P3318 **Final** REVIEW

10 May

THE FACTS : 10 questions, 100 pts.

30: LAST 1/3 OF COURSE

REST is REVIEW of PAST MATERIAL

DIFFICULTY: ROUGHLY SAME AS PRELIMS

(eg no complicated integrals)

Notes & HW to be updated

the PLAN: A few EXAMPLES ON ROTATIONS

↳ where most of the confusion is

FREE PRECESSION of A RIGID BODY

$$\dot{\vec{L}} = 0 = \mathring{\vec{L}} + \vec{\omega} \times \vec{L}$$

↑ ignores time dep. of BODY FRAME BASIS, $\hat{1}, \ldots$

PLUG $\vec{L} = I_1 \omega_1 \hat{1} + \cdots$ (PRINCIPLE AXIS)

$$\Rightarrow \quad I_1 \dot{\omega}_1 = \omega_2 \omega_3 (I_2 - I_3) \qquad + \text{cyclic}$$
$$I_2 \dot{\omega}_2 = \omega_3 \omega_1 (I_3 - I_1)$$
$$I_3 \dot{\omega}_3 = \omega_1 \omega_2 (I_1 - I_2)$$

Concept: stability of a book (thrown in exasperation); 'anti-Goldilocks' thm.

~~'phase space' curves~~

↳ eg TRAJECTORY ON CONST. E. ELLIPSOID

Q: what's being plotted and on what space?

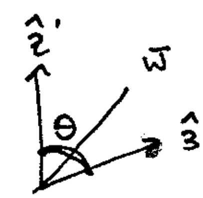
Symmetric top : CASE: $I_1 = I_2 = I$

EULER $\Rightarrow$

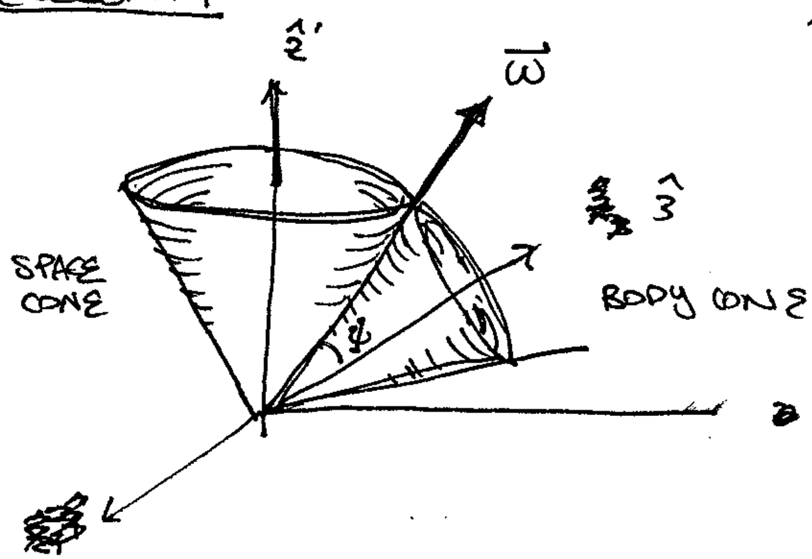
$$\dot{\omega}_1 = -\omega_3\left(\frac{I_3}{I} - 1\right)\omega_2 = -\Omega\omega_2$$
$$\dot{\omega}_2 = +\omega_3\left(\frac{I_3}{I} - 1\right)\omega_1 = +\Omega\omega_1$$
$$\dot{\omega}_3 = 0$$

$$\dot{\vec{\omega}} = \overset{\circ}{\vec{\omega}} = (\Omega\hat{3}) \times \vec{\omega}$$

<u>Concept</u>: W/ $T_{rot} = \frac{1}{2}\vec{\omega}\cdot\vec{L} = $ CONST, CONVINCE YOURSELF THAT THIS IMPLIES PRECESSION



Precession



<u>SPACE FRAME</u>: $\vec{\omega}$ PRECESSES ABOUT $\hat{z}'$

BODY FRAME: —"— ABOUT $\hat{3}$

$\hat{3}$ ROTATES ABOUT $\hat{z}'$, SO BODY ~~CONES~~ CONE ROLLS AROUND SPACE CONE.

IN THE (principle axis) BODY FRAME

$$L_1 = 0$$
$$L_2 = L \sin\theta = I_2 \omega_2 = I_2 \omega \sin\psi$$
$$L_3 = L \cos\theta = I_3 \omega_3 = I_3 \omega \cos\psi$$

$$\Rightarrow \frac{L_2}{L_3} = \boxed{\tan\theta = \frac{I_1}{I_3} \tan\psi}$$

HW 10 4b & c

LARMOR PRECESSION

$$\dot{\vec{\sigma}} = g' \vec{\sigma} \times \vec{B}$$

in rot. frame: $\overbrace{\dot{\vec{\sigma}}\big|_{SPACE}}^{\dot{\vec{\sigma}}} = \dot{\vec{\sigma}}\big|_{body} + \vec{\omega} \times \vec{\sigma}$

$$\rightarrow \dot{\vec{\sigma}}\big|_{body} = \dot{\vec{\sigma}}\big|_{SPACE} - \vec{\omega} \times \vec{\sigma}$$
$$=$$
$$= (-g'\vec{B} - \vec{\omega}) \times \vec{\sigma}$$

if $\vec{\omega} = \vec{\omega}_0 = -g'\vec{B}$, then $\vec{B}$ "disappears"

What if $\vec{B} = B_0 \hat{3} + B_1 \hat{1}$

BODY FRAME ; $\vec{\omega} = \omega \hat{3}$

<u>then</u> : $\overset{\circ}{\vec{\sigma}} = g' \vec{\sigma} \times \vec{B} \underbrace{- \vec{\omega} \times \vec{\sigma}}_{\text{effect of rotating frame}}$

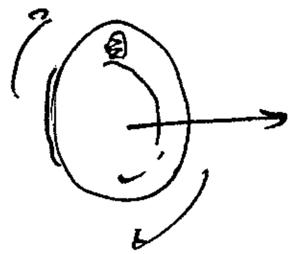
$$\dot{\vec{e}} = \overset{\circ}{\vec{e}} + \vec{\omega} \times \vec{e}$$

$$= \left(-g' B_0 \hat{3} - g' B_1 \hat{1}\right) \times \vec{\sigma} - \omega \hat{3} \times \vec{\sigma}$$

$$= -\underbrace{(g' B_0 + \omega)} \hat{3} \times \vec{\sigma} - g' B_1 \hat{1} \times \vec{\sigma}$$

$\omega = -g' B_0 \Rightarrow$ ONLY PRECESSES ABOUT $\hat{1}$ AXIS.

Feynman wobble



$I_1 = I_2 = I$

$$\dot{\omega}_1 = -\omega_3\left(\frac{I_3}{I} - 1\right)\omega_2$$
$$\dot{\omega}_2 = +\left(\quad '' \quad\right)\omega_1$$

ω_3 IS THE SPIN RATE

$$\Omega = \left(\frac{I_3 - I}{I}\right)\omega_3$$ is WOBBLE RATE

PROLATE OR OBLATE?

FOOTBALL

FAT EARTH

IMPLICATION ON I_3 VS. I?

eg. $I_x = \sum_i m_i (y_i^2 + z_i^2)$

So FAT (PLATE, EARTH) $\Rightarrow$ MORE MASS ALONG x, y

So $I_3 > I$

IN FACT, for DISK: $I_3 = 2I \propto \int y^2 \, d^2x$

So: $\boxed{\Omega = \omega_3}$ FOR IDEAL DISK

But that's the BODY frame (plate frame)
what does it look like in Feynman's frame?

Recall derivation in space frame (football)

$$\vec{L} = I(\omega_1 \hat{1} + \omega_2 \hat{2}) + I_3 \omega_3 \hat{3}$$
$$= \overleftrightarrow{I}\vec{\omega} + (I_3 - I)\omega_3 \hat{3}$$
$$= \overleftrightarrow{I}\vec{\omega} + I\Omega\hat{3} \leftarrow \Omega = \left(\frac{I_3 - I}{I}\right)\omega_3$$

$$\omega_p \hat{L} = \vec{\omega} + \Omega\hat{3} \leftarrow |\vec{L}| = I\omega_p$$

$$(\omega_p \hat{L})^{\cdot} = 0 = \dot{\vec{\omega}} + \underbrace{\Omega(\vec{\omega} \times \hat{3})}$$

$$-(\Omega\hat{3}) \times \vec{\omega} = -(\omega_p \hat{L} - \vec{\omega}) \times \vec{\omega}$$
$$= -\omega_p \hat{L} \times \vec{\omega}$$

$$\dot{\vec{\omega}} = \omega_p \hat{L} \times \vec{\omega}$$

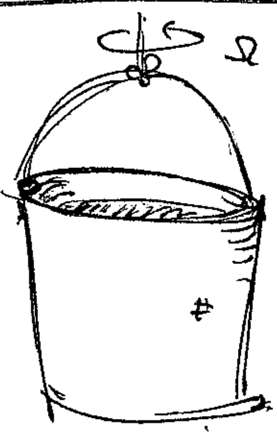
$\vec{\omega}$ PRECESSES ABOUT $\hat{L}$ w/ freq. ω_p

recall: $\cos\theta = \hat{L} \cdot \hat{3} = \frac{I_3 \omega_3}{L} = \frac{I_3 \omega_3}{I \omega_p}$

assume $\cos\theta \approx 1 \Rightarrow \frac{\omega_p}{\omega_3} = \frac{I}{I_3} = \boxed{2}$ famous factor of 2.

Newton's Bucket

punchline to "man from Nantucket" limmerick.
→ 'related but superficially diff."



what is the shape of the surface of the water?

GO TO ROTATING FRAME
FIND SURFACE OF CONST. PRES.

$\Omega = \Omega \hat{z}$

HYDROSTATIC EQUIL: $\rho \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = \vec{F} - \vec{\nabla} P$

DENSITY — PRES.

fictitious force $F = -\rho g \hat{z}$

why this one?. what options?

$$\vec{F}_{BODY} = \vec{F} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v} - m\dot{\vec{\omega}} \times \vec{r}$$

$$\Rightarrow \nabla P = -\rho g \hat{z} - \rho \Omega^2 r \underbrace{\hat{z} \times (\hat{z} \times \hat{r})}$$

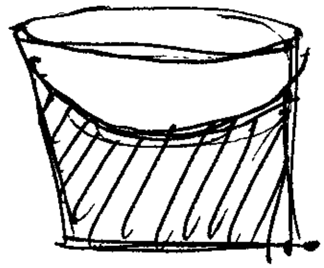
$$= -\rho g z + \rho \Omega^2 r \,\hat{r}$$

$$\Rightarrow P = \rho \left(\tfrac{1}{2} \Omega^2 r^2 - g z \right) + C$$

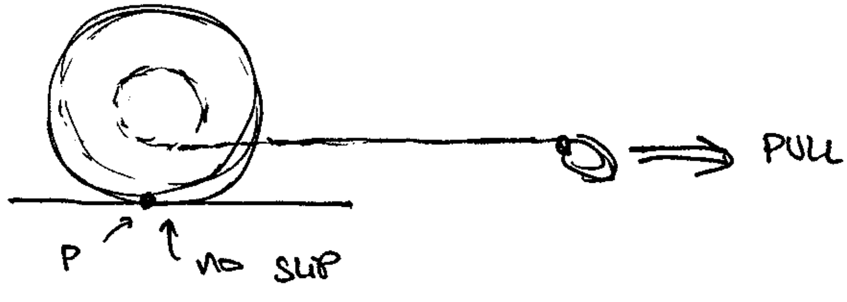
surface of const pressure:

$$\boxed{z = \frac{1}{2g}\Omega^2 r^2 + \text{const}}$$

paraboloid.



<u>Yo-Yo</u>



WHICH WAY DOES IT GO?

POINT P HAS ZERO INSTANTANEOUS VELOCITY
ROTATION ABOUT FIXED (inst.) AXIS OF ROT. P IS
GIVEN BY TORQUE ABOUT P AXIS. ONLY THE
PULL DIR HAS NONZERO TORQUE.

15. SPACE LOCUSTS (or ISOTROPIC DUST FALLING)

THICKNESS $h \ll R_E$

FALL TO THE EARTH
→ LAYER OF DEBRIS
of THICKNESS h

What is the change in the length of the day?

$$I_{\cancel{\text{SPHERE}}\ \text{BALL}} = \frac{2}{5}MR^2 \qquad M = \frac{4}{3}\pi R^3 \rho_E$$

$$I_{\text{SPHERE}} = \frac{2}{3}mR^2 \qquad m = 4\pi R^2 \rho_L h$$

CONSERVATION of ANGULAR MOMENTUM:

$$\underbrace{I_0}_{I_{\text{BALL}}} \omega_0 = \underbrace{I}_{= I_{\text{BALL}} + I_{\text{SPH}}} \omega$$

$$\frac{T}{T_0} = \frac{\omega_0}{\omega} = \frac{I}{I_0} = 1 + \frac{\frac{2}{3}mR^2}{\frac{2}{5}MR^2}$$

$$\frac{5}{3} \cdot \frac{3\rho_L h}{R\rho_E}$$

$$= 1 + \frac{5h}{R}\frac{\rho_L}{\rho_E}$$

$$\boxed{\frac{T - T_0}{T_0} = \frac{5h\rho_L}{\rho_E R}}$$

Review → think of examples done in class & hw

1st PART: LAGRANGIANS
LEAST ACTION → QM LIMIT
STATMECH LIMIT
CONSTRAINTS, DOF
CONSERVED QUANTITIES
CONJUGATE MOMENTA

2nd PART: 2 BODY PROBLEM
REDUCED MASS & RELATIVE MOTION
ORBITS, PRECESSION, KEPLER

NOETHER'S THM
HAMILTONIAN MECH
[Phase SPACE TRAJECTORIES] → LIOUVILLE'S THM ⟷ HEISENBERG
POINCARÉ RECURRENCE

CANONICAL TRANSFORMATIONS
GENERATING FUNCTIONS
POISSON BRACKETS
ACTION-ANGLE VARS
ADIABATIC INVARIANCE → LEVITRON

ROTATING FRAMES → KINEMATICS
→ DYNAMICS
↓
symm. top