P318 : SECTION 1

F Jan 25

TA: FLIP TANEDO

OFFICE PSB 432, OH M 5:15 - 6:15 in PSB 425G

pt267 @

GOAL: IDEAS, "BLOW YOUR MIND", EXTENSIONS
→ YOUR RESPONSIBILITY TO DO READING & PROBLEMS
→ be "PRE-GRAD STUDENTS"

REMARK: 318 vs. 314

same topics, different styles/depth

both will have very good teaching staffs! :)

→ talk to me, or faculty if you're still deciding

GENERAL P318 ADVICE

→ USE STUDY SPACE!
(ask if you want more hrs

- PHYSICS IS A GROUP ACTIVITY → collaborate!
  → mixed jr/soph class: make new friends

- THIS COURSE: foundational (→ stat mech, → quantum), deep
  ... much deeper than we'll get a chance to fully explore. we'll try to mention particularly salient aspects ... BUT YOU ARE ENCOURAGED TO EXPLORE [eg other textbooks, articles, ...]

READ feynman

- GOLDSTEIN
- MARION
- TONG
- MELIA

- DO THE READING (before lecture!) → H&F is a good book, but demands work

- ASK QUESTIONS: in lec, in sec, to each other.
  → think about book questions!

Big Picture

(*) NEWTON: 2^{ND} ⊖ ODE: $\ddot{\vec{x}} \sim \vec{F}$
SOLUTION IS A PATH $\vec{x}(t)$

THIS is NOT P116! MORE focus ON WHY THAN HOW, CONNECTIONS TO STAT & QUANTUM → "grown up" physics!

(*)

P116 PROBLEMS GET HARD WHEN MOTION IS CONSTRAINED.

→ EVEN THOUGH few DoF, NEWTONIAN DESCRIPTION GETS COMPLICATED — LOTS of REDUNDANCY
→ EXTRA COORDS, EXTRA CONSTRAINT FORCES
→ MANY EQNS :(

SAME PHYSICS AS 116

SOLUTION: NEW FORMALISM of MECHANICS

- EQUIVALENT (no more, no less) TO NEWTONIAN
- USES ONLY DOF (not extra coords restr. by CONSTRAINTS)
- NOVEL MATH TO FIND $\vec{x}(t)$ MORE EASILY (eg more 1^{st} ⊖ pDE)

$\vec{x}(q(t))$ ⟨ $\vec{x}(q)$ ← constraint, $q(t)$ ← dynamics

BONUS: FORMULATION WHICH NATURALLY ILLUMINATES THE PATH TO QUANTUM, STATISTICAL MECHANICS.

analogous to current work!

in fact, provides bridge

& also E&M

GENERAL PICTURE: WORK IN PHASE SPACE (why? we'll see)

not new physics... BUT THE RIGHT LANGUAGE.

DEFINE FUNCTIONALS (func of func.)
↳ LAGRANGIAN, ACTION (& HAMILTONIAN)
USE FUNCTIONAL/VARIATIONAL CALCULUS: EXTREMA of THESE FUNCTIONALS GIVE SOLUTION $\vec{x}(t)$ of DYNAMICS!

Section i OH

MAINLY EXAMPLES
& "BIG PICTURE" REMINDERS
... I HAVE A TENDENCY
TO ALSO TALK ABOUT
CONNECTIONS

PSB 425g
MONDAY 5:15-6:15
for HW QUESTIONS
OR GEN COURSE Q's
...OR 432 if nobody there

(OR AFTER SECTION
ALSO BY APPT (PSB 432)
ABOUT ANY COURSE CONCERNS
GENERAL PHYSICS ADVICE
OR OFF TOPIC QUESTIONS

Study Space - R290 M2-3pm

highly SUGGESTED IF YOU CAN MAKE IT.
PROF ESSER WILL BE THERE

WE CAN ARRANGE for ANOTHER HOUR IF
YOU WANT.

I WANT FEEDBACK ABOUT HOW WE SPEND OUR TIME.

sorry, expect many examples motivated by my research

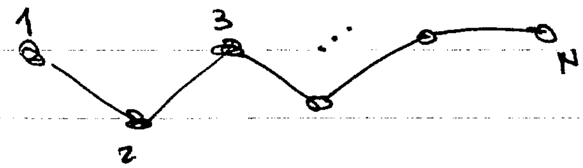
1. XD & DoF

STRING thy: SPACE IS 10 DIM (SPACETIME IS $d = 11$)

HOW MANY DOF IN A PARTICLE? $\longrightarrow \boxed{10}$

WHAT ABOUT A CHAIN?

↳ ~~[illegible]~~ N PARTICLES LINKED BY RIGID RODS of LEN L



"open string" total length IS $(N-1)L$

$$10N - (N-1) = \boxed{9N+1}$$

... WHAT ABOUT A STRING?

TAKE $L \to 0$ WHILE $N \to \infty$ s.t. $(N-1)L = $ const.

comment on worldsheet thy → conformal

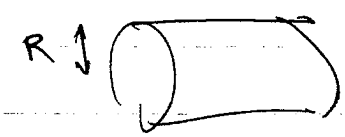
$\hookrightarrow \boxed{\infty} \longrightarrow$ field thy is tricky

↑ MECHANICS w/ CONTINUUM of PARTICLES, NOT JUST DISCRETE #

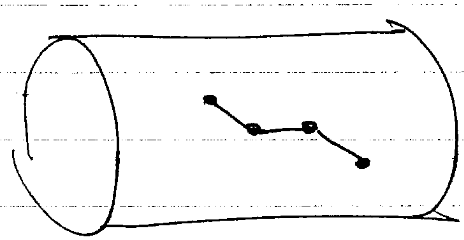
(Newtonian approach FAILS!)

"SMALL" EXTRA DIMENSIONS:

IMAGINE A CYLINDRICAL SPACE.



WHEN $L \ll R$, MIGHT AS WELL BE NON COMPACT

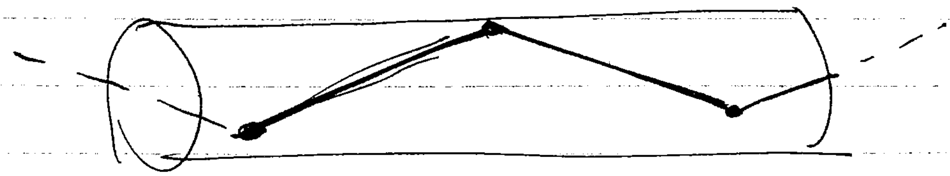


"big, compact XD"

observable

eg send a particle to ∞, but it comes & sneaks up behind you

WHEN $R \ll L$:



HOW MANY DoF WHEN YOU LOOK @ THIS FROM FAR AWAY?

↳ $\boxed{\text{BASICALLY ONE!}}$ EFFECTIVELY 1D

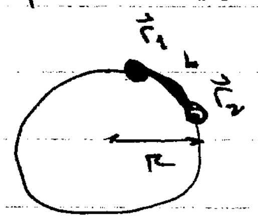
ASIDE: BUT YOU COULD PROBE ENERGY of CHAIN ... eg CHAIN COULD HAVE LOTS of STORED "WIGGLE" ENERGY IN XD ⟶ LOOKS LIKE MASS!!!

↳ OR, IN QUANTUM: EXCITED STATE!

constraints

simpler case: 2 particle chain on a circle IN 2D

WRITE OUT $\vec{r}_1$ & $\vec{r}_2$ IN TERMS OF THE dof IN THE SYSTEM.



eg. $x_1 = R\cos\theta$, $y_1 = R\sin\theta$ } define θ

$\varphi = L/R$ ⟶

$$x_2 = R\cos(\theta-\varphi)$$
$$y_2 = R\sin(\theta-\varphi)$$

can imagine spinning ring on a table, etc.

THE POINT: STILL 1 dof, we can solve for $\theta(t)$ → solve dynamics

2. Beach dog & Quantum & optics — PRELUDE TO VARIATIONAL CALC.

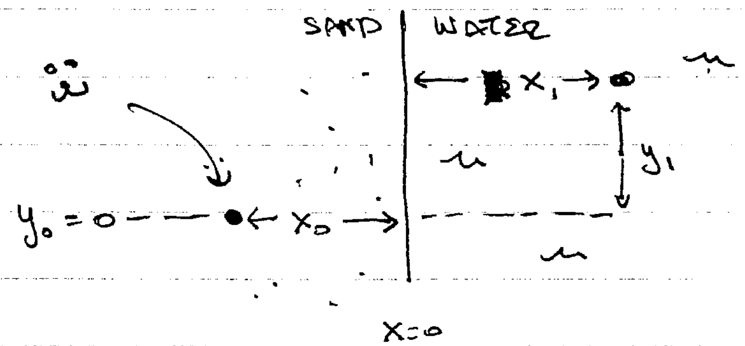
Motivation: *Do Dogs Know Calculus?*

TIM PENNINGS, The College Math Journal V.34, n.3 ('03) 178

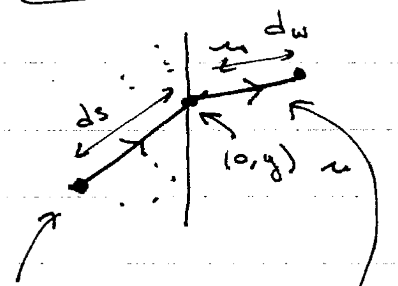
PROBLEM: DOG RUNS FAST ON SAND,

SWIMS SLOWLY IN WATER.

HOW TO MINIMIZE TIME TO BALL?



expect:



in each medium, straight line path to $(0, y)$

POSSIBLE PATHS ARE GIVEN BY POSSIBLE y VALUES.

total time: $T = \dfrac{d_s}{V_s} + \dfrac{d_w}{V_w}$

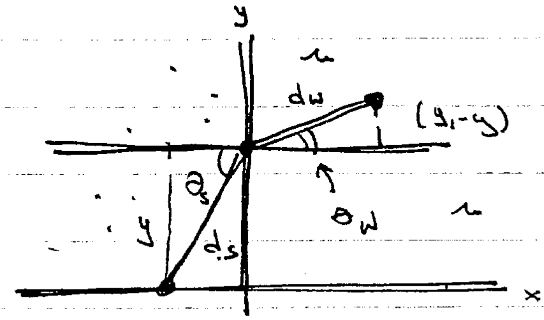
$$T(y) = \frac{\sqrt{x_0^2 + y^2}}{V_s} + \frac{\sqrt{x_1^2 + (y_1 - y)^2}}{V_w}$$

then solve for $T'(y) = 0$ for minimum.

SIMPLE CALCULUS PROBLEM... BUT: INTERPRET AS "SCANNING OVER POSSIBLE PATHS," AS PARAMETERIZED BY y.

$$T'(y) = \frac{1}{V_s}\,\frac{1}{2}\,\frac{2y}{\sqrt{x_s^2+y^2}} + \frac{1}{V_w}\,\frac{1}{2}\,\frac{2(y_1-y)(-)}{\sqrt{x_1^2+(y_1-y)^2}}$$

$$= \frac{1}{V_s}\,\frac{y}{d_s} - \frac{1}{V_w}\,\frac{(y_1-y)}{d_w}$$ ← solve for y.



$$T'(y) = \frac{\sin\theta_s}{V_s} - \frac{\sin\theta_w}{V_w} = 0$$

$$\Rightarrow \boxed{\frac{\sin\theta_s}{V_s} = \frac{\sin\theta_w}{V_w}}$$ SNELL'S LAW!! (optics!)

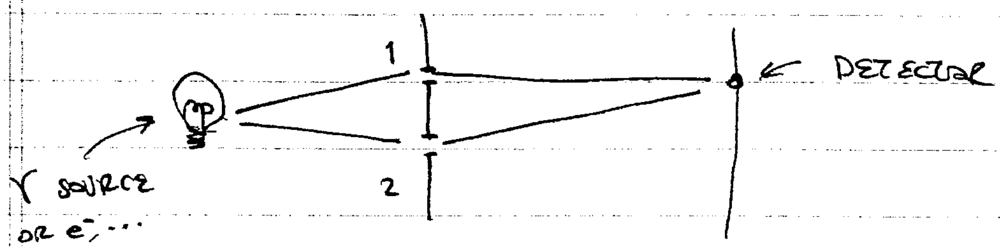
AGAIN: the point is that we sampled all paths.

→ PARTICULARLY SIMPLE, SINCE PATHS ARE PAIRS of STRAIGHT LINE SEGMENTS — JUST ONE VARIABLE.

→ in this course, we'll do the analogous thing for more arbitrary possible paths.

does this sound familiar??

cf. double slit experiment in QM



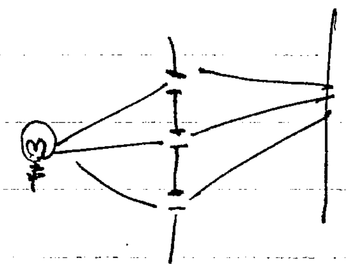
QM: γ TAKES BOTH PATHS

"Prob" of observing γ @ DETECTOR = "$A_1 + A_2$"

↑ AMPLITUDE

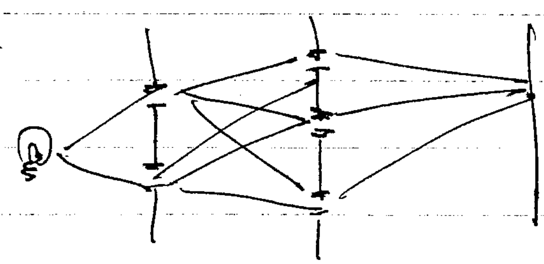
with some weighting

multi slit:



WEIGHTED SUM of 3 PATHS

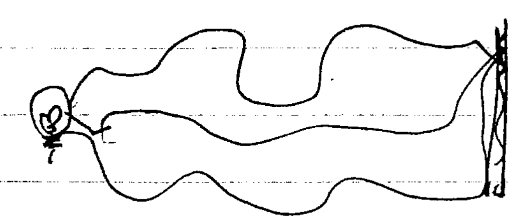
multi barrier



WEIGHTED SUM of 2×3 PATHS

etc.

∞ SLIT, ∞ BARRIER :



sum over all possible paths w/ no restric.

($\infty \times \infty$) such paths!

CLASSICAL PATH: one that extremizes the WEIGHTING.

QUANTUM: sample all.

Weighting ⟷ ACTION

3. Allometry

IN HW: HOMOGENEOUS POTENTIALS

$$U(\alpha\vec{r}_1, \ldots, \alpha\vec{r}_N) = \alpha^k U(\vec{r}_1, \ldots, \vec{r}_N)$$

$$\text{eg } U \sim 1/r \rightarrow k = -1$$

YOU CAN USE <u>DIMENSIONAL ANALYSIS</u> & <u>SCALING</u> TO LEARN A LOT ABOUT A HOMOGENEOUS SYSTEM.

FROM Arnold (in turn from JM Smith, Math. Ideas in Bio).
p.51

- CAMEL HAS TO RUN BTWN SOURCES of WATER. HOW DOES MAX RUN TIME DEPEND ON CHARACTERISTIC LENGTH? $\boxed{\sim L}$

$$\text{stored water} \sim L^3$$
$$\text{evaporation} \sim L^2$$

eg. HEAT $\sim L^2$

- GIVEN THAT POWER $\sim L^2$, HOW DOES MAX RUNNING VELOCITY DEPEND ON L FOR AN ANIMAL?

  a) LEVEL GROUND: AIR RESISTANCE $F \sim v^2 L^2$

  $$\Rightarrow P \sim v^3 L^2$$

  $$\hookrightarrow L^2 \sim v^3 L^2 \Rightarrow \boxed{v \sim L^0}$$ cf RABBIT VS HORSE

  b) UPHILL: GRAVITY $F \sim mgv \sim L^3 v$

  $$\hookrightarrow L^2 \sim L^3 v \Rightarrow \boxed{v \sim L^{-1}}$$

  cf dog vs horse running up hill

- HEIGHT OF MAX JUMP DEP ON L?

ENERGY $\sim L^3 h$
$\uparrow \sim$ mass

WORK BY MUSCLE: (FORCE) L
$\uparrow \sim L^2$ (x-sec of bones, musc.)

$\Rightarrow L^3 h \sim L^3 \Rightarrow h \sim L^0$

HW: not assigned, but I suggest thinking through problem 7 in Ch. 1.
I WILL ANSWER ANY Q's ABOUT IT NEXT WK