LECTURE 1 Physics vs. Math; DIM. ANALYSIS

REMINDER: Do COURSE INTRODUCTION

Physics ≠ Mathematics

WHAT SEPARATES PHYSICS FROM MATH?
∃ many answers, a few are very good.

- CONNECTION to NATURE
  - ↳ empirical methodology

yet: PHYSICS IS UNIQUE IN THE SCIENCES IN THAT THERE'S A STRONG DIVISION BETWEEN Theory & Experiment

Is this "math"? → no.
- SEE OTHER BULLET POINTS

do I need this course? → yes.
1. YOU HAVE TO PASS QUALS
2. THIS IS ABOUT THINKING LIKE A PHYSICIST

- USE / MISUSE / RELIANCE on TAYLOR SERIES

eg. DOMAIN OF VALIDITY MAY FORMALLY BE 0!

more generally: perturbation theory
  - ↳ make the right approximation.

• **UNITS** my favorite answer.

physics relates/connects/predicts measurable DIMENSIONFUL quantities

↑

cm, sec, GeV, watts, ...

ie. these are UNITS

TO BE VERY CLEAR: what is a unit?

DIMENSIONAL QUANTITY:

eg. 3 apples 

3 ← DIMENSIONLESS NUMBER ← math

apples ← PHYSICAL, AGREED UPON STANDARD ← ~PHYSICS

↑ for what?

- eg. COST
- eg. CALORIC INTAKE
- eg. MASS

in fact: the "apple" is a conversion between all of these things.

eg: 1 METER

- DIST TRAVELLED BY LIGHT IN $(299\,792\,458)^{-1}$ sec ← time
- $1/10^6$ DIST FROM EQUATOR TO N. POLE
- SOME # OF WAVELEN OF krypton-86...

DIMENSIONAL ANALYSIS

A PHYSICAL QUANTITY q HAS DIMENSION $[q]$ WHICH WE TYPICALLY WRITE AS

$$[q] = L^a M^b T^c$$

↑ length ↑ mass ↑ time

COULD USE OTHER QUANTITIES (eg PRESSURE); BUT TYPICALLY CAN REDUCE TO THESE

eg. FORCE ... well, we know $F = ma$
$= m\ddot{x}$

<u>then</u>: $[F] = L^1 M^1 T^{-2}$

observe: $[F] = [m] \times [a]$

eg. what about <u>ENERGY</u>?

answ: $E = \frac{1}{2}mv^2$ or mc^2

$[E] = M L^2 T^{-2}$

Zeroth order DIM. Analysis

CHECK THE VALIDITY OF EXPRESSIONS
↑ "sanity check"

eg $(1+x)$ LOOKS FINE
$(1+L)$ DOES NOT!

dim-less.

LESSON: WRITE AS $(1+L/L_0)$
identify dimensionless params.

eg. L is 2 cm. IS THERE A BIG CHANGE IF L is CHANGED TO 4 cm?

DOUBLED!!

↓

not if $L_0 = 10$ m!
("basically zero to L_0.")

even more egregious:

e^L or $\sin(3\text{ cm})$

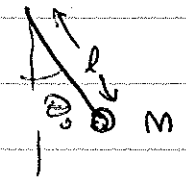
means nothing!

$1 + L + \frac{1}{2!}L^2 + \cdots$ ← each term had better have the same units.

First order DM. Analysis

What is the period of a pendulum?

↳ IDENTIFY RELEVANT QUANTITIES



L	→	l	LENGTH OF PENDULUM
M	→	M	IS THE MASS OF THE WEIGHT
LT^{-2}	→	g	IS THE GRAV. ACCELERATION
(1)	→	θ_0	IS THE INITIAL ANGULAR DISPLACEMENT

WANT AN EXPRESSION FOR A TIME

start w/ $g^{-1/2}$ since $[g^{-1/2}] = T\,L^{-1/2}$

then multiply by $l^{1/2}$ to cancel

$$\tau \sim \sqrt{l/g} \times f(\theta_0)$$

$f(\theta_0)$ ← DIM'LESS FUN. OF θ_0. DON'T KNOW FROM D.A.

INDEP OF M!

So WE GET THE PHYSICS OUT:

l GOES UP, τ GOES UP (LIKE SQRT)

g GOES UP, τ GOES DOWN (⟶)

M GOES UP, τ UNCHANGED

↳ this is real physics

overall prefactors are measurements

Second order analysis

ESTIMATING QUANTITIES

eg the Hierarchy Problem, simplified

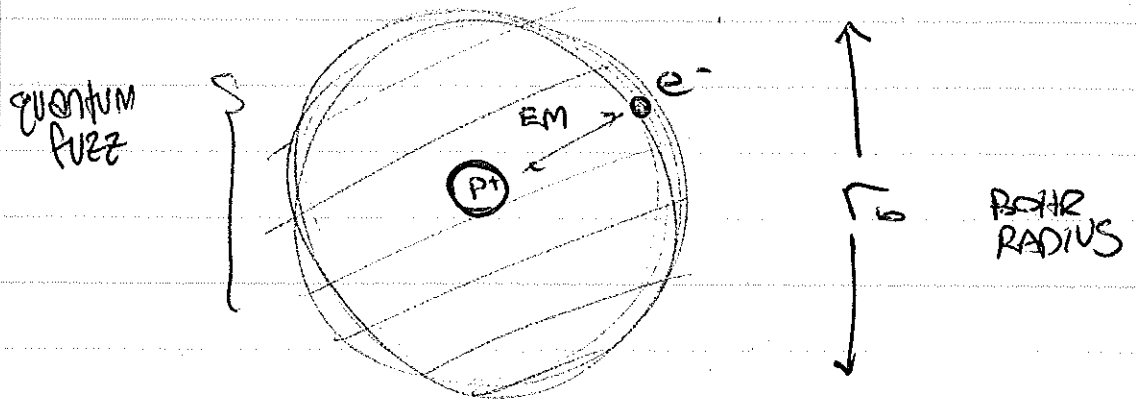
IN FIELD THEORY, EXTERNAL LINES IN A FEYNMAN DIAGRAM CORRESPOND to FACTORS IN A TERM IN A LAGRANGIAN DENSITY.

[redacted] ↑ homework

↱ math.ucr.edu/home/baez/lengths.html

How large is hydrogen?

WHAT DOES THIS MEAN? SIZE IS SO SMALL THAT IT'S A QUANTUM QUESTION.



WHAT COULD THIS DEPEND ON?

- e^- → m_e } e
- (p+) → M_p } ($q_p = -e$ ← not an indep. param
- CONSTANTS → $\hbar$, c, G_N
- OTHERS? → $B_\oplus$, $M_\oplus$, $M_\odot$, ... ?

NOW STOP & THINK:
which of these quantities don't make sense?

M_p: PROTON MASS >> ELECTRON MASS
SO PROBABLY ONLY ONE MATTERS
(& other mass gives a $O(M_e/M_p)$ correction)

THE ELECTRON IS THE ONE IN A "CLOUD" & WE KNOW FROM QM THAT WHAT REALLY ENTERS IN A 2 BODY PROBLEM IS THE REDUCED MASS: $M_p M_e/(M_p+M_e) \approx M_e$

G_N: HAS TO DO W/ GRAVITY, BUT GRAVITY IS MUCH WEAKER THAN ELECTROMAGNETISM, SO LET'S IGNORE THIS.

SIMILARLY - other quantities don't matter.

LEFT W/ M_e, e, $\hbar$

↑ MASS ↑ CHARGE ↑ "QUANTUM-NESS"
MOMENTUM × ~~BEES~~ LENGTH
(recall this from stat mech!)

CLAIM: $[m_e] = M$, $[\hbar] = ML^2T^{-1}$ } easy

$[e] = \underbrace{M^{1/2} L^{3/2} T^{-1}}_{\text{homework}}$

HINT: USE FORCE LAW $[g] = [F]^{1/2} L$

COMBINE THESE INTO SOMETHING W/ DIMENSION OF LENGTH:

$$\left[\frac{\hbar}{e}\right] = M^{1/2} L^{1/2} \quad \text{(get rid of T)}$$

$$\left[\frac{\hbar}{\sqrt{m_e}\, e}\right] = L^{1/2} \quad \text{(get rid of M)}$$

↑

identify this w/ Bohr radius

$$r_b = (\#)\, \frac{\hbar^2}{m_e e^2}$$

↑

coincidentally: $\# = 1$
could have had 2π's ...

THIRD ORDER DIM ANALYSIS

Scaling & similarity

see: Arnold *Math Methods of Classical Mech.* §11

VECTOR PDE: $m\ddot{\underline{r}} = -\partial U/\partial \underline{r}$

↑ time dependence $\left[\frac{d}{dt}\right] = T^{-1}$

↑ operator

(think of df/dt as $\Delta f/\Delta t = \frac{f_1 - f_0}{t_1 - t_0}$

$\Rightarrow \left[df/dt\right] = [f]\, T^{-1}$

$[U] = [F]\, L = ML^2T^{-2}$

$\left[\partial U/\partial r\right] = MLT^{-2}$

SUPPOSE WE HAVE A GRAVITATIONAL SYSTEM

eg ELLIPTICAL ORBITS OF PLANETS ABOUT A STAR

↑

SO WE HAVE A SET OF SOLUTIONS TO THE PDE ABOVE, $\underline{r}_0(t)$

WE CAN USE DIM-ANALYSIS TO UNDERSTAND OTHER SOLUTIONS.

IMAGINE DOING A SCALING TRANSFORM ON (t) ↝ $t \equiv \alpha t'$

OLD VAR t , t' NEW VAR

this is really just choosing units.

OBSERVE: ONLY LHS $(m \ddot{\vec{r}}_0(t))$ CHANGES! WHY? RHS HAS T^{-2} DIM, BUT THIS COMES FROM $[G_N] \sim T^{-2}$, CONSTANT.

SO: $m \ddot{\vec{r}}_0(t) \longrightarrow \underbrace{m \alpha^{-2}}_{\text{can "undo" this scaling if we also change } m \equiv \alpha^2 m'} \vec{r}_0(\alpha t)$

encodes planet { $m \ddot{\vec{r}}_0(t) \rightarrow m' \ddot{\vec{r}}_0(\alpha t)$ ← $0 = \frac{d}{d(\alpha t)}$

encodes system { $-\partial U/\partial r$ $-\partial U/\partial r$

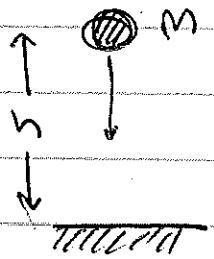
SO: GIVEN ~~SOLUTION~~ TRAJECTORY $\vec{r}_0(t)$ FOR A PLANET OF MASS m; A PLANET THAT IS FOUR TIMES HEAVIER WILL TRAVERSE SAME TRAJECTORY TWICE AS QUICKLY.

FOURTH ORDER DIM. ANALYSIS

USE FOR ERROR ESTIMATE

cf. DIM. ANALYSIS, FALLING BODIES, & THE FINE ART OF NOT SOLVING DIFF EQ. Bohren
Am J. Phys 72 534

HIGH SCHOOL PROBLEM:



WHAT IS THE TIME t_0 FOR AN OBJECT TO DROPPED HIT THE GROUND, FROM HEIGHT h?

HIGH SCHOOL ANSWER:

~~$F = Mg$~~ g = GRAV. ACCEL

~~$M a = F$~~

$$\ddot{x} = g$$

h

$\dot{x}_0$ = init vel

INTEGRATE: $x = \frac{1}{2} g t^2 + \cancel{\dot{x}_0 t} + \cancel{D}$

D PICKS COORD. ORIGIN

$$t_0 = \sqrt{\frac{2h}{g}}$$

this is the easy answer.
often good enough.

IMPORTANT QUESTION:
HOW GOOD IS THIS APPROX?

BETTER: SAME Q, BUT: WITHOUT DOING HARD WORK!
↳ SOLVING ANY MORE DIFF EQS.

OUR "ZEROTH ORDER" ESTIMATE IS $t_0 = \sqrt{2h/g}$
WANT TO KNOW THE ERROR:

$$\frac{t_{\text{realistic}} - t_0}{t_0}$$

DIMENSIONLESS COMBINATION
THAT GIVES FRACTIONAL ERROR
FROM NEGLECTING "MICROPHYSICS"

→ by the way: this is a "deep" idea. this is why a CHEF does not need to know SUBATOMIC PHYSICS.

UNDERLYING IDEA (FORMALIZED)
IN MUCH OF THEORETICAL PHYSICS

MAIN IDEA : ξ IS SMALL
(otherwise to was the wrong thing to calculate in the first place)

$$\frac{t_r - t_o}{t_o} \equiv f(\xi)$$

ξ ← DIMENSIONLESS PARAM CHARACTERIZING UNACCOUNTED FOR MICROPHYSICS

which should also be small!

PICK ξ s.t. $\xi \to 0$ CORRESPONDS TO TURNING OFF THE MICROPHYSICS

THEN $f(0) = 0$

if this is not true, use $\xi' = Y_\xi$

THEN WE MAY TAYLOR EXPAND (super small)

$$f(\xi) = f(0) + \left.\frac{df}{d\xi}\right|_0 \xi + \mathcal{O}(\xi^2)$$

$f(0)$: 0

$\left.\frac{df}{d\xi}\right|_0$: SOME DIM'LESS NUMBER, PRESUMABLY $\mathcal{O}(1)$

TO TO LEADING ORDER IN THE MICROPHYSICS, THE ERROR GOES LIKE

$$\boxed{\frac{t-t_0}{t_0} \sim \xi}$$

eg. g is not constant, varies w/ height (radial distance from center of earth!)

RELEVANT PARAMETERS TO INCLUDE? R, radius of earth!

(WHY NOT G_N? → ALREADY SEEPETH IN G_N)

↳ $m\ddot{x} = \frac{-mg}{(1+z/R)^2}$

TWO CHOICES FOR DIM'LESS ξ: $\frac{h}{R}$, $\frac{R}{h}$

↳ $R \gg h$, so $\xi = h/R$

(when $R \to \infty$, expect naive result to be correct.)

then $\boxed{\frac{t-t_0}{t_0} \sim h/R}$

↑ this symbol encodes physics.

DON'T FORGET MARKERS

NEXT WK: REVIEW OF LINEAR ALG/QM & INTRO TO DIFF EQ.