

1

a taste of

LECTURE 27: GAUGE THEORY

28 NOV

↑ how I use gauge sym.

REMINDERS:

(1) please fill out course evaluations
on iEval

→ feel free to email, feedback / constr.
directly to me DETAILED
crit.

(2) Grading: exact rubric TBD;
but if you've completed most of the
homework, I'll grade from (B+, A)

↳ NEED TO HAVE DONE ENOUGH

to show: Green's func } Comput Int!

→ if not: B.

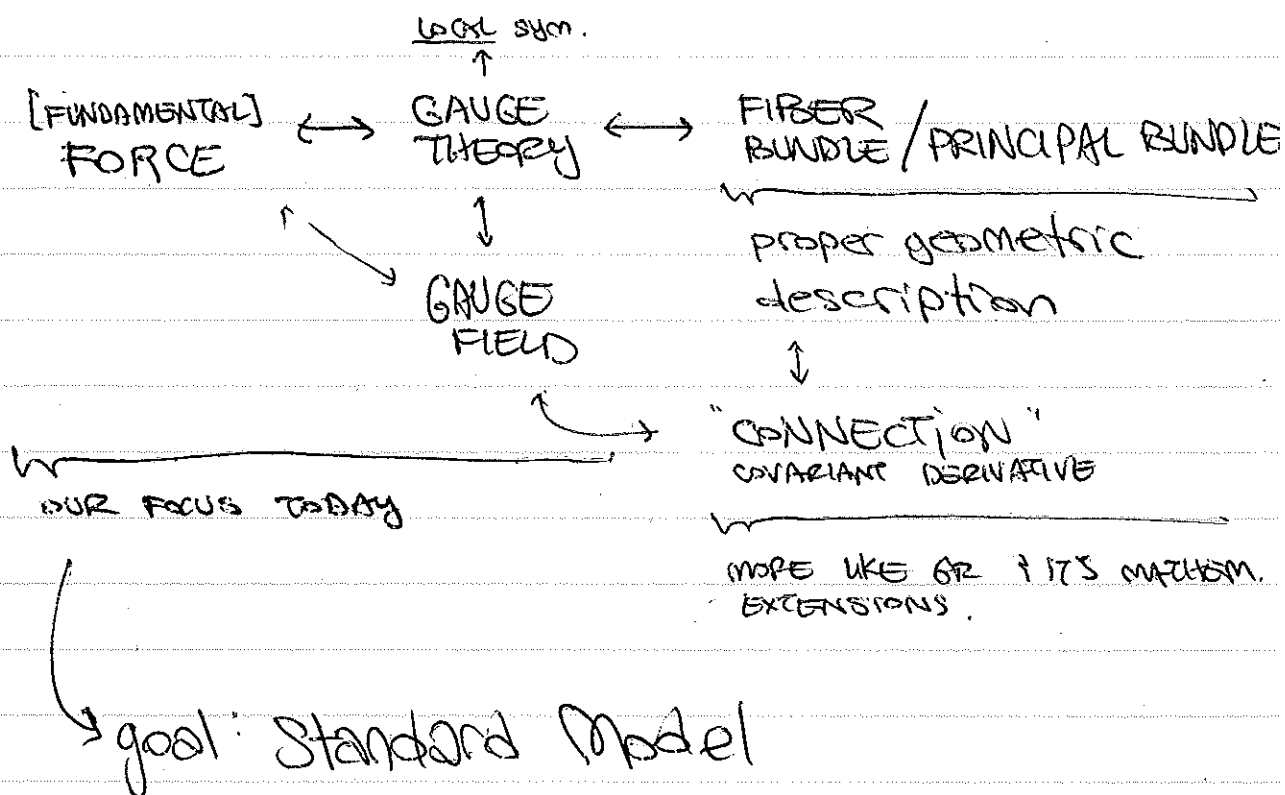
LAST TIME: SPACETIME SYMMETRY

↳ remarkably subtle rep of what?
how we understand
particles of a given spin

WE DIDN'T GO IN DETAIL, BUT POINTED OUT
WHERE THINGS ARE TRICKY → YOU GET
NEAT BEHAVIOR.

TODAY: FORCES (more gen: interactions)

JARGON: some related ideas



ALL SYMMETRIES THAT WE HAVE CONSIDERED
THUS FAR ARE GLOBAL.

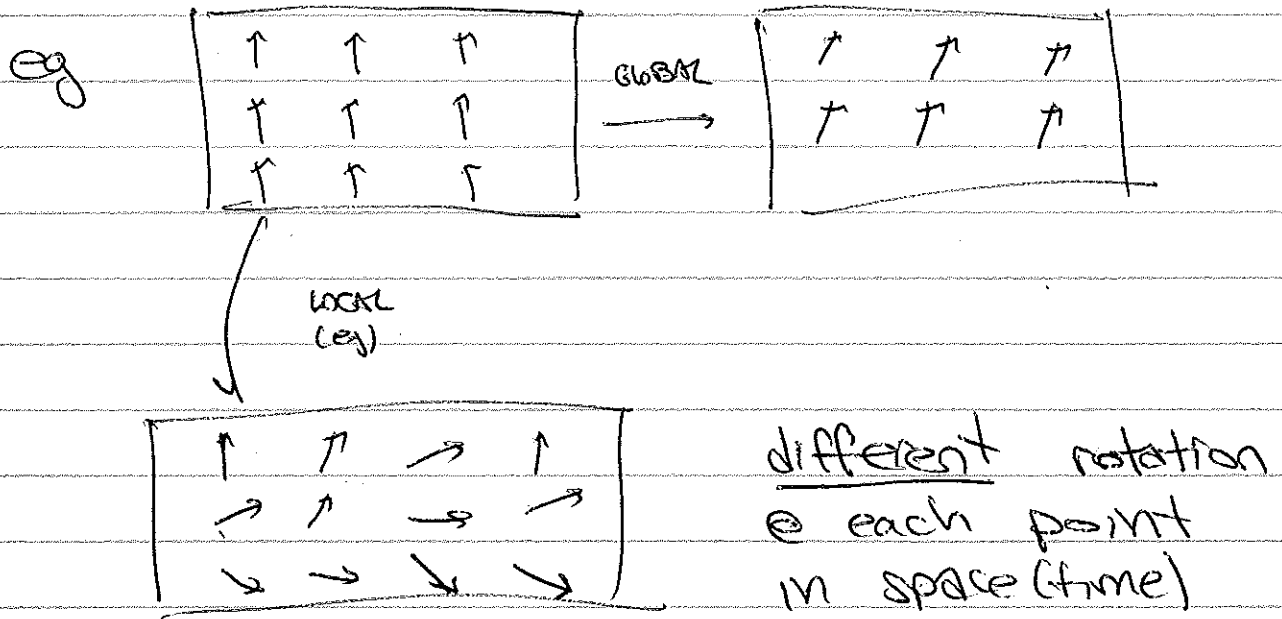
$$e^{it^a T^a}$$

↑
constant parameter
in spacetime

vs.

$$e^{it^a(x) T^a}$$

↑
twist a bit here,
twist a lot there



eg. for a lattice of spins: B-field all messed up!

GAUGE SYMMETRIES ARE SPECIAL SYM.
THAT HOLD LOCALLY.

SIMPLEST EXAMPLE: $U(1) \leftarrow \text{rep: } e^{i\alpha(x)}$
(0 phase)

this is an ∞ # of transformations
→ it's a field full of transformations

GAUGE SYM if $\exists$ some physics that is
"charged" under this $U(1)$ but for
which ~~nature is~~ the laws of physics
are unchanged.

WE HAVE A FAVORITE EXAMPLE OF THIS

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

DOES NOT CHANGE $\underline{E}$ OR $\underline{B}$
 $\swarrow \searrow$
physical observables

WE RECALL WHY: PHYSICAL EFFECTS LIVE
 IN $F = dA$

"cohomology" $\left\{ \begin{array}{l} \text{BUT } dd\alpha = 0 \\ \text{SO } A \rightarrow A + d\alpha \\ \text{LEAVES } F = dA \text{ UNCHANGED} \end{array} \right.$

$A_\mu(x)$ IS A 4-VECTOR FIELD

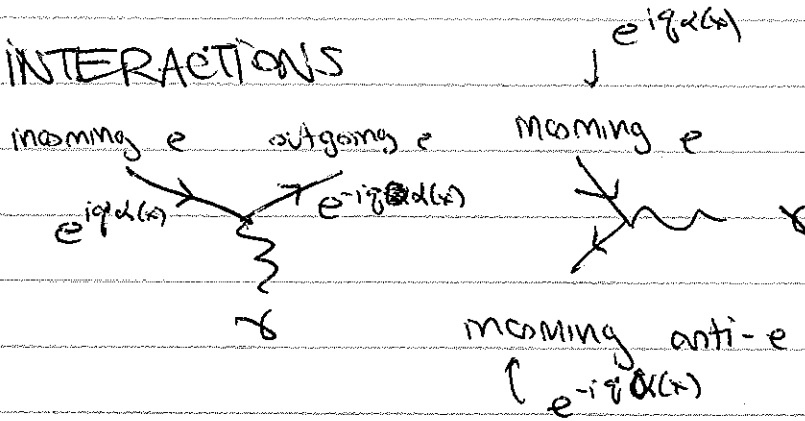
↑ QUANTUM EXCITATIONS OF THIS FIELD
 ARE CALLED PHOTONS

↑ identified w/ green's functions
 in matrix elements.

electrons: have charge q under this $U(1)$
 meaning $\psi \rightarrow e^{iq\alpha} \psi$
 under $U(1)$

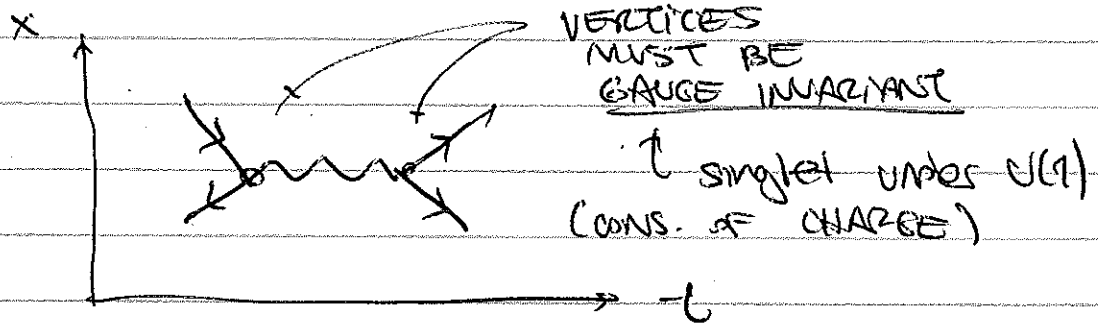
photons: uncharged (just funny shift)

INTERACTIONS



these "Feynman diagrams" encode quantum interactions.

the vertex is some point in spacetime @ which the interaction occurs.



"electron & positron annihilate into a virtual photon, before popping back out w/ re-arranged momenta"

REMARK: the existence of the A_μ (photon)



local/gauge symmetry

↳ in fact: A_μ is the connection
(part of covariant derivative)

WHEN YOU MAKE TRANS. PARAM LOCAL
(a field), you also BRING A_μ TO LIFE.

ALTERNATIVE VIEW: A MASSLESS PARTICLE
THAT IS A SPIN-1
CAN BE DESCRIBED
BY A 4-COMPONENT
FIELD ... BUT
IT'S NOT LORENTZ COVARIANT.

↳ to describe theory in a ^{MANIFESTLY} Lorentz
invariant way, NEED TO
USE A 4-VECTOR.



introduces extra deg. of freedom
that you have to remove
by GAUGE FIXING

CHOICE IN HOW TO DO THIS IS SAME GAUGE SYM.
→ REDUNDANCY in theory.

³
 $SU(3)$: NON-ABELIAN
 commutators don't vanish

MORE GENERAL CASE.

FACT: GAUGE FIELDS IDENTIFIED w/ ADJOINT
 ↑ ↑
 "force mediators" representation of the group

USUALLY: MATTER FIELDS $\Leftrightarrow$ FUNDAMENTAL
 $(T) (:)$
 ↑ matrix

ANTIMATTER $\Leftrightarrow$ ANTIFUNDAMENTAL
 $(T^+) (i)$
 equiv. $(\dots) (T)$

~~FOR $SU(2)$: GENERATORS $\sigma^1, \sigma^2, \sigma^3$~~
 ~~$\sigma^1, \sigma^2, \sigma^3$~~
 ~~$\uparrow \quad \uparrow$~~
 ~~$W \quad Z$~~ ~~bosons~~
 ~~$\uparrow$~~
~~HAPPENS TO HAVE~~
~~EEC. CHARGE...!~~

MATTER : $\underline{3}$ of $SU(3)$

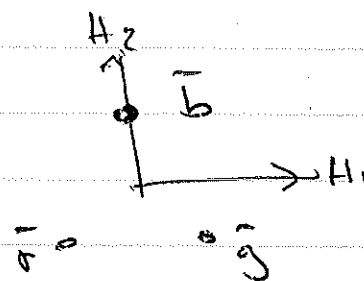
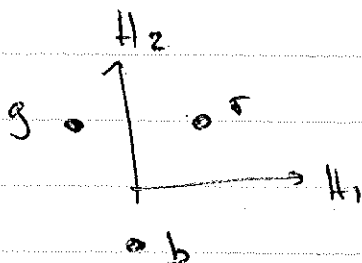
↑
3 DIM VECTOR SPACE :

$\begin{pmatrix} r \\ g \\ b \end{pmatrix} \leftarrow \begin{array}{l} \text{'red' quark} \\ \text{'green' quark} \\ \text{'blue' quark} \end{array}$

ANTI-MATTER : $\underline{\bar{3}}$ of $SU(3)$

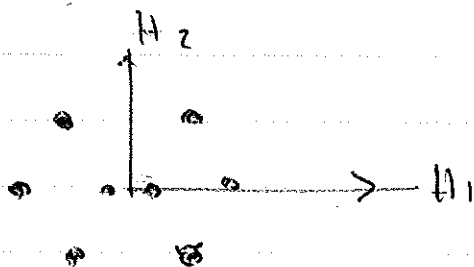
$\begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$

RECALL :

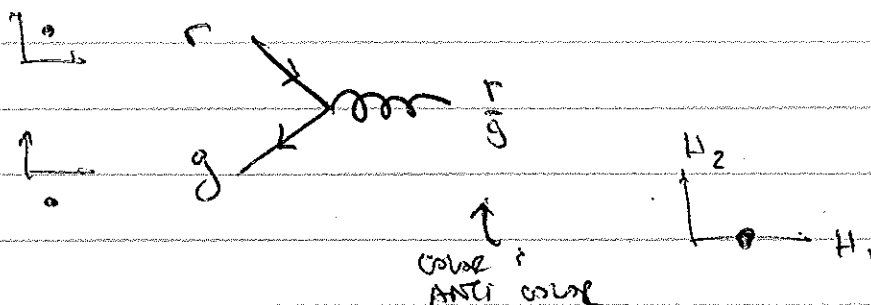


GLUONS : 8 of them, one for each generator.

↑
live in the ADJOINT



I WANT TO UNDERSTAND



"A red quark + anti-green anti-quark interacts w/ a $r\bar{g}$ gluon"

REPS: $\underline{3} \otimes \underline{\bar{3}} = \underline{3} \oplus \underline{1}$

↑

there is some overlap w/
 $\underline{3} \otimes \underline{\bar{3}}$ And the ADJOINT.

⇒ ∃ some ~~matrix~~ matrix
 that connects these
 reps

the generators themselves

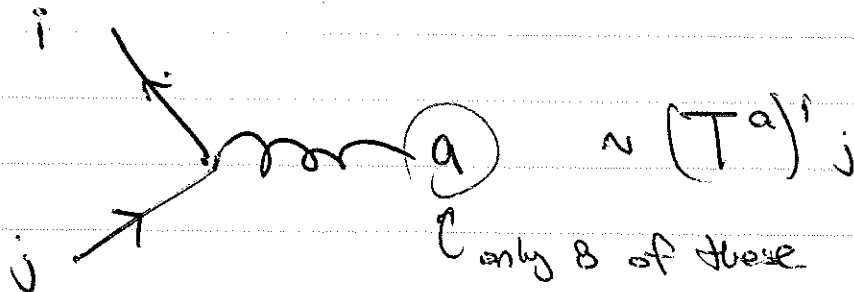
$$(T^a)_{ij}$$

index for $\underline{3}$ index for $\underline{\bar{3}}$

$$\bar{q}_i G^a (T^a)_{ij} q_j$$

↳

all indices contract
s.t. this combination
is a gauge invariant



if I transform, then the vertex
does not do anything.

A starting point for the Standard Model of particle physics.

(THE PERIODIC TABLE)

REAL W^0

REAL Y

8 gluons

$W^\pm, Z$

γ

(1/2, 0)
spinor

$$SU(3) \times SU(2) \times U(1)_Y$$

quarks

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

3

2

$+\frac{1}{6}$

$\bar{u}_R$

$\bar{3}$

$-\frac{2}{3}$

$\bar{d}_R$

$\bar{3}$

$+\frac{1}{3}$

leptons

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$\bar{3}$

2

$\frac{1}{2}$

e_R

$\bar{3}$

$+1$

Higgs

H

2

$-\frac{1}{2}$

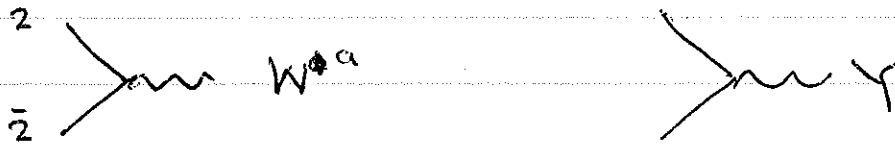
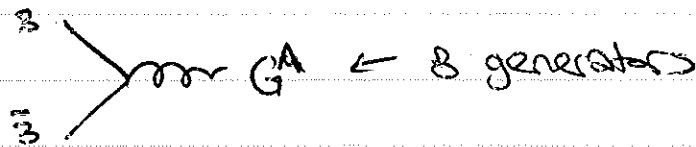
ELECTRIC CHARGE : $T^3 + Y$

eg. $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ has $T^3 \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} -1/2 \nu_L \\ +1/2 e_L \end{pmatrix}$

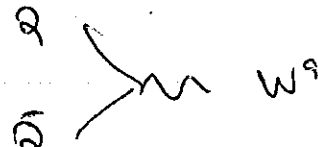
so: $SU(2)$ is kind of special,
 IN FACT: $SU(2) \times U(1)$ is special;
 "electroweak" symmetry is
"broken" to ELECTROMAG.

but this is a question ~~of~~ of
 the phase of our vacuum.

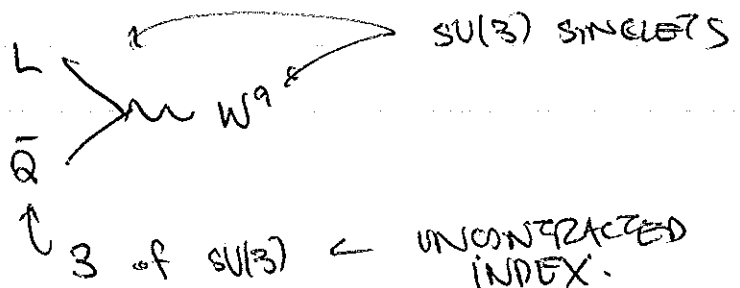
WHAT THE TABLE TELLS US:
 WHAT INTERACTIONS ARE ALLOWED



as long as other quantum numbers allow!



BUT NOT



NB: ALL ~~SU(3)~~
 INDICES
 "CONTRACTED"
 "2 x 2 = 3 + 1"

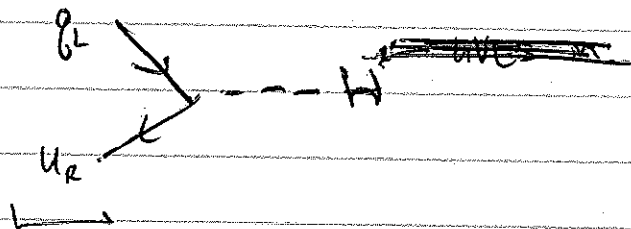
YUKAWA couplings interaction w/ Higgs

↑
SPIN = 0

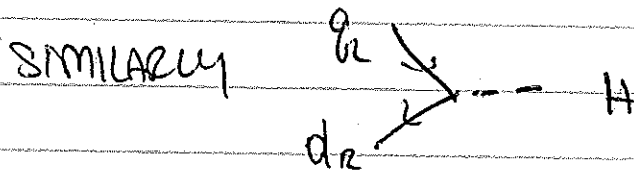
" $2 \times 2 = 3 \rightarrow \textcircled{1}$ "

case $SU(2)$

$$\xrightarrow{Z_Y = 0} (\bar{Q}_L)_{-1/6}^i H_j^+ (U_R)_{2/3}^j \underbrace{\delta_j^i \delta_j^i}_{\text{invariant tensor}}$$



have to be same color



$$(\bar{Q}_L)_{-1/6}^i H_j^+ (d_R)_{-1/3}^j \delta_j^i \rightarrow \neq 0 \quad \times$$

$$(\bar{Q}_L)_{-1/6}^i H_j^+ (d_R)_{-1/3}^j \underbrace{\epsilon^{ij}}_{\text{invariant!}} \rightarrow = 0 \quad \checkmark$$

↑ observe: $\underline{2} = \bar{2}$

$$H^i \text{ vs } H_j^+ \epsilon^{ij}$$

IN GREY DETAIL: ADDING SPIN

$$\begin{array}{ccccccc}
 (\frac{1}{2}, 0) & \otimes & (0, \frac{1}{2}) & = & \text{vector} & \oplus & \text{scalar} \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 \text{LH PARTICLE} & & \text{ANTIPARTICLE} & & & & \\
 \text{RCP} & & \text{REF} & & & & \\
 \alpha & & \alpha & & (\sigma^{\mu})^{\dot{\alpha}}_{\beta} & & \delta^{\dot{\alpha}}_{\beta}
 \end{array}$$

$(\sigma^{\mu})^{\dot{\alpha}}_{\beta}$
 $(\sigma^{\mu})^{\alpha}_{\dot{\beta}}$

$$(\bar{Q}_L)_{I\dot{\alpha}} (G^A_{\mu})^I_J (\sigma^{\mu})^{\dot{\alpha}}_{\beta} (Q_L)^{J\dot{\beta}} \underbrace{\delta^{\dot{\beta}}_{\beta}}_{\text{SU}(2)}$$

$\xrightarrow{\text{spin}} (T^A)^I_J (\sigma^{\mu})^{\dot{\alpha}}_{\beta}$

$$(\bar{Q}_L)_{I\dot{\alpha}} H^J (\psi_R)^{J\dot{\alpha}} \underbrace{\delta^{\dot{\beta}}_J \delta^{\dot{\alpha}}_I \delta^{\dot{\gamma}}_{\alpha}}_{\text{SU}(2)}$$

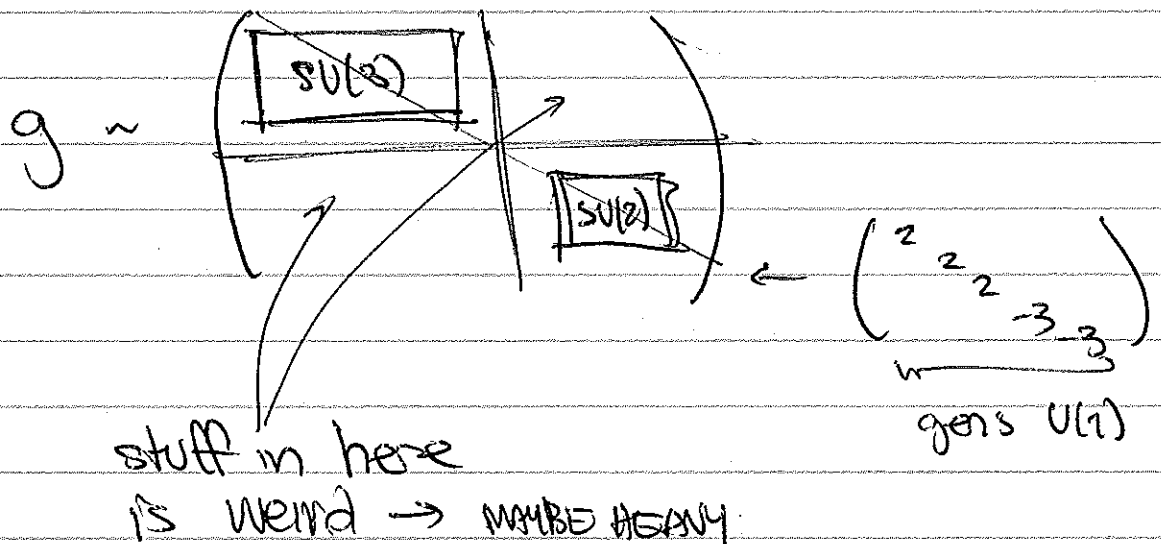
$\xrightarrow{\text{spin}} \delta^{\dot{\beta}}_J \delta^{\dot{\alpha}}_I \delta^{\dot{\gamma}}_{\alpha}$

BONUSUNIFICATION

Why $SU(3) \times SU(2) \times U(1)$?

maybe these all live in
a bigger, simple group?

one candidate: $SU(5)$



NOT MENTIONED : GRAVITY AS GAUGE THY?

$\rightarrow$ see color. THIS THY!
(I think)