LEC 1 (A)

PHYSICS 208: GENERAL RELATIVITY

10 Jan

WEBSITE: faculty.ucr.edu/~flipt/P208_2016.html

LECTURES: PHY 2104 TUE/THR 5:10 - 6:30

can we move to reading or conf. room?

→ PROBLEM: missing 3 lectures

could do make up over exams? OR GOOGLE HANGOUTS? OR no make up, just work?

GRADING / HW / EXAMS

- EVERYONE HERE IS A GRAD STUDENT TAKING THIS CLASS TO LEARN; I DON'T INTEND FOR ANYONE TO BE PENALIZED FOR THAT.

- NO EXAMS
- GRADING (A or B) BASED ON HW (WEEKLY, DUE TUE) (SUBJECT TO CHANGE!)

  PLEASE DO MINIMUM WORK! (>50% HW)

  → WILL BE GRADED LOOSELY, BUT WILL BE IMPORTANT FOR ME TO CALIBRATE

- HOMEWORK WILL INCLUDE READING — PLEASE DO THIS, I WILL EXPECT YOU TO HAVE SEEN THE ASSIGNED MATERIAL AHEAD OF CLASS.

- AS YOU ALREADY KNOW: MOST OF YOUR LEARNING WILL COME FROM DOING PROBLEMS & DISCUSSING W/ EACH OTHER!!

  LECTURE IS A WAY TO GUIDE THIS LEARNING, BUT CANNOT REPLACE IT.

Discussion / OH :

- no discussion sec.
- office hrs: the 30 min before or after class & by appt.

(AFTER 5:30 pm ON MOST DAYS I WILL BE AVAILABLE)

TEXTBOOK : official choice is HARTLE
see website for other suggestions — I DON'T CARE WHICH YOU USE.

TODAY: Assessment ("judgement free")
NO NEED FOR NAMES, FEEL FREE TO DISCUSS.

no gravity, no curvature

REVIEW: SPECIAL RELATIVITY

SPEED OF LIGHT IS CONSTANT

... & IMPLICATIONS OF THIS.

RECALL THE USUAL IDEAS

- LENGTH CONTRACTION & TIME DILATION | $\vec{E} \leftrightarrow \vec{B}$
  "watching a train pass by"
- SIMULTANEITY is NOT ABSOLUTE

gives all sorts of cute "paradoxes" that test our intuition

UNITS: I WILL ALMOST ALWAYS WORK IN UNITS WHERE $\boxed{c = 1}$ (NATURAL UNITS)

YOU CAN USE DIMENSIONAL ANALYSIS TO UNIQUELY RESTORE FACTORS OF c TO GIVE "unnatural units" (eg MKS)

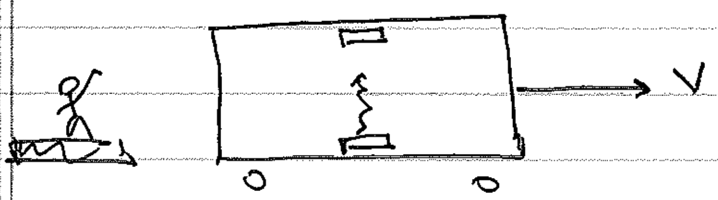
So I can write things like

$$ds^2 = dt^2 - dx^2$$

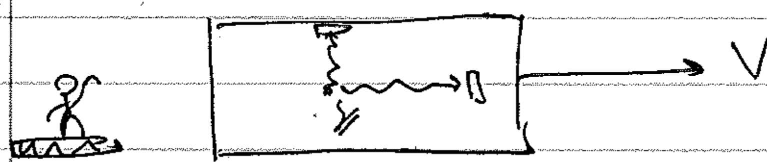
in natural units, these both have dim. of (length)^2

clocks & rulers on moving train

RESULT OF "TRAIN ANALYSIS": (see HW)

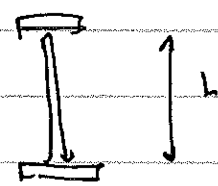


LIGHT CLOCK

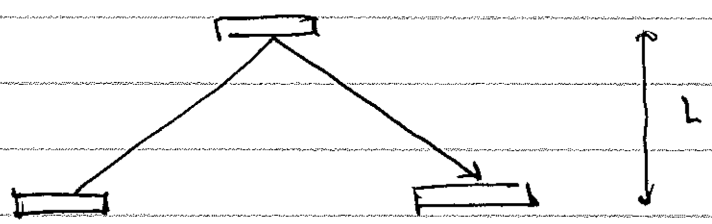


INTERFEROMETER

Key point: in train frame



in train station frame



BUT IN BOTH FRAMES, SPEED OF LIGHT IS CONSTANT.

for v along $\hat{x}$ direction

TIME DILATION — LEN. CONTR?? (GET TO THIS LATER)

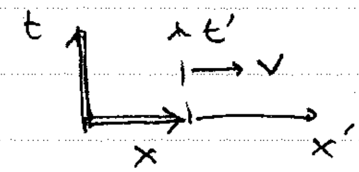
$$\left.\begin{aligned} t' &= \gamma t - \gamma\beta x \\ x' &= -\gamma\beta t + \gamma x \\ y' &= y \\ z' &= z \end{aligned}\right\} \text{LORENTZ BOOST ALONG } \hat{x}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}, \quad \beta = v$$

We've set $c=1$, or else $t \mapsto ct$

WE SIMPLIFY FURTHER & DEFINE

SIGN CONVENTION IS FOR



$$x^0 \equiv t$$
$$x^1 \equiv x$$
$$x^2 \equiv y$$
$$x^3 \equiv z$$

& to understand physics, sufficient to consider 2D (x^0, x^1) plane.

$$\begin{pmatrix} x^{0'} \\ x^{1'} \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}}_{\Lambda} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$

β can have either sign.

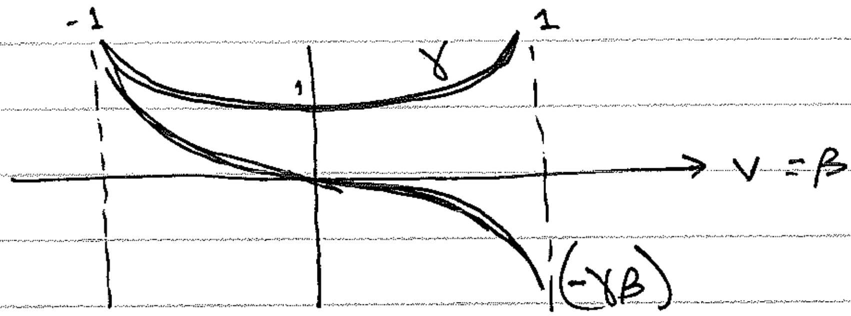
SCALING: AS $v = \beta$ increases, $(0 \le |\beta| < 1)$

γ increases ; singular @ $\beta = 1$

$\gamma\beta$ increases ; & zero @ $\beta = 0$

CHECK: for $\beta = 0$, $\Lambda = \mathbb{1}$. no transf.
"continuously connected to the identity"

SO PURELY FROM ASYMPTOTICS, WE HAVE



SO A BOOST IN $\hat{x}$ DIRECTION LOOKS LIKE

$$\Lambda = \begin{pmatrix} \underbrace{\gamma}_{1 \le \gamma < \infty} & \underbrace{-\gamma\beta}_{-\infty < \gamma\beta < \infty} \\ -\gamma\beta & \gamma \end{pmatrix}$$

infinitesimal boost: $(\beta \ll 1)$

$$\Lambda - 1 \approx \begin{pmatrix} O(\beta^2) & -\beta \\ -\beta & O(\beta^2) \end{pmatrix}$$

this infinitesimal transf. is called a <u>generator</u> of the boost

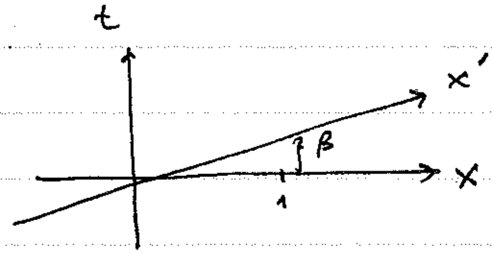
GIVES GROUP STRUCTURE OF LORENTZ SYMMETRY

WHAT DO (x', t') COORDINATES LOOK LIKE ON (x,t) PLANE?

PLOT (x', t') AXES

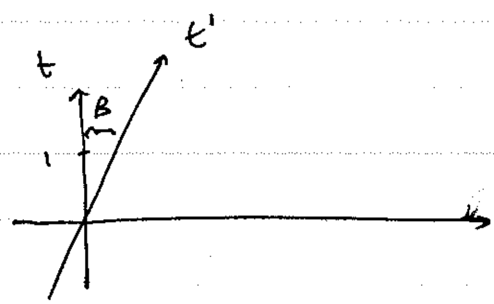
x' AXIS $\longleftrightarrow$ $t' \equiv \underbrace{0 = \gamma t - \gamma \beta x}$

$$t = \beta x$$



t' AXIS $\longleftrightarrow$ $x' \equiv \underbrace{0 = \gamma x - \gamma \beta t}$

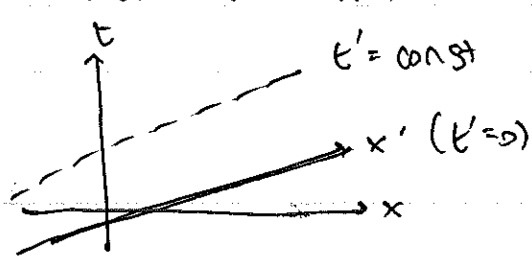
$$x = \beta t \quad \left(\text{or } t = \tfrac{1}{\beta} x\right)$$

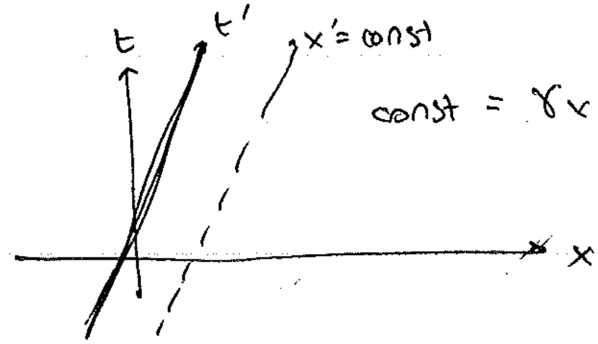


shifts of axes
↓

LINES OF CONSTANT x' OR t' ARE SIMILAR

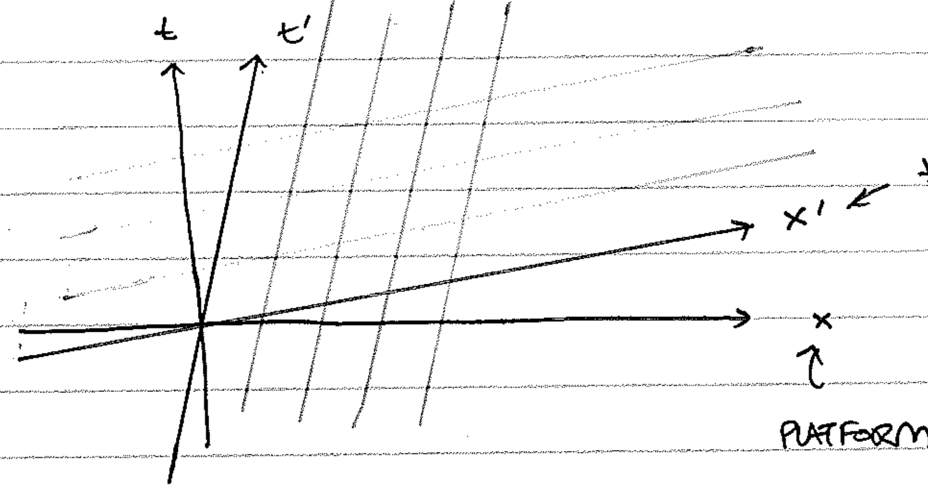
$\text{const} = \gamma t - \gamma \beta x$





$\text{const} = \gamma x - \gamma \beta t$

AND SO, FOR A BOOST IN THE $+\hat{x}$ DIRECTION, A CARTESIAN GRID IS TRANSFORMED TO :

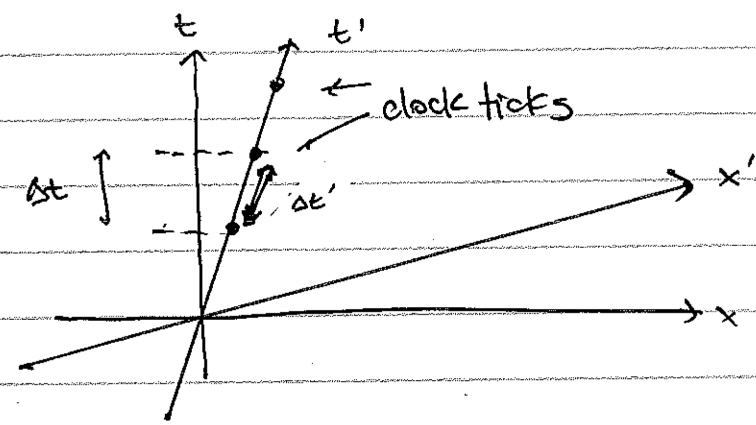


THIS CONTAINS ALL THE INFORMATION OF THE BOOST.

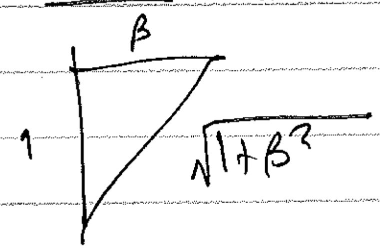
TIME DILATION

LET $\mathcal{O}'$ BE FRAME OF CLOCK (on train)

$\Delta x' = 0$ ← clock is stationary in train



hey - that doesnt look like time dilation?!



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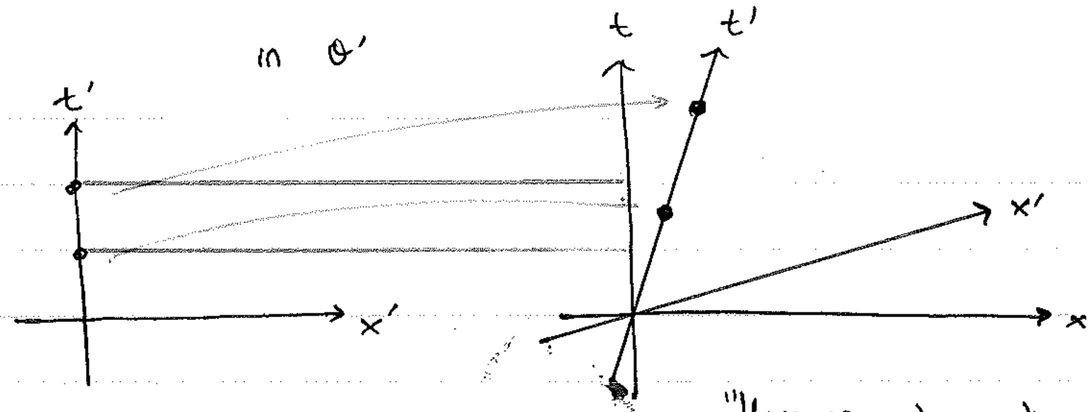


"this is not just a rotation"

SANITY CHECK

t' AXIS

$$x' = 0 = -\gamma\beta t + \gamma x \Rightarrow \gamma x = \gamma\beta t$$

then: $$t' = \gamma t - \gamma\beta x$$
$$= \gamma t - \gamma\beta^2 t$$
$$= \frac{1-\beta^2}{\sqrt{1-\beta^2}}\, t \quad \leftarrow \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$= \frac{1}{\gamma} t$$

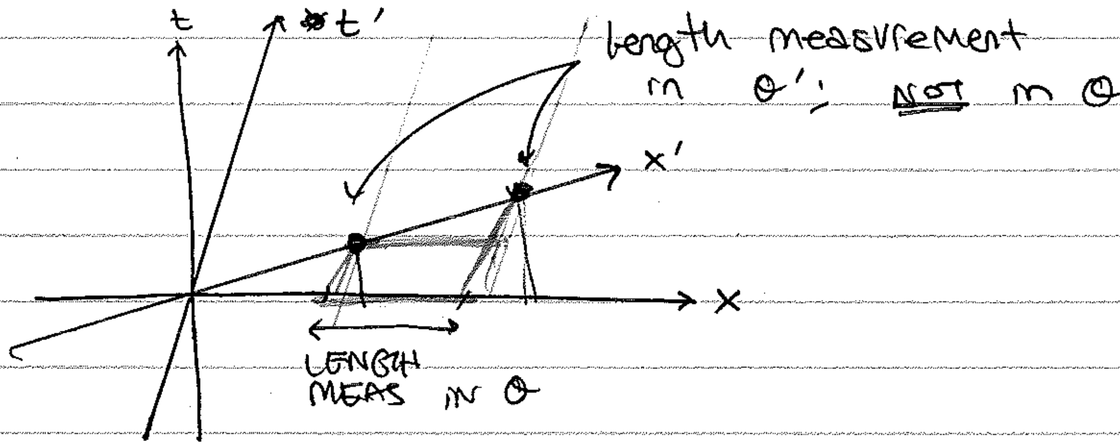
$$\boxed{t = \gamma t'} > t'$$

[see mult. hyperbolae]

LENGTH CONTRACTION : more subtle

"events" in spacetime

LENGTH MEASUREMENT IS 2 POINTS
(t_1, x_1) & (t_2, x_2) s.t. $t_2 - t_1 = 0$



OBSERVE ASYMMETRY

TIME DILATION : $\Delta x' = 0$

LENGTH CONTRACTION : $\Delta t = 0$

not $\Delta t' = 0$

[maybe I should expand this section]

SIMULTANEITY → out the window

↳ this is why we must work in spacetime vs. space & time

WANT TO GET BACK TO THIS FROM GEOMETRIC POV.
FIRST, REVIEW "TRIVIAL" GEOMETRY.

GEOMETRY: REVIEW OF 2D EUCLIDEAN SPACE

"ordinary flat space"

VECTORS: WE'LL USE INDICES (mathematicians hate this)

$$\underline{V} = V^i \underline{e}_{(i)} \quad \longleftarrow \quad \text{SUMMATION NOTATION}$$

ABSTRACT OBJECT REPRESENTING THE (i) UNIT VECTOR

$$V^i \underline{e}_{(i)} \equiv V^1 \underline{e}_{(1)} + V^2 \underline{e}_{(2)} + \cdots$$

eg if $\underline{V} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, then this is:

$$\underline{V} = \underbrace{1}_{V^1} \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\underline{e}_{(1)}} + \underbrace{3}_{V^2} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\underline{e}_{(2)}}$$

the components of a vector have upper indices. sometimes we'll be sloppy & identify V^i with the vector itself.

WE CAN TAKE 2 VECTORS & MAKE A NUMBER
that is: there is a linear map

$$g : V \times V \to \mathbb{R}$$

vector space ($\mathbb{R}^2$ in this case)

EUCLID. FLAT SPACE METRIC

$$g(\underline{v}, \underline{w}) \equiv v_i w^i = v_1 w^1 + v_2 w^2$$

LOWER INDEX OBJECT : dual vector
row vector
etc.

can think of lower index object
as $g(\underline{v}, \cdot) \longleftrightarrow v_i$

function: $V \to \mathbb{R}$

so v_i is a linear function of vectors
$v_i(w^i) = v_i w^i$

gives the usual dot product.

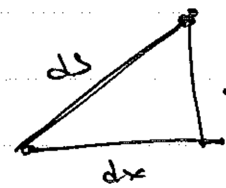
$$\underline{v} \cdot \underline{w} = \underline{v}^T \underline{w} = v_i w^i = v^1 w^1 + v^2 w^2$$

"matrix" mult

stop thinking like this

LET US WRITE THIS IN A PECULIAR WAY

$$ds^2 = dx^2 + dy^2$$



infinitesimal length between 2 points

$$= g_{ij}\, dx^i\, dx^j$$

$$g_{ij} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

<u>METRIC</u> (related to measuring distances)

<u>Then</u>: $V_i = g_{ij} V^j$ METRIC LOWERS INDICES

$$g_{ij} V^i W^j = V^1 W^1 + V^2 W^2$$

METRIC AS A MAP: $V \times V \to \mathbb{R}$

$$g^{ij} g_{jk} = \delta^i{}_k$$ metric & inverse

$$g^{ij} V_j = V^i$$ inverse metric raises indices

<u>VERY</u> loosely can think of lower index object as a ROW vector.

$$V_i W^i = \underbrace{(V_1 \;\; V_2)}_{\vec{V}^T} \begin{pmatrix} W^1 \\ W^2 \end{pmatrix} = V_1 W^1 + V_2 W^2$$

(this is a <u>crutch</u>; BETTER: ROW VECTOR IS AN <u>EXAMPLE</u> OF A LOWER INDEX OBJECT ON EUCLIDEAN SPACE)

GIVEN THE METRIC OF EUCLIDEAN SPACE, THERE ARE SPECIAL TRANSFORMATIONS THAT LEAVE 'DOT PRODUCTS' UNCHANGED:

$$\underbrace{(V_1 \;\; V_2)\begin{pmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{pmatrix}}_{(R_\theta \underline{V})^T} \underbrace{\begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}\begin{pmatrix} W^1 \\ W^2 \end{pmatrix}}_{(R_\theta \underline{W})}$$

$R^j{}_i V_j \rightarrow (R_\theta \underline{V})^T \qquad (R_\theta \underline{W}) \leftarrow R^i{}_j W^j$

<u>ROTATION</u>, R_θ, CAN BE <u>DEFINED</u> AS A TRANSFORMATION THAT LEAVES $V^i W^j g_{ij}$ INVT.

__USEFUL:__ OBJECTS W/ INDICES TRANSFORM

UPPER INDEX i : $R^i{}_j$ [illegible]^j

LOWER INDEX i : $R^j{}_i$ [illegible]$_j$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ ''

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ [illegible]$_j\, R^j{}_i$

__explicitly__
$$R^i{}_j = \begin{pmatrix} R^1{}_1 = c_\theta & R^1{}_2 = s_\theta \\ R^2{}_1 = -s_\theta & R^2{}_2 = c_\theta \end{pmatrix}$$

__"COLUMN VEC"__ :
$$V^i \to R^i{}_j V^j = R^i{}_1 V^1 + R^i{}_2 V^2$$
$$= \begin{pmatrix} R^1{}_1 V^1 + R^1{}_2 V^2 \\ R^2{}_1 V^1 + R^2{}_2 V^2 \end{pmatrix}$$
$$= \begin{pmatrix} c_\theta V^1 + s_\theta V^2 \\ - s_\theta V^1 + c_\theta V^2 \end{pmatrix}$$

__"ROW VEC"__
$$V_i \longrightarrow V_j R^j{}_i = V_1 R^1{}_i + V_2 R^2{}_i$$
$$= \left(V_1 R^1{}_1 + V_2 R^2{}_1 \, , \; V_1 R^1{}_2 + V_2 R^2{}_2 \right)$$
$$= \left(c_\theta V_1 - s_\theta V_2 \, , \; s_\theta V_1 + c_\theta V_2 \right)$$

FOR A MORE GENERAL OBJECT

(p, q) tensor: $\underbrace{T^{i_1 \cdots i_p}{}_{j_1 \cdots j_q}}$

multi-linear map
in the sense that

$$T : \underbrace{V \times \cdots \times V}_{q \text{ vectors}} \times \underbrace{V^* \times \cdots V^*}_{p \text{ dual vec}} \to \mathbb{R}$$

$$T^{i_1 \cdots i_p}{}_{j_1 \cdots j_q} V_{(1)}^{j_1} \cdots V_{(q)}^{j_q} W_{(1) i_1} \cdots W_{(p) i_p}$$

HOW DOES IT TRANSFORM?

$R^i{}_j$ for EACH UPPER INDEX $T^{\cdots j \cdots}{}_{\cdots}$
$R^j{}_i$ for EACH LOWER INDEX $T^{\cdots}{}_{\cdots j \cdots}$

GENERALIZE TO ROTATIONS IN, SAY, $\mathbb{R}^3$

IMPORTANT: THINGS W/ NO FREE INDICES DO NOT TRANSFORM (not frame-dep.)

$V \cdot W = \underbrace{V^i W_i}_{\text{no free indices}}$ is invariant

same under rotations

GEOMETRY & RELATIVITY: MINKOWSKI SPACE

LORENTZ TRANSFORM

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$ ← does not look like a rotation!

WHAT TYPE OF METRIC (what kind of space) IS INVARIANT UNDER Λ?

PROPOSE: $ds^2 = dt^2 - dx^2$

$$\longrightarrow (\gamma dt' - \gamma\beta dx')^2 - (-\gamma\beta dt' + \gamma dx)^2$$

$$= \gamma^2 (dt')^2 - 2\gamma^2\beta dt' dx' + \gamma^2\beta^2 (dx')^2$$
$$- \gamma^2\beta^2 (dt')^2 + \text{——"——} - \gamma^2 (dx')^2$$

note: $\gamma^2(1-\beta^2) \equiv 1$

$$= (dt')^2 - (dx')^2 \checkmark$$

MINKOWSKI SPACE (FLAT SPACETIME)

REDERIVE LORENTZ TRANSFORMATION FROM GEOMETRY

$$ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu \qquad x^\mu = \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \begin{pmatrix} t \\ x \end{pmatrix}$$
$$\equiv dt^2 - dx^2$$

when this was a + sign in euclidean space, we recognized that $ds^2 =$ const gives a circle.

for MINKOWSKI SPACE, ds^2 is a hyperbola.

SO USE HYPERBOLIC TRIG FUNCTIONS

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x})$$
$$\sinh x = \tfrac{1}{2}(e^x - e^{-x})$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

INDEED

$$\begin{pmatrix} t \\ x \end{pmatrix} \longrightarrow \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

is an invariance

η is transformation parameter.

~~[illegible]~~

CHECK: let $q = \begin{pmatrix} t \\ x \end{pmatrix} \xmapsto{\Lambda} \begin{pmatrix} ct + sx \\ st + cx \end{pmatrix}$

where $c = \cosh R$
$s = \sinh R$

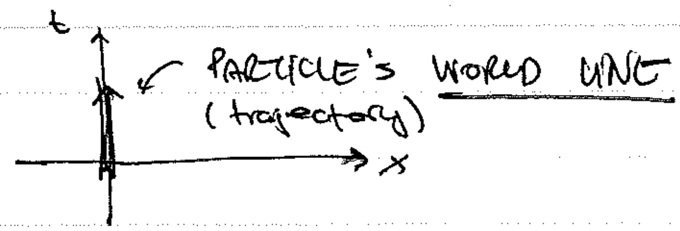
the length is $\|q\|^2 = q \cdot q \quad = t^2 - x^2$

$$= (ct + sx)^2 - (st + cx)^2$$
$$= c^2t^2 + 2cs\,tx + s^2x^2$$
$$- s^2t^2 - 2cs\,tx + c^2x^2$$

$$= t^2 - x^2 \quad \checkmark$$

BUT WHAT TO MAKE OF R?
CONNECT TO PHYSICS

IN THE REST FRAME OF A PARTICLE,
$\Delta x / \Delta t = 0$



now do LORENTZ TRANSFORM:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} t \cosh R + x \sinh R \\ t \sinh R + x \cosh R \end{pmatrix}$$

ORIGIN IS NOT TRANSFORMED.

BUT FOR $\Delta t \neq 0$

$$\Delta x = 0$$

$$\begin{pmatrix} \Delta t' \\ \Delta x' \end{pmatrix} = \begin{pmatrix} \Delta t \cosh R \\ \Delta t \sinh R \end{pmatrix}$$

then $\boxed{\underbrace{\frac{\Delta x'}{\Delta t'}} = \tanh R}$ → R IS RAPIDITY

VELOCITY, $v = \beta$

next use :

$$\cosh^2 R - \sinh^2 R = 1$$

$$1 - \underbrace{\tanh^2 R}_{\beta^2} = \frac{1}{\cosh^2 R}$$ top left elem of Λ.

$$\Rightarrow \cosh^2 R = \frac{1}{1-\beta}$$

$$\Lambda = \begin{pmatrix} \cosh R = \frac{1}{\sqrt{1-\beta}} & 0 \\ 0 & \frac{1}{\sqrt{1-\beta}} \end{pmatrix}$$

then use: $\sinh R = \underset{\pm\beta}{\tanh R}\ \underset{\gamma}{\cosh R}$

$$\Lambda = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix}$$

sign ambiguity corresp. to which way we boost.

(ANALOGOUS to which direction we rotate θ)