

Supplemental Appendix

A Combined Estimator of Regression Models with Measurement Errors

Bai Huang*, Tae-Hwy Lee†, Aman Ullah‡

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There are 10 Figures. Only the first four figures are included in the paper for the space consideration.

1. Figure 1: $n = 100$, $W = I$ and $\sigma_1 = \sigma_2 = \cdots = \sigma_q = \sigma$.
2. Figure 2: $n = 100$, $W = \left(\hat{V}_1 - \hat{V}_0\right)^{-1}$ and $\sigma_1 = \sigma_2 = \cdots = \sigma_q = \sigma$.
3. Figure 3: $n = 200$, $W = I$ and $\sigma_1 = \sigma_2 = \cdots = \sigma_q = \sigma$.
4. Figure 4: $n = 200$, $W = \left(\hat{V}_1 - \hat{V}_0\right)^{-1}$ and $\sigma_1 = \sigma_2 = \cdots = \sigma_q = \sigma$.
5. Figure 5: $n = 100$, $W = I$ and $\sigma_1 = \sigma$, $\sigma_2 = \cdots = \sigma_q = 0$.
6. Figure 6: $n = 100$, $W = \left(\hat{V}_1 - \hat{V}_0\right)^{-1}$ and $\sigma_1 = \sigma$, $\sigma_2 = \cdots = \sigma_q = 0$.
7. Figure 7: $n = 200$, $W = I$ and $\sigma_1 = \sigma$, $\sigma_2 = \cdots = \sigma_q = 0$.
8. Figure 8: $n = 200$, $W = \left(\hat{V}_1 - \hat{V}_0\right)^{-1}$ and $\sigma_1 = \sigma$, $\sigma_2 = \cdots = \sigma_q = 0$.

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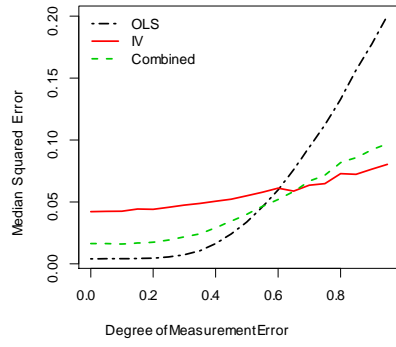
‡Department of Economics, University of California, Riverside, CA 92521. aman.ullah@ucr.edu

There are two additional figures. Figure 9 and Figure 10 replicate Hansen (2016) monte carlo results with some larger values of q .

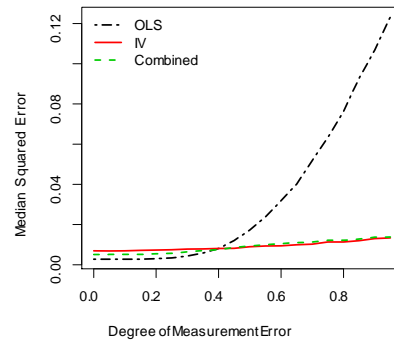
9. Figure 9: $n = 100$, $W = I$ and $\sigma_1 = \sigma_2 = \dots = \sigma_q = \sigma$.

10. Figure 10: $n = 100$, $W = \left(\hat{V}_1 - \hat{V}_0\right)^{-1}$ and $\sigma_1 = \sigma_2 = \dots = \sigma_q = \sigma$.

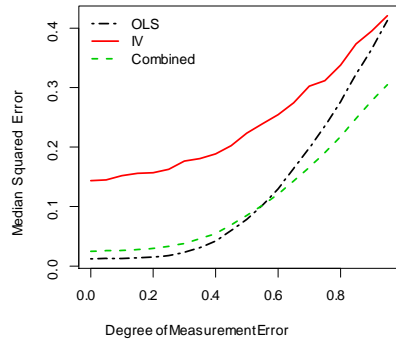
In each figure, there are 12 subfigures. Subfigures 1 and 2 are the case $q = 1$. Subfigures 3 and 4 are the case $q = 2$. Subfigures 5 and 6 are the case $q = 3$. Subfigures 7 and 8 are the case $q = 5$. Subfigures 9 and 10 are the case $q = 10$. Subfigures 11 and 12 are the case $q = 20$. The subfigures with odd numbers are the case $R^2 = 0.1$. The subfigures with even numbers are the case $R^2 = 0.4$.



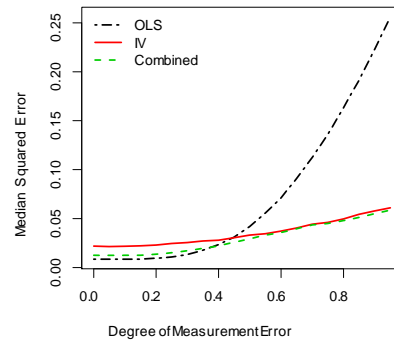
1.1 : $q = 1, R^2 = 0.1$



1.2 : $q = 1, R^2 = 0.4$

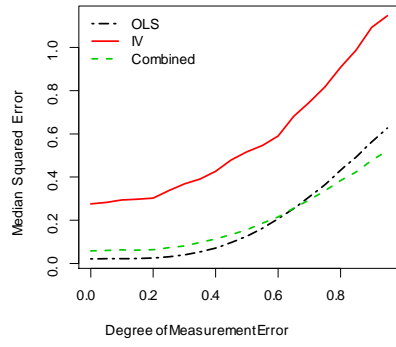


1.3 : $q = 2, R^2 = 0.1$

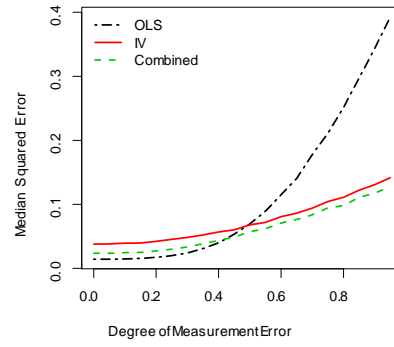


1.4 : $q = 2, R^2 = 0.4$

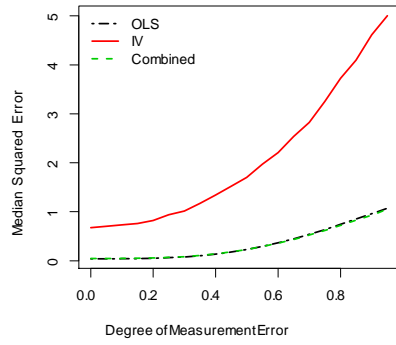
Figure 1: $n = 100, W = I$



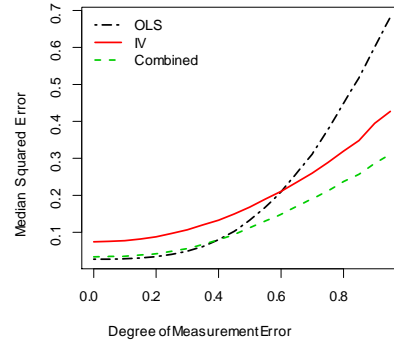
1.5 : $q = 3, R^2 = 0.1$



1.6 : $q = 3, R^2 = 0.4$

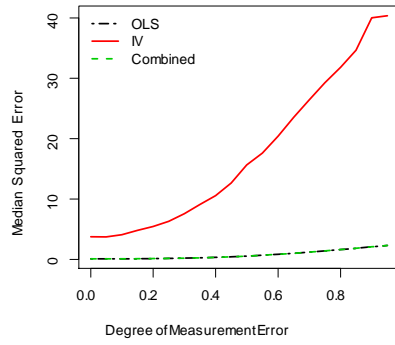


1.7 : $q = 5, R^2 = 0.1$

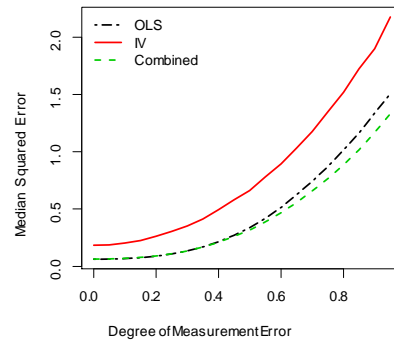


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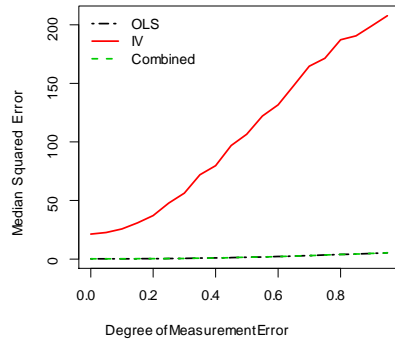
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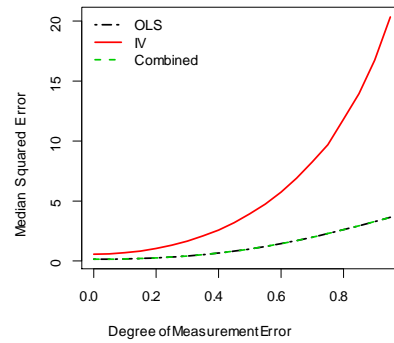
1.9 : $q = 10, R^2 = 0.1$



1.10 : $q = 10, R^2 = 0.4$

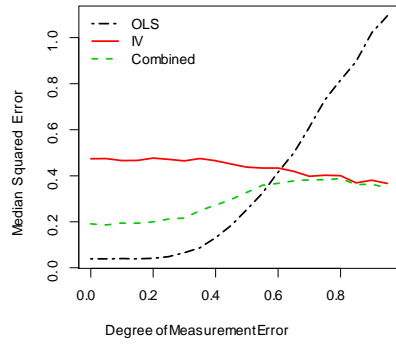


1.11 : $q = 20, R^2 = 0.1$

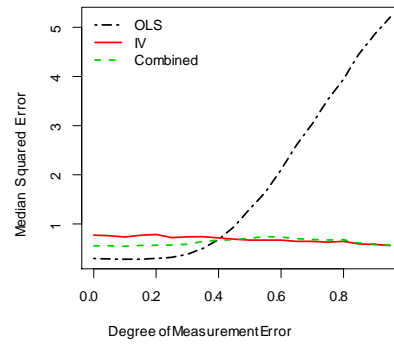


1.12 : $q = 20, R^2 = 0.4$

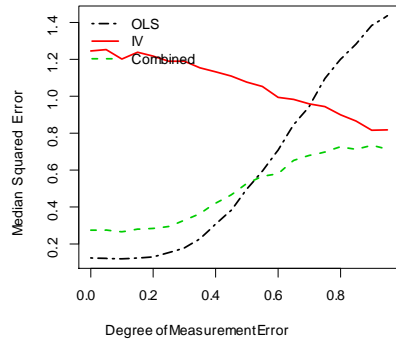
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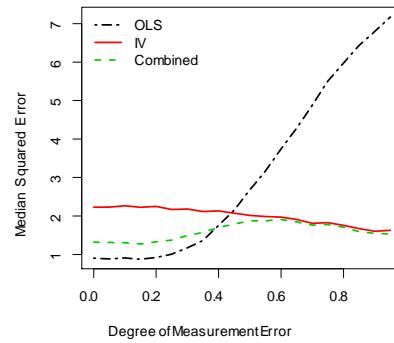
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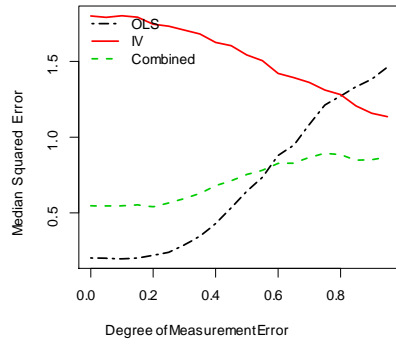


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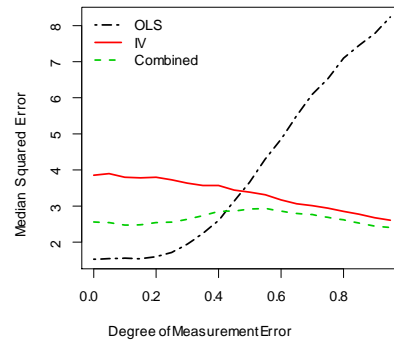


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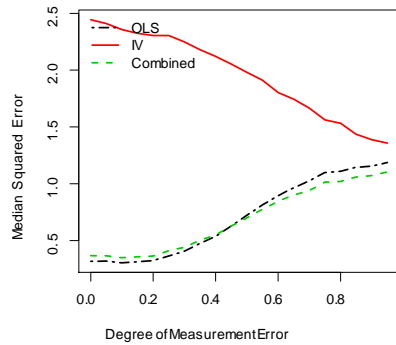
Figure 2: $n = 100, W = (\hat{V}_1 - \hat{V}_0)^{-1}$



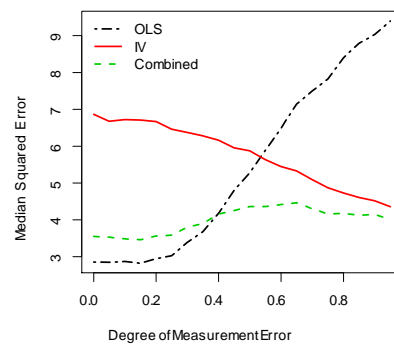
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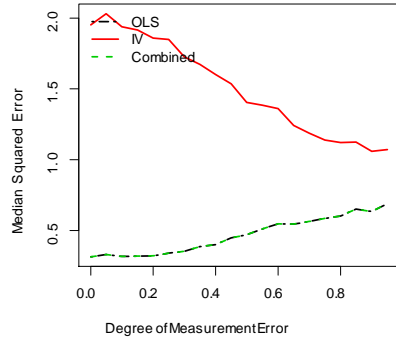


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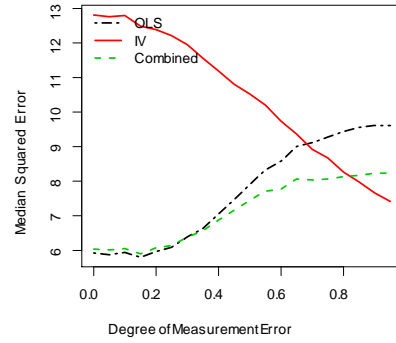


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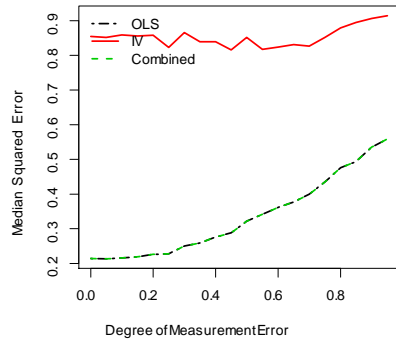
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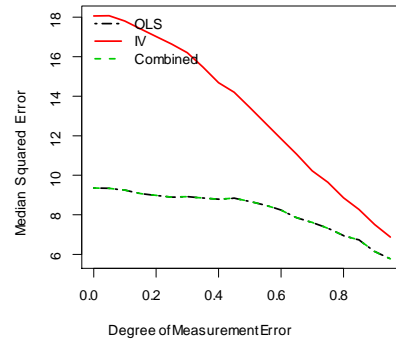
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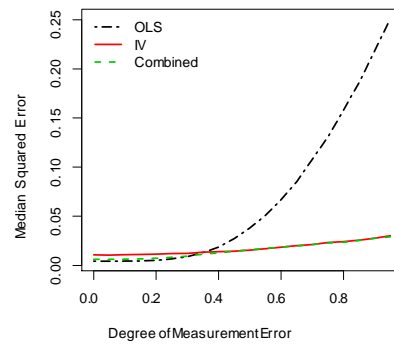
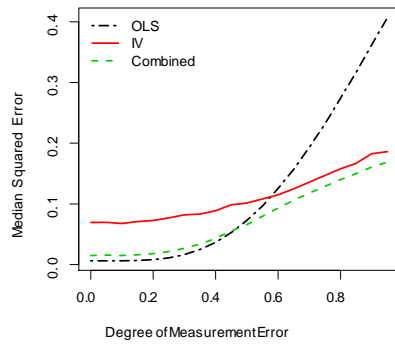
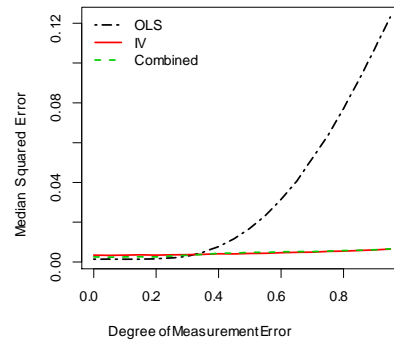
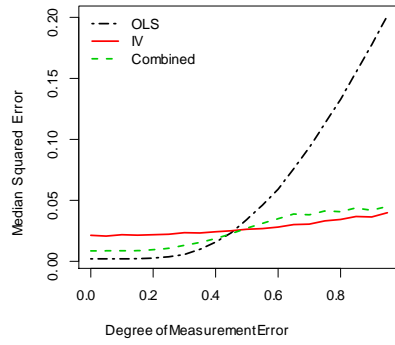


Figure 3: $n = 200, W = I$

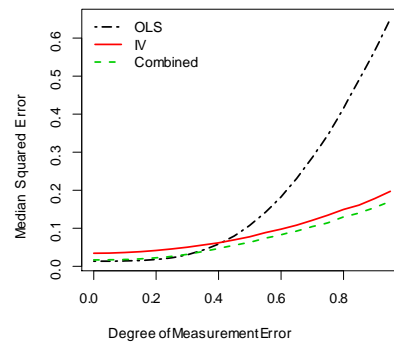
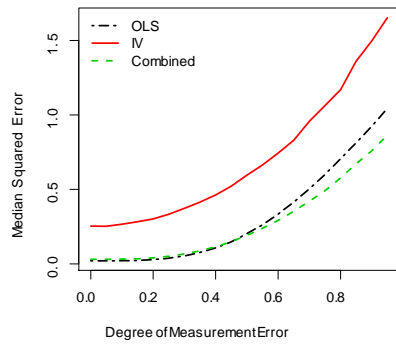
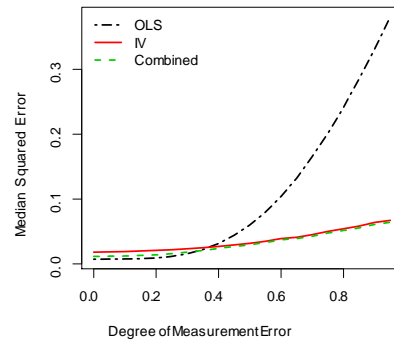
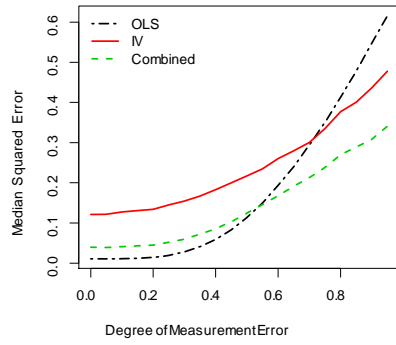
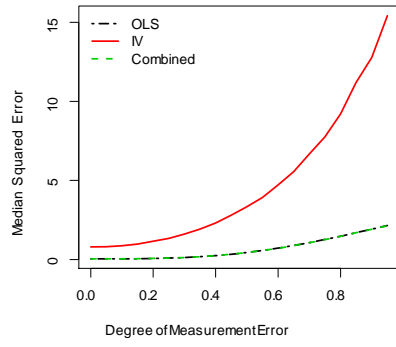
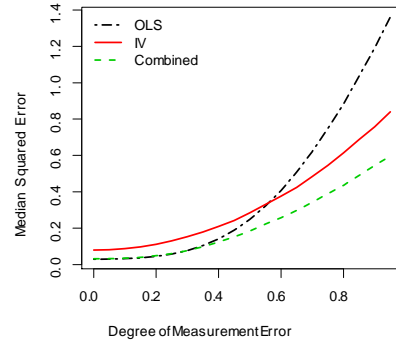


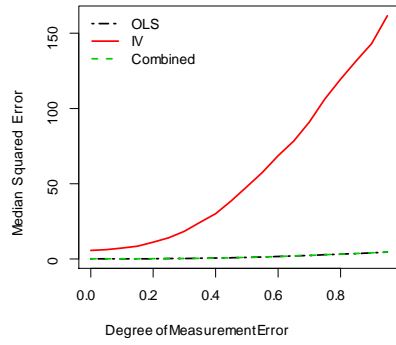
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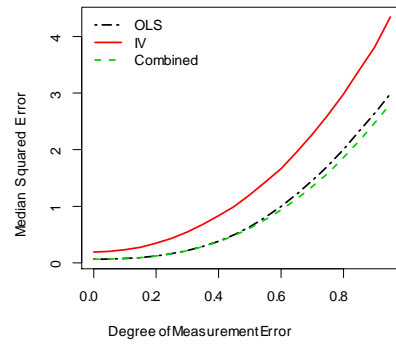
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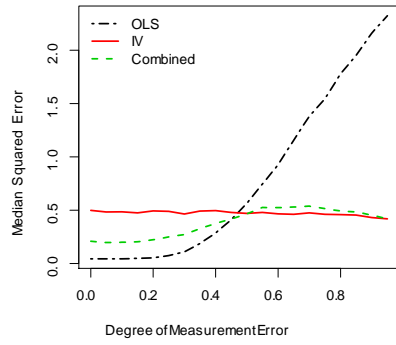


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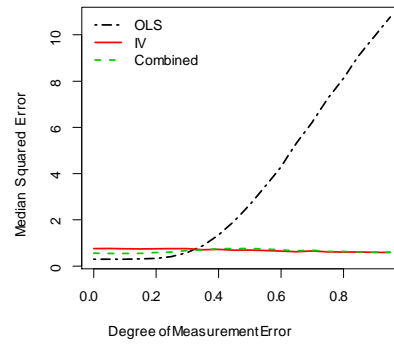


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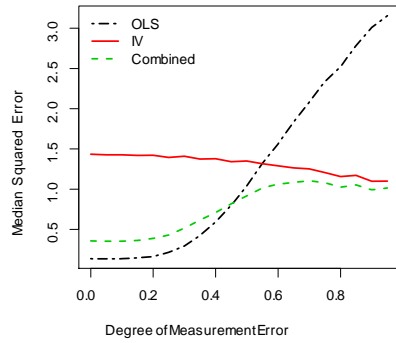
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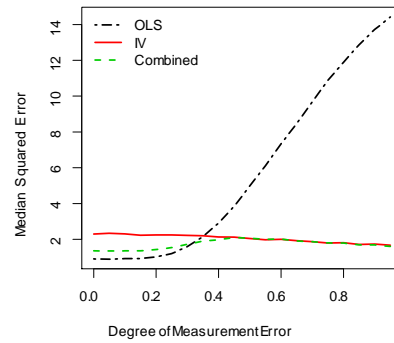
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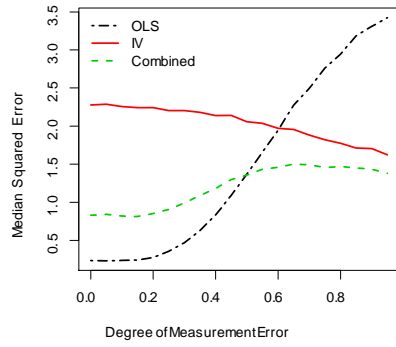


4.3 : $q = 2, R^2 = 0.1$

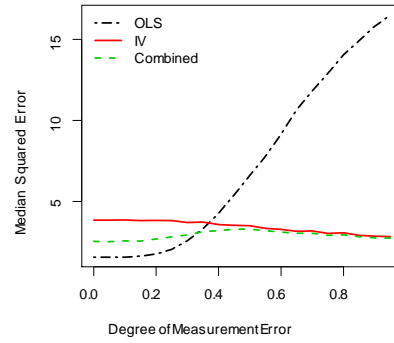


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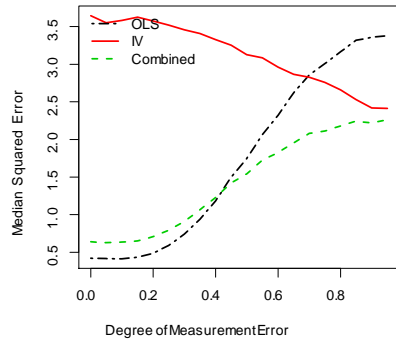
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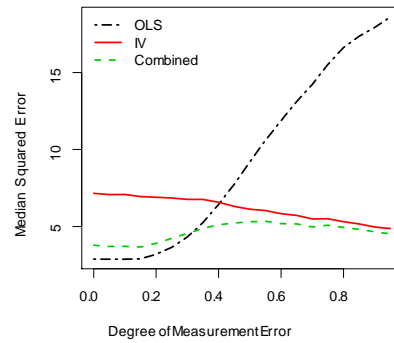
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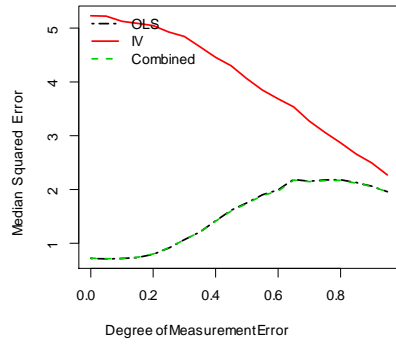


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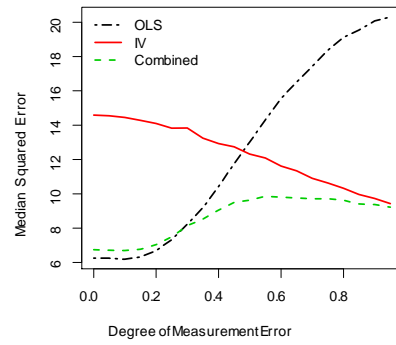


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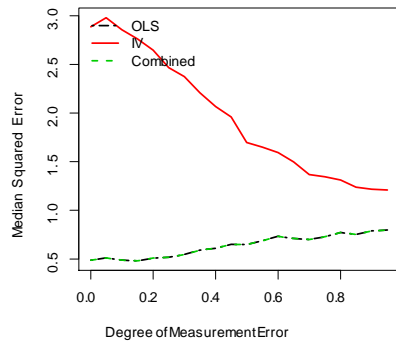
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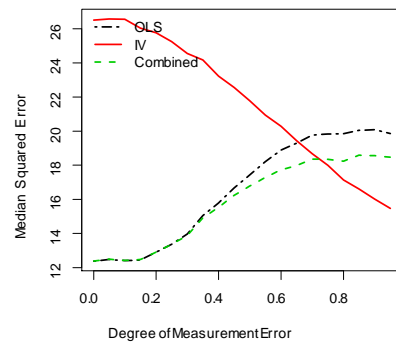
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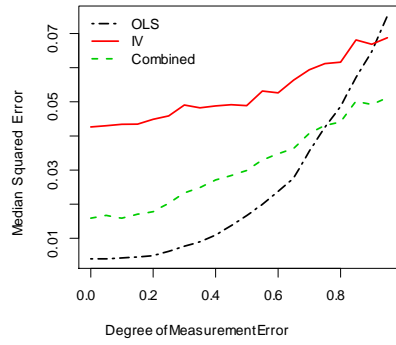


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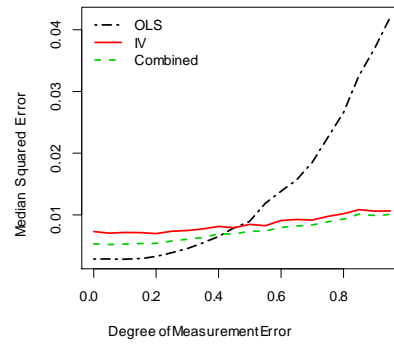


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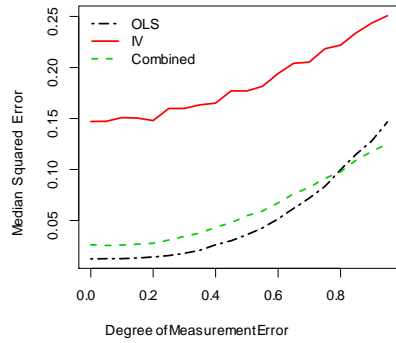
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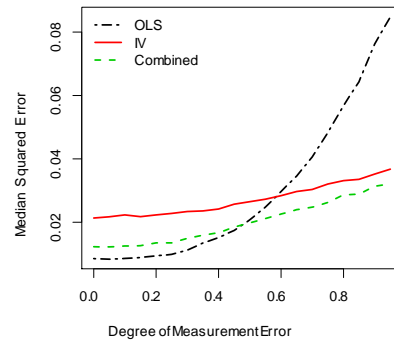
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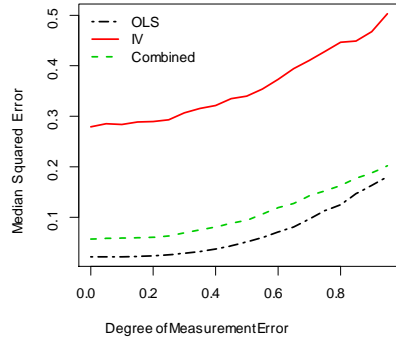


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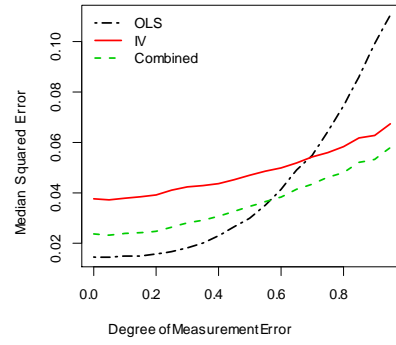


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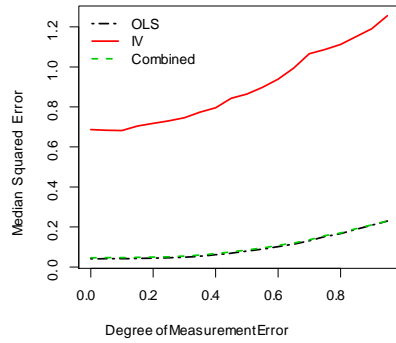
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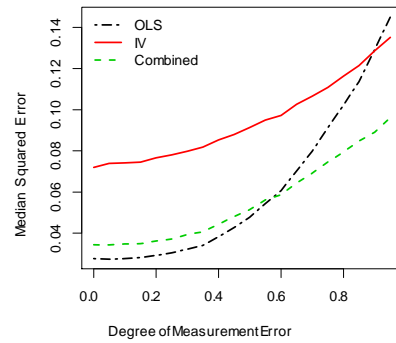
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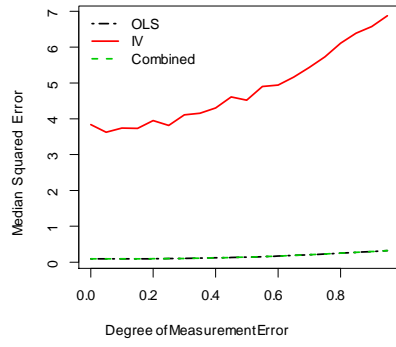


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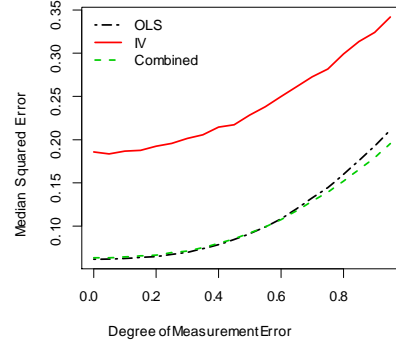


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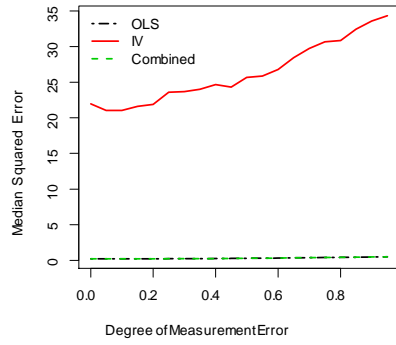
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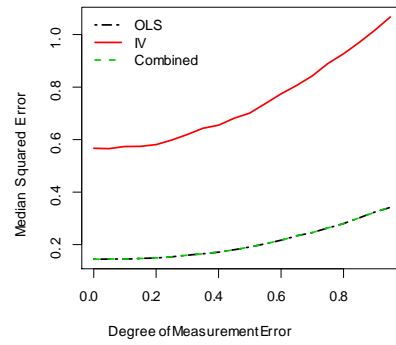
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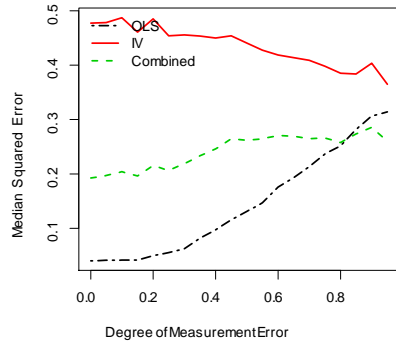


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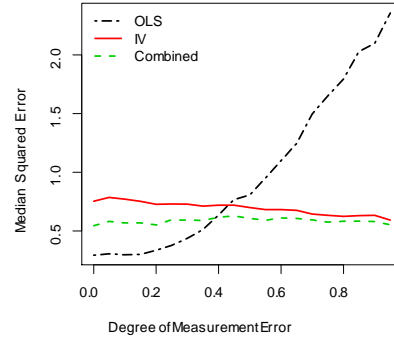


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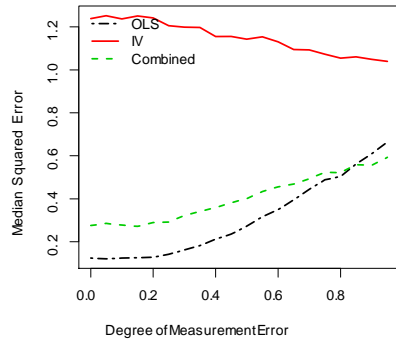
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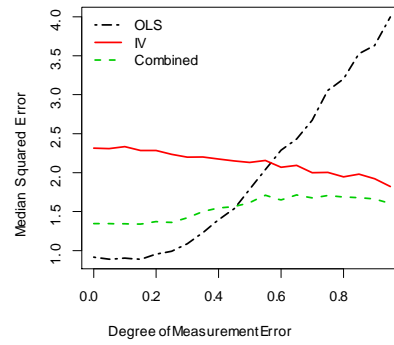
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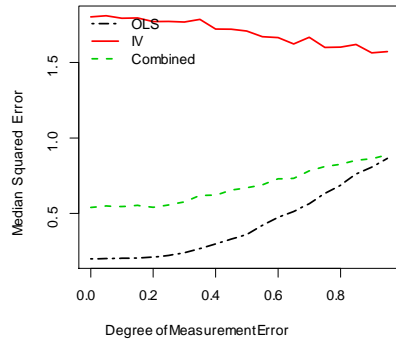


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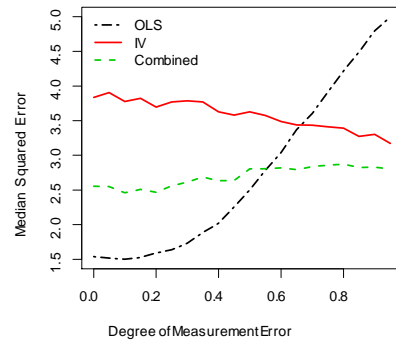


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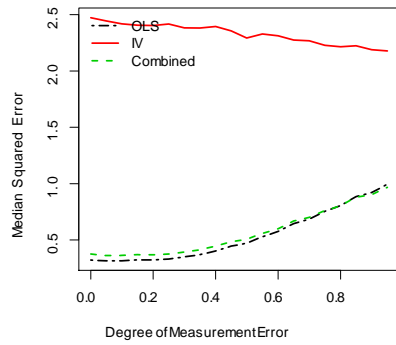
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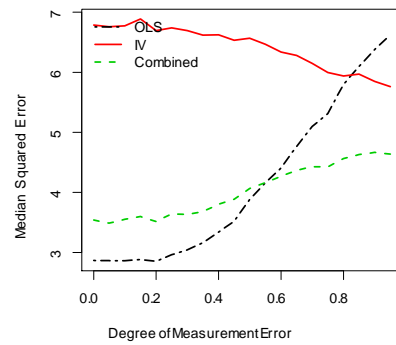
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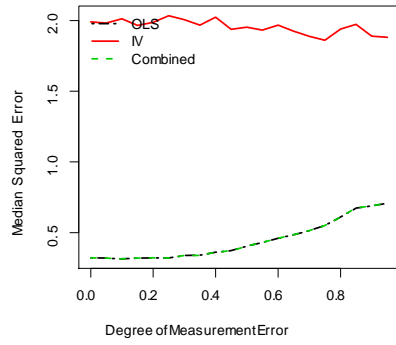


6.7 : $q = 5, R^2 = 0.1$

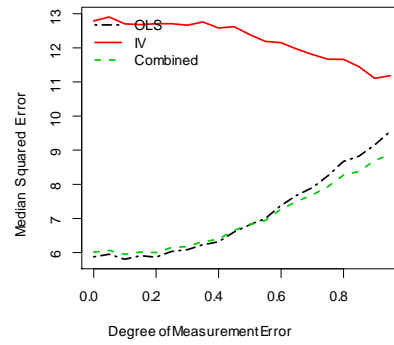


6.8 : $q = 5, R^2 = 0.4$

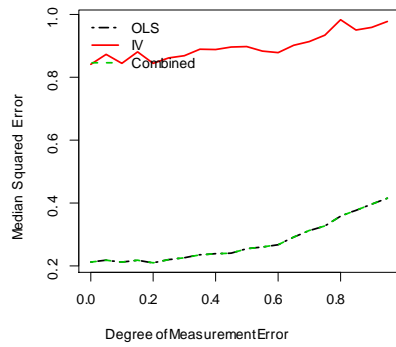
Figure 6: $n = 100, W = (\hat{V}_1 - \hat{V}_0)^{-1}, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



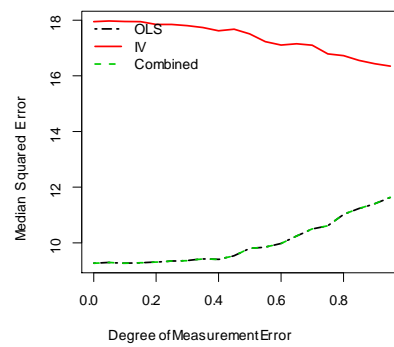
6.9 : $q = 10, R^2 = 0.1$



6.10 : $q = 10, R^2 = 0.4$

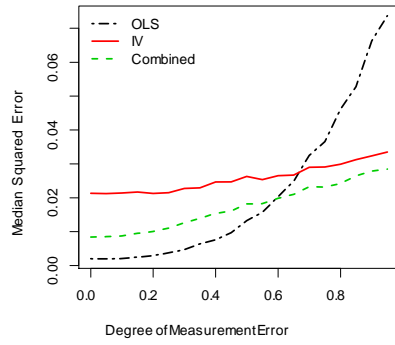


6.11 : $q = 20, R^2 = 0.1$

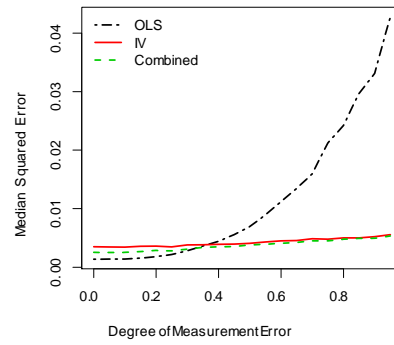


6.12 : $q = 20, R^2 = 0.4$

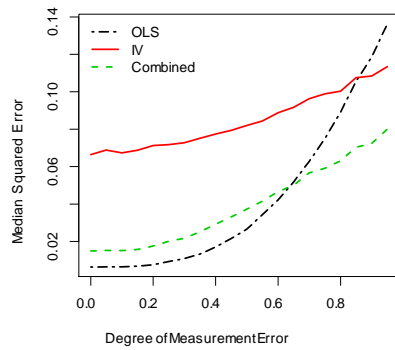
Figure 6: $n = 100, W = (\hat{V}_1 - \hat{V}_0)^{-1}, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



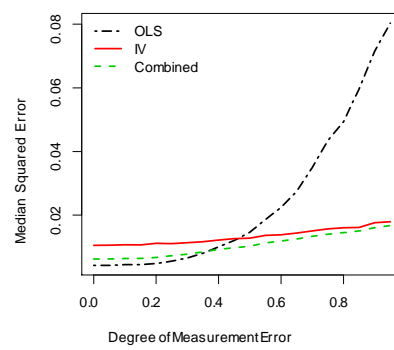
7.1 : $q = 1, R^2 = 0.1$



7.2 : $q = 1, R^2 = 0.4$



7.3 : $q = 2, R^2 = 0.1$



7.4 : $q = 2, R^2 = 0.1$

Figure 7: $n = 200, W = I, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.

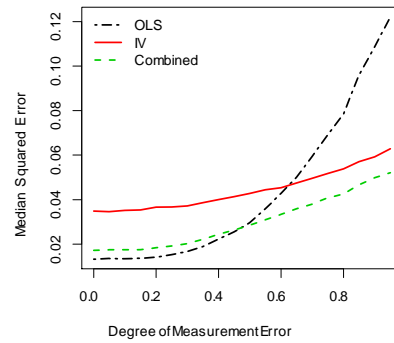
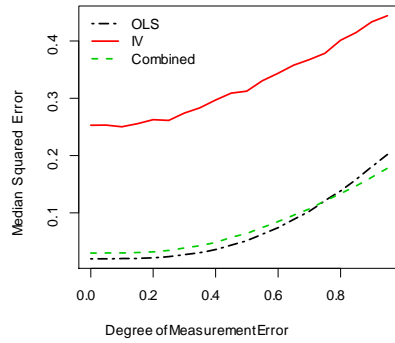
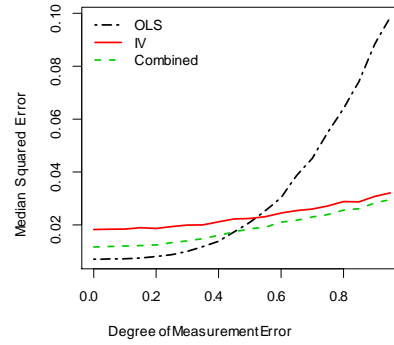
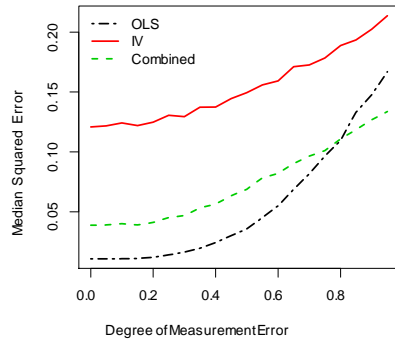
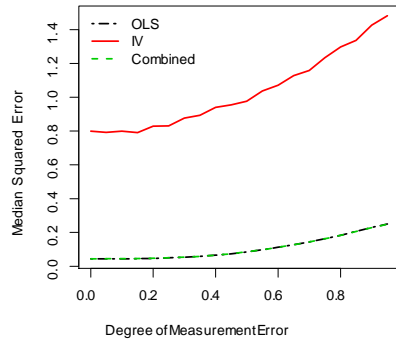
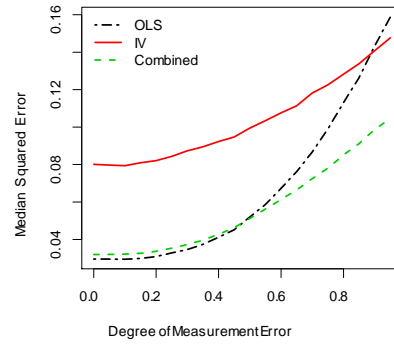


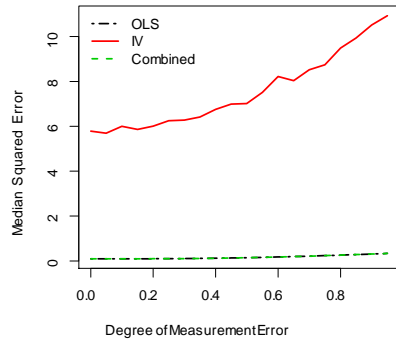
Figure 7: $n = 200, W = I, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



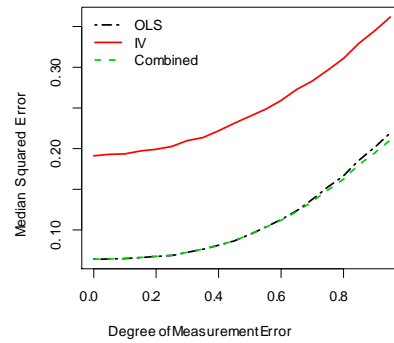
7.9 : $q = 10, R^2 = 0.1$



7.10 : $q = 10, R^2 = 0.4$

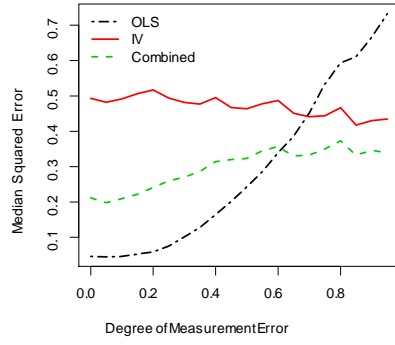


7.11 : $q = 20, R^2 = 0.1$

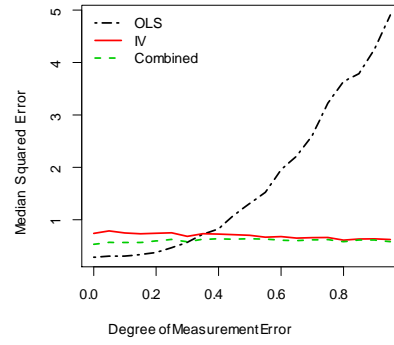


7.12 : $q = 20, R^2 = 0.4$

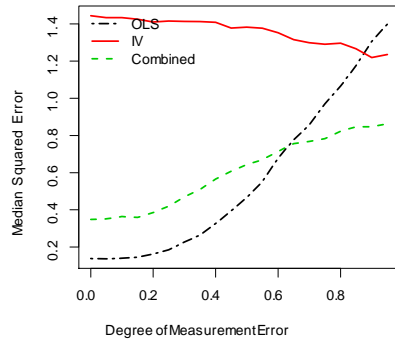
Figure 7: $n = 200, W = I, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



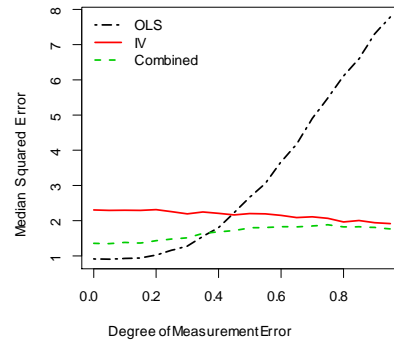
8.1 : $q = 1, R^2 = 0.1$



8.2 : $q = 1, R^2 = 0.4$

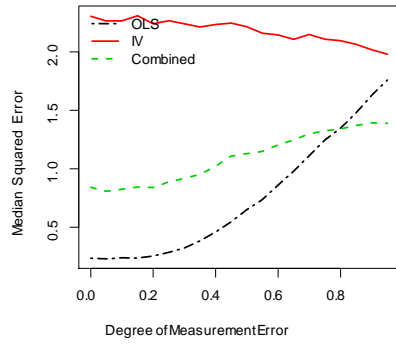


8.3 : $q = 2, R^2 = 0.1$

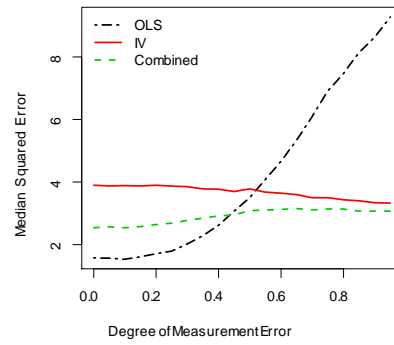


8.4 : $q = 2, R^2 = 0.4$

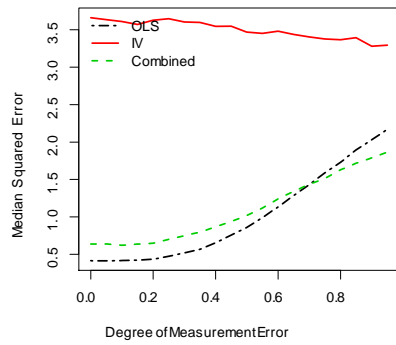
Figure 8: $n = 200, W = (\hat{V}_1 - \hat{V}_0)^{-1}, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



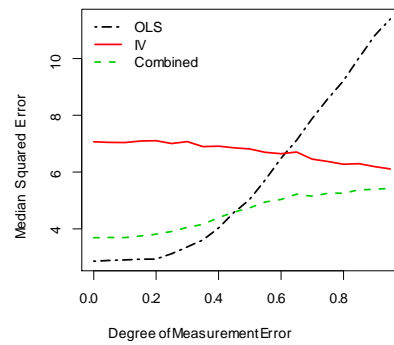
8.5 : $q = 3, R^2 = 0.1$



8.6 : $q = 3, R^2 = 0.4$

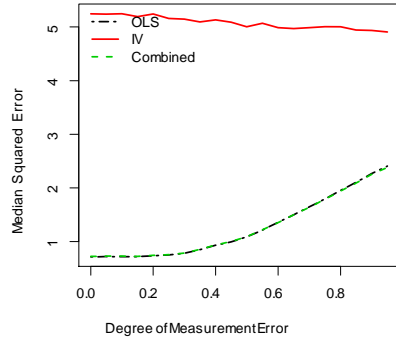


8.7 : $q = 5, R^2 = 0.1$

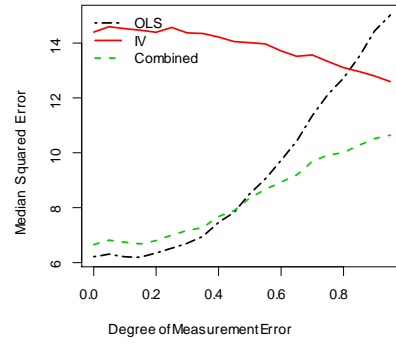


8.8 : $q = 5, R^2 = 0.4$

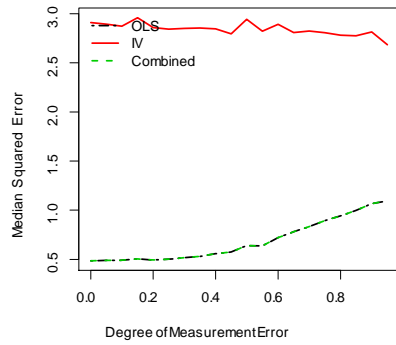
Figure 8: $n = 200, W = (\hat{V}_1 - \hat{V}_0)^{-1}, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



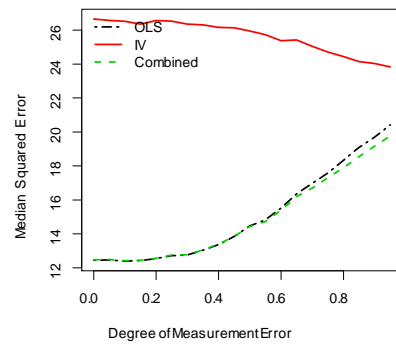
8.9 : $q = 10, R^2 = 0.1$



8.10 : $q = 10, R^2 = 0.4$

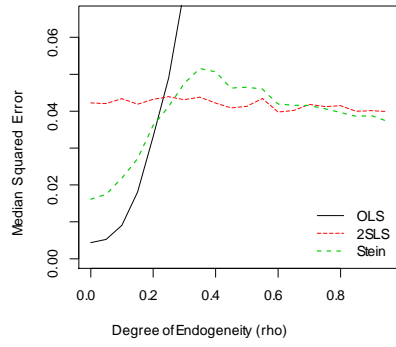


8.11 : $q = 20, R^2 = 0.1$

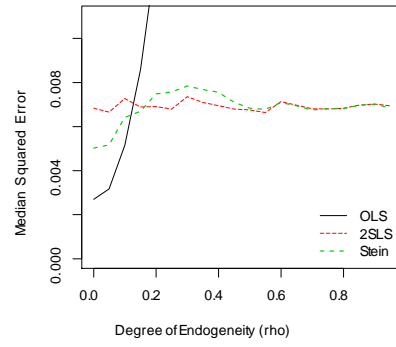


8.12 : $q = 20, R^2 = 0.4$

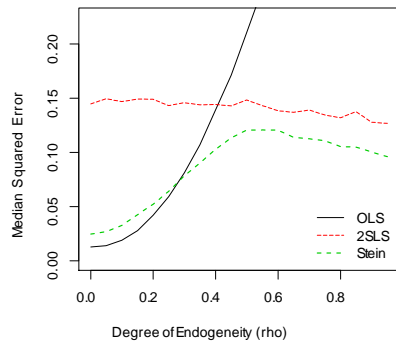
Figure 8: $n = 200, W = (\hat{V}_1 - \hat{V}_0)^{-1}, \rho_1 = \rho$ and $\rho_2 = \dots = \rho_q = 0$.



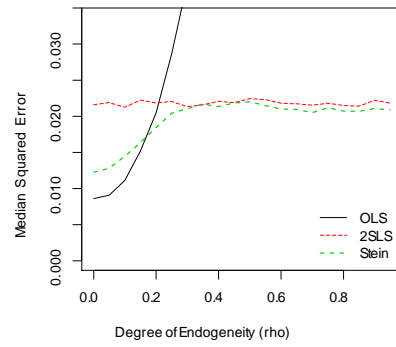
9.1 : $q = 1, R^2 = 0.1$



9.2 : $q = 1, R^2 = 0.4$

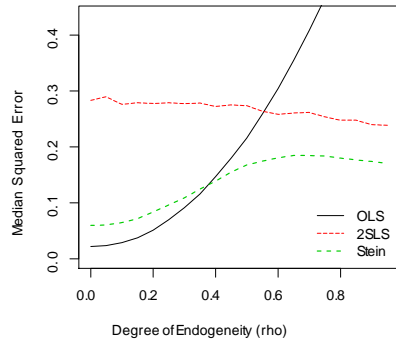


9.3 : $q = 2, R^2 = 0.1$

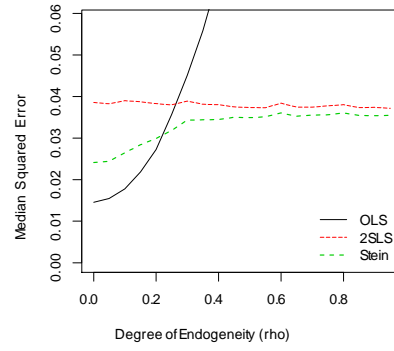


9.4 : $q = 2, R^2 = 0.4$

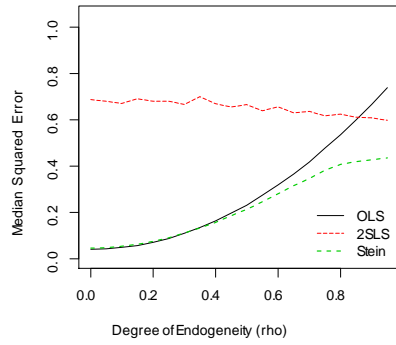
Figure 9: $n = 100, W = I$



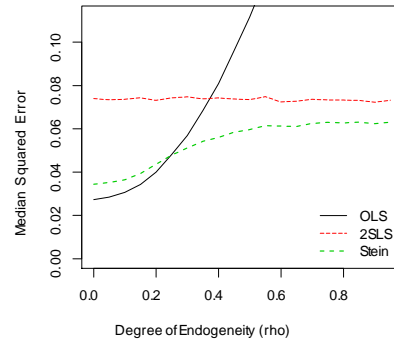
9.5 : $q = 3, R^2 = 0.1$



9.6 : $q = 3, R^2 = 0.4$

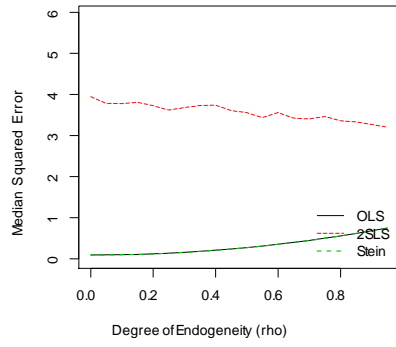


9.7 : $q = 5, R^2 = 0.1$

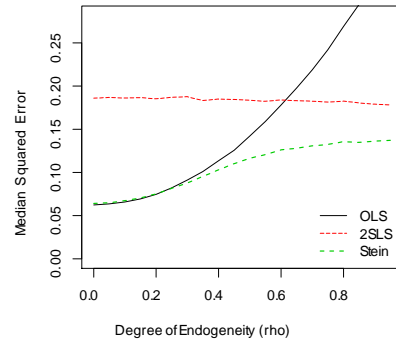


9.8 : $q = 5, R^2 = 0.4$

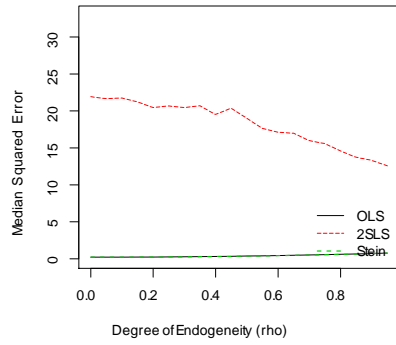
Figure 9: $n = 100, W = I$



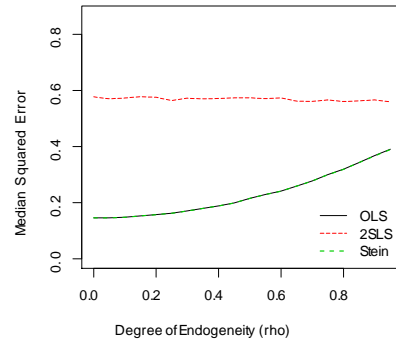
9.9 : $q = 10, R^2 = 0.1$



9.10 : $q = 10, R^2 = 0.4$

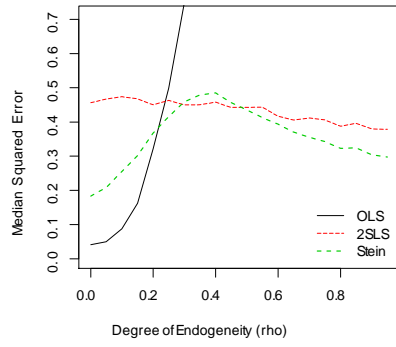


9.11 : $q = 20, R^2 = 0.1$

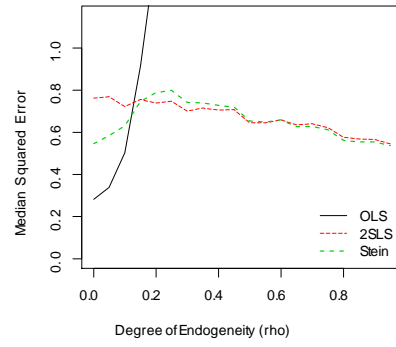


9.12 : $q = 20, R^2 = 0.4$

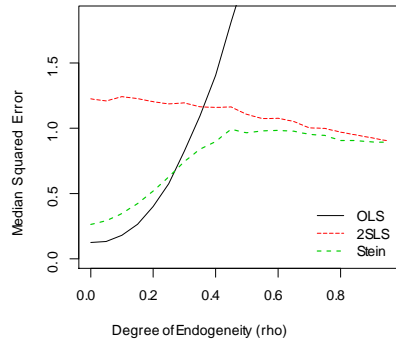
Figure 9: $n = 100, W = I$



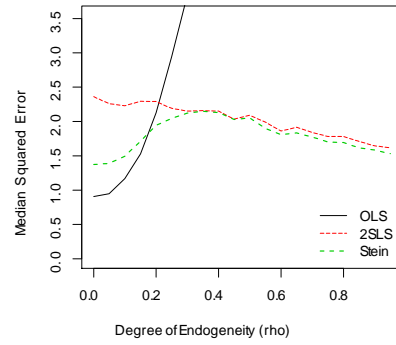
10.1 : $q = 1, R^2 = 0.1$



10.2 : $q = 1, R^2 = 0.4$

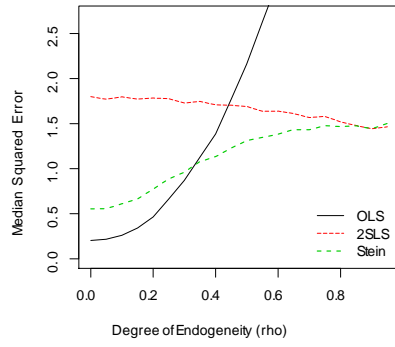


10.3 : $q = 2, R^2 = 0.1$

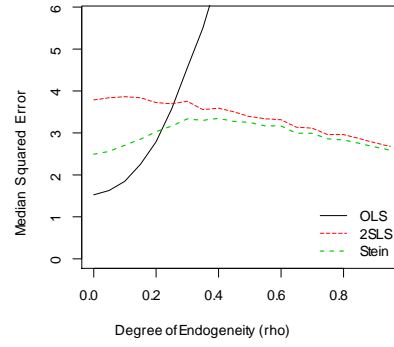


10.4 : $q = 2, R^2 = 0.4$

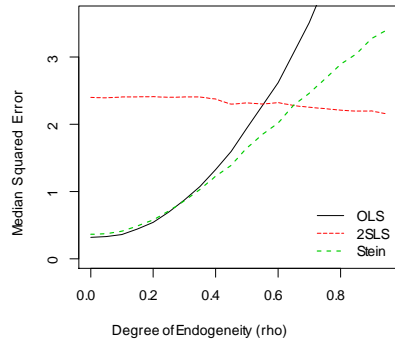
Figure 10: $n = 100, W = (\hat{V}_{2SLS} - \hat{V}_{OLS})^{-1}$



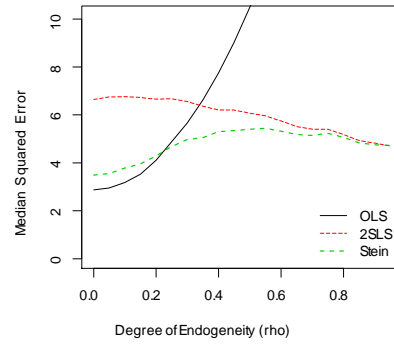
10.5 : $q = 3, R^2 = 0.1$



10.6 : $q = 3, R^2 = 0.4$

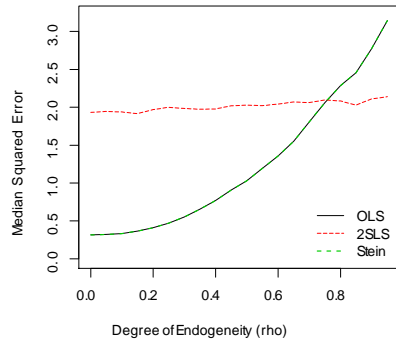


10.7 : $q = 5, R^2 = 0.1$

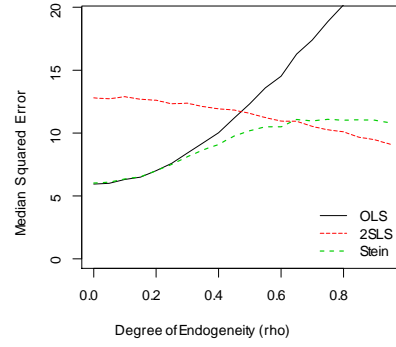


10.8 : $q = 5, R^2 = 0.4$

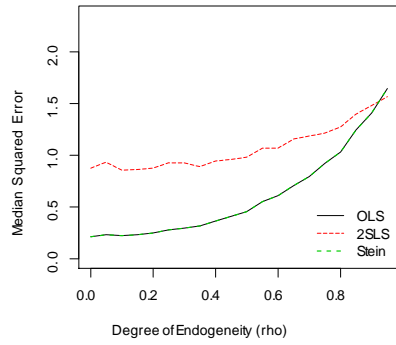
Figure 10: $n = 100, W = \left(\hat{V}_{2SLS} - \hat{V}_{OLS} \right)^{-1}$



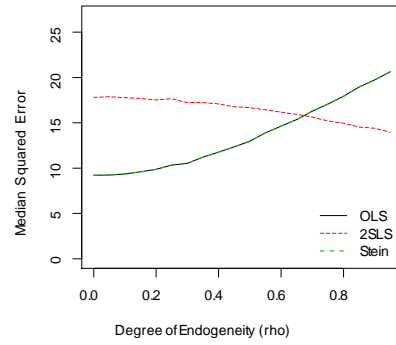
10.9 : $q = 10, R^2 = 0.1$



10.10 : $q = 10, R^2 = 0.4$



10.11 : $q = 20, R^2 = 0.1$



10.12 : $q = 20, R^2 = 0.4$

Figure 10: $n = 100, W = \left(\hat{V}_{2SLS} - \hat{V}_{OLS} \right)^{-1}$