2 - \kappa_1) - \pi_1 \gamma][u(e_2 + \frac{u}{\pi_2}) + \frac{\pi_2}{\pi_2}e_2 - (\frac{u}{\pi_2})^2]}{\pi_2 \kappa_1 + \frac{\pi_2}{\pi_2}[\pi_2(\kappa_2 - \kappa_1) - \pi_1 \gamma]}} = M.
\]  

(29)

**Proposition 3.** There exists a unique optimal probability of tax auditing \( \eta^* \in (0, 1) \) in our model economy.

Using equations (11) and (12), it is straightforward to show that

\[
\frac{\partial^2 r^*}{\partial \eta^2} = \frac{-1}{[1 - 4(\theta - \eta)]^{3/2}} < 0.
\]  

(30)

This finding, together with \( \frac{\partial r^*}{\partial \eta} > 0 \), implies that \( r^*(\eta) \) is an upward-sloping curve which is concave to the origin, as shown in Figure 2.21 On the other hand, Figure 2 also illustrates that the equilibrium condition (29) is depicted as a horizontal line. It follows that their intersection will yield our model’s unique optimal tax-auditing probability \( \eta^* \), and that the corresponding optimal \( \tau^* \) and \( \beta^* \) can then be derived through equations (6) and (8).

We then take total differentiation on the equality between (11) and (29) to obtain

---

20Recall that \( r^* \) is equal to the ratio of firms’ after-tax to before-tax output \( \frac{y_{after-tax}}{y} \), hence it is a positive number which is smaller than one.

21Plugging \( \eta = 0 \) into equation (11) shows that the intercept on the vertical axis of Figure 2 is \( r^* = \frac{1+2(1-\theta)+\sqrt{1-4\theta}}{4} > 0 \).
Figure 2: Optimal Tax-Auditing Probability $\eta^*$ when $\gamma$ Falls

$$
\frac{d\eta^*}{d\gamma} = -\frac{\pi_1\kappa_1}{2} \frac{\sqrt{\frac{u}{\pi_2}(e_2 + \frac{u}{\pi_2}) + \frac{1}{2}e_2^2 - \frac{1}{4}e_2^2}}{2(\kappa_2 - \kappa_1)[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma]^2} \left\{ \frac{2[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma]}{2\kappa_2 + [\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma]} \right\}^\frac{3}{2} < 0, \quad (31)
$$

which indicates that the government in countries with a more developed financial sector, represented by a lower level of agency cost $\gamma$, will adopt a higher tax-detection probability $\eta^*$. Next, using the chain rule leads to the following relationship between firms’ optimal fraction of sales reported to the tax authority (or tax compliance) and financial development:

$$
\frac{d\beta^*}{d\gamma} = \frac{d\beta^*}{d\eta^*} \frac{d\eta^*}{d\gamma} < 0,
$$

where $\frac{d\beta^*}{d\eta^*}$ can easily be derived from equation (8) evaluated at the optimal probability of tax auditing $\eta^*$. It follows that

**Proposition 4.** Financial development, measured by a lower level of the agency cost $\gamma$, leads the government to adopt a higher probability of tax auditing and thus a decrease in the incidence of tax evasion.

The intuition for Proposition 4 can be understood as follows. We note that a reduction in the agency or monitoring cost will generate two opposite effects. On the one hand, using equation (29) shows that a smaller $\gamma$ shifts up the horizontal line in Figure 2 and thus
raises the equilibrium expected after-tax rate of returns from firms’ production and tax evasion decisions to \( r^* = M' > M \). That is, a more developed financial sector mitigates the problem of asymmetric information in credit markets, which in turn will stimulate more investment projects to be implemented. It follows that the economy’s total output becomes higher, and that producing agents’ aggregate utilities will rise because of their higher consumption spending (see equations 26 and 27). On the other hand, a smaller \( \gamma \) alleviates the degree of equilibrium credit rationing in financial markets, hence more less-efficient firms will receive bank loans. This leads to an increase in the total amount of lending/borrowing under costly monitoring given by \( \phi'(\eta) - \phi \) (see equation 24 with \( \frac{\partial \phi'(\eta)}{\partial \gamma} < 0 \)). It follows that the economy’s overall production efficiency will fall in that less-efficient agents are audited more often by the financial intermediary. Figure 2 together with our total-differentiation derivation à la (31) illustrate that after taking into account these counteracting impacts, the government will choose to raise its optimal probability of tax auditing \( (\eta' > \eta^*) \) to maintain the maximum of the social welfare function (25). Since a higher tax-auditing probability leads to more tax compliance, i.e. \( \frac{\partial \eta^*}{\partial \gamma} > 0 \) à la (8), an empirically-realistic negative correlation between the level of financial development and firms’ tax evasion ensues. As a result, it is socially optimal for the government in countries with a less developed financial sector to implement a relatively looser policy of tax enforcement, which in turn will lead to a higher degree of tax evasion.

6 Financial Development and the Taxes-to-GDP Ratio

As pointed out by the review article of Besley and Persson (2014), it is now well known that the proportion of a country’s GDP accounted by the government’s tax revenue is higher in developed economies than their developing counterpart. Since financial markets are relatively more developed in rich nations, this empirical regularity implies that the taxes-to-output ratio is positively correlated with the degree of financial development. In the context of our baseline model economy analyzed above, we note that under the optimal financial contract characterized by equations (19)-(21), the expected amount of (gross) output that a funded agent-\( \phi \) will produce in the good state (with probability \( \pi_2 \)) is equal to either \((\kappa_2 - \kappa_1)\) for \( \phi \in [0, \phi] \) or \((1-p)(\kappa_2 - \kappa_1)\) for \( \phi \in [\phi, \phi'(\eta)] \). It follows that the total tax revenue collected by the government is

\[
TR = \theta \left[ \int_{0}^{\phi} \pi_2(\kappa_2 - \kappa_1)h(\phi)d\phi + \int_{\phi}^{\phi'(\eta)} \pi_2(1-p)(\kappa_2 - \kappa_1)h(\phi)d\phi \right],
\]

where \( 0 < \theta < 1 \) is the fraction of each operating firms’ reported sales that will be levied by the fiscal authority. Moreover, since a producing agent-\( \phi \) needs to expend \( \phi \) units of resources
for generating output, the economy’s GDP or its expected aggregate value-added/net output is

\[ GDP = \int_0^\phi [\mathbb{E}(\kappa_2 - \kappa_1) - \phi] h(\phi)d\phi + \int_{\phi_{max}}^{\phi(\eta)} [\mathbb{E}(\kappa_2(1 - p)(\kappa_2 - \kappa_1) - \phi] h(\phi)d\phi. \]  

(34)

After substituting the equilibrium \( r^* \) from (29) into the expression of \( \phi(\eta) \) given by equation (24), Appendix C proves that the sign of \( \frac{\partial(\text{TR}_GDP)}{\partial \tau} \) is always negative. Hence, this result can be summarized as

**Proposition 5.** Under the optimal policy of tax enforcement, financial development or a lower level of the agency cost \( \gamma \) leads to an increase in the ratio of tax revenue over GDP.

The intuition for this Proposition is straightforward. As discussed earlier, a smaller \( \gamma \) will enable more firms to receive bank loans and produce output. Since a constant fraction of agents’ sales revenue is levied by the government, financial development raises the total amount of taxes collected. However, a smaller \( \gamma \) that induces more less-efficient firms to borrow and produce will also decrease the economy’s overall production efficiency as each of these additional establishments needs to use a relatively larger amount of resources. It follows that their contribution to the value-added GDP will be lower compared to that from more-efficient agents. Proposition 5 analytically shows that financial development generates a relatively higher increase in tax revenue than that in aggregate net output, hence the taxes-to-GDP ratio will rise within our model economy.

As it turns out, this positive correlation between financial development and the output fraction of tax revenue is related to a debate on the role of “state capacity” in economic development. One strand of literature emphasizes that developing countries exhibit weaker ability to impose income taxation, which in turn leads to a lower taxes-to-GDP ratio. In particular, a series of papers by Besley and Persson (2009, 2010, 2013) focus on two aspects of state capacity: (i) fiscal capacity (the power of state to collect taxes) and (ii) legal capacity (the power of state regulation on market activities). Moreover, these authors postulate state capacities to be modeled as the government’s forward-looking investment, and examine the development and growth effects of some determinants of state building, such as the risk of external/internal conflict, political instability and dependence on natural resources. Another strand of literature stresses that developing countries possess lower will to collect tax revenue due to the presence of more stringent informational and/or enforcement constraints. For example, Gordon and Li (2009) stipulate that the government can make use of information gathered from the financial system (e.g., bank records) to help enforce its optimal tax policy. It follows that firms must balance the economic benefits of accessing the financial sector against
the resulting tax liabilities. As a result, a smaller taxes-to-GDP ratio will prevail in poor countries because the advantage from gaining access to formal financial markets within these economies is much more modest. In this paper, we have shown that with a less-developed financial sector, it is socially optimal for developing countries to adopt a lower tax-detection probability and thus a relatively smaller tax proportion out of GDP. Conceptually, our analysis can be regarded as advancing the “will to collect” viewpoint although the underlying economic mechanism is different from those considered in previous studies.

7 Extension: Firm Size and Optimal Tax Compliance

Some recent studies have empirically examined the incidence of tax evasion within developing economies at the firm level. Representative examples include Bigio and Zilberman (2011), Kleven et al. (2016), Carrillo et al. (2017), Zareh and Peichl (2017), and Bachas et al. (2018), among others. A robust piece of evidence documented by this literature is that large firms comply with taxes, whereas small firms do not. Using the number of employees as the indicator for firm size, Bachas et al. (2018) report a strong positive link between an industry’s average firm size and the associated degree of tax compliance. Moreover, Kleven et al. (2016) find a positive correlation between tax compliance and firm size both across countries and across firms within a country.

For the sake of analytical tractability, our baseline model derives a single optimal tax-auditing probability \( \eta^* \) that will be faced by all agents. In order to analyze the aforementioned empirical interrelations between firm size and tax compliance, this section explores an extended setting that allows for size-dependent probabilities of tax detection. We first note that the output level of each operating firm is equal to \( \pi_2(\kappa_2 - T_2) \), where \( T_2 = \kappa_2 - (1 - p)(\kappa_2 - \kappa_1) \) and the bank’s optimal auditing probability \( p \) depends on an agent’s type represented by \( \phi \). In particular, since a low-\( \phi \) establishment is audited less often (see equation 21 with \( \frac{\partial p}{\partial \phi} > 0 \)), the quantity of its output will be higher than that by a high-\( \phi \) counterpart. It follows that among the two groups of agents who are engaged in borrowing and production, those in the first subset with \( \phi \in [0, \bar{\phi}] \) can be classified as “large firms” which produce more output, while the remaining establishments can be categorized as “small firms”. Under these circumstances, the government is postulated to impose separate tax-enforcement probabilities (denoted as \( \eta_L \) and \( \eta_S \)) on large and small firms, respectively. We further assume that public spending is financed by a common \( \theta \in (0, 1) \) percentage of the government’s total revenue (including taxes and

\[ \text{Newbery and Stern (1987) and Gordon (2009) have collected studies analyzing developing countries from this perspective.} \]
penalties), hence its balanced-budget equation (5) will hold for either (large or small) group of agents.

Following the same procedure of section 3, we find that the size-specific tax rates and fractions of producing agents’ reported output are

\[
\tau^*(\eta_i) = \frac{1 + 2\eta_i - \sqrt{1 - 4(\theta - \eta_i)}}{2} \quad \text{and} \quad \beta^*(\eta_i) = \frac{1 + \sqrt{1 - 4(\theta - \eta_i)}}{2},
\]

and that the resulting expected after-tax rate of returns from their production and tax evasion decisions \(r^*(\eta_i)\) are given by

\[
\left(1 - \eta_i\right) \left[1 - \sqrt{1 - 4(\theta - \eta_i)}\right] + \frac{\left[1 + \sqrt{1 - 4(\theta - \eta_i)}\right]}{4} \left[1 - 2\eta_i + \sqrt{1 - 4(\theta - \eta_i)}\right] - \frac{\left[1 - \sqrt{1 - 4(\theta - \eta_i)}\right]^2}{8},
\]

where \(i = L, S\). As in our baseline model, it can be shown that \(\frac{\partial r^*(\eta_i)}{\partial \eta_i} > 0\) and \(\frac{\partial^2 r^*(\eta_i)}{\partial \eta_i^2} < 0\).

Next, since firm size does not affect any derivations of the financial optimal contract between the bank and operating establishments, Proposition 2 will continue to hold and the expression of \(\phi \ a \ la \ (22)\) remains unchanged; whereas the critical level of \(\phi\) that governs whether a small firm borrows/produces or not is

\[
\phi^*(\eta_S) = \frac{\kappa_1 + \left[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma\right]}{R_{aw} - \frac{\frac{\kappa_1^2 + 2\kappa_1}{\tau^*(\eta_S)(\kappa_2 - \kappa_1)}}}. \quad (37)
\]

In what follows, we first examine the setting in which firm size (being large or small) is known to the government. In this case, there is no incentive for a large firm to pretend as a small establishment, and vice versa. We then study the formulation in which firm size is private information, which may therefore create an incentive for a large/small establishment to misreport as small/large.

### 7.1 When Firm Size is Known to the Government

We begin with the benchmark environment in which the tax authority is postulated to know each producing agent’s size (large or small) when its sales revenue is reported. In this case, a benevolent government selects two tax-detection probabilities to maximize the economy’s social welfare function \(\tilde{SW}\) given by
The first-order condition with respect to $L$ is

$$\frac{\partial g_{SW}}{\partial L} = \pi_2 (\kappa_2 - \kappa_1) \frac{\rho_r}{R_w} \frac{\partial r^*(\eta_L)}{\partial \eta_L} > 0,$$

which implies that the optimal $\eta_L^*$ should be set as high as possible such that each large firm will report the true level of its output, thus there is no tax evasion with $\beta_L^* = 1$. Using equations (35) and (36), we can then easily obtain $\eta_L^* = \tau_L^* = \theta$ and $r_L^* = 1 - \theta$. On the other hand, the corresponding first-order condition on $S$ is

$$\frac{\partial g_{SW}}{\partial S} = \frac{\partial r^*(\eta_S)}{\partial \eta_S} \left( \frac{\rho_r}{R_w} \frac{\partial r^*(\eta_L)}{\partial \eta_L} \right) \left( \frac{\pi_2}{r^*(\eta_S)(\kappa_2 - \kappa_1)^2} \right) \left( \frac{\pi_2}{2} \left[ \left( \frac{\rho_r}{\pi_2} \right)^2 - \frac{\rho_r}{\pi_2} + \frac{\rho_r}{\pi_2} + e_2 \right] \right).$$

Setting $\frac{\partial g_{SW}}{\partial \eta_S} = 0$ yields that the equilibrium expected after-tax rate of returns from small firms' production and tax evasion decisions will be a constant given by

$$r_S^* = \frac{e_2 + \frac{\rho_r}{\pi_2}}{\kappa_2 - \kappa_1} = M_S \in (0, 1).$$

It follows that the optimal $\eta_S^*$ can be derived through the equality of $M_S$ and $r^*(\eta_S)$ as in equation (36).\textsuperscript{23}

To gain further insights of our findings, Figure 3 compares $\eta_L^*$ and $\eta_S^*$ versus $\eta^*$ from our baseline model under a single tax-auditing probability. Since the degree of firms' tax compliance is monotonically increasing in the aggressiveness of the government’s optimal tax enforcement policy, together with the result that $\beta_L^*$ takes on the highest possible value of one, it is immediately clear that $\eta_L^*$ is higher than $\eta_S^*$ and $\eta^*$. In addition, using equation (41) and the restriction that $\frac{\rho_r}{\kappa_2 - \kappa_1} \in (0, 1)$ per footnote 18, it is straightforward to show that $r^* - r_S^* > 0$, which in turn implies that $\eta^* > \eta_S^*$.\textsuperscript{23}

\textsuperscript{23}Notice that the level of agency/monitoring cost does not enter the equilibrium expression of $r_S^* = M_S$ because large and small firms are separated by $\phi$ that is independent of $\gamma$. It follows that changes in the degree of financial development will not affect $\eta_S^*$, and thus $\beta_S^*$ as well.
Proposition 6. When each producing firm’s being large or small is known to the government under size-dependent probabilities of tax detection, the optimal tax-auditing probability on large firms is higher than that on small firms, \( \eta^*_{L} > \eta^*_{S} \). Moreover, large firms fully comply with taxes with \( \beta^*_{L} = 1 \), whereas small firms evade taxes with \( \beta^*_{S} \in (0, 1) \).

The underlying intuition for this Proposition is straightforward. Recall that large firms are efficient agents whose \( \phi \)'s lie within the interval between 0 and \( \phi \left( \frac{\alpha_1}{\pi w} \right) \), and that they are not subject to the problem of asymmetric information in credit markets. Since an increase in \( \eta_L \) exerts no impact on \( \phi \) but will raise \( r^*(\eta_L) \) per equation (36), the socially optimal tax-auditing probability on large firms should be set as high as possible, leading to \( \eta^*_{L} = \theta \) and \( \beta^*_{L} = 1 \). On the contrary, not only an increase in \( \eta_S \) yields a higher \( r^*(\eta_S) \), it also raises \( \phi^c(\eta_S) \) (see equation 37 with \( \frac{\partial \phi^c(\eta_S)}{\partial \eta_S} > 0 \)) thus more less-efficient firms will be induced to enter into financial contracts. While a higher \( r^*(\eta_S) \) generates an increase in the aggregate expected utilities, a higher \( \phi^c(\eta_S) \) decreases the economy’s overall production efficiency. By balancing these two opposite effects, we obtain an interior solution for the optimal \( \eta^*_S \) that will be lower than \( \eta^*_L \). It follows that the degree of small firms’ tax compliance is relatively lower, \( 0 < \beta^*_S < \beta^*_L = 1 \). Finally, Figure 3 shows that as standard in the signalling literature, the optimal “pooling” probability of tax detection \( \eta^* \) (derived in section 5 as shown in Figure 2) will appear in between those under the “separating” optimum (derived in this subsection), i.e. \( \eta^*_S < \eta^* < \eta^*_L \).
7.2 When Firm Size is Private Information

When each producing agent’s size is private information, we will encounter another type of asymmetric information since a large firm may have incentive to mimic a small establishment, and vice versa. In particular, if a large firm with \( \phi \in [0, \tilde{\phi}] \) would like to misreport, it must pretend to be the agent with \( \phi^c(\eta_S) \) because this specific small establishment’s before-tax output (and thus its tax burden) in the good state is the lowest:

\[
y^c_S = \pi_2 (1 - p^c) (\kappa_2 - \kappa_1), \quad \text{where} \quad p^c = \frac{\phi^c(\eta_S) R_w - \kappa_1}{\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma}
\]

and the critical \( \phi^c(\eta_S) \) is given by (37). On the other hand, if a large firm does not pretend to be small, then its before-tax output in the good state is

\[
y_L = \pi_2 (\kappa_2 - \kappa_1).
\]

Since \( y_L \) is higher than the output level of any small firm and each operating establishment’s reported sales are subject to the same collection rate \( \theta \), a “mimicking” small firm will pay a higher amount of taxes than that under truthful reporting. As a result, the incentive-compatibility constraint for small establishments is always slack and can be omitted.

In order to induce large firms to reveal their true size to the fiscal authority, our subsequent analysis will examine two mechanisms that can achieve this desired outcome. First, we consider a scenario in which the government collects an additional penalty (denoted as \( \delta \)) when pretending large firms are detected as small establishments. In this case, the resulting expected after-tax output is given by

\[
y^\text{after-tax}_L = \beta^*_S (1 - \tau^*_S) y^*_S + (1 - \beta^*_S)(1 - \eta_S) y^c_S - \frac{(1 - \beta^*_S)^2}{2} y^*_S + (1 - \eta_S) \{ \theta(y_L - y^c_S) \} - \eta_S \* \delta,
\]

where \( (y^*_S)^\text{after-tax} \) denotes the expected after-tax sales revenue that the agent with \( \phi^c(\eta_S) \) is able to generate. Moreover, a “mimicking” large firm can consume an extra amount of \( \theta(y_L - y^*_S) \) if undetected, and will be additionally punished with \( \delta \) if audited. We also note that when a large firm does not act as if it were small, its expected after-tax output is

\[
y^\text{after-tax}_L = \beta^*_L (1 - \tau^*_L) y_L + (1 - \beta^*_L)(1 - \eta_L) y_L - \frac{(1 - \beta^*_L)^2}{2} y_L.
\]

It follows that the requisite level of penalty to equate \( y^\text{after-tax}_L \) and \( y^\text{after-tax}_L \) is
\[
\delta^* = \frac{1}{\eta_S} \left\{ (y_S^c)^{after-tax} + (1 - \eta_S) \left[ \theta(y_L - y_S^c) \right] - y_L^{after-tax} \right\},
\]

under which large establishments do not have incentive to pretend as small firms. Consequently, the government’s optimization problem will be the same as that analyzed in the preceding subsection when firm size is not private information.

**Proposition 7.** When firm size is private information and the government imposes an additional penalty \( \delta^* \) on pretending large firms if detected, the optimal tax-auditing probabilities on large and small establishments are identical to those reported in Proposition 6.

Second, we consider a scenario in which additional punishment \( \delta^* \) is not available to the fiscal authority for distinguishing small from “mimicking” large establishments. As a result, a pretending large firm can enjoy the bonus consumption of \( \theta(y_L - y_S^c) \), regardless of whether it is audited or not; and thus its expected after-tax output is

\[
\tilde{y}_{L}^{after-tax} = \beta_{S}^* (1 - \tau_{S}^*) y_S + (1 - \beta_{S}^*) (1 - \eta_{S}) y_S - \frac{(1 - \beta_{S}^*)^2}{2} y_S^c + \theta(y_L - y_S^c).
\]

In this environment, the government will maximize the economy’s social welfare function \( \tilde{SW} \) as in (38), subject to the large firm’s incentive-compatibility constraint given by

\[
\pi_2 \left( y_{L}^{after-tax} - e_2 \right) \geq \pi_2 \left( \tilde{y}_{L}^{after-tax} - e_2 \right),
\]

which states that the expected utility of a truth-telling large establishment must be greater than or equal to what it could obtain from pretending as a small firm.

Using \( \lambda \) to denote the Lagrange multiplier associated with the constraint (48), it is straightforward to show that the first-order condition with regard to \( \eta_L \) is given by

\[
\pi_2 \frac{\partial \tilde{SW}_a}{\partial \eta_L} + \lambda \left[ \frac{\partial y_{L}^{after-tax}}{\partial \eta_L} - \frac{\partial \tilde{y}_{L}^{after-tax}}{\partial \eta_L} \right],
\]

where \( \frac{\partial y_{L}^{after-tax}}{\partial \eta_L} = \frac{\partial \tilde{y}_{L}^{after-tax}}{\partial \eta_L} = 0 \) through the envelope theorem. Hence, this optimality condition is reduced to

\[
\pi_2 \frac{\partial \tilde{SW}_a}{\partial \eta_L} = \pi_2 (\kappa_2 - \kappa_1) \frac{\kappa_1}{R_w} \left[ \frac{\partial r^*(\eta_L)}{\partial \eta_L} \right]_{positive} > 0,
\]
which is exactly the same as equation (39). It follows that as in the previous settings, \( \eta_L^* = \tau_L^* = \theta, r_L^* = 1 - \theta \) and \( \beta_L^* = 1 \), i.e. large firms will truthfully report their sales revenue and fully comply with taxes. Next, the corresponding first-order condition on \( \eta_S \) is

\[
\pi_2 \frac{\partial \tilde{SW}_b}{\partial \eta_S} + \frac{\partial \tilde{SW}_c}{\partial \eta_S} + \lambda \left[ \frac{\partial y_{Lafter-tax}}{\partial \eta_S} - \frac{\partial \tilde{y}_{Lafter-tax}}{\partial \eta_S} \right],
\]

(51)

where \( \frac{\partial \tilde{y}_{Lafter-tax}}{\partial \eta_S} = 0 \) through the envelope theorem. Setting (51) to zero yields that

\[
\lambda^* = \frac{\pi_2 \frac{\partial \tilde{SW}_b}{\partial \eta_S} + \frac{\partial \tilde{SW}_c}{\partial \eta_S}}{-\theta \frac{\partial y_S}{\partial \eta_S}},
\]

(52)

which represents the optimum value of marginal utility burden coming from (48). Finally, the binding incentive-compatibility constraint for large firms leads to

\[
y_{Lafter-tax} = \tilde{y}_{Lafter-tax},
\]

(53)

which can then be rewritten as

\[
(1 - \theta)y_L = r^*(\eta_S)y_S^c + \theta(y_L - y_S^c),
\]

(54)

where \( r^*(\eta_S) \) is given by (41), together with \( y_S^c \) and \( y_L \) being taken from equations (42)-(43). It follows that the resulting optimal \( \eta_S^* \) will prevent a large firm from pretending as small.\(^{24}\)

Since equation (54) is highly nonlinear in \( \eta_S \), we are unable to derive the analytical expression(s) of its solution(s). As a result, numerical experiments are conducted to quantitatively explore the existence and uniqueness of the optimal tax-auditing probability on small firms. Due to space limitation, we will report a representative set of results as follows. Under the parameterization with \( \pi_1 = 0.2, \theta = 0.55, e_2 = u = 0.5, \kappa_1 = 1 \) and \( \kappa_2 = 4 \), it can be shown that there are two solutions to (54): \( \eta_S^* = 0.404 \) and \( \eta_S^* = 0.759 \). However, the latter solution is ruled out because plugging it into equation (36) leads to \( \beta_S^* = 1.177 \), which is not feasible. We also note that given the unique \( \eta_S^* = 0.404 < \eta_L^* = \theta = 0.55 \),\(^{25} \) the associated tax compliance rates are \( \beta_S^* = 0.822 < \beta_L^* = 1 \), i.e. large firms fully comply with taxes, whereas small

\(^{24}\)Substituting \( \phi'(\eta_S) \) from (37) into the bank’s auditing probability (42) shows that \( p^c \) is independent of \( R_w \) and \( \gamma \). These two parameters thus do not affect \( y_S^c \) and will not enter the large firm’s binding incentive-compatibility constraint (54). It follows that \( R_w \) and \( \gamma \) do not play any role in our quantitative analysis below for numerically solving \( \eta_S^* \) and \( \beta_S^* \).

\(^{25}\)Based on this interior solution of \( \eta_S^* \), the numerical values for all the remaining endogenous variables, including \( \tau_S^*, r_S^* \) and \( \lambda^* \), can then be easily obtained.
firms evade taxes. Moreover, these findings turn out to be qualitatively robust to a wide range of alternative combinations over the parameter space of \( \{\pi_1, \theta, \epsilon_2, \kappa_1, \kappa_2\} \).\(^{26}\)

**Proposition 8.** When firm size is private information under a binding incentive-compatibility constraint on large establishments, the optimal tax-auditing probability on small firms, as well as their tax compliance rate, are lower than those for large firms: \( \eta^*_S < \eta^*_L = \theta \) and \( \beta^*_S < \beta^*_L = 1 \).

In sum, this section examines three formulations within our small-open-economy model that allows for size-dependent probabilities of tax enforcement: (i) when firm size is known to the government; (ii) when firm size is private information and the government imposes additional penalty on pretending large establishments; and (iii) when firm size is private information and the government must satisfy the large firm’s incentive-compatibility constraint. Our analysis finds that in each of these settings, large firms fully comply with taxes while small establishments evade taxes. Since large firms always report the true outcome of their capital acquisition to the bank, and they are able to pay back the loan’s full amount even in the bad state, the socially optimal tax-auditing probability on them will be set as high as possible. As a result, these efficient agents choose to truthfully report their sales revenue to the government. By contrast, the socially optimal tax-detection probability on small firms needs to balance two opposite effects: an increase in the expected after-tax rate of returns from their production and tax evasion decisions versus a decrease in the economy’s overall production efficiency because more less-efficient agents are induced to enter the financial market. Therefore, small establishments will conceal a portion of their output from the fiscal authority. These results together indicate an empirically-realistic positive correlation between firm size and tax compliance within developing countries, as documented by Zilberman (2011), Kleven et al. (2016), Carrillo et al. (2017), Zareh and Peichl (2017), and Bachas et al. (2018).

8 Conclusion

In this paper, we begin with integrating asymmetric information in credit markets with the government’s optimal policy of tax enforcement to examine the theoretical interrelations between financial development and tax evasion within a small open economy. Heterogenous agents/firms seek to obtain state-contingent working capital from competitive banks for producing output, and the government imposes a tax rate on firms’ total reported sales. Both the true outcome of production and the accurate amount of sales are each firm’s private in-

\(^{26}\)In addition, we conduct comparative statics exercise and find that \( \frac{\partial \eta^*_S}{\partial \pi_1} < 0; \frac{\partial \eta^*_S}{\partial \theta} > 0; \frac{\partial \beta^*_S}{\partial \epsilon_2} > 0; \frac{\partial \beta^*_S}{\partial \kappa_1} > 0; \frac{\partial \beta^*_S}{\partial \kappa_2} > 0; \frac{\partial \eta^*_L}{\partial \pi_1} > 0; \) and \( \frac{\partial \eta^*_L}{\partial \theta} < 0. \) All the quantitative results from our sensitivity analysis are available upon request.
formation. Therefore, agents may misreport their capital possession to the bank as well as underreport their sales to the fiscal authority. We first analyze the firm’s optimal decisions on output production and tax evasion, and find that raising the government’s tax-detection probability leads to more tax compliance by agents. Next, we derive the optimal financial contract that determines the equilibrium measure of establishments that will receive bank loans and produce output. We then solve the optimization problem for a benevolent government, and show that a lower agency cost in credit markets will generate an increase in the socially optimal tax-auditing probability. These findings altogether imply that countries with a more developed financial sector are associated with a less degree of tax evasion on firms’ sales. Our analysis therefore provides a theoretical explanation for the observed negative correlation between financial development and tax evasion documented by recent empirical studies. We also analytically show that a higher probability of tax enforcement will lead to an increase in the proportion of tax revenue over GDP within our baseline model economy. This result in turn implies an empirically-realistic positive correlation between financial development and the taxes-to-GDP ratio. Finally, in an extended setting which allows for size-dependent probabilities of tax detection, we show that in accordance with the empirical evidence, large firms fully comply with taxes while small establishments evade taxes, regardless of whether firm size is private information or not.

Our paper can be extended in several directions. For example, Beck et al. (2014) report that the degree of tax evasion is lower in countries with a better information sharing system. To shed light on this stylized fact, it would be worthwhile to analyze how information sharing (e.g. Pagano and Jappelli [1993] and Padilla and Pagano [2000]) may affect firms’ decision to evade taxation. In addition, it would be worthwhile to extend our static analytical framework to a dynamic setting in which the government may or may not credibly commit to continue with its optimal tax enforcement policy over time. These possible extensions will allow us to examine the robustness of this paper’s theoretical results and policy implications. We plan to pursue these research projects in the near future.
9 Appendix A

Based on the revelation principle, a risk-neutral agent’s expected utility will be equal to zero in the bad state, regardless of whether it is audited by the bank or not. In this Appendix, we will compare the agent’s utility in the good state, which is given by $(1 - p) r^*(\eta) \left[ (\kappa_1 - T_1) + (\kappa_2 - \kappa_1) \right] - e_2$ from the binding incentive-compatibility constraint (15), under (i) $T^a = T_1 = \kappa_1 - \frac{e_1}{r^*(\eta)}$, where $e_1 > 0$ versus that under (ii) $T^a = T_1 = \kappa_1$ and $e_1 = 0$.

In the former case (i), the total loan payment made by a funded firm and the resulting optimal bank-auditing probability are

$$T_2 (e_1 > 0) = \kappa_2 - (1 - p) \left[ \kappa_2 - \kappa_1 + \frac{e_1}{r^*(\eta)} \right], \quad \text{and}$$

$$p (e_1 > 0) = \frac{\phi R_w - \kappa_1 + \frac{e_1}{r^*(\eta)}}{\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma + \frac{e_1}{r^*(\eta)}}. \quad (A.2)$$

It follows that the agent’s utility in the good state is given by

$$U (e_1 > 0) = \frac{[r^*(\eta) (\kappa_2 - \kappa_1) + e_1] \left\{ \pi_1 \kappa_1 + \pi_2 \kappa_2 - \phi R_w - \pi_1 \left[ \gamma + \frac{e_1}{r^*(\eta)} \right] \right\}}{\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma + \frac{e_1}{r^*(\eta)}} - e_2. \quad (A.3)$$

In the latter case (ii), it is straightforward to derive that the corresponding utility in the good state is

$$U (e_1 = 0) = \frac{r^*(\eta) (\kappa_2 - \kappa_1) \left[ \pi_1 (\kappa_1 - \gamma) + \pi_2 \kappa_2 - \phi R_w \right]}{\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma} - e_2. \quad (A.4)$$

Taking the difference between (A.4) and (A.3) shows that $U (e_1 = 0) - U (e_1 > 0)$

$$= \frac{\pi_1 \gamma \left[ \pi_1 (\kappa_1 - \gamma) + \pi_2 \kappa_2 - \phi R_w \right]}{[\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma] \left[ \pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma + \frac{\pi_2 e_1}{r^*(\eta)} \right]} + \frac{\pi_1 e_1 \left[ r^*(\eta) (\kappa_2 - \kappa_1) + e_1 \right]}{r^*(\eta) \left[ \pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma + \frac{\pi_2 e_1}{r^*(\eta)} \right]} > 0. \quad (A.5)$$

As a result, the optimal financial contract between banks and firms will be characterized by $T^a = T_1 = \kappa_1$ and $e_1 = 0$, i.e. agents’ working capital is completely taken away and thus no output is produced in the bad state.

10 Appendix B

Using equation (26) for the aggregate welfare of subgroup (a), it is straightforward to show that
\[
\frac{\partial SW_a}{\partial \eta} = \frac{\partial r^*(\eta)}{\partial \eta} \left\{ \frac{\pi_2 \kappa_1 (\kappa_2 - \kappa_1)}{R_w} \right\}, \tag{B.1}
\]

where \(\frac{\partial r^*(\eta)}{\partial \eta} > 0\) is given by (12). Similarly, we use equation (27) for the aggregate welfare of subgroup (b) to derive that

\[
\frac{\partial SW_b}{\partial \eta} = \frac{\partial r^*(\eta)}{\partial \eta} \left\{ \frac{\pi_2 (\kappa_2 - \kappa_1)(\kappa_2 - \kappa_1) - \pi_2 \gamma}{2R_w} \right\} \left\{ 1 - \frac{e_2^2 - \left( \frac{\mu}{\pi_2} \right)^2}{[r^*(\eta)(\kappa_2 - \kappa_1)]^2} \right\}. \tag{B.2}
\]

Finally, using equation (28) leads to

\[
\frac{\partial SW_c}{\partial \eta} = \frac{\partial r^*(\eta)}{\partial \eta} \left\{ -\frac{u[\pi_2 (\kappa_2 - \kappa_1) - \pi_2 \gamma]}{R_w} \left( \frac{\kappa_2 - \kappa_1}{[r^*(\eta)(\kappa_2 - \kappa_1)]^2} \right) \right\}. \tag{B.3}
\]

The government’s optimal tax-auditing probability \(\eta^*\) is selected through \(\frac{\partial SW_a}{\partial \eta} + \frac{\partial SW_b}{\partial \eta} + \frac{\partial SW_c}{\partial \eta} = 0\). After taking the summation of (B.1), (B.2) and (B.3), we find that this requisite condition yields the following quadratic equation in \(r^*(\eta)\):

\[
[r^*(\eta)(\kappa_2 - \kappa_1)]^2 = \frac{[\pi_2 (\kappa_2 - \kappa_1) - \pi_2 \gamma]u(e_2 + \frac{\mu}{\pi_2}) + \frac{\pi_2 e_2^2 - \left( \frac{\mu}{\pi_2} \right)^2}{\pi_2 \kappa_1 + \frac{\pi_2}{\pi_2} \left( \pi_2 (\kappa_2 - \kappa_1) - \pi_2 \gamma \right)}}. \tag{B.4}
\]

The positive solution to (B.4) results in equation (29) in the main text.

### 11 Appendix C

We use equations (33) and (34) to find that

\[
GDP = \frac{TR}{\theta} - \int_0^{\phi^c(\eta)} \phi h(\phi) d\phi = \frac{TR}{\theta} - \frac{1}{2} [\phi^c(\eta)]^2, \tag{C.1}
\]

which in turn leads to

\[
\frac{GDP}{TR} = \frac{1}{\theta} - \frac{1}{2} \frac{[\phi^c(\eta)]^2}{TR}. \tag{C.2}
\]

Under the bank’s optimal auditing probability \(p\) given by (21), together with \(\phi^c(\eta)\) and \(\phi^c(\eta)\) from (22) and (24), we can express the government’s tax revenue as follows:

\[
TR = \frac{\theta \pi_2 (\kappa_2 - \kappa_1)}{R_w} \left\{ \kappa_1 + \frac{1}{2} [\pi_2 (\kappa_2 - \kappa_1) - \pi_2 \gamma] \left\{ 1 - \left[ \frac{e_2 + \frac{\mu}{\pi_2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \right]^2 \right\} \right\}. \tag{C.3}
\]
where $r^*(\eta)$ is set by the optimal tax-enforcement policy (29). Next, combining equations (24) and (C.3) yields that

$$\frac{[\phi^c(\eta)]^2}{TR} = \frac{(\kappa_1 + \Omega)^2}{\theta \pi_2 (\kappa_2 - \kappa_1) \left\{ \kappa_1 + \Omega \left[ 1 + \frac{e_2 + \frac{n}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \right] \right\}}, \quad \text{(C.4)}$$

where $\Omega \equiv [\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma] \left[ 1 - \frac{e_2 + \frac{n}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \right] > 0$. After some tedious but manageable algebra, it can be shown that

$$\frac{\partial \left\{ \frac{[\phi^c(\eta)]^2}{TR} \right\}}{\partial \gamma} = \frac{\phi^c(\eta) \left\{ 2TR \frac{\partial \phi^c(\eta)}{\partial \gamma} - \phi^c(\eta) \frac{\partial TR}{\partial \gamma} \right\}}{TR^2} = \frac{\Psi}{TR^2}, \quad \text{(C.5)}$$

where

$$\Psi \equiv \theta \pi_2 (\kappa_2 - \kappa_1)(\kappa_1 + \Omega) \left\{ \frac{3}{2} \kappa_1 + \frac{1}{2} \Omega + \frac{1}{2} (\Omega - \kappa_1) \left[ \frac{e_2 + \frac{n}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \right] \right\} \frac{\partial \Omega}{\partial \gamma} + \theta \pi_2 \Omega \left[ (\kappa_2 - \kappa_1)(\kappa_1 + \Omega) \right] \frac{\partial r^*(\eta)}{\partial \gamma}.$$

Given $\frac{e_2 + \frac{n}{2}}{r^*(\eta)(\kappa_2 - \kappa_1)} \in (0, 1)$ per footnote 18, the terms inside the curly brackets altogether will be strictly positive. In addition, since a decrease in the agency cost $\gamma$ raises $r^*(\eta)$ as well as $[\pi_2 (\kappa_2 - \kappa_1) - \pi_1 \gamma]$, we obtain that $\frac{\partial r^*(\eta)}{\partial \gamma} < 0$ and $\frac{\partial \Omega}{\partial \gamma} < 0$. These results thus lead to $\Psi < 0$, which in turn implies that $\frac{\partial (\frac{TR}{GDP})}{\partial \gamma} = -\frac{\partial (\frac{GDP}{TR})}{\partial \gamma}$ is always negative.
References


