LEC 24 : SU(3) 18 NOV.

ALGEBRA : 3x3 TRACELESS HERMITIAN

OR ANTIHERM. DEP ON e^{ix} vs e^{x}

3x3 GENERALIZATION OF SU(2) PAULI MATRICES

BASIS: $\left(\begin{array}{c|c} \sigma_i & \\ \hline & 0 \end{array}\right) = \lambda_i$

LIKE $\sigma_{1,2}$ for 1-3

$\left(\begin{array}{c|c} & 1 \\ \hline 1 & 0 \end{array}\right), \quad \left(\begin{array}{c|c} & -i \\ \hline i & 0 \end{array}\right)$

LIKE $\sigma_{1,2}$ for 2-3

$\left(\begin{array}{c|c} 0 & \\ \hline & \begin{matrix} & 1 \\ 1 & \end{matrix} \end{array}\right), \quad \left(\begin{array}{c|c} 0 & \\ \hline & \begin{matrix} & -i \\ i & \end{matrix} \end{array}\right)$

$\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$

DIAGONAL GENERATOR

HOW MANY ? $\left(\begin{array}{c|c} \sigma_3 & \\ \hline & 0 \end{array}\right), \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$

only 2 b/c has to be traceless

$$J_{\bullet} = \begin{pmatrix} & \frac{1}{\sqrt{2}} \\ 0 & \end{pmatrix}, \begin{pmatrix} & 0 \\ \frac{1}{\sqrt{2}} & \end{pmatrix}, \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

BUT JUST LIKE $\mathcal{L}(SU(2))$, WE CAN COMBINE THESE INTO DIAGONAL & RAISING/LOWERING

$$h_1 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \qquad h_2 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$$e_+^1 = \frac{1}{\sqrt{2}}\left(\begin{array}{cc|c} 0 & 1 & \\ \hline & 0 & \\ & & 0 \end{array}\right) \qquad e_-^1 = e_+^{1\dagger}$$

$$e_+^2 = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & & 0 \\ \hline & 0 & 1 \\ & & 0 \end{array}\right) \qquad e_-^2 = (e_+^2)^\dagger$$

$$e_+^3 = \frac{1}{\sqrt{2}}\begin{pmatrix} & & 1 \\ & 0 & \\ 0 & & \end{pmatrix} \qquad e_-^3 = (e_+^3)^\dagger$$

NOW WRITE H_i, $E_\pm^A$ for REP

COMMUTATION RELATIONS

$$[H_1, H_2] = 0 \longleftarrow \text{2 COMMUTING GEN'S.}$$

see next pg

$$\begin{aligned} &[H_1, E_\pm^1] = \pm E_\pm^1 && [H_2, E_\pm^1] = 0 \\ &[H_1, E_\pm^2] = \mp \tfrac{1}{2} E_\pm^2 && [H_2, E_\pm^2] = \pm \tfrac{\sqrt{3}}{2} E_\pm^2 \\ &[H_1, E_\pm^3] = \pm \tfrac{1}{2} E_\pm^3 && [H_2, E_\pm^3] = \pm \tfrac{\sqrt{3}}{2} E_\pm^3 \end{aligned}$$

weird normaliz, but basically same structure.

$$[E^1_+, E^1_-] = H_1$$
$$[E^2_+, E^2_-] = \frac{\sqrt{3}}{2} H_2 - \frac{1}{2} H_1$$
$$[E^3_+, E^3_-] = \frac{\sqrt{3}}{2} H_2 + \frac{1}{2} H_1$$

$\mathcal{C}$ also very similar.

<u>ALMOST</u> 3 COPIES OF $\mathcal{L}(SU(2))$, BUT DON'T HAVE 3 DIAG. GENS.

$$[E^1_+, E^2_+] = \frac{1}{\sqrt{2}} E^3_+$$
$$[E^1_-, E^2_-] = -\frac{1}{\sqrt{2}} E^3_-$$

$$[E^1_+, E^3_-] = -\frac{1}{\sqrt{2}} E^2_-$$
$$[E^1_-, E^3_+] = \frac{1}{\sqrt{2}} E^2_+$$

$$[E^2_+, E^3_-] = \frac{1}{\sqrt{2}} E^1_-$$
$$[E^2_-, E^3_+] = -\frac{1}{\sqrt{2}} E^1_+$$

the "3 copies of $\mathcal{L}(SU(2))$" don't commute!

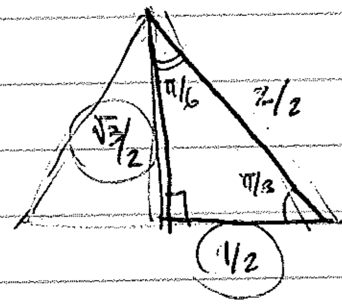
all other commutators vanish.

$[E^2_+, E^2_-] = \dots - \dots$

$$(*) \Rightarrow \left[\frac{\sqrt{3}}{2} H_2 - \frac{1}{2} H_1 , E^2_\pm\right] = \pm E^2_\pm \quad \&$$
$$\left[\frac{\sqrt{3}}{2} H_2 + \frac{1}{2} H_1 , E^3_\pm\right] = \pm E^3_\pm$$

$[E^3_+, E^3_-] = \dots + \dots$

HERE'S A USEFUL DIAGRAM



SO: $\left[\frac{\sqrt{3}}{2} H_2 - \frac{1}{2} H_1 , E^2_{\pm} \right] = \pm E^2_{\pm}$

" E^2_+ RAISES H_2 BY $\frac{\sqrt{3}}{2}$

LOWERS H_1 BY $1/2$

E^2_- LOWERS H_2 — " —

RAISES H_1 — " —

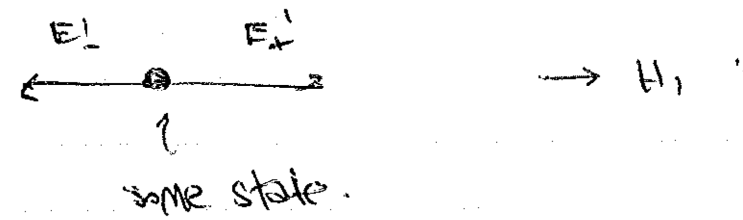
E^3_+ RAISES H_2 BY $\sqrt{3}/2$

RAISES H_1 BY $1/2$

E^3_- LOWERS — " —

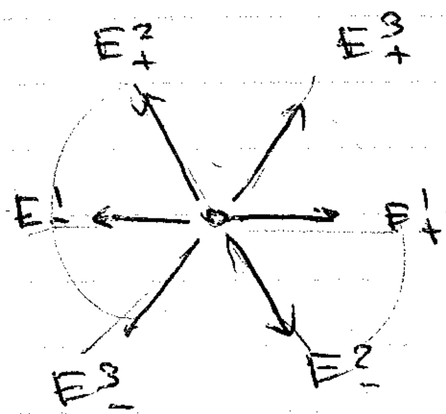
LOWERS — " —

THIS GIVES A SET OF RAISING/LOWERING DIRECTIONS



SO FAR: JUST LIKE $\mathcal{L}(SU(2))$

BUT NOW OTHER DIRECTIONS



NOW PROCEED AS BEFORE

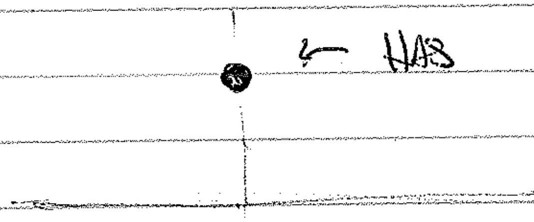
$\exists$ SOME HIGHEST WEIGHT STATE

$$|p, q\rangle \qquad \text{s.t. } E^i_+ |p, q\rangle = 0$$

p: H_1 EIGVAL.
q: H_2 EIGVAL.

CASE 1

HIGHEST WEIGHT STATE IS ON POS H_2 AXIS



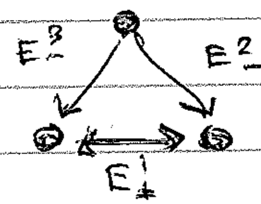
← HAS $H_1|0,q\rangle = 0$

↑

SPIN-0 w/rt H_1

no states to left OR right.

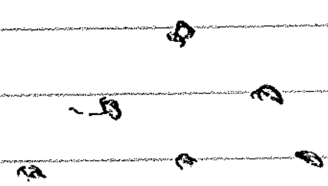
SO ACT W/ LOWERING:



in fact: this is a REP!

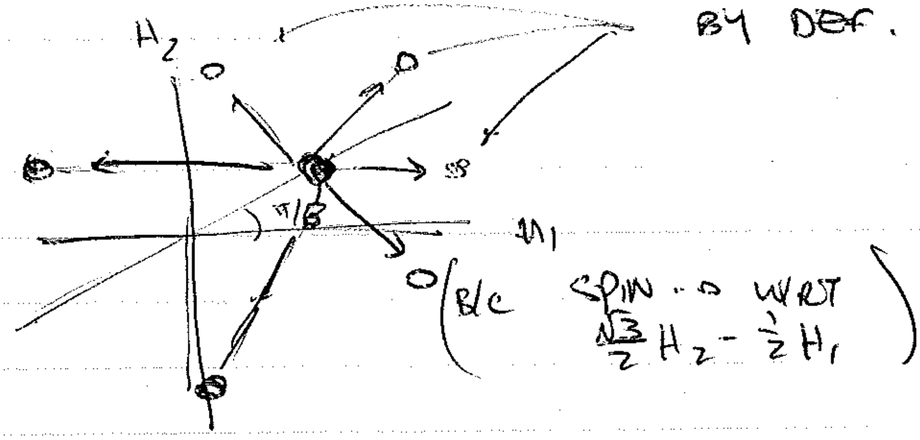
WE CALL IT $\bar{3}$ (bar: upward pyramid)

CAN ALSO HAVE ~~HIGHER~~ LARGER REPS



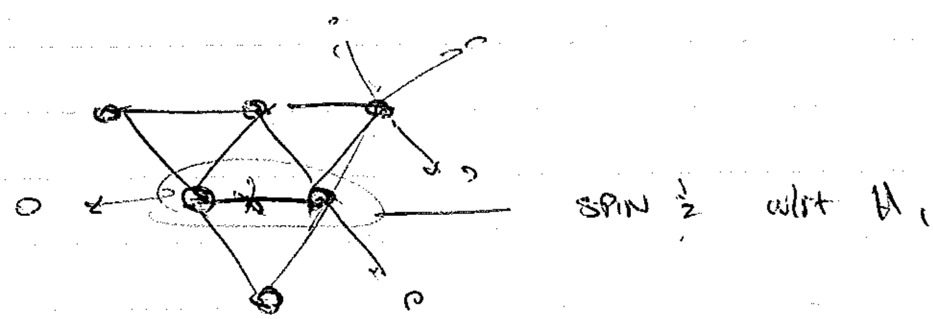
DEPENDS ON HIGHEST WEIGHT

CASE 2 : LOWEST STATE IS ON $-H_2$ AXIS
ie. HIGHEST WEIGHT IS ON
$\pi/6$ LINE.

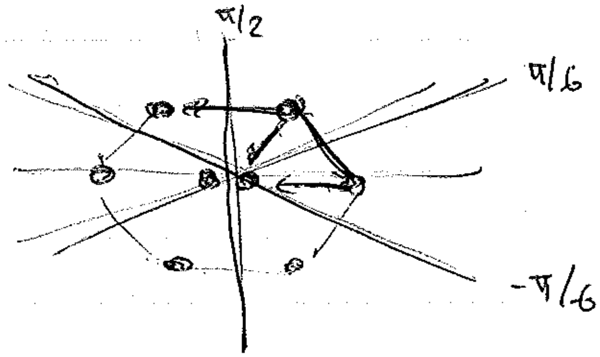


gives $\underline{3}$

similarly, can make bigger reps

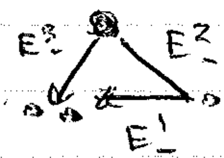


CASE 3 : GENERAL: $\pi/6 < \theta < \pi/2$



sym axes about which you have ladders

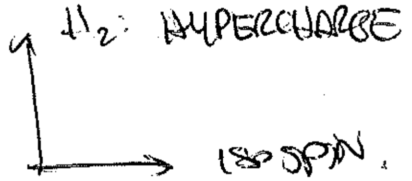
MULTIPLICITY:



two unique ways to get to $|0,0\rangle$ using only lowering ops

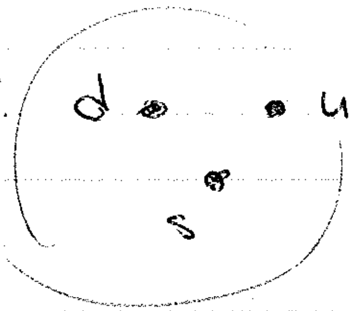
$\rightarrow$ 2 distinct states.

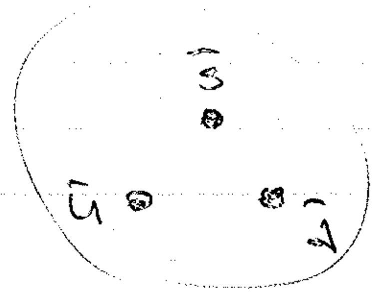
INTERPRETATION



} SU(3) FLAVOR

QUARKS:

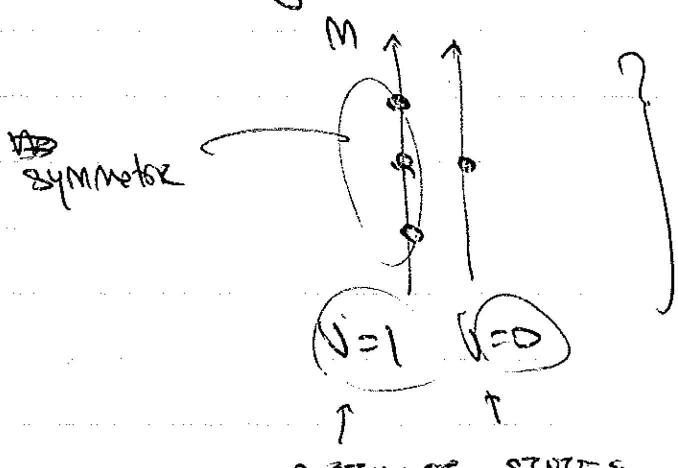




ANTI QUARKS

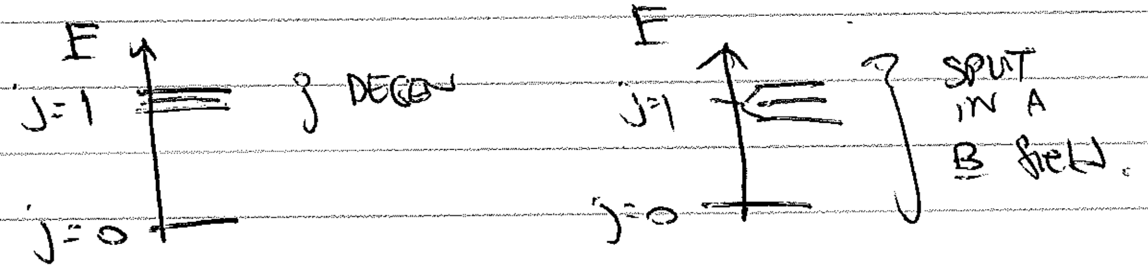
(q's come in SU(3) color reps, BUT YOU CAN'T TELL THEM APART B/C THAT'S A GOOD SYMMETRY)

analog: HYDROGEN



$\underline{2} \times \underline{2} = \underline{3} + 1$

HYDROGEN, CONT'D



you only resolve these when the sym. is broken.

SU(3) flavor is broken because $M_u \neq M_d \neq M_s$ so those states aren't symmetric.

BUT: SU(3) COLOR IS CONFINING, SO YOU NEVER OBSERVE QUARKS, JUST BOUND STATES.

IN ORDER TO BE COLOR NEUTRAL,
QUARKS <u>PAIR UP</u> (can also form triplets)

e.g. $u, d, s = \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

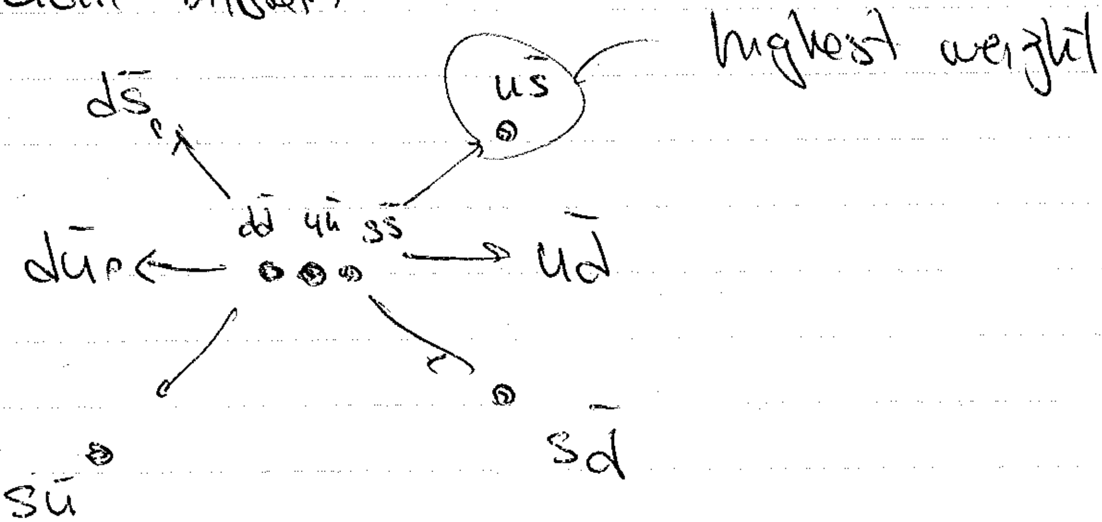
symmetry acts as $e^{i\alpha \cdot T}\begin{pmatrix}1\\0\\0\end{pmatrix}$

QUARK w/ ANTI QUARK

$3 \times \bar{3}$

$\begin{pmatrix} d & & u \\ & & \\ & s & \end{pmatrix} \otimes \begin{pmatrix} & \bar{s} & \\ & & \\ \bar{u} & & \bar{d} \end{pmatrix}$

WEIGHT DIAGRAM:



$3 \otimes \bar{3} = \underline{8} \oplus 1$

actually see A MESON <u>OCTET</u>
@ LOW ENERGIES.