Chapter 5

Conditional CAPM

5.1 Conditional CAPM: Theory

5.1.1 Risk According to the CAPM

The CAPM is not a perfect model of expected returns. In the 40+ years of its history, many systematic deviations from it have been documented. (We will talk more about those deviations, or *anomalies*, later in the course). However, the CAPM made the point that still remains the central concept in modern finance theory: risk is not return variance. Rather, risk is the covariance of returns with an economy-wide source of volatility (namely, the market).

Indeed, the variance created by the independent realizations of random outcomes can always be eliminated through *diversification*. For example, playing heads-and-tails once and betting $100 on heads is a very risky thing to do. But playing heads-and-tails 10000 times betting 1 cent is not risky at all, because the number of bets you win and lose will be approximately equal, and on average, according to the Law of Large Numbers, you will break even almost surely.

The CAPM points out that not all risks in the economy can be handled this way.

Surely, the risk that the company will lose its visionary leader is huge if you hold one stock only, but the same risk is small if you hold hundreds of stocks in your portfolio. However, the risk of economy sliding into another recession will impact all stocks in any portfolio. No matter how diversified the portfolio is, it will be hit by the economy-wide shock. The question is only how much it will be hit.

The CAPM says that risk is exactly this "how much they will be hit", and only this type of risk is compensated in the market. The stock that does not react much to ups and downs in the economy has low risk (low beta) and will offer low expected return.

Risk according to the CAPM: Any asset that appreciates when the market goes up and loses value when the market goes down, is risky and has to earn more than the risk-free rate.
Risk is when the value drops when market goes down.

The predictability of market returns provides important information that the amount of risk and the investors' reluctance to bear it both change over time. In recessions, investors are very risk-averse and require a high return for bearing risk. In expansions, they are willing to take bigger risks for smaller return. It is therefore natural that the investors would wish to limit their risk exposure during recessions and increase the risk exposure during booms. However, the CAPM is a one-period model, in which such preferences cannot exist. This could be one of the reasons the CAPM fails to explain many regularities in returns.

5.1.2 Risk According to the Conditional CAPM

Here is an example of how the Conditional CAPM works. Assume that recessions are three times shorter than expansions, that is, the economy spends 25% of time in a recession and 75% of time in an expansion. Assume also that the expected market risk premium is 4% during expansions and 12% during recessions. The average market risk premium is then $\frac{1}{4} \cdot 12\% + \frac{3}{4} \cdot 4\% = 6\%$, just as we observe in the data.

Consider two stocks. One stock has the beta of 2 in recession and the beta of $\frac{2}{3}$ in expansion. The other has the beta of $\frac{1}{2}$ in recession and the beta of $\frac{7}{6}$ in expansion. The CAPM will see only the average beta of each stock: $2 \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{3}{4} = 1$ for the first one and $\frac{1}{2} \cdot \frac{1}{4} + \frac{7}{6} \cdot \frac{3}{4} = 1$ for the second one. Hence, the CAPM will predict that the excess return to both stocks will be, on average, 6%.

	Stock A: Countercyclical Beta			Stock B: Countercyclical Beta		
	Market risk premium	Stock beta	Stock risk premium	Market risk premium	Stock beta	Stock risk premium
Recession (p=1/4)	12%	2	24%	12%	1/2	6%
Expansion (p=3/4)	4%	2/3	8/3%	4%	7/6	14/3%
Average	6%	1	8%	6%	1	5%
Static CAPM	6%	1	6%	6%	1	6%

However, in reality the first stock is expected to earn $2 \cdot 12\% = 24\%$ in recessions and $\frac{2}{3} \cdot 4\% = \frac{8}{3}\%$ in expansions, which makes its expected excess return $\frac{8}{3}\% \cdot \frac{3}{4} + 24\% \cdot \frac{1}{4} = 8\%$ on average. The second stock is expected to earn $\frac{1}{2} \cdot 12\% = 6\%$ during recessions and $\frac{7}{6} \cdot 4\% = \frac{14}{3}\%$ during expansions, which makes its expected excess return $\frac{14}{3}\% \cdot \frac{3}{4} + 6\% \cdot \frac{1}{4} = 5\%$ on average.

The first stock is the one that exhibits the undesirable behavior: its risk exposure (aka market beta) increases in recessions, when bearing risk is especially painful, and decreases in expansions, when investors do not mind bearing more risk. Hence, the first stock is riskier than what the CAPM would lead us to think and earns higher return, which cannot be explained by the CAPM. The CAPM would estimate that the first stock has an abnormal return (aka the alpha) of 2% per year, suggesting this is a good investment. The Conditional CAPM retorts that the extra reward comes to the owners of the first stock in return for bearing the extra risk of undesirable beta changes, and the first stock is in fact as good investment as anything else.

The second stock, to the contrary, exhibits the desirable behavior by lowering the beta when risks are high and raising the beta when risks are low. Thus, the second stock is less risky than what the CAPM says, and earns a lower return. The CAPM would attribute the negative alpha of -1% per year to the second stock, suggesting that it is a poor investment choice, but the Conditional CAPM shows that the second stock is a reasonable conservative investment.

Risk according to the Conditional CAPM: In addition to the market risk as described by the CAPM, the countercyclicality of the market beta (i.e., higher beta in recessions) is another source of risk. **Risk is when the risk exposure increases in bad times.**

5.1.3 How Important is the Risk Captured by the Conditional CAPM

In this subsection, we will try to estimate the amount of additional risk created by the changing beta. That is, we will try to answer the question: is it likely that the difference in the expected return as estimated by the CAPM and the Conditional CAPM substantial? To do that, we need to look at what went wrong with the static CAPM from the statistical standpoint. Let's recall the definition of covariance:

$$Cov(X,\ Y) \equiv E[(X - E(X)) \cdot (Y - E(Y))] \tag{5.1}$$

That is, covariance is the expected value of the product between the two deviations from the average.

Now, let's use the fact that expectation is linear, that is, the expected sum is the sum of expectations ($E(X+Y) = E(X) + E(Y)$) and $E(aX) = a \cdot E(X)$. $E(X)$ and

$E(Y)$ in (5.1) above are numbers, not random variables, therefore

$$E[(X - E(X)) \cdot (Y - E(Y))] = E[X \cdot Y - E(X) \cdot Y - E(Y) \cdot X + E(X) \cdot E(Y)] =$$

$$= E(X \cdot Y) - E(E(X) \cdot Y) - E(E(Y) \cdot X) + E(E(X) \cdot E(Y)) =$$

$$= E(X \cdot Y) - E(X) \cdot E(Y) - E(Y) \cdot E(X) + E(X) \cdot E(Y) = E(X \cdot Y) - E(X) \cdot E(Y)$$

I conclude that the definition of the covariance implies

$$Cov(X,\ Y) = E(X \cdot Y) - E(X) \cdot E(Y) \Rightarrow \tag{5.2}$$

$$\Rightarrow E(X \cdot Y) = Cov(X,\ Y) + E(X) \cdot E(Y) \tag{5.3}$$

Assume now that both the beta and the market risk premium are random variables. What we are interested in is the risk premium of the stock,

$$E(\beta_S \cdot R_M) = E(\beta_S) \cdot E(R_M) + Cov(\beta_S,\ R_M) \tag{5.4}$$

That is, we are interested in the average product of the stock beta and the market risk premium in all states of the world.

The static CAPM does not distinguish between the states of the world and assumes that if we see any variation in the stock beta and the market risk premium, this variation is pure noise. Therefore, the static CAPM suggests that we compute the product of the average beta and the average market risk premium.

In reality, however, the variation in the stock beta and the market risk premium is not purely random. It is correlated, and the covariance term measures the bias in the estimate of the expected risk premium of the stock provided by the static CAPM. That is, if the covariance is positive (i.e, in most states of the world the stock beta and the market risk premium are either both high or both low, see Stock A from the numerical example above), then the static CAPM will underestimate the risk and the expected risk premium of the stock. Conversely, if the correlation is negative (i.e., in most states of the world either the stock beta is high and the market risk premium is low, or vice versa), the static CAPM will overestimate the risk and the expected risk premium of the stock.

The covariance term in (5.4) does much more than express the simple example in the previous subsection in the complex mathematical symbols. In fact, it gives the estimate of the bias in the static CAPM compared to the Conditional CAPM, that is, exactly what we are looking for in this section. We can now do some comparative statics, i.e., look at what the bias depends on.

By definition again, the covariance is the product of the correlation and the two standard deviations,

$$Cov(\beta_S,\ R_M) \equiv Corr(\beta_S,\ R_M) \cdot \sigma(\beta_S) \cdot \sigma(R_M) \tag{5.5}$$

We draw two important conclusions from (5.5). First, the bias in the static CAPM and, therefore, the usefulness of the Conditional CAPM increases in the correlation between the stock beta, β_S, and the market risk premium, R_M. If they are tightly related (for example, driven by the same variables like the four variables in Chapter 4, Section 1), the bias is large and the Conditional CAPM is important. If they vary pretty much independently of each other, there is little difference between the Conditional CAPM and the static CAPM. Once again: time variation in the market risk premium and the stock beta is a necessary, but insufficient condition for the Conditional CAPM usefulness. The Conditional CAPM is only useful if those two move together, not just move.

Second, the usefulness of the Conditional CAPM increases in the variation in both the market risk premium and the stock beta. If both of them vary by a lot, the difference between the static CAPM and the Conditional CAPM is large, and using the correct model (the Conditional CAPM) is important.

But do the market risk premium and the stocks' betas vary enough to make the Conditional CAPM significantly different in its predictions from the static CAPM? Let's turn one last time to the numerical example from the previous section. Even if the risk premium in recession and in expansion is very different (in our example it triples from 4% to 12%, about what we figured out it would do - see Chapter 4, Section 1) and the betas vary by a lot (the beta differential between the first and second stocks changes from 1.5 in recession to -0.5 in expansion, which is unbelievably large compared to what we see in the data), the return differential is only 3%, way smaller than the most important deviations from the CAPM we will consider later in the course. This is called the Lewellen-Nagel (JFE 2006) critique: the variations in betas and the expected market risk premium are too small to make the Conditional CAPM successful in explaining the return differentials between different groups of stocks we observe in the real-life data.

5.2 Conditional CAPM in the Data

5.2.1 First Look at the Value Effect

Petkova and Zhang (JFE 2005) use the Conditional CAPM in the effort to explain one of the most important and stubborn anomalies in finance - the value effect. The simple CAPM cannot explain why value firms (mature companies with low market-to-book) earn on average higher returns than growth firms (young fast-growing companies with high market-to-book).

Market-to-book is the ratio of the firm's market capitalization to its book value of equity. Market-to-book higher than 1 signals the existence of growth potential, market-to-book lower than 1 is the sign of poor prospects. High market-to-book ratios are characteristic of risky high tech firms, for which a high proportion of the value is their future that has not yet found its way to the books. Low market-to-book firms are generally mature companies with few expansion opportunities, but steady cash flows and high dividends. Hence, it is pretty surprising that value firms beat growth firms, since most people would say that growth firms are riskier.

The following regression illustrates the value effect:

$$HML_t = \underset{(0.12)}{0.58} - \underset{(0.03)}{0.27} \cdot (MKT_t - RF_t) \tag{5.6}$$

The variable on the left-hand side is the zero-investment portfolio that sells growth firms and uses the proceeds to buy value firms[1]. It is regressed on the excess return to the market portfolio (value-weighted CRSP index is used as a proxy). According to the CAPM, the intercept of the regression (aka the alpha), which measures abnormal return, should be zero.

It is important to understand why the CAPM predicts that the alpha in (5.6) should be zero. It will allow us to understand why the alpha is the measure of abnormal returns and enable us to use it to measure the deviations from the CAPM (anomalies) and the performance of actively managed portfolios (e.g., mutual funds).

[1]Data footnote: The sample period in the regression is from August 1963 to December 2006. The HML portfolio is one of the Fama-French factors we will discuss later. To form it, we sort all firms into three market-to-book groups (top 30%, middle 40%, bottom 30%) and two size groups (below median and above median). Then in each size group we buy value firms (bottom 30% on market-to-book) and sell growth firms (top 30% on market-to-book). The returns to the two value minus growth portfolios are value-weighted. The HML return is the simple average of the returns to the two value minus growth portfolios (value minus growth for small firms and value minus growth for large firms). The portfolio is called HML as an acronym for "high minus low", because Fama and French in their paper sorted firms on book-to-market, not market-to-book, and value firms have high book-to-market.

The CAPM implies that for each asset (a stock, a bond, an option)

$$E(R_t) = RF_t + \beta \cdot (E(MKT_t) - RF_t) \Rightarrow E(R_t) - RF_t = \beta \cdot (E(MKT_t) - RF_t) \quad (5.7)$$

The rearranging of the terms on the right implies that the asset's risk premium is exactly proportional to the market risk premium, and the beta is the coefficient of proportionality. Therefore, the CAPM predicts that in the regression of *excess* returns to any asset on the *excess* return to the market the intercept (the alpha) is zero. If the alpha is not zero, the CAPM is violated. The positive alpha implies that the asset earns the return above what is a fair compensation for risk $\beta \cdot (E(MKT_t) - RF_t)$, and vice versa.

Also, pay attention to the fact that the regression prescribed by the CAPM has excess returns both on the left and on the right. Why is (5.6) different? It is different because we are looking at the zero-investment portfolio that shorts growth firms and uses the proceeds to finance the purchases of value firms.[2] Essentially, we are looking at the difference in the (excess) returns between value firms and growth firms. You can interpret (5.6) as two regressions

$$E(H_t) - RF_t = \beta_H \cdot (E(MKT_t) - RF_t) \quad (5.8)$$

$$E(L_t) - RF_t = \beta_L \cdot (E(MKT_t) - RF_t) \quad (5.9)$$

Now deduct (5.9) from (5.8), and you will get (5.6) - the risk-free rate cancels in the process, and thus does not appear in (5.6).

Now, let's go back to the alpha, our measure of abnormal return and the CAPM validity. In regression (5.6), the alpha far exceeds zero: it is 0.59% (59 bp) per month

[2]By law, you have to leave the proceeds from the short sale with the lender, and the lender will pay you close to the risk-free rate on those. That is, when you short (e.g., growth stocks), the government makes you go long in the Treasury bill. To complete the zero-investment strategy (HML) in real life, you have to do an extra deal: borrow at the risk-free rate or close and buy the long part (value stocks). Then you will be long in value stocks, short in Treasuries, short in growth stocks, and long in Treasuries, all for the same sum of money. The Treasuries positions cancel out, and you hold the HML portfolio with no cash invested.

and highly significant ($0.58/0.12 = 4.99 \gg 2$). This is the value effect: even though value firms have lower beta than growth firms (by -0.27 in the regression), they earn higher return. The return differential between value and growth beats the CAPM prediction by 7% ($0.59\% \cdot 12$) per year. (We will talk more about the value effect and the HML portfolio when we get to the Fama-French model and anomalies).

5.2.2 Empirical Setup

Petkova and Zhang allow the beta to change over time by making it a linear function of the four macroeconomic variables we discussed when we talked about long-run return predictability - the default premium (DEF), the dividend yield (DIV), the Treasury bill rate (TB), and the term premium ($TERM$). That is, Petkova and Zhang assume that

$$\beta_t = \gamma_0 + \gamma_1 \cdot DEF_{t-1} + \gamma_2 \cdot DIV_{t-1} + \gamma_3 \cdot TB_{t-1} + \gamma_4 \cdot TERM_{t-1} \tag{5.10}$$

and

$$MKT_t - RF_t = \lambda_0 + \lambda_1 \cdot DEF_{t-1} + \lambda_2 \cdot DIV_{t-1} + \lambda_3 \cdot TB_{t-1} + \lambda_4 \cdot TERM_{t-1} \tag{5.11}$$

Notice that Petkova and Zhang assume that the beta of HML depends on the same four variables which we found to be the predictors of the market risk premium in Chapter 4, Section 1. This is not a coincidence: as discussed in Section 1.3 of this chapter, the extra explanatory power of the Conditional CAPM compared to the static CAPM is proportional to the correlation between the asset's beta and the market risk premium (see equation (5.4)). Petkova and Zhang are trying to capture as much of this correlation as possible.

Empirical Advice: When you estimate Conditional CAPM, assume that the beta depends only on the macroeconomic variables that predict the market risk premium. Putting the variables that do not in (5.10) gets you no extra mileage.

The Conditional CAPM Petkova and Zhang estimate requires running the regression

$$HML_t = \alpha + \beta_t \cdot (MKT_t - RF_t) \tag{5.12}$$

If the intercept α is insignificantly different from zero, then the Conditional CAPM is capable of explaining the value effect.

Given their specification of β_t, Petkova and Zhang estimate the following regression:

$$HML_t = \alpha + (\gamma_0 + \gamma_1 \cdot DEF_{t-1} + \gamma_2 \cdot DIV_{t-1} + \gamma_3 \cdot TB_{t-1} + \gamma_4 \cdot TERM_{t-1}) \cdot (MKT_t - RF_t) \tag{5.13}$$

or, after rearranging,

$$\begin{aligned} HML_t = \alpha + \gamma_0 \cdot (MKT_t - RF_t) + \gamma_1 \cdot DEF_{t-1} \cdot (MKT_t - RF_t) \\ + \gamma_2 \cdot DIV_{t-1} \cdot (MKT_t - RF_t) + \gamma_3 \cdot TB_{t-1} \cdot (MKT_t - RF_t) \\ + \gamma_4 \cdot TERM_{t-1} \cdot (MKT_t - RF_t) \end{aligned} \tag{5.14}$$

which means regressing the HML return not only on the excess market return, as in the simple CAPM, but also on the products of the excess market return with the macro variables.

When Petkova and Zhang estimate (5.15), they get, with rearranging back to the form of (5.13), the following result that we discuss in the next subsection:

$$\begin{aligned} HML_t = \underset{(0.115)}{0.45} + (-\underset{(0.09)}{0.36} - \underset{(0.08)}{0.11}\ DEF_{t-1} + \underset{(0.03)}{0.21}\ DIV_{t-1} - \\ - \underset{(0.02)}{0.07}\ TB_{t-1} - \underset{(0.03)}{0.01}\ TERM_{t-1}) \cdot (MKT_t - RF_t) \end{aligned} \tag{5.15}$$

5.2.3 Interpreting the Results

What do we conclude from the regression above? First, the good news: the macro variables we use predict the market beta of the value minus growth strategy in a reasonable way. If the Conditional CAPM is to explain the value effect, the market beta of the value minus growth strategy has to increase during recessions.

In recessions, the dividend yield, the default premium, and the term premium are high, so they should be positively related to the market beta of HML and have positive coefficients in the regression estimated above. The default premium and the term premium, as we see in (5.15), have negative, but insignificant coefficients. In regression (5.15), the only significant coefficient out of the three is the coefficient on the dividend yield, and it has the correct positive sign. Its magnitude suggests that if the dividend yield increases by 1%, the market beta of the value minus growth strategy increases by 0.2. Given that the dividend yield changes by up to 2% from peak to trough, the impact of dividend yield on the beta seems economically sizeable.

Also, the Treasury bill rate (the measure of expected inflation) is low during recessions, hence it should be negatively related with the market beta of HML, if the market beta is to increase in recessions. In (5.15), the coefficient on the Treasury bill rate is significantly different from zero and economically large. It implies that the decrease in the Treasury bill rate by 1% per year triggers, on average, the increase in the market beta differential between value and growth by 0.07. Given that the Treasury bill rate can vary easily by 3% to 5% within one business cycle, the coefficient in (5.15) suggests that the impact of expected inflation can cause the beta differential between value firms and growth firms change by up to 0.35.

Now the bad news: the alpha is still highly significant ($0.45/0.115 = 3.9 \gg 2$), meaning that the Conditional CAPM cannot handle the value effect. Compared to (5.6), where we estimate the ordinary CAPM, the alpha decreases by only 13 bp per month (1.6% per year). This is consistent with the Lewellen-Nagel critique - it seems that, despite the seemingly strong dependence of the beta on the macro variables in (5.15), the variation of the beta and the expected risk premium are not enough to make a big impact.

To show that the variation in the beta is not large enough, I perform the following experiment: first, I use (5.11) to estimate expected market risk premium and partition the sample into the months with the expected market risk premium (predicted values from (5.11)) in the top 25% (recession) and the months with the expected market risk premium in the bottom 25% (boom). I use (5.10) with coefficients from (5.15) to form the series of the market beta of the value minus growth strategy and average the beta in the recession months and the boom months (as defined above). I find that during recessions the market beta of the HML is -0.135, and during booms it is -0.41, consistent with the estimates in (5.15).

5.2.4 Conditional CAPM Works, but Not by Enough

The results in the previous paragraph bring us to two conclusions. First, while it is true that following the value minus growth strategy is riskier in recessions than in booms, growth is always riskier than value, and therefore growth stocks should have higher average returns (whereas in the data they earn by 46 bp per month less than value firms). Hence, there is no way the Conditional CAPM can explain the value effect.

Second, the variation in the market betas is indeed too small. We should not add up the coefficients in (5.15) and conclude that if both expected inflation and dividend yield can move the market beta of HML by 0.4, it will change by 0.8 from recession to expansion. In many instances, the macro variables pull in different directions - e.g., in the 70s both the dividend yield and the Treasury bill rate were high.

We can even perform a back-of-envelope calculation based on the beta differential computed in the end of the last subsection. Recessions and booms in the definition of this paragraph take 50% of the sample, the expected risk premium varies between recessions and booms by 1% per month at most (see our exercises in the long-run predictability sections or just obtain the expected risk premium from (5.11) and average it in recessions and booms). Hence, if we are to explain the value effect of about 0.6% per month, the difference in the market beta of the HML portfolio between recessions and booms should be 1.2 ($0.5 \cdot 1\% \cdot 1.2 = 0.6\%$). The beta differential of -0.275 we observe in the data gives us the chance to explain at most 15 bp of the value effect, close to the change in the alphas we observe as we go from (5.6) to (5.15).

What have we learned about the Conditional CAPM? Good news: the Conditional CAPM shows that following the value minus growth strategy is relatively more risky during recessions, which is an important observation. We can perform the same sort of estimation for any asset we consider (e.g., a mutual fund offered in 401(k)), and it will give us a heads-up on the potential risks we may face. Bad news: according to the Conditional CAPM, risk and risk exposure do not change by enough to make an economically large impact. In particular, the slopes in (5.15) are sizeable, but the model as a whole does not work well. Hence, if we need a precise quantitative answer about whether the risk discovered by the Conditional CAPM is fairly compensated by the observed return, we may want to try something else, which will be the point of the next sections.