

SUPPLEMENTARY APPENDIX

Jumps in Cross-Sectional Rank and Expected Returns: A Mixture Model

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July 2007

In this supplementary appendix we provide detailed results and further discussion, which is not presented in the paper due to constraints in the length of the manuscript.

We report the following six items:

1. Table S.1: Descriptive statistics of the weekly returns of the constituents of the SP500 index: Cross-sectional frequency distribution (500 firms) of the unconditional moments (time series mean, standard deviation, skewness, and kurtosis) and cross-sectional summary statistics of the unconditional jump from January 1, 1990 to December 27, 2000.
2. Table S.2: Estimation results of the duration model $f_1(J_t|\mathcal{F}_{t-1};\theta_1)$
3. Table S.3: Estimation results of the model for conditional returns $f_2(y_t|J_t, \mathcal{F}_{t-1};\theta_2)$
4. Section 4: Technical trading rules
5. Section 5: Wang's (2001) Monte Carlo method to compute VaR for the VCR-Mixture trading Rule
6. Section 6: White's (2000) reality check and Hansen's (2005) extension.

1 Table S.1: Descriptive statistics

In Table S.1, we summarize the unconditional moments (mean, standard deviation, skewness, and kurtosis) of all 500 firms, in the estimation sample. The frequency distribution of the unconditional mean is unimodal with a weekly mean return of 0.029%. For the unconditional standard deviation, the median value is 5.25%. The coefficient of skewness is predominantly negative with a median value of -0.12 . All the firms have a large coefficient of kurtosis with a median value of 10.34. We calculate the Box-Pierce statistics up to the fourth order to test for autocorrelation in returns and we find mild autocorrelation for about one-third of the firms. However, the Box-Pierce test up to fourth order for autocorrelation in squared returns indicates strong dependence in second moments for all the firms in the SP500 index.

2 Table S.2: Estimation results for $f_1(J_t|\mathcal{F}_{t-1};\theta_1)$

In Table S.2, we report the cross-sectional frequency distributions of the estimates $\hat{\theta}_1 \equiv (\hat{\alpha}, \hat{\beta}, \hat{\delta}')$ for the duration model

$$\begin{aligned} p_t &= [\Psi_{N(t-1)} + \delta' X_{t-1}]^{-1} \\ \Psi_{N(t)} &= \alpha D_{N(t)-1} + \beta \Psi_{N(t)-1} \\ \delta' X_{t-1} &= \delta_1 + \delta_2 y_{t-1} \mathbf{1}(z_{t-1} \leq 0.5) + \delta_3 y_{t-1} \mathbf{1}(z_{t-1} > 0.5) \end{aligned}$$

for all the 466 firms in the estimation sample.

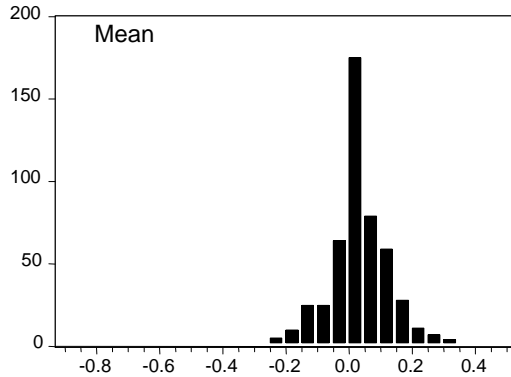
3 Table S.3: Estimation results for $f_2(y_t|J_t, \mathcal{F}_{t-1};\theta_2)$

In Table S.3, we report the estimation results corresponding to the model

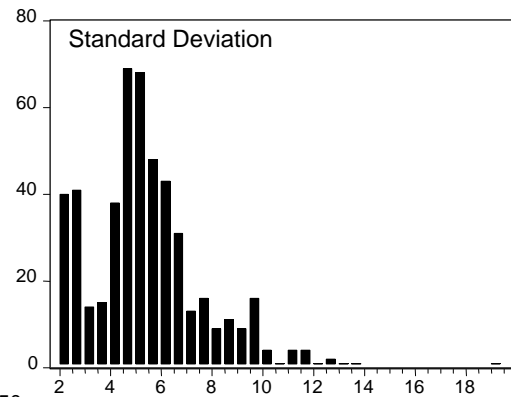
$$\begin{aligned} f_2(y_t|J_t, \mathcal{F}_{t-1};\theta_2) &= \begin{cases} N(\mu_{1t}, \sigma_{1t}^2) & \text{if } J_t = 1 \\ N(\mu_{0t}, \sigma_{0t}^2) & \text{if } J_t = 0 \end{cases}, \\ \mu_{1t} &\equiv E(y_t|\mathcal{F}_{t-1}, J_t = 1) = \nu_1 + \gamma_1 y_{t-1} + \eta_1 z_{t-1}, \\ \mu_{0t} &\equiv E(y_t|\mathcal{F}_{t-1}, J_t = 0) = \nu_0 + \gamma_0 y_{t-1} + \eta_0 z_{t-1}, \\ \sigma_{1t}^2 &= \sigma_{0t}^2 = \sigma_t^2 = E(\epsilon_t^2|\mathcal{F}_{t-1}, J_t) = \omega + \rho \epsilon_{t-1}^2 + \tau \sigma_{t-1}^2, \\ \epsilon_{t-1} &= (y_{t-1} - \mu_{1,t-1})J_{t-1} + (y_{t-1} - \mu_{0,t-1})(1 - J_{t-1}) \\ \theta_2 &= (\nu_1, \gamma_1, \eta_1, \nu_0, \gamma_0, \eta_0, \omega, \rho, \tau)'. \end{aligned}$$

for the 466 firms. We report the cross-sectional frequency distributions of the parameters estimates in the conditional mean and conditional variance.

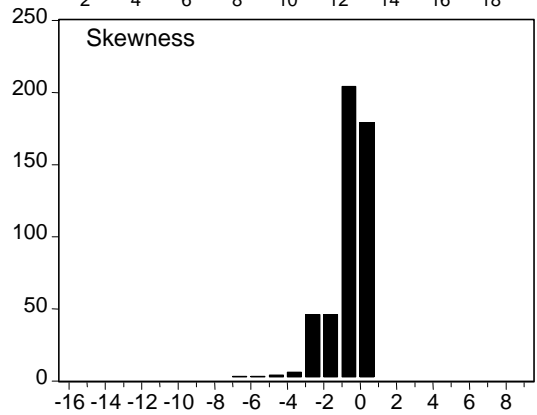
Table S.1
Descriptive statistics of weekly returns of the SP500 firms
January 1, 1990-December 27, 2000
Cross-sectional frequency distribution of unconditional moments



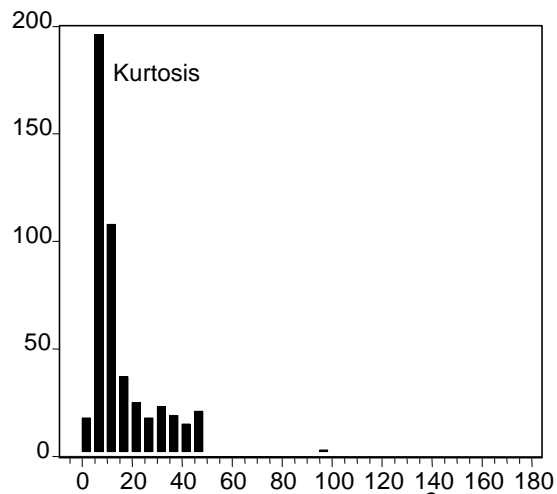
Series: MEAN	
Sample 1 500	
Observations 500	
Mean	0.029868
Median	0.017000
Maximum	0.463000
Minimum	-0.877000
Std. Dev.	0.114643
Skewness	-1.332909
Kurtosis	13.23695
Jarque-Bera	2331.285
Probability	0.000000



Series: STD	
Sample 1 500	
Observations 500	
Mean	5.524196
Median	5.256000
Maximum	19.14200
Minimum	2.020000
Std. Dev.	2.286806
Skewness	1.158602
Kurtosis	5.945845
Jarque-Bera	292.6550
Probability	0.000000



Series: SKEW	
Sample 1 500	
Observations 500	
Mean	-0.625946
Median	-0.118000
Maximum	8.636000
Minimum	-15.97200
Std. Dev.	1.636561
Skewness	-3.109677
Kurtosis	28.00459
Jarque-Bera	13831.45
Probability	0.000000



Series: KURT	
Sample 1 500	
Observations 500	
Mean	19.26952
Median	10.34500
Maximum	178.8320
Minimum	-0.324000
Std. Dev.	21.46704
Skewness	3.972950
Kurtosis	24.23618
Jarque-Bera	10710.68
Probability	0.000000

Table S.1 (cont.)
Descriptive statistics of the SP500 firms
January 1, 1990-December 27, 2000

Cross-sectional summary statistics of the unconditional jump:

$$\bar{p}_i(h) \equiv \frac{\sum_{t=1}^R J_{i,t}(h)}{R}$$

where $J_{i,t}(h) \equiv 1(z_{i,t} - z_{i,t-1} \geq h)$ for $h = 0.25, 0.50, 0.75, 0.90$

$\bar{p}_i(h) \equiv$ percentage number of jumps over a maximum of 573 weeks for firm i

Cross-sectional moments (over 466 firms)	$h = 0.25$	$h = 0.50$	$h = 0.75$	$h = 0.90$
Mean	56%	27%	9%	2%
Median	56%	25%	6%	0.9%
Max.	70%	49%	32%	20%
Min.	30%	2%	0%	0%

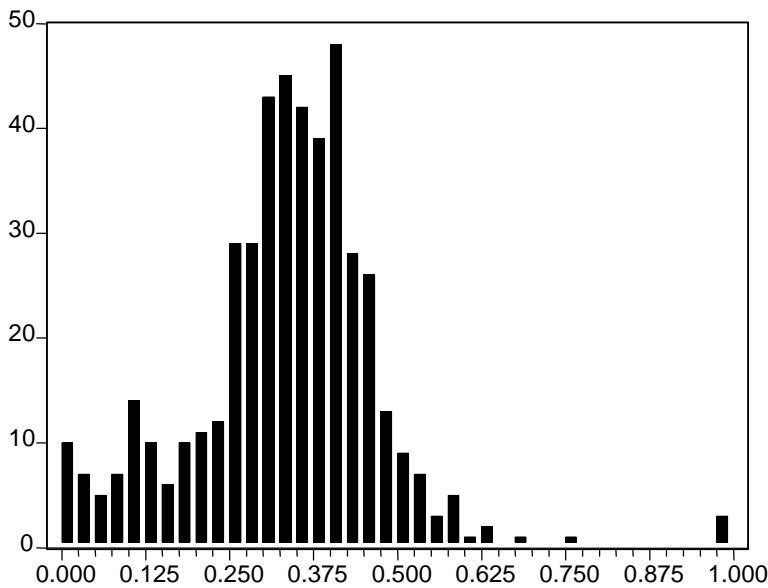
Entry explanation: for instance, choose the number in bold “Median = 56%” (second column, third row). It means that 50% of the SP500 firms have had a jump of at least 0.25 in the cross-sectional ranking of returns in 56% of the weeks between Jan.1 1990 and Dec. 27, 2000 or alternatively, there is a jump of at least 0.25 every 1.8 weeks. The smaller the jump, the larger the summary statistic is.

Table S.2
Cross-sectional frequency distribution of the estimates of the duration model
when $J_t = 1(|z_t - z_{t-1}| \geq 0.5)$

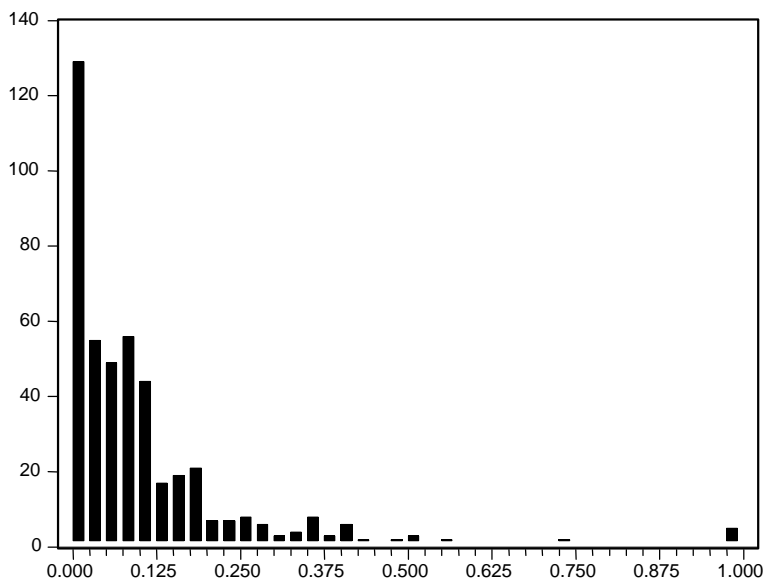
$$p_t = [\Psi_{N(t-1)} + \delta' X_{t-1}]^{-1},$$

$$\Psi_{N(t)} = \alpha D_{N(t)-1} + \beta \Psi_{N(t)-1},$$

$$\delta' X_{t-1} = \delta_1 + \delta_2 y_{t-1} \mathbf{1}(z_{t-1} \leq 0.5) + \delta_3 y_{t-1} \mathbf{1}(z_{t-1} > 0.5)$$



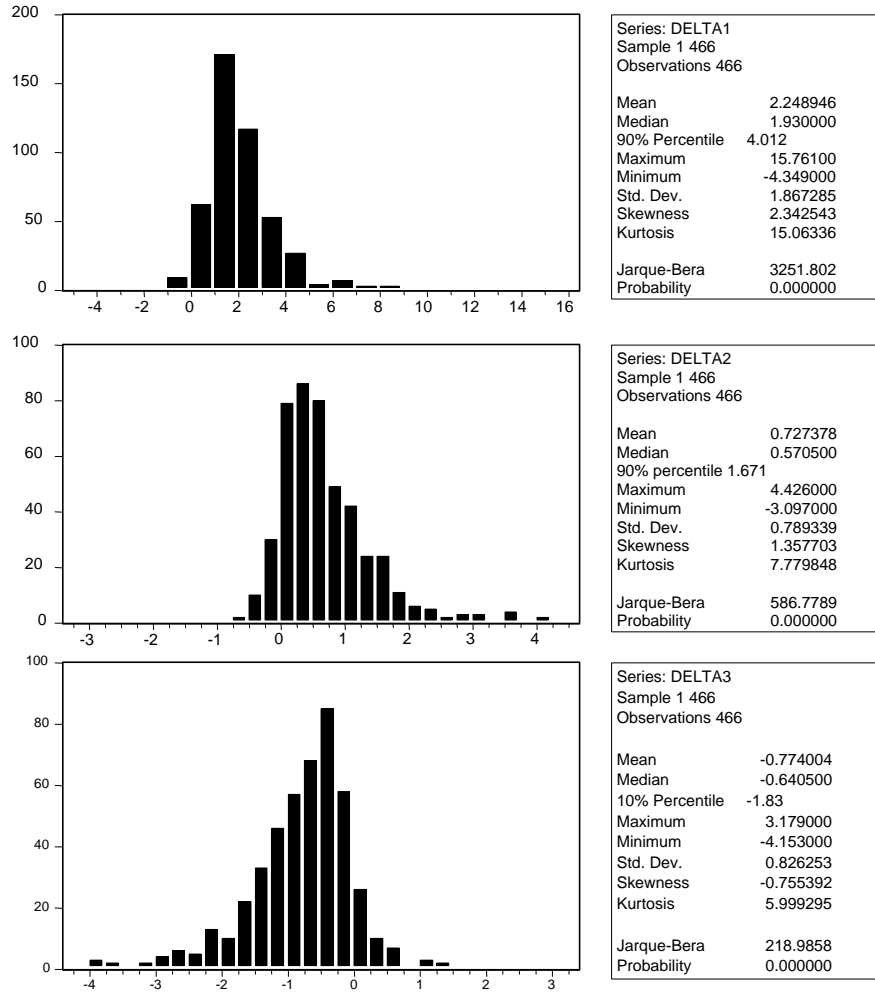
Series: ALPHA	
Sample 1 466	
Observations 466	
Mean	0.336601
Median	0.347000
Maximum	0.989000
Minimum	0.011000
Std. Dev.	0.137334
Skewness	0.243171
Kurtosis	5.830162
Jarque-Bera	160.1165
Probability	0.000000



Series: BETA	
Sample 1 466	
Observations 466	
Mean	0.124251
Median	0.074500
Maximum	0.989000
Minimum	0.010000
Std. Dev.	0.165666
Skewness	2.853888
Kurtosis	12.80408
Jarque-Bera	2498.901
Probability	0.000000

Table S.2 (cont.)

Cross-sectional frequency distribution of the estimates of the duration model



Median Values of Parameter Estimates of q_1 in the ACH Model $f_1(J_t | \mathcal{S}_{t-1}; q_1)$

Industry Sectors in the SP500 index	% of firms	$\hat{a} + \hat{b}$	\hat{d}_2	\hat{d}_3	\hat{p}	Unconditional \bar{p}
Consumer Goods	25.2	0.456	0.542	-0.573	0.257	0.246
Energy	5.5	0.359	0.769	-0.840	0.347	0.297
Finance	16.5	0.444	0.959	-0.488	0.257	0.239
Health Care	11.2	0.387	0.501	-0.672	0.326	0.295
Industrials	11.2	0.451	0.586	-0.946	0.238	0.215
Information Technology	17.7	0.334	0.318	-0.413	0.447	0.380
Material	6.4	0.437	0.587	-0.690	0.290	0.248
Utilities	6.4	0.497	0.851	-1.101	0.132	0.137
All sectors	100.0	0.422	0.571	-0.641	0.267	0.253

Note: \hat{p}_t is the conditional probability of jumping obtained from equation (10) for every firm in the sample; \hat{p} is the median value of \hat{p}_t calculated over firms and over time; and

$\bar{p} \equiv \frac{\sum_{t=1}^R J_t}{R}$ is the unconditional probability of jumping for which we report the median values calculated over the cross-section of firms.

Table S.3
Cross-sectional frequency distribution of the estimates of the nonlinear model for
expected returns when $J_t = 1(|z_t - z_{t-1}| \geq 0.5)$

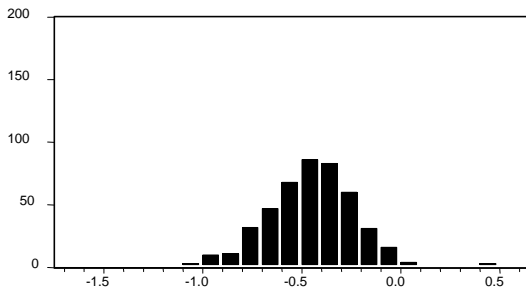
$$f_2(y_t | J_t, \mathcal{S}_{t-1}; \mathbf{q}_2) = \begin{cases} N(\mathbf{m}_{1,t}, \mathbf{s}_{1,t}^2) & \text{if } J_t = 1 \\ N(\mathbf{m}_{0,t}, \mathbf{s}_{0,t}^2) & \text{if } J_t = 0 \end{cases}$$

$$\mathbf{m}_{1,t} = \mathbf{n}_1 + \mathbf{g}_1 y_{t-1} + \mathbf{h}_1 z_{t-1} \quad \mathbf{m}_{0,t} = \mathbf{n}_0 + \mathbf{g}_0 y_{t-1} + \mathbf{h}_0 z_{t-1}$$

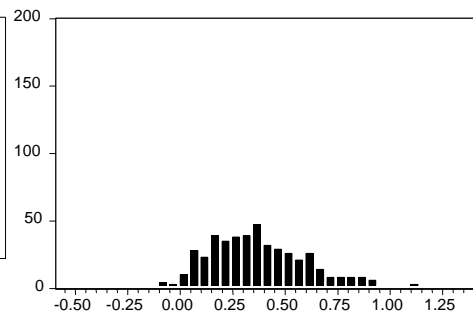
$$\mathbf{s}_{1,t}^2 = \mathbf{w} + \mathbf{r} \mathbf{e}_{t-1}^2 + \mathbf{t} \mathbf{s}_{t-1}^2$$

$$\text{with } \mathbf{e}_{t-1} = (y_{t-1} - \mathbf{m}_{1,t-1})J_{t-1} + (y_{t-1} - \mathbf{m}_{0,t-1})(1 - J_{t-1})$$

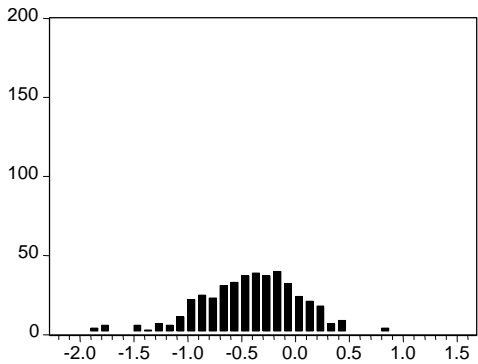
Conditional mean parameter estimates



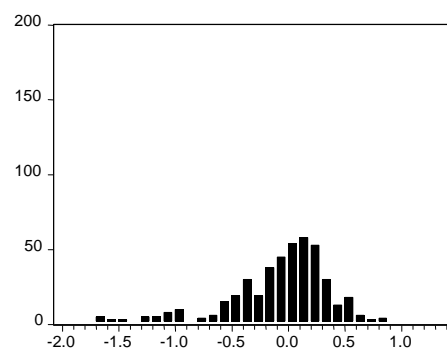
Series: GAMMA1	
Sample 1 466	
Observations 466	
Mean	-0.447371
Median	-0.440000
Maximum	0.518000
Minimum	-1.643000
Std. Dev.	0.256279
Skewness	-0.261153
Kurtosis	5.523249
Jarque-Bera	128.9187
Probability	0.000000



Series: GAMMA0	
Sample 1 466	
Observations 466	
Mean	0.377494
Median	0.358000
Maximum	1.369000
Minimum	-0.536000
Std. Dev.	0.248578
Skewness	0.424047
Kurtosis	3.862003
Jarque-Bera	28.39326
Probability	0.000001



Series: ETA1	
Sample 1 466	
Observations 466	
Mean	-0.409373
Median	-0.377500
Maximum	1.539000
Minimum	-2.149000
Std. Dev.	0.553127
Skewness	-0.260720
Kurtosis	3.972640
Jarque-Bera	23.64812
Probability	0.000007



Series: ETA0	
Sample 1 466	
Observations 466	
Mean	-0.076479
Median	0.029500
Maximum	1.334000
Minimum	-1.930000
Std. Dev.	0.513634
Skewness	-1.043064
Kurtosis	4.735894
Jarque-Bera	143.0088
Probability	0.000000

Median Values of Parameter Estimates of \mathbf{q}_2 in the Non-linear Model

$$f_2(y_t | J_t, \mathcal{S}_{t-1}; \mathbf{q}_2)$$

Industry Sectors in the SP500 index	% of firms	$\hat{\mathbf{g}}_1$	$\hat{\mathbf{g}}_0$	$\hat{\mathbf{h}}_1$	$\hat{\mathbf{h}}_0$	$\hat{\mathbf{r}} + \hat{\mathbf{t}}$
Consumer Goods	25.2	-0.445	0.345	-0.375	-0.010	0.909
Energy	5.5	-0.591	0.384	-0.252	0.022	0.940
Finance	16.5	-0.422	0.347	-0.369	0.058	0.972
Health Care	11.2	-0.319	0.365	-0.658	0.031	0.899
Industrials	11.2	-0.415	0.338	-0.358	0.056	0.885
Information Technology	17.7	-0.419	0.480	-0.422	0.015	0.920
Material	6.4	-0.444	0.353	-0.473	-0.036	0.965
Utilities	6.4	-0.575	0.136	-0.069	0.208	0.909
All sectors	100.0	-0.44	0.358	-0.378	0.029	0.925

4 Technical Trading Rules

In addition to the three model based trading rules (VCR-Mixture Trading Rule, VCR Trading Rule, and Buy-and-Hold-the-Market Trading Rule), discussed in the paper, we also consider four classes of *technical* trading rules considered by Sullivan, Timmermann and White (1999): *Filter-Rule*, *Moving-Average-Rule*, *Channel-Break-Out-Rule*, and *Support-and-Resistance-Rule*. All of these four trading rules are based on the SP500 index and they can be considered as rules that exploit the momentum in returns. For each of the four technical trading rules, we consider four parameterizations as explained below.

Filter-Rule(x): If the weekly closing price of a particular security moves up at least x per cent, buy and hold the security until its price moves down at least x per cent from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the weekly closing price rises at least x per cent above a subsequent low at which time one covers and buys. The neutral position is obtained by liquidating a long position when the price decreases y percent from the previous high, and covering a short position when the price increases y percent from the previous low. We apply one of the rules of Sullivan *et al.* (1999) to define subsequent high (low). A subsequent high (low) is the highest (lowest) closing price achieved while holding a particular long (short) position. We also allow for the holding of the asset for c weeks ignoring any signals generated from the market. We consider $\{x : 0.05, 0.10, 0.20, 0.50\}$, $y = 0.5x$, and $c = 1$.

Moving-Average-Rule(l, s): This rule involves going long (short) when the short period moving average (s) rises above (falls below) the long period moving average (l). Its idea is to smooth out the series and locate the initiation of trend (when s penetrates l). We consider four sets of local moving averages with $\{(l, s) : (10, 2), (20, 2), (10, 4), (20, 4)\}$, a fixed percentage band filter to rule out false signals with the band $b = 0.05$ for all cases, and $c = 1$ as for the filter rule.

Channel-Break-Out-Rule(n, x): A channel is said to occur when the high over the previous n time periods is within x percent of the low over the previous n time periods, not including the current price. The strategy is to buy when the closing price exceeds the channel, and to go short when the price moves below the channel. Long and short positions are held for a fixed number of days, $c = 1$. A fixed percentage band, $b = 0.05$, is applied to the channel as a filter. We consider the four sets of parameters $\{(n, x) : (4, 0.05), (10, 0.05), (4, 0.10), (10, 0.10)\}$.

Support-and-Resistance-Rule(n): Buy when the closing price exceeds the maximum price over the previous n time periods, and sell when the closing price is less than the minimum price over the previous n time periods. We consider $\{n : 2, 4, 8, 16\}$, and a fixed percentage band $b = 0.05$.

5 Wang's (2001) Monte Carlo Method

For the VCR-Mixture Trading Rule, where we are interested in the VaR of a portfolio of K asset, each one following a mixture of conditional normal distributions, the computation of the VaR is not straightforward because a mixture of normals does not belong to the location-scale family. We implement the analytical Monte Carlo method of Wang (2001), which is described in some detail here.

We follow proposition 4.1 of Wang (2001) to calculate $VaR_{t+1}^\alpha(\hat{\theta}_t)$ for a portfolio consisting of assets whose weekly returns is a conditionally mixed normal distribution. A brief exposition of the procedure follows.

Let $Y_t = (Y_{1t}, \dots, Y_{nt})'$ is a random vector of weekly return where Y_{rt} ($r = 1, \dots, n$) are (conditionally) univariate mixed normal distribution with k_r component (in our case $k_r = 2$ for all r). Let the conditional variance covariance matrix of Y_t be Σ_{Y_t} , and $\sigma_{ijt}(Y_t)$ the ij^{th} element of that matrix. While we parametrically model the diagonal terms of Σ_{Y_t} , the off-diagonal elements are obtained by calculating time varying sample covariance (that is $\sigma_{ijt}(Y_t) = \frac{1}{t-2} \sum_{s=1}^{t-1} (Y_{is} - \bar{Y}_i)(Y_{js} - \bar{Y}_j)$, where $\bar{Y}_r = \frac{1}{t-2} \sum_{s=1}^{t-1} Y_{rs}$). The conditional density of Y_{rt} is

$$f(y_{rt}|\mathcal{F}_{t-1}) = \sum_{h=1}^{k_i} p_{rt_h} \frac{1}{2\pi\sigma_{rt_h}} \exp\left(-\frac{1}{2} \left[\frac{y_{rt} - \mu_{rt_h}}{\sigma_{rt_h}}\right]^2\right)$$

where μ_{rt_h} and σ_{rt_h} represent the conditional mean and conditional variance of the r^{th} component of normal mixture and p_{rt_h} is the probability associated with r^{th} component of the mixture (in our setting $p_{rt_1} =$ probability of sharp jump, and $p_{rt_2} = 1 - p_{rt_1}$). We are interested in the behavior of the random variable $D_t = n^{-1} \sum_{r=1}^n \omega_r Y_{rt}$, where ω_r is the weight attached to the r^{th} asset in the portfolio.

The conditional VaR is obtained from the following numerical function.

$$\alpha = \Pr(D_t \leq VaR_{t+1}^\alpha(\hat{\theta}_t)) = \sum_{h_1=1}^{k_1} \dots \sum_{h_n=1}^{k_n} p_{1h_1} \dots p_{nh_n} \Phi\left(\frac{VaR_{t+1}^\alpha(\hat{\theta}_t) - \mu_{h_1 \dots h_n t}}{\sigma_{h_1 \dots h_n t}}\right)$$

where $\Phi(\cdot)$ is the cdf of a standard normal distribution, $\mu_{h_1 \dots h_n t} = \sum_{r=1}^n \omega_r \mu_{rt_{h_r}}$, $\sigma_{h_1 \dots h_n t} = \sum_{r=1}^n \sum_{s=1}^n \omega_r \omega_s \sigma_{rt_{h_r}} \sigma_{st_{h_s}} \sigma_{rst}(Z_t)$, where

$$\sigma_{rst}(Z_t) = \left(\frac{\sigma_{rst}(Y_t) - \sum_{h=1}^{k_r} \sum_{l=1}^{k_s} p_{rt_h} p_{st_l} (\mu_{rt_h} - \mu_{rt})(\mu_{st_l} - \mu_{st})}{\sum_{h=1}^{k_r} \sum_{l=1}^{k_s} p_{rt_h} p_{st_l} \sigma_{rt_h} \sigma_{st_l}} \right)$$

for $r \neq s$, and $\sigma_{rst}(Z_t) = 1$ for $r = s$, and μ_{rt} and μ_{st} are the conditional mean of Y_{rt} and Y_{st}

respectively. Note that right hand side of above equation is a monotonic increasing function in $VaR_{t+1}^\alpha(\hat{\theta}_t)$ thus it can be calculated numerically.

6 Reality Check

Let l be the number of competing trading rules ($k = 1, \dots, l$) to compare with the benchmark rule (indexed as $k = 0$). For each trading rule k , one-step predictions are to be made for P periods from $R + 1$ through T using a rolling sample, as explained in the previous sections. Consider a generic loss function $L(Y, \theta)$ where Y consists of variables in the information set. In our case, we have six forecast evaluation functions: MTR, SR, MSR, V_1, V_2 , and V_3 . As the first three forecast evaluation functions, MTR, SR, MSR , are to be maximized, while the last three forecast evaluation functions based on VaR, V_1, V_2 , and V_3 , are to be minimized, the generic loss function $L(Y, \theta)$ denotes one of the six *loss* functions, $-MTR, -SR, -MSR, V_1, V_2$, and V_3 , so that the following discussion is based on minimization of the objective function $L(Y, \theta)$.

The best trading rule is the one that minimizes the expected loss. We test a hypothesis about an $l \times 1$ vector of moments, $E(\mathbf{f})$, where $\mathbf{f} \equiv \mathbf{f}(Y, \theta)$ is an $l \times 1$ vector with the k^{th} element $f_k = L_0(Y, \theta) - L_k(Y, \theta)$, $\theta \equiv \text{plim } \hat{\theta}_T$, and $L_0(\cdot, \cdot)$ is the loss under the benchmark rule, $L_k(\cdot, \cdot)$ is the loss provided by the k^{th} trading rule. A test for a hypothesis on $E(\mathbf{f})$ may be based on the $l \times 1$ statistic $\bar{\mathbf{f}} \equiv P^{-1} \sum_{t=R}^{T-1} \hat{\mathbf{f}}_{t+1}$, where $\hat{\mathbf{f}}_{t+1} \equiv \mathbf{f}(Y_{t+1}, \hat{\theta}_t)$.

Our interest is to compare all the trading rules with a benchmark. An appropriate null hypothesis is that all the trading rules are no better than a benchmark, i.e., $\mathbb{H}_0 : \max_{1 \leq k \leq l} E(f_k) \leq 0$. This is a multiple hypothesis, the intersection of the one-sided individual hypotheses $E(f_k) \leq 0$, $k = 1, \dots, l$. The alternative is that \mathbb{H}_0 is false, that is, the best trading rule is superior to the benchmark. If the null hypothesis is rejected, there must be at least one trading rule for which $E(f_k)$ is positive. Suppose that $\sqrt{P}(\bar{\mathbf{f}} - E(\mathbf{f})) \xrightarrow{d} N(0, \Omega)$ as $P(T) \rightarrow \infty$ when $T \rightarrow \infty$, for Ω positive semi-definite. White's (2000) test statistic for \mathbb{H}_0 is formed as $\bar{V} \equiv \max_{1 \leq k \leq l} \sqrt{P} \bar{f}_k$, which converges in distribution to $\max_{1 \leq k \leq l} G_k$ under \mathbb{H}_0 , where the limit random vector $G = (G_1, \dots, G_l)'$ is $N(0, \Omega)$. However, as the null limiting distribution of $\max_{1 \leq k \leq l} G_k$ is unknown, White (2000, Theorem 2.3) shows that the distribution of $\sqrt{P}(\bar{\mathbf{f}}^* - \bar{\mathbf{f}})$ converges to that of $\sqrt{P}(\bar{\mathbf{f}} - E(\mathbf{f}))$, where $\bar{\mathbf{f}}^*$ is obtained from the stationary bootstrap of Politis and Romano (1994). By the continuous mapping theorem this result extends to the maximal element of the vector $\sqrt{P}(\bar{\mathbf{f}}^* - \bar{\mathbf{f}})$ so that the empirical distribution of $\bar{V}^* = \max_{1 \leq k \leq l} \sqrt{P}(f_k^* - \bar{f}_k)$ is used to compute the p-value of \bar{V} (White, 2000, Corollary 2.4). This p-value is called the "Reality Check p-value".

The inclusion of \bar{f}_k in $\bar{V}^* = \max_{1 \leq k \leq l} \sqrt{P}(\bar{f}_k^* - \bar{f}_k)$ guarantees that the statistic satisfies the null hypothesis $E(\bar{f}_k^* - \bar{f}_k) = 0$ for all k . This setting makes the null hypothesis the least favorable to the alternative and consequently, it renders a very conservative test. When a highly misspecified model is introduced, the reality check p-value becomes very large and, depending on the variance of \bar{f}_k , it may remain large even after the inclusion of better models. Hence, the White's reality check p-value may be considered as an upper bound for the true p-value. In Hansen (2005) the statistic $\bar{V}^* = \max_{1 \leq k \leq l} \sqrt{P}(\bar{f}_k^* - \bar{f}_k)$ is modified as

$$\bar{V}^* = \max_{1 \leq k \leq l} \sqrt{P}(\bar{f}_k^* - g(\bar{f}_k))$$

Different $g(\cdot)$ functions will produce different bootstrap distributions that are compatible with the null hypothesis. If $g(\bar{f}_k) = \max(\bar{f}_k, 0)$, the null hypothesis is the most favorable to the alternative, and the p-value associated with the test statistic under the null will be a lower bound for the true p-value. Hansen (2005) recommended setting $g(\cdot)$ as a function of the variance of \bar{f}_k , i.e.

$$g(\bar{f}_k) = \begin{cases} 0 & \text{if } \bar{f}_k \leq -A_k \\ \bar{f}_k & \text{if } \bar{f}_k > -A_k \end{cases}$$

where $A_k = \frac{1}{4}P^{1/4}\sqrt{\text{var}(\bar{f}_k)}$ with the variance estimated from the bootstrap resamples.