Money-Income Granger-Causality in Quantiles*

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Abstract

The causal relationship between money and income (output) has been an important topic and has been extensively studied. However, those empirical studies are almost entirely on Granger-causality in the conditional mean. Compared to conditional mean, conditional quantiles give a broader picture of an economy in various scenarios. In this paper, we explore whether forecasting conditional quantiles of output growth can be improved using money growth information. We compare the check loss values of quantile forecasts of output growth with and without using past information on money growth, and assess the statistical significance of the loss-differentials. Using U.S. monthly series of real personal income or industrial production for income and output, and M1 or M2 for money, we find that out-of-sample quantile forecasting for output growth is significantly improved by accounting for past money growth information, particularly in tails of the output growth conditional distribution. On the other hand, money-income Granger-causality in the conditional mean is quite weak and unstable. These empirical findings in this paper have *not* been observed in the money-income literature. The *new* results of this paper have an important implication on monetary policy, because they imply that the effectiveness of monetary policy has been under-estimated by merely testing Granger-causality in conditional mean. Money does Granger-cause income more strongly than it has been known and therefore information on money growth can (and should) be more utilized in implementing monetary policy.

Keywords : Money-income Granger-causality, Conditional mean, Conditional quantiles, Conditional distribution
JEL Classification : C32, C5, E4, E5

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1 Introduction

Granger-causality (GC), introduced by Granger (1969, 1980, 1988), is one of the important issues that have been much studied in empirical macroeconomics and empirical finance. Particularly the causal relationship between money and income is one of the most widely studied subject in economics. In this paper, we extend the literature in two ways. The literature on money-income causality is studied for the conditional *mean* and most papers have used the *in-sample* significance of money variables in the output growth equation. (In this paper, the terms, income and output, will be used interchangeably.) First, we go beyond the conditional mean, and examine the conditional distribution and conditional *quantiles*. Second, we examine the *out-of-sample* predictive contents of money variables in forecasting output growth.

While GC is naturally defined in terms of conditional distribution (see Granger and Newbold 1986), almost all the papers in this literature have focused on GC-in-mean (GCM). The GC-in-distribution (GCD) has been less studied empirically perhaps because it is in fact about independence and so it may be too broad to be directly linked to a policy implication. More useful may be the particular quantiles of the conditional distribution because by inverting the conditional distribution we obtain the conditional quantiles. Hence, we may examine directly the GC in distribution, or indirectly via GC in conditional quantiles (GCQ). Granger (2003) notes that the study of the time series of quantiles is relevant as the predictive distribution can be expressed in terms of the CDF, the density, the characteristic function, or quantiles.

Vast empirical literature on the money-income causality has very mixed results on GCM – usually unstable and sensitive to the choice of sample periods, data sets and variables (e.g., M1 or M2 for money, personal income (PI) or industrial production (IP) for income, with or without including some other variables such as interest rates and business cycle indicators in the regression, etc.). Different countries, sample periods and variables are studied in those empirical research, but no consensus has been reached. The results in this paper for GCD and GCQ are much more stable and stronger.

The aim of this paper is to study the GC beyond the conditional mean between money and income, which is in line with the suggested directions of Granger (2003, 2005, 2006).[1] Forecasting conditional quantiles is important in economic policy when a particular scenario of the economy is

[1] Granger (2006) remarks, in the 20th anniversary issue of the *Advances in Econometrics*, “For most of its history time series theory considered conditional means, but later conditional variances. The next natural development would be conditional quantiles, but this area is receiving less attention than I expected. The last stages are initially conditional marginal distributions, and finally conditional multivariate distributions. Some interesting theory is starting in these areas but there is enormous amount to be done.”

concerned. For instance, in asset valuation, different scenarios of output growth are extremely useful in sensitivity analysis, scenario analysis, and risk management. For industries greatly influenced by overall macroeconomic conditions, forecasting output growth helps to evaluate the industry exposure in different scenarios. The fan chart of Bank of England for different quantile forecasts of inflation rate is another example.

Although it is not stable in the U.S. data on whether money growth helps to improve forecasting of the conditional mean of output growth, we find that there is much stronger evidence that it helps to improve forecasting of its conditional quantiles. Forecasting the conditional quantiles of output growth depends on its conditional distribution, and so we may also test for GCD. GCD implies GCQ in some quantiles, although GCD does not necessarily imply GCQ in each quantile. GCD is a necessary but not sufficient condition for GCQ in specific quantile. GCQ in a specific quantile exists if the lagged money variables help to improve forecasting the output growth at that quantile. Two quantile regression models for output growth with and without money growth information are estimated and the out-of-sample average of the "check" loss values of the two quantile models are compared. Because these two quantile forecasting models are nested, the unconditional predictive ability test proposed by Diebold and Mariano (1995) and West (1996) fails in that its asymptotic distribution degenerates. We therefore utilize the *conditional* predictive ability test proposed by Giacomini and White (GW henceforth, 2006).

Our empirical study uses several different data sets over various sample periods. We find the following results. First, for the causality in the conditional mean, differently from Chao, Corradi and Swanson (2001) who test out-of-sample Granger-causality in mean using moment conditions, we compare the squared forecast error loss values of two conditional mean forecasts of output growth with and without money. The result is very weak for the GCM (as expected from the existing literature). We find that the predictive ability of a model with including money as a predictor for the conditional mean of output growth could be even worse than a model without money (due to parameter estimation error), and the result varies sensitively over time as pointed out by Eichenbaum and Singleton (1986), Stock and Watson (1996), Swanson (1998) and Thoma (1994). Second, for the money-income GC in the conditional distribution, we use a nonparametric copula function, and find a more stable and significant result for GCD in many subsamples even when there exists no significant GCM. Third, for the GCQ, two conditional quantile regression models with and without money are estimated and their quantile forecasts are compared for their out-of-sample check loss values. We find that GCQ is significant in tail quantiles in most subsamples and most data sets, while it is not significant in the center of the distribution. Forth, comparing results across different data sets (which consist of different variables for money and income), it seems that GCQ between money and industrial production (IP) is more significant that between money and real personal income (PI).

The structure of this paper is as follows. In Section 2, we discuss GC in mean, GC in distribution, and GC in quantiles. Section 3 reports the empirical findings. Section 4 concludes.

2 Granger-causality

We use the following notation. Let R denote the sample size for estimation (for which we use a rolling scheme), P denote the size of the out-of-sample period for forecast evaluation, and $T = R+P$. Let x be money growth and y the output growth. Consider the distribution functions conditional on the information set $\mathcal{F}_t$ as $F_{t+1}(x|\mathcal{F}_t) = \Pr(x_{t+1} < x|\mathcal{F}_t)$, $G_{t+1}(y|\mathcal{F}_t) = \Pr(y_{t+1} < y|\mathcal{F}_t)$, and $H_{t+1}(x, y|\mathcal{F}_t) = \Pr(x_{t+1} < x \text{ and } y_{t+1} < y|\mathcal{F}_t)$. Let $f_{t+1}(x|\mathcal{F}_t)$, $g_{t+1}(y|\mathcal{F}_t)$, and $h_{t+1}(x, y|\mathcal{F}_t)$ be the corresponding densities. Let $u = F_{t+1}(x|\mathcal{F}_t)$ and $v = G_{t+1}(y|\mathcal{F}_t)$. Let $C_{t+1}(u, v|\mathcal{F}_t)$ and $c_{t+1}(u, v|\mathcal{F}_t)$ be the conditional copula function and the conditional copula density function respectively. See Appendix A for a brief introduction on the copula theory. Let $E(y_{t+1}|\mathcal{F}_t)$ be the conditional mean of y_{t+1}. Let $X_t = (x_t, \ldots, x_{t+1-q})'$ and $\mathcal{G}_t$ be the information set excluding X_t, i.e., $\mathcal{G}_t = \mathcal{F}_t/\{X_t\}$.

2.1 Money-Income Granger-causality in Mean

Starting with Friedman (1956) the debate about role of money on income attracts attention of a lot of economists. Numerous studies have been devoted to the interaction between money and income. Theoretical models are constructed to explore the roles of aggregate demand fluctuation and money demand fluctuation, such as in Kaldor (1970), Modigliani (1977), Meltzer (1963), among others. Along with the theoretical development, many empirical studies have been made following the seminal research of Sims (1972, 1980). Sims (1972) shows money Granger-causes income, but his results were criticized due to the bias caused by hidden factors. Sims (1980) applies a vector autoregressive (VAR) model to handle a vector of variables and reports that money does not Granger cause income after the World War II. After Sims, Granger-causality and VAR models have become generally accepted instruments for studying the money and income relationship. Stock and Watson (1989) contend that the deterministic trend plays important roles and use detrended money in the analysis. They find more significant money-income causality using the detrended money growth rate. Friedman and Kuttner (1992, 1993) and Thoma (1994) also report limited evidence for the money-income causality. However, they find money-income causality is time-varying with regard to different sample periods or with regard to different variables.[2] Swanson (1998) tests money-income Granger-causality in an error-correction model. Dufour and Renault (1998) and Dufour, Pelletier and Renault (2006) test the long horizon causality.

Definition 1. (Non Granger-causality in mean, NGCM): *X_t does not Granger-cause y_{t+1} in mean if and only if $E(y_{t+1}|X_t, \mathcal{G}_t) = E(y_{t+1}|\mathcal{G}_t)$ almost surely (a.s.).*

To test for Granger-causality in mean (GCM), we can utilize either an in-sample test or an out-of-sample test. In the literature, most tests of money-income causality focus on in-sample conditional mean in a VAR model. The in-sample Granger-causality test is to test the null hypothesis that coefficients of money are all insignificant in the output equation. A Wald-type test is often used in an in-sample test of GCM. Following Ashley, Granger, and Schmalensee (1980) who argue Granger-causality makes more sense in a predictive setting, we conduct an out-of-sample test for GCM based on two nested models. The first model does not account for money-income GCM (referred as Model 1 or "NGCM") and the second does (referred as Model 2 or "GCM"):

$$\text{Model 1 (NGCM)} \quad : \quad y_{t+1} = E(y_{t+1}|\mathcal{G}_t) + \varepsilon_{1,t+1} = V_t'\theta_1 + \varepsilon_{1,t+1}, \tag{1}$$

$$\text{Model 2 (GCM)} \quad : \quad y_{t+1} = E(y_{t+1}|X_t, \mathcal{G}_t) + \varepsilon_{2,t+1} = W_t'\theta_2 + \varepsilon_{2,t+1}, \tag{2}$$

where $V_t \in \mathcal{G}_t$ and $W_t = (X_t' \; V_t')$ are vectors of regressors. V_t includes a constant term. The parameters $\{\theta_i\}$ are estimated by minimizing the squared error loss using the rolling sample of the most recent R observations at time t $(t = R, \ldots, T-1)$:

$$\hat{\theta}_{1,t} = \arg\min_{\theta_1} \sum_{s=t-R+1}^{t} (y_s - V_{s-1}'\theta_1)^2, \tag{3}$$

$$\hat{\theta}_{2,t} = \arg\min_{\theta_2} \sum_{s=t-R+1}^{t} (y_s - W_{s-1}'\theta_2)^2. \tag{4}$$

Denote $\hat{y}_{1,t+1}(\hat{\theta}_{1,t}) = V_t'\hat{\theta}_{1,t}$ and $\hat{y}_{2,t+1}(\hat{\theta}_{2,t}) = W_t'\hat{\theta}_{2,t}$, the forecasts of y_{t+1} from Model 1 and Model 2, respectively, and let $\hat{\varepsilon}_{i,t+1}(\hat{\theta}_{i,t}) = y_{t+1} - \hat{y}_{i,t+1}(\hat{\theta}_{i,t})$ be the forecast error of Model i. In the empirical analysis of Section 3, we choose $X_t = (x_t, \ldots, x_{t+1-q})'$ and $V_t = (Y_t', I_t, B_t)'$ where $Y_t = (y_t, \ldots, y_{t+1-q})'$, $q = 12$, I_t is the 3 month T-bill interest rate, and B_t is the business cycle coincident index. See Table 1A for details.

[2] For instance, by replacing the three month T-bill rate by commercial paper rate, the money-income causality become less significant. In general, these empirical studies give us relatively controversial results on the money-income causality.

As the models are nested, we can not use the tests of Diebold and Mariano (1995) and West (1996). A test for Granger-causality is to compare the loss functions of forecasts conditional on two information sets, $\mathcal{G}_t$ and $\mathcal{F}_t$. As we are interested in comparing the loss of forecasting output growth y_{t+1} without and with using the information on past money growth X_t, we use the conditional predictive ability test of GW (2006). Let $L_{t+1}(\cdot)$ be a loss function. The null hypothesis of NGCM is therefore

$$H_0 : E[L_{t+1}(y_{t+1}, \hat{y}_{1,t+1}) - L_{t+1}(y_{t+1}, \hat{y}_{2,t+1})|\mathcal{F}_t] = 0, \qquad t = R, \ldots, T-1. \tag{5}$$

Under the H_0 the loss differential $\Delta L_{t+1} \equiv L_{t+1}(y_{t+1}, \hat{y}_{1,t+1}) - L_{t+1}(y_{t+1}, \hat{y}_{2,t+1})$ is a martingale difference sequence (MDS), which implies $E(h_t \Delta L_{t+1}) = 0$ for any h_t that is $\mathcal{F}_t$-measurable. Denoting $Z_{t+1} = h_t \Delta L_{t+1}$, the GW (2006) statistic is

$$GW_{R,P} = P\bar{Z}'_{R,P}\hat{\Omega}_P^{-1}\bar{Z}_{R,P}, \tag{6}$$

where $\bar{Z}'_{R,P} = \frac{1}{P}\sum_{t=R}^{T-1} h_t \Delta L_{t+1}$ and $\hat{\Omega}_P = \frac{1}{P}\sum_{t=R}^{T-1} Z_{t+1}Z'_{t+1}$. Under some regularity conditions, $GW_{R,P} \to^d \chi^2_q$ as $P \to \infty$ under H_0.[3] We choose the "test" function, h_t, such that it is $\mathcal{F}_t$-measurable but not $\mathcal{G}_t$-measurable. For simplicity, we choose $h_t = X_t = (x_t, \ldots, x_{t+1-q})'$.[4]

To test money-income Granger-causality in mean, we choose the squared error loss $L_{t+1}(y_{t+1}, \hat{y}_{i,t+1}) = \hat{\varepsilon}^2_{i,t+1}$ $(i = 1, 2)$ for the out-of-sample forecast evaluation because the conditional mean is the optimal forecast under the squared error loss. We also minimize the same loss for in-sample parameter estimation as shown in (3) and (4). Therefore, $Z_{t+1} = h_t \Delta L_{t+1} = h_t(\hat{\varepsilon}^2_{1,t+1} - \hat{\varepsilon}^2_{2,t+1})$. To be consistent with the literature using monthly series, we choose h_t using 12 lags of money growth rate, i.e., $h_t = X_t = (x_t \ \ldots \ x_{t-11})'$ with $q = 12$.

Because the GW statistic is for equal conditional predictive ability test, the rejection of the null hypothesis only implies that the two models are not equal in conditional predictive ability. To choose one model over the other, we follow the decision rule suggested by GW (2006) to construct

[3] Chao, Corradi and Swanson (2001) propose an out-of-sample test using following test statistic

$$CCS_{R,P} = \frac{1}{P}\sum_{t=R}^{T-1} \hat{\varepsilon}_{1,t+1} h(X_t),$$

which follows zero-mean normal distribution asymptotically with its asymptotic variance incoorporating estimation error.

[4] Two possible ways to improve the power of the test are (i) to choose q in a way to maximize the test power and (ii) to choose h_t from transforms of X_t as suggested in Bierens (1990), Stinchcombe and White (1998), or Hong (1999). We do not consider these extensions in this paper for simplicity and also to match the choice of h_t with the vast literature on GCM. Following Lee, White and Granger (1993) and Stinchcombe and White (1998), h_t will be called a *test* function.

a statistic as

$$I_P = \frac{1}{P} \sum_{t=R}^{T-1} \mathbf{1}(\hat{\alpha}'_P h_t < 0), \tag{7}$$

where $\mathbf{1}(\cdot)$ is the indicator function and $\hat{\alpha}_P$ is the coefficient of h_t by regressing ΔL_{t+1} on h_t ($t = R, \ldots, T-1$). As the rejection of H_0 occurs when the test function h_t can predict the loss difference ΔL_{t+1} in out-of-sample, $\hat{\alpha}'_P h_t \approx E(\Delta L_{t+1}|\mathcal{F}_t)$ will be the out-of-sample predicted loss differences. If I_P is greater than 0.5, Model 1 (NGCM) will be selected; otherwise Model 2 (GCM) will be selected.

2.2 Asymmetric GCM versus GCQ

Hayo (1999) nicely summarizes five stylized facts found in the empirical literature on the existence and strength of GCM between money and output using U.S. data: (a) In a model with only two variables, money Granger-causes output (Sims 1972). (b) The statistical significance of the effect of money on output will be lower when including other variables in a multivariate test such as prices and interest rates (Sims 1980). (c) The use of narrow money is less likely to support GC from money to output than broad money (King and Plosser 1984). (d) Assuming that variables are trend stationary and modelling them in (log-) levels with a deterministic trend is more likely to lead to significant test results than assuming difference stationary and employing growth rates (Christiano and Ljungquist 1988, Stock and Watson 1989, Hafer and Kutan 1997). (e) Allowing asymmetric effects of money on output growth and including the business cycle greatly influences results and strengthens the causal effect of money (Cover 1992, Thoma 1994, Weise 1999, Lo and Piger 2005, Ravn and Sola 2004, Psaradakis, Ravn, and Sola 2005).

Hayo (1999) revisited the above U.S. stylized facts using a broad data base of 14 EU countries plus Canada and Japan. It is found that very few of the above, particularly (b) and (d), can be sustained. Also found in the literature is that GCM is unstable, changing with sample periods, data to use (variables and frequency), and countries. Psaradakis, Ravn, and Sola (2004) provide some summary on this instability evidence from the literature. Davis and Tanner (1997) also find the instability of the GCM across countries.

What appears to be robust is (e). Thoma (1994) shows for monthly data with M1 that the state of business cycle has considerable influence on the results and strengthens of the GCM of money. When real activity declines the effect of money on output becomes stronger, while the opposite takes place during a recovery. Numerous papers in the literature have found that the evidence for the GCM becomes more evident when some asymmetry has been introduced. Weise (1999) and

Lo and Piger (2005) classify the three forms of asymmetry studies in a large body of empirical literature on money-income causality.

A1 (sign asymmetry): asymmetry related to the direction of the monetary policy action (Cover 1992, Dolado, Pedrero and Ruge-Murcia 2004)

A2 (size asymmetry): asymmetry related to the size of the policy action. (Ravn and Sola 2004, Dolado, Pedrero and Ruge-Murcia 2004)

A3 (business cycle asymmetry): asymmetry related to the existing business cycle business cycle phase (Thoma 1994, Weise 1999, Lo and Piger 2005, Garcia and Schaller 2002)

Weise (1999) finds no evidence for A1, some evidence for A2, and strong evidence for A3. Bernanke and Gertler (1995) and Galbraith (1996) explain A3 via credit rationing and its threshold effects on the relationship between money and output. Lo and Piger (2005) examine A3 using a regime-switching model in the response of U.S. output to monetary policy and find that policy actions during recessions have larger output effects than those taken during expansions. To deal with the instability and the asymmetry in GCM between money and income, many researchers have used split subsamples or rolling samples, or nonlinear models such as regime-switching models and threshold models.

The objective of this paper is to study GCQ, which is useful for scenario analysis in implementing monetary policy. Our empirical results (in Section 3) for GCQ are "symmetric", in that GCQ is insignificant in or near the center of the predicted distribution of the output growth while it is strongly significant in both tails. (The results of Section 3 show that GCQ is significant in both tails.) The difference between the asymmetric GCM and GCQ is that the former refers to the empirical fact that the predictive power of past money growth to predict the mean of output growth is stronger when the *past output growth* is negative (in recession), while the (symmetric) GCQ refers to the fact the predictive power of past money growth to predict the quantiles of output growth is stronger when the scenario of our interest is the *future output growth* in tails of its predicted distribution. Hence, the asymmetric GCM prescribes a monetary policy based on the past information, while the GCQ enables a monetary policy to be based on the forward looking scenarios of output growth. The GCQ can indicate how/whether the past and current money growth affects the various future states (i.e., quantiles) of the output growth.

The GCQ is also different from the Granger-causality in Risk (GCR) proposed by Hong, Liu, and Wang (2009). They extend the causality in variance in the literature (volatility spillover) to causality in extreme distribution defined on the platform of Value-at-Risk. However, the focus of

GCR is the risk spillover, i.e., the causality from occurrence of one extreme event to the occurrence of another extreme event, while that for GCQ is the forecasting of quantiles based on all available information of the causal variable, not merely the tail distribution of the causal variable.

2.3 Money-Income Granger-causality in Quantiles

Most empirical studies on money-income causality focus on Granger-causality in mean. As discussed above, in many cases, one may care about conditional distribution of output growth. Even without significant Granger-causality in mean, Granger-causality in distribution (GCD) may still be significant.

Definition 2. (Non Granger-causality in distribution, NGCD): *X_t does not Granger-cause y_{t+1} in distribution if and only if* $\Pr(y_{t+1} < y|X_t, \mathcal{G}_t) = \Pr(y_{t+1} < y|\mathcal{G}_t)$ *a.s. for all y.*

Remark: Note that we can write for $y \in \mathbb{R}$,

$$G_{t+1}(y|\mathcal{F}_t) = \Pr(y_{t+1} < y|\mathcal{F}_t) = E[\mathbf{1}(y_{t+1} < y|\mathcal{F}_t)] = E(z_{t+1}|\mathcal{F}_t), \quad (8)$$

$$G_{t+1}(y|\mathcal{G}_t) = \Pr(y_{t+1} < y|\mathcal{G}_t) = E[\mathbf{1}(y_{t+1} < y|\mathcal{G}_t)] = E(z_{t+1}|\mathcal{G}_t), \quad (9)$$

where $z_{t+1} = \mathbf{1}(y_{t+1} < y)$. Therefore, Definition 2 is equivalent to

$$E(z_{t+1}|\mathcal{F}_t) = E(z_{t+1}|\mathcal{G}_t) \text{ } a.s. \text{ for } all \text{ } y. \quad (10)$$

Hong, Liu, and Wang (2009) use this to test for Granger-causality in risk for *a fixed* value of y between two financial markets (X_t and y_{t+1}). The GCD between X_t and y_{t+1} can be viewed as GCM between X_t and z_{t+1} for *all* y. □

There is GCD if $\Pr(y_{t+1} < y|X_t, \mathcal{G}_t) \neq \Pr(y_{t+1} < y|\mathcal{G}_t)$ for *some* y. X_t does not Granger-cause y_{t+1} in distribution if $G_{t+1}(y|X_t, \mathcal{G}_t) = G_{t+1}(y|\mathcal{G}_t)$ $a.s.$ or $g_{t+1}(y|X_t, \mathcal{G}_t) = g_{t+1}(y|\mathcal{G}_t)$ $a.s.$

As the conditional distribution can be inverted to conditional quantiles, we test for Granger-causality in conditional distribution via testing for Granger-causality in conditional quantiles. Let the conditional quantile of y_{t+1} be denoted $q_\alpha(y_{t+1}|\mathcal{F}_t)$ such that $G_{t+1}(q_\alpha(y_{t+1}|\mathcal{F}_t)|\mathcal{F}_t) = \alpha$. The conditional quantile $q_\alpha(y_{t+1}|X_t, \mathcal{G}_t)$ can be obtained by inverting the conditional distribution $G_{t+1}(y|\mathcal{F}_t) = \alpha$. Recall that $\mathcal{G}_t$ is the information set excluding X_t, i.e., $\mathcal{G}_t = \mathcal{F}_t/\{X_t\}$. We now define GC in conditional quantile (GCQ).

Definition 3. **(Non Granger-causality in quantile):** *X_t does not Granger-cause y_{t+1} in α-quantile if and only if $q_\alpha(y_{t+1}|X_t, \mathcal{G}_t) = q_\alpha(y_{t+1}|\mathcal{G}_t)$ a.s.*

GC in conditional quantile refers to the case that $q_\alpha(y_{t+1}|X_t, \mathcal{G}_t) \neq q_\alpha(y_{t+1}|\mathcal{G}_t)$. If X_t does not Granger-cause y_{t+1} in distribution, $q_\alpha(y_{t+1}|X_t, \mathcal{G}_t) = q_\alpha(y_{t+1}|\mathcal{G}_t)$ since $g_{t+1}(y|X_t, \mathcal{G}_t) = g_{t+1}(y|\mathcal{G}_t)$. Therefore, non-Granger-causality in distribution implies non-Granger-causality in conditional quantiles. On the contrary, GC in distribution does not necessarily imply GC in each quantile, while significant GC in any conditional quantile implies significant GC in distribution. For some quantiles, X_t may Granger-cause y_{t+1}, while for other quantiles it may not. Granger (2003, p. 700) notes that some quantiles may differ from other quantiles in time series behavior (such as long memory and stationarity). For example, different parts of the distribution can have different time series properties; one tail could be stationary and the other tail may have a unit root.

While the quantile forecast $q_\alpha(y_{t+1}|X_t, \mathcal{G}_t)$ can be derived from inverting the density forecast, in this paper we use linear quantile regression. An out-of-sample test for GCQ is based on two nested linear models. The first model does not account for money-income GC in α-quantile (referred as Model 1 or "NGCQ") and the second does (referred as Model 2 or "GCQ"):

$$\text{Model 1} \quad : \quad y_{t+1} = q_\alpha(y_{t+1}|\mathcal{G}_t) + e_{1,t+1} = V_t'\theta_1(\alpha) + e_{1,t+1}, \tag{11}$$

$$\text{Model 2} \quad : \quad y_{t+1} = q_\alpha(y_{t+1}|X_t, \mathcal{G}_t) + e_{2,t+1} = W_t'\theta_2(\alpha) + e_{2,t+1}, \tag{12}$$

where $V_t \in \mathcal{G}_t$ and $W_t = (X_t' \ V_t')$ are vectors of regressors and V_t includes a constant term. The parameters $\theta_i(\alpha)$ are estimated by minimizing the "check" function discussed in Koenker and Bassett (1978) using the rolling sample of the most recent R observations at time t ($t = R, \ldots, T-1$) :

$$\hat{\theta}_{1,t}(\alpha) \quad = \quad \arg\min_{\theta_1(\alpha)} \sum_{s=t-R+1}^{t} \rho_\alpha(y_s - V_{s-1}'\theta_1(\alpha)), \tag{13}$$

$$\hat{\theta}_{2,t}(\alpha) \quad = \quad \arg\min_{\theta_2(\alpha)} \sum_{s=t-R+1}^{t} \rho_\alpha(y_s - W_{s-1}'\theta_2(\alpha)), \tag{14}$$

where $\rho_\alpha(e) \equiv [\alpha - \mathbf{1}(e < 0)]e$. Denote $\hat{q}^1_{\alpha,t+1}(\hat{\theta}_{1,t}(\alpha))$, $\hat{q}^2_{\alpha,t+1}(\hat{\theta}_{2,t}(\alpha))$ for the α-quantile forecasts of y_{t+1} from Model 1 and Model 2 respectively, and let $\hat{e}_{i,t+1}(\hat{\theta}_{i,t}(\alpha)) = y_{t+1} - \hat{q}^i_{\alpha,t+1}(\hat{\theta}_{i,t}(\alpha))$.

Denote $\hat{\theta}_{2X,t}(\alpha)$ as parameters in the GCQ quantile regression model for X_t' and $\hat{\theta}_{2V,t}(\alpha)$ as parameters in that model for V_t'. Thus $\hat{\theta}_{2,t}(\alpha) = (\hat{\theta}_{2X,t}'(\alpha), \hat{\theta}_{2V,t}'(\alpha))$.A convenient test for NGCQ is therefore to test $H_0 : \hat{\theta}_{2X,t}(\alpha) = \mathbf{0}$. Koenker and Basset (1982) show that under some regularity conditions, the parameter estimates has asymptotic normal distribution, i.e.,

$$n^{1/2}\left(\hat{\theta}_{2,t}(\alpha) - \theta_{2,t}(\alpha)\right) \xrightarrow{d} N(\mathbf{0}, \omega^2(\alpha, G)\Omega^{-1}), \tag{15}$$

where $\Omega = \lim_{R\to\infty} R^{-1}\sum w_t w_t'$, $\omega^2(\alpha, G) = \alpha(1-\alpha)/g^2(G^{-1}(\alpha))$ and $G(\cdot)$ and $g(\cdot)$ are the distribution function and density function of y respectively. The covariance matrix Ω can be estimated by its sample estimate $\hat{\Omega}$. $g(G^{-1}(\alpha))^{-1}$ is the reciprocal of the density function and is called the sparsity function. It can be estimated by the difference quotient of the empirical quantile function with a chosen bandwidth (such as the Bofinger bandwidth or the Hall-Sheather bandwidth).

Koenker and Machado (1999) propose two tests for $H_0 : \hat{\theta}_{2X,t}(\alpha) = \mathbf{0}$, a Wald-type test or a likelihood ratio test. Let the covariance matrix be partitioned accordingly in the GCQ model with Ω_{XX} and Ω_{VV} as the two diagonal terms and Ω_{XV} and Ω_{VX} as the two off-diagonal terms. The Wald-type test statistic is computed by

$$\text{Wald}_t(\alpha) = \hat{\theta}'_{2X,t}(\alpha)\hat{\Sigma}(\alpha)^{-1}\hat{\theta}_{2X,t}(\alpha), \tag{16}$$

where

$$\hat{\Sigma}(\alpha) = R^{-1}\hat{\omega}^2(\alpha, G)(\Omega_{XX} - \Omega_{XV}\Omega_{VV}^{-1}\Omega_{VX})^{-1},$$

and $\hat{\omega}(\alpha, G)$ is computed using the estimated sparsity function. The likelihood ratio test is based on the difference between the in-sample check loss. Denote $L_1(\alpha) = \sum_{s=t-R+1}^{t} \rho_\alpha(y_s - V'_{s-1}\theta_1(\alpha))$ and $L_2(\alpha) = \sum_{s=t-R+1}^{t} \rho_\alpha(y_s - W'_{s-1}\theta_2(\alpha))$. The LR test statistic is calculated by

$$\text{LR}_t(\alpha) = 2[\alpha(1-\alpha)\hat{g}(\hat{G}^{-1}(\alpha))^{-1}]^{-1}(L_1(\alpha) - L_2(\alpha)), \tag{17}$$

where $\hat{g}(\hat{G}^{-1}(\alpha)))^{-1}$ is the estimated sparsity function. Koenker and Machado (1999) show that under some regularity conditions, $\text{Wald}_t(\alpha)$ and $\text{LR}_t(\alpha)$ are equivalent and follow a χ^2_k distribution asymptotically where k is the dimension of X.

The above-mentioned two test statistics are relatively easy to compute and have good asymptotic property, but they have some deficiencies. For instance, the estimation of the sparsity function requires the Gaussian distribution and the nice asymptotic property of those test statistics is not robust with non i.i.d. cases. Some alternative methods have been put forward to deal with such deficiencies, for instance, the bootstrap resampling proposed by Parzen, Wei and Ying (1994), or the Markov chain marginal bootstrap by He and Hu (2002), or the Huber sandwich local estimate of the sparsity function proposed by Koenker and d'Orey (1993). But no consensus has been reached on those issues. Moreover, it is merely an in-sample test and can not be used to compare the predictive power of different models, which, unfortunately, is exactly the essence of Granger-causality. Therefore, we propose to testing GCQ using a predictive ability test.

A test for Granger-causality is to compare the check-loss functions of forecasts conditional on two information sets, $\mathcal{G}_t$ and $\mathcal{F}_t$. We again use the conditional predictive ability test of GW (2006) using the check-loss function, i.e., $L_{t+1}(y_{t+1}, \hat{y}_{i,t+1}) = \rho_\alpha(\hat{e}_{i,t+1}(\hat{\theta}_{i,t}(\alpha)))$. The null hypothesis of NGCQ is therefore

$$H_0 : E[\rho_\alpha(\hat{e}_{1,t+1}(\hat{\theta}_{1,t}(\alpha))) - \rho_\alpha(\hat{e}_{2,t+1}(\hat{\theta}_{2,t}(\alpha)))|\mathcal{F}_t] = 0, \qquad t = R, \ldots, T-1. \tag{18}$$

Under the H_0 the loss differential $\Delta L_{t+1} \equiv \rho_\alpha(\hat{e}_{1,t+1}(\hat{\theta}_{1,t}(\alpha))) - \rho_\alpha(\hat{e}_{2,t+1}(\hat{\theta}_{2,t}(\alpha)))$ is an MDS, which implies $E(h_t \Delta L_{t+1}) = 0$ for any h_t that is $\mathcal{F}_t$-measurable. Denoting $Z_{t+1} = h_t \Delta L_{t+1}$, the GW (2006) statistic is of the same form as in (6) with $\bar{Z}'_{R,P} = \frac{1}{P}\sum_{t=R}^{T-1} h_t \Delta L_{t+1}$ and $\hat{\Omega}_P = \frac{1}{P}\sum_{t=R}^{T-1} Z_{t+1} Z'_{t+1}$. Under some regularity conditions, $GW_{R,P} \to^d \chi^2_q$ as $P \to \infty$ under H_0. We choose the same test function $h_t = X_t = (x_t, \ldots, x_{t+1-q})'$ as before with $q = 12$. When the null hypothesis of the equal conditional predictive ability is rejected, the forecast model selection rule is the same as in (7) in Section 2.1.

In Section 3 for the empirical analysis, we choose $X_t = (x_t, \ldots, x_{t+1-q})'$ and $Y_t = (y_t, \ldots, y_{t+1-q})'$ with $q = 12$, and let $\mathcal{G}_t = \sigma(V_t)$ be the σ-field generated by $V_t = (Y'_t, I_t, B_t)'$ where I_t denotes the 3 month T-bill interest rate and B_t denotes the business cycle coincident index. See Table 1A.

3 Empirical Analysis

In the literature, empirical studies of Granger-causality in mean commonly apply VAR models with exogenous variables. Different exogenous variables, such as treasury bill rates, federal funds rates, commercial paper rates and business cycle indicators are used. Real Personal Income or Industry Production is used as the proxy for income, and M2 is used as the proxy for money stock. We report the results only with M2 for space as the results with M1 are similar. We use monthly data of real personal income, industrial production index, M2 money stock, 3-month T-bill rate and the Stock and Watson experimental coincident index in the empirical study. The sample period is from 1959:04 to 2001:12 (513 observations). The source of the Stock and Watson experimental coincident index is the website of James Stock, while source for all other data is the Federal Reserve Economic Database (FRED) of Federal Reserve Bank of St. Luis.

We construct two data sets with two different income variables. Data Set 1 uses Real Personal Income for income and M2 for money. Data Set 2 uses Industrial Production for income and also uses M2 for money. The description of those data sets is listed in Table 1A. Noting that output, money and interest rate series are all non-stationary processes, we take the log-difference of output and money series and the first difference of interest rate series. Business cycle index (the Stock and Watson experimental coincident index) is a stationary process itself. Denote y_t as the output growth rate at time t, m_t as the money growth rate at time t, I_t as change of interest rate at time t and B_t as the business cycle indicator at time t.

As discussed earlier in Section 1, it is well documented in the literature that the results for GCM are generally weak and sensitive over different sample periods. While we find that the results for GCQ in tails are much stronger, we also find that the GCQ results are robust over different sample periods. For the robustness check, we consider 10 different subsample periods constructed as follows. In each of Subsample 1 to 6, we set $T = 360$ (30 years), with $R = 240$ (20 years) and $P = 120$ (10 years). Forecasting horizon is 1, and a recursive method is used in each subsample. We shift the subsample period by two years to get another subsample. We also construct Subsample 7 to 10 with whole sample ($T = 500$), but with different combinations of R and P. A description of those subsamples is listed in Table 1B. As a referee pointed out, we can consider recent work by Rossi and Inoue (2011) and Hansen and Timmermann (2012) for testing predictive ability over a collection of sample splits. We leave this for other paper as the GCQ results presented in Table 4 are very strong and stable over all of the 10 subsamples, indicating the results are not due to data snooping over different sample splits.

3.1 Money-Income Granger-causality in Mean

For the forecasting setting, as discussed in Section 2, an out-of-sample Granger-causality test is more appropriate. We estimate two nested models, one model without money-income Granger-causality in mean (Model 1), and the other with money-income Granger-causality in mean (Model 2):

$$\text{Model 1} \quad : \quad y_t = \beta_0 + \sum_{l=1}^{12} \beta_{y,l} y_{t-l} + \beta_I I_{t-1} + \beta_B B_{t-1} + \varepsilon_{1,t}, \tag{19}$$

$$\text{Model 2} \quad : \quad y_t = \beta_0 + \sum_{l=1}^{12} \beta_{y,l} y_{t-l} + \sum_{l=1}^{12} \beta_{m,l} m_{t-l} + \beta_I I_{t-1} + \beta_B B_{t-1} + \varepsilon_{2,t}. \tag{20}$$

The unconditional out-of-sample mean quadratic losses of these two models are reported in Table 2A. In Data Set 1, the unconditional mean squared forecast error (MSFE) of Model 2 is less than that of Model 1, while in Data Set 2 the MSFE of Model 1 are generally smaller.

The p-values of $GW_{R,P}$ and I_P statistics are listed in Table 2B. The p-values of $GW_{R,P}$ indicate that the null hypothesis of the equal conditional predictive ability can not be rejected for all subsamples and for both data sets at a reasonable significance level.

Comparing Data Set 1 to Data Set 2, GCM remains insignificant whether Real Personal Income or Industrial Production is used. (Similarly GCM is not significant with M1 or M2 although the results with M1 are not shown to save space.) The results of the different sample periods (Subsample 1 to 6) are very robust, showing that with the shift of the sample window, money-income causality in mean remains insignificant across all the data sets. With the increase of ratio of P/R from Subsamples 7 to 10, GCM still remains insignificant.

In a forecasting model, using so many lagged money variables in Model 2 may cause the "over-fit" of the model and damage the forecasting performance. Therefore, in order to reduce the number of parameters in the large model, we also check the robustness of our GCM results by using a weighed moving average of $(x_t, \ldots, x_{t+1-q})$ for estimation and forecasting, e.g., $\sum_{l=1}^{q} w_l x_{t+1-l}$ with weights w_l such that $\sum_{l=1}^{q} w_l = 1$. We use three such different weight functions, namely a linear declining weight, a equal weight ($w_l = q^{-1}$), and a beta polynomial function which creates flexible nonlinear declining weights as introduced in Ghysels, Sinko and Valkanov (2006). We use these weighted moving average (a scalar) in place of the q-vector X_t in estimation, forecasting, and testing. It is found that Model 2 (GCM) is still no better than Model 1 (NGCM) in terms of predictive ability. Hence, we find that out-of-sample GCM is not significant. Adding the information on lagged money growth rate is not very useful to improve the conditional mean forecasting of U.S. output growth over the various sample periods and different choices of the variables.

3.2 Money-Income Granger-Causality in Quantile

As discussed in Section 2, significant GCD does not imply GCQ in each conditional quantile. Therefore, in our empirical study, we choose 11 quantiles (α = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95). We check the GCQ in these different quantiles of the conditional distribution of output growth.

First, we use the check loss function to compare the *unconditional* predictive ability of the GCQ and the NGCQ models.[5] The unconditional check loss values of GCQ and NGCQ models are reported in Table 3. The ratios of the unconditional check loss values of GCQ to NGCQ can be easily obtained from Table 3 for each α. (The ratios are not presented for space.) The ratio less

[5] Besides the standard check loss, as a robust check, we also use the loss functions of the tick-exponential family introduced in Komunjer (2005). The results using these generalized check functions were essentially the same as those reported here with the standard check loss function and therefore not reported for space.

than 1 indicates the money-income Granger-causality in quantile. In terms of check losses and loss ratios, the GCQ model performs better than the NGCQ model in almost all subsamples of both data sets in the tails. In the central region, however, the GCQ model has lower check losses than the NGCQ model only in a few subsamples. The same pattern is observed in the rolling subsamples (Subsample 1 to 6). This implies that GCQ is stable across the sample period and data sets. In the whole data subsamples (Subsample 7 to 10), we find more significant GCQ with the increase of P. The loss ratios are much smaller than 1 in the tails than in the central regions.

Next, to compare the *conditional* check loss values, the p-values of $GW_{R,P}$ and I_P statistics are reported in Table 4. According to the p-values of $GW_{R,P}$ and I_P for the conditional predictive ability test, the GCQ model is significantly better to the NGCQ model in the tails. After accounting for money-income Granger-causality, quantile forecasting of output is improved at tails. The Granger-causality in quantile seems to be more significant between money and Industrial Production than that between money and Personal Income. Money does significantly improve the forecasting of output/income tail quantiles. However, money does not improve forecasting of the output growth in conditional mean and the conditional quantiles close to median.

4 Conclusions

The relationship between money and income is a much-studied but controversial topic in the literature. This paper follows a VAR framework and applies an out-of-sample test for money-income Granger-causality. We find that money-income Granger-causality in mean is not significant for all data sets and all subsample periods that we considered.

We define Granger-causality in quantile and compare two quantile forecasts with or without money-income Granger-causality in quantile. Empirical results show the potential of improving quantile forecasting of output growth rate by incorporating information on money-income causality in quantile, especially in the tails. Causality between money and Industrial Production seems to be more significant than that between money and Personal Income (while M2 has stronger causality in quantiles to Personal Income than M1 does). However, money is not very useful for forecasting near the center quantiles of the conditional distribution of output growth.

The empirical findings of this paper on the money-income Granger-causality in quantiles is entirely new and have never been documented in the money-income literature. The new results on GCQ have an important implication on monetary policy, showing that the effectiveness of monetary policy has been under-estimated by merely testing Granger-causality in mean. Money does Granger-cause income more strongly than it has been known and therefore the information on money growth can (and should) be more utilized in implementing monetary policy.

What does the lack of GC in mean tell us that is different from the presence of GC in the extreme quantiles? As a referee points out, the answer may be "risk management" in the sense of Kilian and Manganelli (2008), who derive a generalization of the Taylor rule (that links changes in the interest rate to the balance of the risks implied by the dual objective of sustainable economic growth and price stability) which reconciles economic models of expected utility maximization with the risk management approach to central banking. The results of Kilian and Manganelli (2008) suggest that Fed policy decisions under Greenspan were better described in terms of the Fed weighing upside and downside risks to their objectives rather than simply responding to the conditional mean of output growth (or output gap) and inflation.

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Table 1. Description of Data Sets and Samples

Panel A. Description of Data Sets

	Income *y*	Money *m*	Interest Rate *I*	Business Cycle Index *B*
Data Set 1	Real Personal Income	M2	3-Month T-bill Rate	Coincident Index
Data Set 2	Industrial Production Index	M2	3-Month T-bill Rate	Coincident Index

Notes:

(1) To make these series stationary, we take log-difference of income and money variables, and take the first difference of interest rate.

(2) The business cycle index series are taken from James Stock's web page, http://www.economics.harvard.edu/faculty/stock/links.htm, while the other data are obtained from the Federal Reserve Economic Database (FRED) of Federal Reserve Bank of St. Luis.

(3) All data are monthly data, with sample period of 1959:04 to 2001:12, with 513 observations.

Panel B. Description of Subsamples in Out-of-sample Tests

	Starting Month	Ending Month	*T*	*R*	*P*
Subsample 1	1960:05	1991:04	360	240	120
Subsample 2	1962:05	1993:04	360	240	120
Subsample 3	1964:05	1995:04	360	240	120
Subsample 4	1966:05	1997:04	360	240	120
Subsample 5	1968:05	1999:04	360	240	120
Subsample 6	1970:05	2001:04	360	240	120
Subsample 7	1960:05	2001:12	500	380	120
Subsample 8	1960:05	2001:12	500	320	180
Subsample 9	1960:05	2001:12	500	260	240
Subsample 10	1960:05	2001:12	500	200	300

Notes:

(1) Subsample 1 to 6 have a fixed window of 30 years, with 20 years as in-sample period and 10 years as out-of-sample period. Subsamples are moving forward by two years each time.

(2) Subsample 7 to 10 are the samples that contain all observations but with different combination of *R* and *P*. Due to the 12 lags used in the model and log-difference of money and income, there are 500 observations.

Table 2. Out-of-Sample Test for Granger-causality in Mean

Panel A. Comparing Unconditional Predictive Ability (Squared error loss)

	Data Set 1		Data Set 2	
Loss	Model 1	Model 2	Model 1	Model 2
Subsample 1	0.0778	0.0744	0.0415	0.0419
Subsample 2	0.0927	0.0884	0.0378	0.0377
Subsample 3	0.0823	0.0780	0.0460	0.0468
Subsample 4	0.0820	0.0776	0.0435	0.0436
Subsample 5	0.0818	0.0768	0.0509	0.0516
Subsample 6	0.1124	0.1036	0.0452	0.0455
Subsample 7	0.1252	0.1162	0.0406	0.0405
Subsample 8	0.0926	0.0881	0.0418	0.0419
Subsample 9	0.1014	0.0956	0.0414	0.0420
Subsample 10	0.0993	0.0937	0.0416	0.0420

Notes: Quadratic loss values for two models are reported. "Model 1" refers to the model without Granger-causality in mean, while "Model 2" refers to the model with Granger-causality in mean. The loss value of Model 2 is shaded when it is smaller than that of Model 1.

Panel B. Test for Conditional Predictive Ability

	Data Set 1		Data Set 2	
	P_{GW}	I_{GW}	P_{GW}	I_{GW}
Subsample 1	0.5427	0.5583	0.4873	0.5417
Subsample 2	0.5396	0.5250	0.5154	0.5000
Subsample 3	0.5588	0.4750	0.6943	0.5417
Subsample 4	0.6817	0.4583	0.8779	0.5583
Subsample 5	0.5015	0.4083	0.7118	0.6083
Subsample 6	0.4113	0.3250	0.6066	0.5500
Subsample 7	0.6436	0.4167	0.8561	0.4833
Subsample 8	0.5155	0.4500	0.8597	0.5778
Subsample 9	0.3409	0.4583	0.7694	0.5833
Subsample 10	0.2876	0.4233	0.8342	0.5900

Notes: P_{GW} refers to the asymptotic p-value of the nR^2 version of the Wald statistics of Giacomini and White (2005). We choose a linear test function which contains 12 lags of money growth rate. The asymptotic p-values of the Giacomini and White statistics are obtained from a chi-square distribution with 12 degrees of freedom. I_{GW} refers to the I_P statistic in Giacomini and White (2005). See Section 2.1. At 10% level, if $P_{GW} < 0.10$ and $I_{GW} < 0.5$, we may prefer Model 2 (GCM) over the Model 1 (NGCM); if $P_{GW} < 0.10$ and $I_{GW} > 0.5$, we may prefer Model 1 to Model 2. None of the cases satisfies ($P_{GW} < 0.10$ and $I_{GW} < 0.5$) or ($P_{GW} < 0.10$ and $I_{GW} > 0.5$). In fact all p-values are very large.

Table 3. Out-of-Sample Test for Granger-causality in Quantiles
Comparing Unconditional Predictive Ability (Check loss)

Panel A. Data Set 1

Sub-sample	α=0.05		α=0.10		α=0.20		α=0.30		α=0.40		α=0.50		α=0.60		α=0.70		α=0.80		α=0.90		α=0.95	
	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ
1	0.0399	0.0277	0.0538	0.0535	0.0613	0.0641	0.0721	0.0715	0.0766	0.0743	0.0789	0.0745	0.0756	0.0738	0.0734	0.0733	0.0651	0.0690	0.0483	0.0491	0.0345	0.0301
2	0.0361	0.0259	0.0586	0.0573	0.0719	0.0720	0.0828	0.0851	0.0932	0.0930	0.0952	0.0920	0.0928	0.0899	0.0868	0.0832	0.0731	0.0713	0.0498	0.0463	0.0311	0.0296
3	0.0327	0.0300	0.0418	0.0415	0.0594	0.0566	0.0716	0.0738	0.0826	0.0793	0.0829	0.0833	0.0829	0.0782	0.0790	0.0710	0.0650	0.0615	0.0463	0.0390	0.0285	0.0223
4	0.0305	0.0277	0.0406	0.0394	0.0580	0.0573	0.0701	0.0733	0.0782	0.0829	0.0804	0.0821	0.0790	0.0800	0.0784	0.0752	0.0665	0.0655	0.0466	0.0420	0.0277	0.0218
5	0.0280	0.0274	0.0425	0.0379	0.0602	0.0572	0.0730	0.0764	0.0802	0.0838	0.0814	0.0846	0.0814	0.0826	0.0797	0.0792	0.0664	0.0653	0.0415	0.0392	0.0260	0.0173
6	0.0419	0.0339	0.0602	0.0537	0.0772	0.0701	0.0868	0.0891	0.0937	0.0986	0.0988	0.1006	0.0999	0.1010	0.0936	0.0950	0.0792	0.0762	0.0521	0.0478	0.0336	0.0259
7	0.0409	0.0346	0.0664	0.0639	0.0865	0.0869	0.0987	0.1010	0.1053	0.1093	0.1093	0.1099	0.1051	0.1077	0.1027	0.0993	0.0840	0.0838	0.0588	0.0483	0.0335	0.0256
8	0.0346	0.0324	0.0525	0.0503	0.0692	0.0705	0.0800	0.0821	0.0858	0.0896	0.0894	0.0898	0.0860	0.0879	0.0834	0.0824	0.0699	0.0707	0.0490	0.0450	0.0306	0.0241
9	0.0379	0.0320	0.0604	0.0593	0.0743	0.0748	0.0855	0.0865	0.0912	0.0922	0.0943	0.0922	0.0909	0.0908	0.0889	0.0859	0.0749	0.0757	0.0535	0.0503	0.0346	0.0280
10	0.0378	0.0326	0.0589	0.0572	0.0720	0.0728	0.0829	0.0845	0.0879	0.0904	0.0906	0.0895	0.0874	0.0880	0.0856	0.0836	0.0715	0.0728	0.0513	0.0470	0.0313	0.0255

Panel B. Data Set 2

Sub-sample	α=0.05		α=0.10		α=0.20		α=0.30		α=0.40		α=0.50		α=0.60		α=0.70		α=0.80		α=0.90		α=0.95	
	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ	NGCQ	GCQ
1	0.0197	0.0155	0.0338	0.0306	0.0569	0.0550	0.0693	0.0718	0.0778	0.0779	0.0807	0.0811	0.0780	0.0764	0.0677	0.0664	0.0535	0.0543	0.0332	0.0342	0.0196	0.0174
2	0.0181	0.0149	0.0316	0.0278	0.0515	0.0506	0.0637	0.0607	0.0751	0.0708	0.0747	0.0731	0.0726	0.0722	0.0656	0.0625	0.0505	0.0500	0.0335	0.0356	0.0212	0.0191
3	0.0223	0.0157	0.0379	0.0313	0.0580	0.0607	0.0737	0.0744	0.0828	0.0825	0.0837	0.0859	0.0804	0.0822	0.0707	0.0713	0.0553	0.0541	0.0356	0.0346	0.0214	0.0192
4	0.0206	0.0165	0.0398	0.0310	0.0584	0.0564	0.0734	0.0728	0.0784	0.0784	0.0785	0.0801	0.0758	0.0786	0.0695	0.0687	0.0561	0.0536	0.0344	0.0331	0.0221	0.0201
5	0.0207	0.0168	0.0417	0.0337	0.0659	0.0619	0.0776	0.0774	0.0859	0.0841	0.0855	0.0870	0.0824	0.0833	0.0751	0.0743	0.0623	0.0601	0.0406	0.0373	0.0231	0.0231
6	0.0195	0.0167	0.0421	0.0323	0.0618	0.0578	0.0769	0.0731	0.0821	0.0855	0.0835	0.0827	0.0799	0.0792	0.0724	0.0702	0.0578	0.0556	0.0399	0.0324	0.0197	0.0187
7	0.0206	0.0172	0.0414	0.0331	0.0607	0.0553	0.0722	0.0708	0.0770	0.0789	0.0813	0.0792	0.0745	0.0723	0.0663	0.0645	0.0552	0.0535	0.0350	0.0297	0.0188	0.0163
8	0.0195	0.0164	0.0382	0.0318	0.0592	0.0553	0.0719	0.0703	0.0782	0.0799	0.0815	0.0810	0.0757	0.0761	0.0671	0.0663	0.0538	0.0531	0.0339	0.0312	0.0197	0.0176
9	0.0200	0.0162	0.0367	0.0312	0.0568	0.0543	0.0705	0.0701	0.0776	0.0796	0.0827	0.0813	0.0773	0.0758	0.0684	0.0667	0.0556	0.0560	0.0346	0.0325	0.0193	0.0175
10	0.0195	0.0161	0.0377	0.0321	0.0582	0.0550	0.0714	0.0702	0.0774	0.0793	0.0820	0.0810	0.0771	0.0759	0.0683	0.0676	0.0557	0.0557	0.0347	0.0317	0.0192	0.0173

Notes: (1) The numbers in the first column is referring to the 10 subsamples. See Table 1, Panel B.

(2) "NGCQ" refers to Model 1, the quantile forecasting model without money-income Granger-causality in quantile, i.e, not including the lagged money growth rate as independent variables.

(3) "GCQ" refers to Model 2, the quantile forecasting model with money-income Granger-causality in quantile, i.e, including the lagged money growth rate as independent variables.

(4) A check loss function proposed by Koenker and Bassett (1978) is used to evaluate the out-of-sample performance of the two quantile forecasting models. The out-of-sample average of the loss values are reported in this table. The loss value of Model 2 is shaded when it is smaller than that of Model 1.

Table 4. Out-of-Sample Test for Granger-causality in Quantiles
Test for Conditional Predictive Ability

Panel A. Data Set 1

Sub-sample	α=0.05		α=0.10		α=0.20		α=0.30		α=0.40		α=0.50		α=0.60		α=0.70		α=0.80		α=0.90		α=0.95	
	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}
1	**0.0117**	**0.4500**	0.7422	0.4667	0.0471	0.6083	0.3107	0.5167	0.8232	0.6167	0.9025	0.4833	0.8333	0.5417	0.4207	0.6500	0.0002	0.6750	0.0085	0.6417	**0.0015**	**0.4000**
2	**0.0030**	**0.4667**	0.9390	0.4667	0.7655	0.5583	0.7137	0.6250	0.8008	0.7417	0.5908	0.6417	0.4173	0.5000	0.2084	0.5083	**0.0739**	**0.4667**	0.0192	0.5000	**0.0691**	**0.4583**
3	0.1779	0.3500	0.4385	0.4917	0.5595	0.4083	0.5189	0.5333	0.7671	0.5250	0.7387	0.6333	0.4010	0.4333	0.2000	0.4083	0.6170	0.3750	**0.0859**	**0.3917**	**0.0496**	**0.4083**
4	**0.0090**	**0.2833**	**0.0540**	**0.4583**	0.4654	0.4750	0.3460	0.5917	0.3853	0.6250	0.7581	0.5750	0.6841	0.4750	0.3531	0.4500	0.6698	0.5167	0.4106	0.3750	0.3230	0.3333
5	**0.0308**	**0.4333**	**0.0463**	**0.1583**	0.4546	0.4000	0.2683	0.5583	0.0689	0.5583	0.5733	0.6083	0.8052	0.5750	0.5605	0.4833	0.5908	0.4250	0.1066	0.3833	**0.0577**	**0.2917**
6	**0.0276**	**0.4167**	0.1218	0.1917	0.5791	0.3000	0.2310	0.5083	0.1766	0.6667	0.5141	0.4917	0.7996	0.4917	0.4245	0.4833	0.2632	0.3000	**0.0051**	**0.2667**	**0.0140**	**0.2417**
7	0.1685	0.4583	**0.0846**	**0.4167**	0.2196	0.3833	0.1963	0.5500	0.1008	0.6000	0.7883	0.6750	0.4584	0.6167	0.2115	0.4417	0.2124	0.4250	0.2393	0.2833	**0.0408**	**0.2500**
8	0.1136	0.5056	0.2827	0.4056	0.3981	0.5000	0.2287	0.6111	0.1810	0.6778	0.7713	0.6944	0.5418	0.6000	0.3520	0.4944	0.2462	0.4944	0.2197	0.3889	**0.0011**	**0.1889**
9	**0.0852**	**0.4958**	0.3348	0.4333	0.5132	0.4958	0.3477	0.5917	0.3066	0.6375	0.6190	0.6333	0.4214	0.5833	0.2017	0.4833	0.0854	0.5458	**0.0676**	**0.3917**	**0.0000**	**0.2375**
10	**0.0105**	**0.4567**	**0.0902**	**0.3967**	0.1426	0.5100	0.1308	0.6100	0.0696	0.6600	0.3022	0.5867	0.2714	0.6133	0.1773	0.4867	0.0378	0.5267	**0.0237**	**0.3300**	**0.0000**	**0.2433**

Panel B. Data Set 2

Sub-sample	α=0.05		α=0.10		α=0.20		α=0.30		α=0.40		α=0.50		α=0.60		α=0.70		α=0.80		α=0.90		α=0.95	
	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}	P_{GW}	I_{GW}
1	**0.0002**	**0.3417**	0.1628	0.3417	0.2214	0.4833	0.5189	0.6500	0.6532	0.4917	0.5541	0.6083	0.6286	0.5250	0.7982	0.4167	0.4725	0.5333	0.0689	0.5583	**0.0020**	**0.4333**
2	**0.0335**	**0.4083**	**0.1007**	**0.4083**	0.1330	0.4583	0.6568	0.3333	0.3921	0.3917	0.3471	0.5083	0.3360	0.5250	0.6631	0.2583	**0.0740**	**0.3750**	0.0083	0.5333	**0.0360**	**0.4000**
3	0.1249	0.2750	**0.1017**	**0.2417**	0.7646	0.6750	0.8068	0.4750	0.5166	0.5083	0.2172	0.6000	0.0282	0.5667	0.3202	0.5000	**0.0340**	**0.3667**	**0.0020**	**0.4250**	**0.0085**	**0.4167**
4	**0.0539**	**0.2083**	**0.1045**	**0.1417**	0.6332	0.4167	0.5413	0.4167	0.5070	0.4583	0.4126	0.6083	0.3790	0.5750	0.3531	0.4917	0.4377	0.3917	**0.0129**	**0.4417**	**0.0031**	**0.3333**
5	**0.0049**	**0.2333**	**0.0296**	**0.1583**	0.6305	0.3417	0.7290	0.5250	0.5032	0.4333	0.4176	0.5833	0.6161	0.4833	0.6840	0.4583	0.1679	0.4083	**0.0216**	**0.3833**	**0.0014**	**0.4500**
6	**0.0019**	**0.3000**	**0.0164**	**0.1333**	0.3927	0.3417	0.1993	0.4167	0.5847	0.6833	0.1255	0.4750	0.4161	0.4500	0.7979	0.4417	0.4001	0.3333	**0.0037**	**0.2417**	**0.1045**	**0.3833**
7	**0.0048**	**0.3167**	**0.0881**	**0.2167**	0.1320	0.3250	0.7178	0.4833	0.2517	0.5417	0.4681	0.4500	0.2813	0.4000	0.6677	0.4333	0.1654	0.4333	**0.0580**	**0.1833**	**0.0075**	**0.4333**
8	**0.0019**	**0.2556**	0.1205	0.2056	0.2170	0.3333	0.9851	0.4500	0.7164	0.6556	0.4882	0.5222	0.3393	0.5722	0.6736	0.4778	0.3259	0.4889	**0.0875**	**0.2722**	**0.0001**	**0.4111**
9	**0.0014**	**0.2042**	0.1190	0.1917	**0.0914**	**0.3917**	0.8946	0.5208	0.7946	0.6583	0.3457	0.4833	0.2288	0.4833	0.8215	0.4292	0.2571	0.5125	**0.0545**	**0.3167**	**0.0000**	**0.4375**
10	**0.0002**	**0.2167**	**0.0226**	**0.1933**	0.1229	0.3200	0.8991	0.4567	0.4114	0.6500	0.4170	0.5067	0.1150	0.5400	0.5778	0.5200	**0.0722**	**0.4800**	**0.0036**	**0.2533**	**0.0000**	**0.4333**

Notes: (1) The numbers in the first column is referring to the 16 subsamples. See Table 1, Panel B.

(2) P_{GW} refers to the asymptotic p-value of the nR^2 version of the Wald statistics of Giacomini and White (2005). We choose a linear test function which contains 12 lags of money growth rate. The asymptotic p-values of the Giacomini and White statistics are obtained from a chi-square distribution with 12 degrees of freedom.

(3) I_{GW} refers to the I_P statistic in Giacomini and White (2005). See Section 2.4.

(4) At 10% level, if $P_{GW} < 0.10$ and $I_{GW} < 0.5$, we prefer Model 2 (GCQ) over Model 1 (NGCQ). These cases are reported in bold font. If $P_{GW} < 0.10$ and $I_{GW} > 0.5$, we prefer Model 1 to Model 2.