Solutions to Final Exam

AEC 504 - Summer 2007

Fundamentals of Economics

1 Veni, vidi, vici (30 points)

Two firms with constant marginal costs serve two markets for two different goods. The demand function for good 1 is $Q_1 = 20 - p_1 + 0.5p_2$, where p_1 and p_2 are the prices of good 1 and good 2, respectively. The demand function for good 2 is $Q_2 = 25 + p_1 - p_2$.

To answer this problem, you need not perform any calculations. (You certainly may if you want, but the answers have to contain economic intuition).

i. *(5 points)* Are those two goods substitutes or complements?

The goods are substitutes, since cutting the price of the one reduces the demand for the other and vice versa.

ii. *(10 points)* Suppose that we start from the competitive outcome. Will the firms want to collude? What will happen to the prices if the firms collude?

Hint: Think about the externalities that firm 1 creates for firm 2 if firm 1, say, increases the price of its product.

If firm 1 cuts its price, it produces negative externalities for firm 2, because the demand firm 2 faces shrinks. However, under competition firm 1 does not care about the externality and thus is likely to charge too low price compared to what it would charge under collusion. Hence, the collusion raises the prices. It should come as natural, as the firms produce substitutable goods, i.e., compete.

The firms will surely want to collude to eliminate the externality problem.

iii. *(10 points)* How would your answer to (ii) change if the demand functions were $Q_1 = 20 - p_1 - p_2$ and $Q_2 = 25 - p_1 - p_2$?

Here the externality runs the other way: the price cuts firm 1 makes are good news for firm 2 (the goods are now complements). Under competition firms do not care about externalities and the prices will be too high compared to the collusive outcome. Hence, the collusion now brings about price decline. It happens because now the goods the firms produce are complimentary, and their rivalry is detrimental.

Clearly, the firms will want to collude in this case as well, because the externality is still around, even though it goes the other way.

It is easy to check that in general, even if collusion does not bring about changes in the efficiency of production, collusion of substitutes producers lowers social welfare and collusion of complements producers raises social welfare. You (and Antitrust committee) would not mind the fact that Dell may produce mice, printers, etc., but you both will be disappointed if Dell and Compaq merge.

iv. *(5 points)* Suppose the demand functions change to $Q_1 = 20 - p_1$ and $Q_2 = 25 - p_2$. Will the firms want to collude?

No, they will not benefit from that, because the markets are completely independent and are monopolized at the very start. You can call this monopoly in two markets two different names or call it one name – it will not change anything.

2 Hard Love (25 points)

Sugar Kowalczyk enjoys money, M, and saxophone music, S. Her utility function is $U = S^2 + M$ and her budget constraint is $pS + M = I$.

i. *(15 points)* Derive Sugar's demand for saxophone music and money. Is the law of demand satisfied for Sugar's demand for saxophone music? What is the economic intuition behind your answer? What change, if any, should be made in the utility function to make the consumer behave in line with the law of demand?

$$\frac{MU_S}{MU_M} = \frac{p_S}{p_M} \Rightarrow S = \frac{p}{2},$$

and it is the demand function for saxophone music.

To get the demand for money, substitute the demand for saxophone music into the budget constraint: $M = I - p^2/2$

The demand function for S violates the law of demand, as the demand for S increases with price.

The reason is that the marginal utility MU_S ***for this utility function also increases with S (for standard utility functions it would decrease). Clearly, Sugar is addicted to saxophone music and wants more and more of it after she hears some, eventually ending up paying a high price, as she actually confesses in the movie ('Some Like It Hot'). Alas, there is nothing irrational in it: if the price Sugar has to pay for the music is high, she has to consume a lot and get addicted, otherwise she will have to give up saxophone music (and saxophone players) altogether, and we know from the movie she just cannot do it.***

To eliminate addiction, we need to lower the power of S***. You can check for yourself that any utility function of the form*** $U = S^\alpha + M, \quad \alpha < 1$ ***will yield negatively sloped demand for*** S***.***

ii. *(10 points)* Is Sugar risk-averse with respect to random fluctuations in income? What about fluctuations in the price of music?

Hint: Find the indirect utility function, i.e. express Sugar's utility in terms of her income and the price of music. Then determine if Sugar is risk-averse with respect to fluctuations in I treating p as a constant, and vice versa, that is, would she rather take a fixed income (price) or a gamble with the same mathematical expectation.

$$x = \frac{p}{2}, \; y = I - \frac{p^2}{2} \Rightarrow V = I - \frac{p^2}{4}$$

The indirect utility function V is linear in I, so Sugar is risk-neutral with respect to fluctuations in income. V is concave in p, so Sugar is risk-averse with respect to fluctuations in the price of music.

You can also perform a thought experiment, asking yourself if Sugar would prefer the price fixed at 1 or the price, taking values 2 and 0 with equal probabilities. In the first case, her expected utility will be $I - 0.25$, whereas in the second case, it will be $I - 0.5 \cdot 0 - 0.5 \cdot 1 = I - 0.5 < I - 0.25$. Since Sugar prefers the fixed price over the fair bet, she is risk-averse. (You can perform the same thought experiment with respect to income, which is quite trivial).

3 A Standard Monopoly Problem (35 points)

Consider a monopoly which faces the demand curve $P = 150 - 2Q$ and has *MC=30*. Assume *FC=0*.

i. *(5 points)* What is the optimal quantity for the monopoly? What profit does it make at this quantity?

$$TR = PQ = (150-2Q)\cdot Q \Rightarrow MR = 150-4Q;\ MR = MC \Rightarrow 150-4Q = 30 \Rightarrow Q = 30$$

$$\Pi = TR - TC = (150 - 2 \cdot 30) \cdot 30 - 30 \cdot 30 = 1800$$

ii. *(5 points)* What is the point elasticity of demand at the optimal quantity? What is the markup (defined as $\frac{P - MC}{P}$) at the optimal quantity? What is the relation between the two?

Notice that the demand function you are given (commonly referred to as inverse demand function) expresses P in terms of Q, and to compute elasticity you need it to do otherwise. So, the demand function equation you need is

$$Q = 75 - \frac{P}{2} \Rightarrow \eta = -\frac{dQ}{dP} \cdot \frac{P}{Q} = \frac{P/2}{75 - P/2}$$

$$Q = 30 \Rightarrow P = 150 - 2 \cdot 30 = 90 \Rightarrow \frac{P - MC}{P} = \frac{90 - 30}{90} = \frac{2}{3}$$

$$Q = 30 \Rightarrow \eta = \frac{90/2}{75 - 90/2} = \frac{3}{2}$$

The optimal markup is the inverse of the demand elasticity at the optimum point. The derivation in (iii) shows that it is always true.

<u>Remark</u>: Notice that you could take the elasticity of the inverse demand function straight away, defining it as $\varepsilon = -\frac{dP}{dQ} \cdot \frac{Q}{P}$***. Just remember that what you will get is not the demand elasticity*** $-\ \varepsilon = 1/\eta$***.***

iii. *(10 points)* Assuming your answer to (ii) is true in general, will a monopoly ever produce on the inelastic portion of its demand curve? To get full credit, provide a rigorous proof that it will or will not.

On the inelastic part of the demand curve the inverse of the elasticity is greater than 1, which would imply $\frac{P - MC}{P} > 1 \Rightarrow MC < 0$***, which does not make sense. So, the monopoly will never produce on the inelastic part of the demand curve.***

To prove rigorously that the result in (ii) is always true, consider a monopoly facing demand function $P = P(Q)$***.***

$$TR = P(Q) \cdot Q \Rightarrow MR = P'(Q) \cdot Q + P(Q) = MC \Rightarrow$$

$$\frac{P(Q) - MC}{P(Q)} = -\frac{dP(Q)}{dQ} \cdot \frac{Q}{P(Q)} = \frac{1}{\eta},\ QED$$

Here is a lengthy piece of intuition behind the result:

If a monopolist is producing on the inelastic portion of its demand curve, it can increase revenue by increasing price. The corresponding decrease in quantity also implies that costs will decrease. Recall that with a linear demand curve, marginal revenue equals zero at the mid-point of the demand curve, which is also the point at which total revenue is maximized and the elasticity of demand is equal to 1. Since marginal cost will not be negative, the point at which marginal revenue equals marginal cost must occur at or to the left of this point (since marginal

revenue is decreasing in quantity). It is also true that the elasticity of demand is always greater than 1 to the left of this point.

iv. *(5 points)* What would be the competitive long-run outcome?

In this problem MC=AC is always true, so we only have to set

$$MC = p \Rightarrow 150 - 2Q = 30 \Rightarrow Q = 60 \Rightarrow P = 30$$

v. *(10 points)* What is the gain/loss to the society from switching to the competitive outcome?

Under monopoly, social welfare (SW) consists of consumer surplus (CS), which is the area of triangle 150-90-A and the profit of the monopoly, which is the area of quadrangle 90-A-D-C. CS=0.5*(150-30)*30=900, Π***=(90-30)*30=1800, SW=900+1800=2700.***

Under competition, the firm gets zero profit, but CS is now equal to the area of the triangle 150-B-C (and equal to SW). So, under competition SW=CS=0.5*(150-30)*60=3600, and the gain to the society from switching to competition is 3600-2700=900. You may notice that the area of triangle ABD is equal exactly 900 – it is the dead-weight loss triangle.

4 Monopoly and Entry Prevention (65 points)

Consider a firm that faces the demand curve $P = 150 - Q$ and has $MC = \$20$.

i. *(5 points)* If the firm is a monopoly, what is its optimal price and quantity? What is the profit?

$$MR = 150 - 2Q; \quad MR = MC \Rightarrow 150 - 2Q = 20 \Rightarrow Q = 65 \Rightarrow$$

$$\Rightarrow p = \$85 \Rightarrow \Pi = 65 \cdot (\$85 - \$20) = \$4225$$

ii. *(10 points)* Suppose another firm enters the industry and the two firms compete a-la Cournot. The entrant has $MC = \$40$ and has to pay \$500 to enter the industry.

Compute the optimal quantity for the entrant and the incumbent, their profits and the price.

$$\begin{cases} MR_I = 150 - 2Q_I - Q_E = MC_I = 20 \\ MR_E = 150 - Q_I - 2Q_E = MC_E = 40 \end{cases} \Rightarrow \begin{cases} 2Q_I + Q_E = 130 \\ Q_I + 2Q_E = 110 \end{cases} \Rightarrow \begin{cases} Q_I = 50 \\ Q_E = 30 \end{cases} \Rightarrow$$

$$\Rightarrow p = \$70 \Rightarrow \Pi_I = 50 \cdot (\$70 - \$20) = \$2500,\ \Pi_I = 30 \cdot (\$70 - \$40) - \$500 = \$400$$

iii. *(10 points)* Suppose the incumbent can credibly commit to producing and selling 80 units of the good. Will the entrant still want to enter? Will the incumbent want to commit to selling 80 units of the good?

If the incumbent commits to selling 80 units, the entrant will be a monopoly on the residual demand of $P = 70 - Q_E$***. So, it will choose its quantity and the price as follows:***

$$MR_E = 70 - 2Q_E; \quad MR_E = MC_E \Rightarrow 70 - 2Q_E = 40 \Rightarrow Q_E = 15 \Rightarrow$$

$$\Rightarrow p = \$55 \Rightarrow \Pi_E = 15 \cdot (\$55 - \$40) - \$500 = -\$275,\ \Pi_I = 80 \cdot (\$55 - \$20) = \$2800$$

Since the entrant's profit is negative, it will not enter. Therefore, the incumbent will want to commit to producing and selling 80 units.

iv. *(15 points)* Suppose the incumbent still commits to produce 80 units, but the entrant does not have to pay anything to enter. Will it enter? What will its quantity choice be? Are the quantity choices of the entrant and the incumbent a Nash equilibrium in the Cournot game?

If there are no entry costs, the entrant will enter the industry – in (iii) it makes a positive profit before entrance costs. It will still choose to produce 15 units, because fixed costs do not influence pricing.

The quantity choices are not a Nash equilibrium. The entrant will not want to deviate given the incumbent's choice, because it already has maximum profit given the choice. The incumbent, though, will be tempted to revise the choice given that the entrant sticks to producing 15 units. The incumbent then faces the residual demand of $P = 135 - Q_I$***. So,***

$$MR_I = 135 - 2Q_I; \quad MR_I = MC_I \Rightarrow 135 - 2Q_I = 20 \Rightarrow Q_I = 57.5 \Rightarrow$$

$$\Rightarrow p = \$77.5 \Rightarrow \Pi_I = 57.5 \cdot (\$77.5 - \$20) = \$3306.25 > \$2800$$

v. *(25 points)* Suppose the entrant pays nothing to enter and offers to form a cartel. The cartel is supported by the usual trigger strategy with the threat to revert to the Cournot equilibrium forever. Will the cartel be sustainable if they divide the monopoly profit 50-50? What if they divide it 70-30 in favor of the incumbent? What is the maximum and the minimum amount of money the incumbent can receive as a cartel member so that the cartel is sustainable? Assume that the firms use the discount rate of 15% to compute the present value of the future profits.

Hint: Remember that if the cartel is sustainable, nobody wants to deviate.

Because $MC_I < MC_E, \forall Q$ ***all production in the cartel will be made by the incumbent, and the entrant will sit back and receive the checks. If the profit is divided 50-50, the cartel will never be sustainable, because in the Cournot equilibrium the incumbent gets \$2500, which is more than a half of the monopoly profit (\$4225/2=\$2112.5).***

If the monopoly profit is divided 70-30 in the incumbent's favor, the incumbent will compare the present value of payoffs from not deviating - $0.7 \cdot 4225/0.15 \approx 19717$ ***and the profit from deviating and then getting the Cournot outcome forever -*** $4225 + 2500/(0.15 \cdot 1.15) \approx 18718$***. The optimal deviation, of course, is to sell the whole monopoly output and give nothing to the entrant, who sits back. So, at the discount rate of 15% the incumbent is willing to give the entrant \$1267.5 each period, and even more, up to*** $\$18718 \cdot 0.15 \approx \$1417$***, so that it stayed out (keeping the entrant out is what this cartel is really about). Hence, the minimum amount the incumbent has to leave to itself is \$4225-\$1417=\$2808***

If the entrant deviates, it will just work on the residual demand after the incumbent sells the monopoly output, $P = 150 - 65 - Q = 85 - Q$

$$MR_E = MC_E \Rightarrow 85 - 2Q = 40 \Rightarrow Q = 22.5,\ P = 62.5,\ \Pi = 22.5^2 = 506.25$$

Now it is time to make assumptions. We can assume that the incumbent figures out instantly that the entrant has deviated. The incumbent will

not be able to change the output, but will be able to hold back the check for \$1237.5 it would have owed the entrant this period if the entrant had been true to the cartel agreement. Then the entrant will never deviate, because it loses both when it deviates and afterwards. In fact, the incumbent can pay as few as $0.15 \cdot (506.25 + 900/(0.15 \cdot 1.15 \approx 858.5$ ***to the entrant and still keep him in the cartel (i.e., out of the market). Notice, by the way, that if the incumbent pays the entrant less than \$900, the entrant will actually want to be "punished" by the Cournot equilibrium, but will refrain from breaking the cartel agreement because deviating means punishing itself quite severely.***

Alternatively, we can assume that the incumbent is a bit slow and will deliver to the entrant the 30% of its monopoly profit even in the period the entrant deviates. Of course, the incumbent will not be able to send the entrant \$1267.5, rather, it will send $0.3 \cdot 65 \cdot (62.5 - 20) = 828.75$***, so the deviating entrant will get*** $\$506.25 + \$828.75 = \$1335$***. The entrant now compares*** $1267.5/0.15 = 8450$ ***from being in the cartel and*** $1335 + 900/(0.15 \cdot 1.15) \approx 6552.4$ ***from deviating and chooses to stay in the cartel. The minimum fraction of the monopoly profit the incumbent has to offer the entrant to make it stay out solves***

$$x \cdot \frac{4225}{0.15} = 506.25 + x \cdot 65 \cdot 42.5 + \frac{900}{0.15 \cdot 1.15} \Rightarrow x = 0.2253$$

So, the incumbent has to offer the entrant at least $0.2253 \cdot 4225 = 951.9$ ***to keep it out of the market.***

NB: In your exam, you could make any of the two assumptions and proceed relying on it. You did not need to discuss what happens under both assumptions.

5 Two-Part Tariff (25 points)

Consider a firm with *MC=AC=1* trading with two buyers, whose demand functions are $Q_1 = 5 - 2p_1$ and $Q_2 = 7 - 3p_2$. The firm can distinguish the buyers.

i. *(5 points)* What is the optimal price-discrimination strategy?

$$\max_{P_1}(5-2p_1)\cdot(p_1-1) \Rightarrow p_1 = 1.75; \quad \max_{P_2}(7-3p_2)\cdot(p_2-1) \Rightarrow p_2 = 5/3$$

ii. *(15 points)* Determine the optimal two-part tariff for each buyer. Compare the sales and social welfare under the two-part tariffs with those in (i).

The optimal two-part tariff means fixing P=MC and charging each buyer the fixed fee equal to his consumer surplus at P=1.

$$CS_1(P=1) = \frac{1}{2}\cdot 3\cdot(\frac{5}{2}-1) = 2.25;\ CS_2(P=1) = \frac{1}{2}\cdot 4\cdot(\frac{7}{3}-1) = \frac{8}{3}.$$

So, the optimal two-part tariff is

$$P_1 = 1,\ F_1 = 2.25; \quad P_2 = 1,\ F_2 = \frac{8}{3} \quad \Pi_{(ii)} = SW_{(ii)} = \frac{59}{12}$$

$$Q_{(ii)} = 7,\ Q_{1(i)} = 1.5,\ Q_{2(i)} = 2 \Rightarrow Q_{(i)} = 3.5 < Q_{(ii)} = 7$$

$$\Pi_{1(i)} = 1.5\cdot 0.75 = 1.125;\ \Pi_{2(i)} = 2\cdot 2/3 = 4/3.\ \Pi_{(i)} = \frac{9}{8}+\frac{4}{3} = \frac{59}{24}$$

$$CS_{1(i)}(P=1.75) = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{4} = \frac{9}{16};\ CS_{2(i)}(P=5/3) = \frac{1}{2}\cdot 2\cdot\frac{2}{3} = \frac{2}{3},\ CS_{(i)} = \frac{59}{48}$$

$$SW_{(i)} = CS_{(i)} + Pi_{(i)} = \frac{59}{48}+\frac{59}{24} = \frac{59}{16} < \frac{59}{12} = SW_{(ii)}$$

In fact, the sales and social welfare under two-part tariff are equal to the ones under perfect competition and thus are higher that under price-discrimination.

iii. *(5 points)* Suppose firm cannot distinguish buyers, but still wants to use the two-part tariff. What will be the common two-part tariff the firm will offer to both buyers?

If the firm decides to cater only to the second buyer, who pays a higher fixed fee, its profit will be $\Pi = 8/3$***. To keep both buyers, the firm has to offer*** $P = 1,\ F = 9/4$ ***to both. It will then make*** $\Pi = 2\cdot 9/4 = 9/2 > 8/3$***. So, the optimal two-part tariff is*** $P = 1,\ F = 9/4$***.***