# Predicting the Long-term Stock Market Volatility: A GARCH-MIDAS Model with Variable Selection

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**Summary**

We consider a GARCH-MIDAS model with the short-term and long-term volatility components, where the long-term volatility component depends on many macroeconomic and financial variables. We select the variables that exhibit the strongest effects on the long-term stock market volatility, via maximizing the penalized log-likelihood function with an Adaptive-Lasso penalty. The GARCH-MIDAS model with variable selection enables us to incorporate many variables in a single model without estimating a large number of parameters. In the empirical analysis, four variables (namely, unemployment level, housing starts, PPI and default spread) are selected among a large set of macroeconomic and financial variables. The post-selection estimation results show negative impacts of unemployment level, housing starts and PPI, and a positive impact of default spread, on the long-term stock market volatility component. The recursive out-of-sample forecasting evaluation shows that the variable selection significantly improves the predictive ability of the GARCH-MIDAS model for the long-term stock market volatility.

**Key words**: Stock market volatility, GARCH-MIDAS model, Variable selection, Penalized maximum likelihood, Adaptive-Lasso

**JEL classification**: C32, C51, C53, G12

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# 1. INTRODUCTION

Movements in aggregate financial volatility have significant impacts on capital investment, consumption, and even economic activities (Fornari and Mele, 2013). So what triggers the changes of aggregate financial volatility has drawn much attention among academics and practitioners. Many researchers relate aggregate volatility to macroeconomic variables (Officer, 1973; Schwert, 1989). Engle and Rangel (2008) find that links between economic fundamentals and aggregate financial volatility exist, but they are much weaker than seems reasonable. In addition to macroeconomic indicators, financial market variables including past volatility are also considered in predicting volatility (Ghysels et al., 2006; Christiansen et al., 2012). Recent studies are Engle et al. (2013), Asgharian et al. (2013) and Conrad and Loch (2015). Engle et al. (2013) propose a GARCH-MIDAS model, with the long-term component directly driven by inflation and industrial production. Asgharian et al. (2013) and Conrad and Loch (2015) both employ the GARCH-MIDAS framework to investigate the relationships between macroeconomic indicators and aggregate financial volatility.

The GARCH-MIDAS model is a component model of volatility, which is proposed by Engle et al. (2013). The component GARCH models have been researched for more than 20 years. Ding and Granger (1996) consider a two component model, with an IGARCH(1,1) specification for the long-memory component, and a GARCH(1,1) process for the short-term component. Engle and Lee (1999) propose an additive component GARCH model, where the conditional variance is specified as the sum of two components: one is persistent with a near unit root, and the other component is mean-reverting with rapid time decay. They also indicate that decomposition of volatility into several components is useful in tests of economic and asset pricing hypothesis. Bauwens and Storti

(2009) generalize the model of Ding and Granger (1996) by modeling the volatility as a convex combination of unobserved components where the combination weights are time varying. Engle and Rangel (2008) relax the assumption that the trend in the volatility process reverts to a constant level, and introduce the Spline-GARCH model, where the two components are separated using a multiplicative decomposition. The short-term component is modeled as a GARCH process evolving around a long-term component which reflects macroeconomic conditions, with the long-term component being specified using an exponential quadratic spline. The Spline-GARCH model is useful to understand the long-term or low-frequency volatility in a macroeconomic environment. However, these models do not relate macroeconomic variables with the long-term volatility component. Engle et al. (2013) propose a GARCH-MIDAS model, which directly incorporates low-frequency macroeconomic variables in the long-term volatility component. The GARCH-MIDAS model has been the most popular model that is used to investigate the relationships between aggregate financial volatility and macroeconomic or financial variables (Asgharian et al., 2013; Conrad et al., 2014; Conrad and Loch, 2015; Pan et al., 2017; Su et al., 2017).

This paper characterizes the relationships between the long-term stock market volatility and macroeconomic & financial indicators, and employs a GARCH-MIDAS model that includes a variety of possibly high-dimensional explanatory variables in the long-term volatility component. For a GARCH-MIDAS model with a large number of macroeconomic & financial variables, the "Adaptive-Lasso" of Zou (2006) is applied to determine which variables exhibit the strongest effects on the future long-term stock market volatility. Then we estimate the impacts of the selected variables on the long-term stock market volatility and analyze their "Beta weighting" schemes of the MIDAS model (Ghysels et al., 2007). We also compare the volatility predictive ability of our model with other GARCH-MIDAS models, such as a GARCH-MIDAS model with a single explanatory variable as in Conrad and Loch (2015), a GARCH-MIDAS model with no explanatory variable, and a GARCH-MIDAS model with all of the available explanatory variables.

Our contribution to the literature on stock market volatility predictability is twofold. First, we introduce variable selection in the long-term volatility component of the GARCH-MIDAS model, which helps to determine the most important variables in predicting the long-term stock market volatility. Inspired by Engle et al. (2013) and Boffelli et al. (2017), we consider many covariates in a single model. However, the model with many covariates involves a large number of parameters, which increases estimation complexity and reduces estimation efficiency, and it is difficult to give accurate interpretations on the parameter estimates. Therefore, we combine the Adaptive-Lasso with the log-likelihood function of the GARCH-MIDAS model for variable selection, and estimate the parameters by maximizing the penalized log-likelihood function. The model with variable selection helps us to determine the most important variables in predicting volatility without estimating a large number of parameters.

Second, we select the variables that play the most important roles in predicting the long-term stock market volatility, and provide further evidence on the countercyclical pattern of financial volatility. We choose the optimal tuning parameter using Generalized Information Criteria (GIC), and select the variables with non-zero parameter estimates. Then we estimate the GARCH-MIDAS model with the selected variables (Post-selection estimation), and analyze the parameter estimates and the dynamic structure of the estimated Beta weights.

Our main results are summarized as follows:

(i) Four variables, which are unemployment level, housing starts, PPI and default spread, are selected. Real GDP and industrial production that have been always considered in previous studies are surprisingly not selected.

(ii) Post-selection estimation results show that unemployment level, housing starts and PPI have negative impacts on the long-term stock market volatility. The negative impacts of housing starts and PPI confirm the countercyclical phenomenon of stock market volatility.

(iii) Forecasting results show that the model with selected variables significantly outperform the other GARCH-MIDAS models, except for the model with housing starts. Housing starts is the most powerful predictor among all the variables considered in this paper.

The remainder of this paper is organized as follows. Section 2 introduces the GARCH-MIDAS model with variable selection, and discusses the choice of the tuning parameter in variable selection. Section 3 presents the dataset. Section 4 reports the empirical results, including the variable selection, post-selection estimation results, out-of-sample forecast evaluations and some robustness checks. Section 5 concludes.

# 2. GARCH-MIDAS MODEL AND VARIABLE SELECTION

## 2.1 The GARCH-MIDAS model

We first introduce the GARCH-MIDAS model proposed by Engle et al. (2013) and Conrad and Loch (2015). The model extracts two components of volatility, a short-term component following a mean reverting high-frequency daily GARCH process, and a long-term component incorporating explanatory variables of low frequency using the Beta weighting schemes.

The stock market daily log return $r_{i,t}$ at day $i = 1, \ldots, N_t$ in a period $t = 1, \ldots, T$ (e.g., month, quarter) is represented in the following specification:

$$r_{i,t} - E_{i-1,t}(r_{i,t}) = \sqrt{g_{i,t}\tau_t}\varepsilon_{i,t}, \tag{1}$$

where $E_{i-1,t}(\cdot)$ is the conditional expectation given $\Gamma_{i-1,t}$, the information set up to day $(i-1)$ of period *t*, and $\varepsilon_{i,t}|\Gamma_{i-1,t} \sim N(0,1)$. Daily expected returns are set to be constant $\mu$. The total number of daily observations is denoted as $N_0 = \sum_{t=1}^{T} N_t$. Equation (1) shows that stock market volatility has the two components: the short-term volatility component $g_{i,t}$, and the long-term volatility component $\tau_t$.

The short-term volatility component that accounts for daily fluctuations follows a mean-reverting asymmetric GARCH(1,1) process:

$$g_{i,t} = (1-\alpha-\beta-\gamma/2) + \left(\alpha + \gamma \cdot \mathbf{1}_{\{r_{i-1,t}-\mu<0\}}\right) \cdot \frac{(r_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t}, \tag{2}$$

with the constraints of $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. This model ensures that $E[g_{i,t}] = 1$. The parameter $\gamma$ contains the information of asymmetry.

The long-term volatility component with a single explanatory variable is given by:

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}, \tag{3}$$

where $\log(\tau_t)$ is considered rather than $\tau_t$ in order to ensure the positivity of the long-term volatility and $\varphi_k(\omega_1, \omega_2)$ is the Beta weighting scheme:

$$\varphi_k(\omega_1, \omega_2) = \frac{(k/(K+1))^{\omega_1 - 1} \cdot (1-k/(K+1))^{\omega_2 - 1}}{\sum_{l=1}^{K} (l/(K+1))^{\omega_1 - 1} \cdot (1-l/(K+1))^{\omega_2 - 1}}. \tag{4}$$

The weights $\varphi_k$ are completely determined by two parameters $\omega_1$ and $\omega_2$. It is easy to find that $\varphi_k \geq 0$ for $k = 1, \cdots, K$, and $\sum_{k=1}^{K} \varphi_k = 1$. The Beta weighting schemes can generate decaying, hump-shaped, or U-shaped weights (Ghysels et al., 2007).

The GARCH-MIDAS model has been the most popular methodology for investigating the relationships between stock market volatility and economic variables of low frequency (Asgharian et al., 2013; Conrad et al., 2014; Conrad and Loch, 2015; Su et al., 2017; Pan et al., 2017; Boffelli et al., 2017). However, most of these studies focus on the effects of one variable at a time on the stock market volatility, while many economic and financial variables can lead to changes of stock market volatility. It would be desirable to include all these potentially useful predictors at once in a single model. In this regards, Engle et al. (2013) estimate a single model that combines four variables (the level and volatility of PPI, and the level and volatility of industrial production). Also, Boffelli et al. (2017) use six variables to estimate a GARCH-MIDAS model. Inspired these two papers, we consider including a "large" number of variables in a single GARCH-MIDAS model with modifying Equation (3) as follows:

$$\log(\tau_t) = m + \sum_{j=1}^{J} \theta_j \sum_{k=1}^{K} \varphi_k(\omega_{j,1}, \omega_{j,2}) X_{j,t-k}, \tag{5}$$

where $J$ is the number of explanatory variables ($J = 20$ in this paper), and $\theta_j$ measures the impact of the *j*th variable on the long-term stock market volatility.

Therefore, our GARCH-MIDAS model consists of Equation (1), (2), (4) and (5), with *J* being large. The log-likelihood function is given by Equation (6), and $\Phi$ denotes all the parameters that are to be estimated. The model is usually estimated through the quasi-maximum likelihood estimation, and $\hat{\Phi}$ denotes all the parameter estimates. The asymptotic standard errors are estimated consistently under the assumption of conditional normality.

$$LLF(\Phi) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\left[\log(2\pi) + \log\left(g_{i,t}(\Phi)\tau_t(\Phi)\right) + \frac{(r_{i,t}-\mu)^2}{g_{i,t}(\Phi)\tau_t(\Phi)}\right]. \tag{6}$$

**2.2 GARCH-MIDAS with variable selection**

The number of parameters is $3J + 1$ in Equation (5), which will be a very large number if $J$ is large. With a large number of parameters to be estimated, it may become difficult to identify variables that exhibit the strongest effects. In this paper, we employ variable selection in the long-term volatility component in Equation (5), and use the Adaptive-Lasso of Zou (2006) for the penalized log-likelihood function:

$$PLLF_{\lambda}(\Phi) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\left[\log(2\pi) + \log\left(g_{i,t}(\Phi)\tau_t(\Phi)\right) + \frac{(r_{i,t}-\mu)^2}{g_{i,t}(\Phi)\tau_t(\Phi)}\right] - \lambda\sum_{j=1}^{J}\widehat{w}_j\left|\theta_j\right| \quad (7)$$

where $\lambda > 0$ is the tuning parameter, and $PLLF_{\lambda}(\Phi)$ denotes the penalized log-likelihood function for a given $\lambda$. We use the Adaptive-Lasso as the penalty function. For the first step, we estimate a GARCH-MIDAS model with all the $J$ variables and get the preliminary estimates $\hat{\theta}_j$ from maximizing Equations (6) under the constraints of $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. We calculate the adaptive weight as $\widehat{w}_j = 1/\left|\hat{\theta}_j\right|^{\eta}$. In the simulation of Zou (2006), the probability of containing the true model is the highest when $\eta = 2$, so we take this value in this paper.

For a given tuning parameter $\lambda$, we maximize $PLLF_{\lambda}(\Phi)$ subject to the linear constraints of $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. The Broyden-Fletcher-Goldfard-Shanno (BFGS) algorithm is used for this optimization. $\widehat{\Phi}_{\lambda}$ denotes the parameter estimates from the maximization problem for the given $\lambda$.

### 2.3 Choosing the tuning parameter

Choosing the tuning parameter $\lambda$ in the penalized log-likelihood estimation is important for determining the underlying true model. Cross-validate (CV) and information criteria (AIC and BIC) are widely used in model selection. Yang (2005) indicates that CV is asymptotically equivalent to AIC, implying that CV behaves similarly to AIC. Wang et al. (2009) propose a modified BIC which works for the tuning parameter selection. In this paper, the tuning parameter is determined by Generalized Information Criteria (GIC), which is proposed by Fan and Tang (2012). Similar to AIC and BIC, GIC also contains two components, the first component is used to evaluate the goodness of fit, and the second component is a penalty on the model complexity, which implies GIC trades off between model fitting and model complexity. $GIC_{\lambda}$ denotes this information criteria for a given

$\lambda$, and a GIC applied to the penalized log-likelihood function is shown in Equation (8):

$$GIC_\lambda = \frac{1}{N_0}\{2[LLF(\hat{\Phi}) - PLLF_\lambda(\hat{\Phi}_\lambda)] + a(N_0, p)|\hat{\theta}_\lambda|\}, \tag{8}$$

where $a(N_0, p)$ is a positive value depending on the number of total observations $N_0$, and the number of parameters $p = 3J + 1$ in the long-term volatility component. $LLF(\hat{\Phi})$ is the value of maximized log-likelihood function without variable selection, and $PLLF_\lambda(\hat{\Phi}_\lambda)$ is the value of maximized penalized log-likelihood function. $2[LLF(\hat{\Phi}) - PLLF_\lambda(\hat{\Phi}_\lambda)]$ indicates the scaled deviation measure that is used to evaluate goodness of fit. $|\hat{\theta}_\lambda|$ denotes the number of non-zero elements in $\hat{\theta}_\lambda$, where $\hat{\theta}_\lambda$ is estimated from Equation (8) given the tuning parameter $\lambda$. Fan and Tang (2012) propose a uniform choice of $a(N_0, p) = \log\{\log(N_0)\} \cdot \log(p)$. For practical implementation, the tuning parameter is considered over a range from $\lambda_{\min}$ to $\lambda_{\max}$, and we take the value of $\lambda$ corresponding to the minimum $GIC_\lambda$ as the optimal tuning parameter.

# 3. DATA

In our dataset, we focus on the S&P500, and U.S. macroeconomic & financial data for 1968Q1-2018Q2 period. We consider daily stock market log returns and quarterly macroeconomic & financial variables. The S&P500 index data are obtained from the Center for Research in Security Prices (CRSP), while macroeconomic & financial data are obtained from the FRED database at the Federal Reserve Bank of St Louis, the Federal Reserve Bank of Chicago (FRBC), Quandl.com, the Survey of Consumers from University of Michigan (SCUM), and the personal website of K. R. French and A. Manela. For the data available at a monthly or daily frequencies, we take quarterly averages. Descriptive statistics are reported in Table 1.[1]

**[INSERT TABLE 1 HERE]**

[1] Quandl.com is a database that offers financial and economic data.

### 3.1 Macroeconomic variables

We consider the following macroeconomic variables that have been considered in Christiansen et al. (2012), Engle et al. (2013), Asgharian et al. (2013) and Conrad and Loch (2015): real GDP growth rate, industrial production growth rate, unemployment level, housing starts, nominal corporate profits after tax, real personal consumption, CPI, PPI, the Chicago Fed national activity index (CFNAI), the new orders index of the Institute of Supply Management (ISM), monetary base, and the University of Michigan consumer sentiment index. Unemployment level is the number of persons who are not employed within a quarter. CFNAI is an index designed to gauge overall economic activity and related inflationary pressure, which can be a proxy of business cycles. The new orders index measures changes in new orders, supplier deliveries, inventories, production and employment, and it is a proxy of future activity in any industry, which can be seen as a leading economic indicator.

Real GDP growth rate, industrial production growth rate, unemployment level, housing starts, corporate profits, personal consumption and new orders index are seasonally adjusted. We consider the CFNAI and the new orders index in levels, and take the log difference of unemployment level, housing starts, corporate profits, real personal consumption, CPI, PPI, monetary base, and consumer sentiment.

Volatilities of macroeconomic variables are also important determinants of stock market volatility (Schwert, 1989; Engle et al., 2013; Asgharian et al., 2013), and a GARCH(1,1) model is used to estimate quarterly volatility of macroeconomic variables as mentioned in Engle et al. (2013). We consider the volatility of macroeconomic activity measured by volatility of real GDP growth rate, and the volatility of inflation measured by volatility of CPI.[2]

### 3.2 Financial variables

We employ six financial variables in this paper: term spread, default spread, equity market returns (MKT), short-term reversal factor (STR), implied volatility (IV) and realized volatility (RV). The term spread is calculated as the difference between the 10-year Treasury bond yield and the 3-month Treasury bill rate. Default spread is the yield spread between BAA and AAA rated bonds, which should affect aggregate volatility, according to Merton (1974). Equity market returns (MKT) in Fama and French (1992) can capture the leverage effect (Black, 1976; Glosten et al., 1993; Christiansen et al., 2012). Nagel (2012) finds that the short-term reversal factor (STR) can be related to market volatility. Realized volatility is also considered in volatility forecasts (Andersen et al., 2003; Ghysels et al., 2006; Andersen et al., 2011). Quarterly realized volatility in this paper is calculated as:

$$RV_t = \sum_{i=1}^{N_t} r_{i,t}^2. \tag{9}$$

The implied volatility indices CBOE VIX and VXO are used to measure market expectation of volatility conveyed by stock index option prices, and they are also important in forecasting future financial volatility (Busch et al., 2011). The implied volatility is also a proxy of financial market uncertainty (Chung and Chuwonganant, 2014). Becker et al. (2009) find that VIX subsumes information relating to past jump contributions to aggregate volatility, and reflects information of future jump activity. Bekaert and Hoerova (2014) indicate that VIX has a high predictive power for financial instability. We also consider the implied volatility as an explanatory variable in the long-term volatility component. However, VIX and VXO are available only since 1990 and 1986

[2] The volatility estimated by GARCH(1,1) model is also seen as macroeconomic or inflation uncertainty (Caporale and McKiernan, 1998; Fountas et al., 2006). In this paper, inflation volatility is estimated using the CPI data that is taken log difference.

respectively, and they could not match the time period of the other variables. Manela and Moreira (2017) propose a news-based implied volatility index (NVIX) that captures investors' perception of future uncertainty, and it is actually the estimated VXO index using front-page articles of the *Wall Street Journal*. NVIX is confirmed to be a source of financial aggregate volatility (Su et al., 2017), and we use it as a proxy of implied volatility before 1986. We consider term spread, default risk, MKT and STR in levels, and take log difference of NVIX.

# 4. EMPIRICAL ANALYSIS

### 4.1 Variable selection using Adaptive-Lasso

For practical implementation, we consider the tuning parameter $\lambda$ on a 126-point grid on [1, 26] with an increment of 0.2. Following Tibshirani (1996), we standardize all the macroeconomic & financial variables for variable selection. We estimate the model with every $\lambda \in [1,26]$, and choose the tuning parameter corresponding to the minimum GIC.[3] Figure 1 shows GIC as a function of $\lambda$, and we take $\lambda = 13.0$ as the optimal value of the tuning parameter.

**[INSERT FIGURE 1 HERE]**

Figure 2 presents the parameter estimates of $\theta$ for each value of $\lambda$. When $\lambda = 13.0$, we maximize $PLLF_{\lambda=13.0}(\Phi)$ under linear constraints, and find that the parameter estimates of four variables are not shrunk to zero, which are unemployment level, housing starts, PPI and default spread.[4] These four variables are selected from all of the 20 variables.

**[INSERT FIGURE 2 HERE]**

The variable selection for the GARCH-MIDAS model provides us a new perspective to find out which variable is more important in predicting the long-term stock market volatility. Conrad

[3] The lag length $K = 12$ in Equation (5) is determined following Conrad & Loch (2015).
[4] The results of variable selection are similar when $\lambda \geq 13$.

and Loch (2015) find that real GDP and industrial production growth rate are negatively associated with the future long-term aggregate volatility, which is the well-known countercyclical pattern mentioned by Officer (1973) and Schwert (1989). Choudhry et al. (2016) also reveal a strong relationship between industrial production growth rate and stock market volatility. However, interestingly, these two variables are not selected in our estimation. Real GDP and industrial production growth rate have been emphasized in predicting volatility for more than 40 years, the variable selection results indicate that their roles in predicting the stock market volatility may have been overestimated.

Although previous studies show that the implied volatility has significant impacts on the stock market volatility (Bekaert and Hoerova, 2014; Su et al., 2017), the implied volatility is not selected. The volatility of real GDP and inflation, also known as economic and inflation uncertainty respectively, are not considered to be important variables in predicting the long-term stock market volatility. However, we are not saying that uncertainty is not a key variable in volatility prediction. There are many uncertainty indices that we do not consider in this paper due to data availability, including Economic Policy Uncertainty (Baker et al., 2016), macro uncertainty (Jurado et al., 2015), and financial uncertainty (Ludvigson et al., 2018). Whether they are selected needs further research.

Unemployment level, housing starts, PPI, and default spread are selected from all of the 20 variables. Unemployment and PPI are undoubtedly important macroeconomic indicators for explaining volatility prediction (Engle et al., 2013; Conrad and Loch, 2015; Boffelli et al., 2017). The literature on the relationships between housing starts and financial volatility is quite limited. The most similar one is Löffler (2013), who considers the role of skyscrapers construction starts in U.S. stock return prediction. Although skyscrapers construction activity is different from housing constructions, the research of Löffler (2013) gives us inspirations on the reasons why housing starts matters for the stock market volatility prediction: (1) Housing starts can be seen as a leading indicator of economic activities, and more house constructions imply good expectations of future economy, which motivates more financial investments. (2) The house market is closely associated with the credit market, and a higher supply of new houses indicates an expansion of credit market, which is a driven force of economic growth. (3) Housing starts is found to be heavily and positively affected by the monetary base (Huang, 1973), which also stimulates the economy.

### 4.2 Post-selection estimation

After the variable selection, we estimate a GARCH-MIDAS model with the selected variables (which we refer to as the post-selection estimation). The long-term volatility component with the four selected variables is given by:

$$\log(\tau_t) = m + \theta^{UE} \sum_{k=1}^{12} \varphi_k(\omega_1^{UE}, \omega_2^{UE}) UE_{t-k} + \theta^{HS} \sum_{k=1}^{12} \varphi_k(\omega_1^{HS}, \omega_2^{HS}) HS_{t-k} + \theta^{PPI} \sum_{k=1}^{12} \varphi_k(\omega_1^{PPI}, \omega_2^{PPI})\, PPI_{t-k} + \theta^{DEF} \sum_{k=1}^{12} \varphi_k(\omega_1^{DEF}, \omega_2^{DEF}) DEF_{t-k}, \quad (10)$$

where *UE*, *HS*, *PPI*, and *DEF* denote unemployment level, housing starts, PPI and default spread, respectively. With respect to the post-selection estimator, Belloni et al. (2012) and Belloni and Chernozhukov (2013) show that the post-selection estimator is consistent for the true parameter.

For the inference, we need to assess the significance of the parameter estimates. The variable selection can have a detrimental impact on subsequently constructed inference procedures like confidence intervals, if these are constructed in the "naïve" way where the presence of model selection is ignored (Berk et al., 2013; Leeb et al., 2015). The asymptotically valid confidence intervals may be obtained following Belloni et al. (2016) and Belloni et al. (2018). In this paper, we use the naïve confidence intervals that is constructed as if the model with the selected variables were correct and fixed a priori (thus ignoring the presence of variable selection). Nevertheless, Leeb et al. (2015) show that the actual coverage probability of the naïve confidence interval deviates only moderately from the desired nominal coverage probability, which supports the use of the naïve confidence intervals in the post-selection inference.

**[INSERT TABLE 2 HERE]**

The post-selection estimation results are reported in Table 2. We are interested in the parameter estimates of $\theta$, which measures the impact of the selected variables on the long-term stock market volatility. The estimated parameter on unemployment level is negative and significant at 5% level, which shows that higher unemployment level is associated with lower long-term stock market volatility. The estimated parameter on housing starts is -20.3750, significant at 1% level, indicating that housing starts is a powerful predictor on the long-term stock market volatility. More new house construction activities will lead to lower future aggregate volatility. Higher PPI will decrease the long-term volatility, with a parameter estimate of -17.6853 at 1% significant level. The parameter estimate on default spread is 0.5715, positive and significant at 1% level. The result for default spread is same as in Christiansen et al. (2012). Default spread is associated with market leverage, which is a direct driver of stock volatility (Merton, 1974).

Previous studies find that aggregate financial volatility is countercyclical (Officer, 1973; Schwert, 1989; Engle et al., 2013; Conrad and Loch, 2015). Our results also confirm this conclusion. Housing starts is an indicator that reflects good expectations of future economy, and higher PPI implies a period of economic expansion. Since both of them have negative impacts on the stock market volatility, we provide a new evidence on the countercyclical pattern of aggregate volatility from a perspective of housing starts and inflation.

**[INSERT FIGURE 3 HERE]**

In addition to parameter estimates, we plot the Beta weighting schemes for the four selected variables in Figure 3. The Beta weighting schemes help us find out which lag of the variables has the strongest effect. For unemployment level, housing starts and PPI, the weighting schemes are hump-shaped. The maximum weights are on the 4th, 5th, and 5th lag of unemployment level, housing starts and PPI, respectively. However, the weighting schemes for unemployment level and housing starts are smoother than PPI. The weight on the 5th lag of PPI is 0.5741, much larger than any of other weights, indicating the 5th lag of PPI plays a dominant role among all the lags. The weighting scheme for default spread is extreme and simple, putting almost all the weight on the 1st lag. If we use default spread for predicting the long-term stock market volatility, one quarter lagged default spread is sufficient.

### 4.3 Out-of-sample forecast evaluations

To evaluate the out-of-sample forecast performance of the GARCH-MIDAS model with the selected variables, we consider 1-/2-/3-/4-quarter-ahead forecasts. All the models considered in the out-of-sample forecast evaluations are describe as:

- Model 1: GARCH-MIDAS model with the four selected variables.
- Model 2: GARCH-MIDAS model, which incorporates one variable at a time in the long-term component (Conrad and Loch, 2015).[5]
- Model 3: GARCH-MIDAS model with all the macroeconomic & financial variables, which is a model without variable selection.
- Model 4: GARCH-MIDAS model with no explanatory variables. The long-term volatility

[5] There will be 20 different models for Model 2, with each of the 20 variables considered in this paper.

component is constant, and it can be seen as a GARCH-MIDAS model with variable selection, when the tuning parameter is extremely large that all the parameters on the variables are shrunk to zero.

The full sample is divided into the estimation period from 1968Q1 to 2012Q2 (178 quarters), and the out-of-sample forecasting period from 2012Q3 to 2018Q2 (24 quarters). To evaluate volatility forecasts, we compare the predicted volatility with true conditional variance. Since the true conditional variance is unobservable, a proxy for it is required. The squared daily return is one of the commonly used proxies, however it has been seen as a noisy one. Patton (2011) indicates that realized volatility is better than squared daily return. Besides, some studies find that microstructure noise can be ignored by using higher frequency data (Awartani et al., 2009). Ghysels and Sinko (2011) also show that the 5-minute frequency can be considered as a low-noise environment. In this paper, we apply the S&P500 daily realized volatility $RV^{5\text{min}}$, which is calculated from 5-minute S&P500 intraday returns, and evaluate the volatility forecasts by comparing the predicted volatility with $RV^{5\text{min}}$.[6]

A recursive out-of-sample forecast is employed. The $s$-quarter-ahead long-term volatility component forecast $\hat{\tau}_{t+s}$ for $s = 1,2,3,4$ remains the same within one quarter, and the short-term volatility component forecast $\hat{g}_{i,t+s}$ for $s = 1,2,3,4$ can be calculated iteratively by Equation (2). The daily volatility forecast is $\hat{\tau}_{t+s}\hat{g}_{i,t+s}$, and the forecast error is $RV_{i,t+s}^{5\text{min}} - \hat{\tau}_{t+s}\hat{g}_{i,t+s}$. The mean squared forecast error (MSFE) is given by:

$$MSFE = \frac{1}{N_0 - \sum_{t=1}^{178} N_t} \sum_{t=179}^{T} \sum_{i=1}^{N_t} \left(RV_{i,t}^{5\text{min}} - \hat{\tau}_t \hat{g}_{i,t}\right)^2, \quad (11)$$

where $N_0$ = 12,674 is the total number of daily observations, and $T$ = 202 is the total number of

[6] The S&P500 daily realized volatility data are calculated by 5-minute intraday returns, and the data are obtained from Oxford-Man Institute's realized library (Version 0.3), University of Oxford.

quarters as shown in Table 1.

To compare the forecasting performance of a model over the benchmark, we present the ratio of the corresponding MSFE: $MSFE/MSFE^{benchmark}$. A ratio lower than one indicates a forecasting improvement over the benchmark model. We consider Model 1 as the benchmark. In addition to MSFE, we also compare the conditional predictive ability between Model 1 and Models 2-4 via the Giacomini and White (GW, 2006) test. For each horizon, we conduct the tests of conditional predictive ability of Models 2-4 over Model 1 using a squared error loss function.[7] In the GW test results, a positive sign indicates that Models 2-4 outperform the benchmark, while a negative sign indicates the opposite.

**[INSERT TABLE 3 HERE]**

The forecasting evaluation results are reported in Table 3. We first take a look at the MSFE ratio. Our model outperforms Models 2-4 for volatility forecasts at 1/2/3/4 forecast horizons, except for the model with housing starts. The MSFE ratio comparisons indicate that housing starts is the most powerful stock market volatility predictor, and support the findings in Section 4.1 and 4.2. Model 3 has the highest MSFE ratio among all the models, which implies that incorporating all the macroeconomic and financial information may destroy the out-of-sample predictive ability. In addition, the MSFE ratio is lower when the forecasting horizon is longer, almost for all the models. For example, the ratios for term spread at 1/2/3/4 forecast horizons are 1.0316, 1.0265, 1.0126 and 1.001, respectively. When we do 4-quarter-ahead volatility forecast, the GARCH-MIDAS model with term spread has similar out-of-sample performance to the benchmark model, and the model with consumer sentiment has a ratio that is even lower than 1. The results of MSFE ratio indicate

[7] The loss function can be square error loss, absolute error loss, lin-lin loss, linex loss, etc. (Giacomini and White, 2006). We also use an absolute error loss, and get the similar results to the squared error loss function.

that Model 1 performs the best when forecasting volatility at a shorter horizon.

The GW results are similar to the results of MSFE ratio. For all the forecast horizons, the GW statistics are significant with negative signs, except for housing starts at all the horizons, CPI at the 2-quarter-ahead horizon, and consumer sentiment at the 4-quarter-ahead horizon. Model 1 significantly outperforms almost all the other models considered in this paper. Based on the GW results, housing starts is still the most powerful volatility predictor for any forecast horizons.

In summary, the benchmark model, which considers the four selected variables, significantly outperforms most of the other models at 1-/2-/3-/4-quarter-ahead out-of-sample forecast horizons, and the MSFE ratio and GW results both confirm that housing starts is the most powerful volatility predictor among all the macroeconomic & financial variables.

### 4.4 Robustness checks

In this section, we provide the robustness checks by conducting subsample analysis, and we also estimate the GARCH-MIDAS model with variable selection under the constraint of $\omega_1 = 1$, which guarantees a decaying pattern of the Beta weighting schemes (Restricted Beta weighting schemes).

#### 4.4.1 Subsample analysis

We estimate the GARCH-MIDAS model with the four selected variables through a subsample from 1968Q1 to 2007Q4, in order to exclude the impacts of the financial crisis.[8] The estimation results are reported in Table 4. The parameter estimates of $\theta$ are all significant, and the signs are the same as the results in Table 2. The parameter for housing starts is -13.2804 at 1% significant level. The subsample analysis results do not change the conclusions in Section 4.2.

[8] The financial crisis period is from 2008Q1, which is determined by the NBER business cycle data.

**[INSERT TABLE 4 HERE]**

### 4.4.2 Estimation with restricted Beta weighting schemes

Under the constraint of $\omega_1 = 1$, the Beta weighting scheme which generates a decaying pattern of weights is specified as follows:

$$\varphi_k(\omega_2) = \frac{(1-k/(K+1))^{\omega_2-1}}{\sum_{l=1}^{K}(1-l/(K+1))^{\omega_2-1}}. \quad (12)$$

Conrad and Loch (2015) test the null hypothesis $H_0\colon \omega_1 = 1$ with a likelihood ratio test. However, we are more likely to estimate the weighting functions with more choices of shapes via an unrestricted weighting function. So we estimate the model with variable selection under the constraint of $\omega_1 = 1$ in robustness checks instead of Section 4. The variable selection results show that housing starts, default spread, MKT and RV are selected. Housing starts and default spread remain the most important variables. The results of Section 4.2 show that the 4-6$^{th}$ lagged PPI are more important in predicting the long-term volatility. Since the restricted weighting schemes only generate decaying weights, PPI with higher weights on the 1$^{st}$, 2$^{nd}$, and 3$^{rd}$ lag may not outperform other variables like MKT and RV, and it is reasonable that PPI is not selected. We also find that the parameter for housing starts is significantly different from zero, when the parameter estimates of other 19 variables are shrunk to zero with a large tuning parameter. The finding that housing starts is the most powerful long-term volatility predictor is robust.

## 5. CONCLUSIONS

This paper estimates a GARCH-MIDAS model with variable selection by combining the log-likelihood function with the Adaptive-Lasso penalty. Through maximizing the penalized log-likelihood function under linear constraints, the model could determine the variables that play the most important roles in predicting the long-term stock market volatility.

Four variables are selected, which are unemployment level, housing starts, PPI and default spread. The post-selection estimation results show negative impacts of unemployment level, housing starts and PPI, and a positive impact of default spread. The negative impacts of housing starts and PPI confirm the countercyclical pattern of the stock market volatility. The MFSE ratio and Giacomini and White (2006) test are used in the out-of-sample forecasting evaluations. The results indicate that the predictive ability of the model with selected variables outperforms the other model considered in this paper, except for the model with housing starts. The overall empirical results show that housing starts is the most powerful predictor of the long-term stock market volatility. This paper provides a deeper understanding on the movements of long-term stock market volatility.

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**TABLE 1** Descriptive statistics

| Variable | Obs. | Min. | Max. | Mean. | Std. | Skew. | Kurt. | Database |
|---|---|---|---|---|---|---|---|---|
| *Stock market data* | | | | | | | | |
| S&P 500 returns | 12,674 | -0.0995 | 0.0476 | 0.0001 | 0.0046 | -0.9980 | 25.7123 | CRSP |
| | | | | | | | | |
| *Macroeconomic data* | | | | | | | | |
| real GDP | 202 | -0.0095 | 0.0165 | 0.0030 | 0.0035 | -0.2827 | 2.1819 | FRED |
| industrial production | 202 | -0.0297 | 0.0180 | 0.0023 | 0.0066 | -1.3916 | 4.6192 | FRED |
| unemployment level | 202 | -0.0395 | 0.0993 | 0.0016 | 0.0222 | 1.5223 | 3.0058 | FRED |
| housing starts | 202 | -0.1142 | 0.1192 | -0.0003 | 0.0369 | -0.3278 | 0.9145 | FRED |
| corporate profits | 202 | -0.2303 | 0.1763 | 0.0074 | 0.0307 | -1.5765 | 21.4934 | FRED |
| personal consumption | 202 | -0.0099 | 0.0103 | 0.0033 | 0.0028 | -0.6743 | 2.6145 | FRED |
| CPI | 202 | -0.0101 | 0.0168 | 0.0043 | 0.0034 | 0.6607 | 2.8100 | FRED |
| PPI | 202 | -0.0515 | 0.0298 | 0.0039 | 0.0082 | -1.3514 | 11.0013 | FRED |
| CFNAI | 202 | -3.8724 | 1.8746 | -0.0029 | 0.8480 | -1.4902 | 4.5951 | FRBC |
| new order | 202 | -0.2411 | 0.2459 | 0.0002 | 0.0502 | -0.0897 | 6.6441 | Quandl |
| monetary base | 202 | -0.0241 | 0.2045 | 0.0092 | 0.0171 | 7.9779 | 85.6368 | FRED |
| consumer sentiment | 202 | -0.2134 | 0.2342 | 0.0003 | 0.0676 | 0.0741 | 1.8904 | SCUM |
| read GDP volatility | 202 | 0.0018 | 0.0066 | 0.0034 | 0.0012 | 0.8348 | -0.3658 | FRED* |
| inflation volatility | 202 | 0.0013 | 0.0136 | 0.0030 | 0.0024 | 2.1992 | 4.7509 | FRED* |
| | | | | | | | | |
| *Financial data* | | | | | | | | |
| term spread | 202 | -2.1962 | 3.4691 | 1.3815 | 1.2557 | -0.5641 | -0.1561 | FRED* |
| default spread | 202 | 0.5600 | 3.0233 | 1.0753 | 0.4357 | 1.7872 | 4.0258 | FRED* |
| MKT | 202 | -9.0200 | 7.7667 | 0.9048 | 2.8316 | -0.6646 | 1.0542 | French |
| STR | 202 | -8.6400 | 7.6667 | 0.4666 | 1.8733 | -0.1084 | 4.2576 | French |
| IV | 202 | -0.1320 | 0.2189 | 0.0009 | 0.0506 | 0.8666 | 3.0315 | FRED&Manela |
| RV | 202 | 0.0152 | 2.1566 | 0.1302 | 0.2146 | 6.8839 | 57.0593 | CRSP* |

*Note*: This table reports descriptive statistics for daily returns and quarterly macroeconomic & financial variables, including number of observations (Obs.), minimum (Min.), maximum (Max.), mean (Mean.), standard deviation (Std.), Skewness, (Skew.) and Kurtosis (Kurt.). The database with * indicates that the corresponding variable is calculated by the authors based on the data from the corresponding database.

**TABLE 2** Post-selection estimation results

| | | | | | |
|---|---|---|---|---|---|
| **GARCH parameters and constant estimates** | | | | | |
| $\mu$ | $\alpha$ | $\beta$ | $\gamma$ | $m$ | |
| 0.0118*** | 0.0102** | 0.8980*** | 0.1281*** | -2.3148*** | |
| (0.0031) | (0.0049) | (0.0147) | (0.0205) | (0.1692) | |
| **Parameter estimates for the long-term component** | | | | | |
| $\theta^{UE}$ | $\omega_1^{UE}$ | $\omega_2^{UE}$ | $\theta^{HS}$ | $\omega_1^{HS}$ | $\omega_2^{HS}$ |
| -16.3396** | 1.5840 | 2.2826 | -20.3750*** | 1.7881** | 2.2050 |
| (6.9365) | (1.8950) | (3.4591) | (5.0708) | (0.8913) | (1.7696) |
| $\theta^{PPI}$ | $\omega_1^{PPI}$ | $\omega_2^{PPI}$ | $\theta^{DEF}$ | $\omega_1^{DEF}$ | $\omega_2^{DEF}$ |
| -17.6853*** | 30.6879*** | 44.5625*** | 0.5715*** | -29.7486* | 17.1662 |
| (6.7021) | (8.0499) | (11.1567) | (0.1306) | (17.0166) | (38.6631) |

*Note*: This table reports the post-variable-selection estimation results. The long-term volatility component is given by Equation (10). *UE*, *HS*, *PPI*, and *DEF* indicate unemployment level, housing starts, PPI and default spread. The numbers in parentheses are the robust standard errors, and ***, **, * indicate 1%, 5% and 10% significant levels, respectively.

**TABLE 3** Out-of-sample forecast evaluations

| Models | 1-quarter-ahead MSFE ratio [GW] | 2-quarter-ahead MSFE ratio [GW] | 3-quarter-ahead MSFE ratio [GW] | 4-quarter-ahead MSFE ratio [GW] |
|---|---|---|---|---|
| **Model 1 (Benchmark)** | 1 | 1 | 1 | 1 |
| **Model 2** | | | | |
| real GDP | 1.0716<br>[156.35(-)] | 1.0658<br>[94.15(-)] | 1.0580<br>[60.47(-)] | 1.0461<br>[36.94(-)] |
| industrial production | 1.0883<br>[172.38(-)] | 1.0755<br>[120.70(-)] | 1.0655<br>[99.53(-)] | 1.0527<br>[53.66(-)] |
| unemployment level | 1.1397<br>[385.57(-)] | 1.1353<br>[277.99(-)] | 1.1235<br>[227.94(-)] | 1.1119<br>[115.30(-)] |
| housing starts | 0.9801<br>[98.27(+)] | 0.9749<br>[88.21(+)] | 0.9668<br>[72.14(+)] | 0.9551<br>[51.36(+)] |
| corporate profits | 1.1157<br>[368.35(-)] | 1.1098<br>[250.66(-)] | 1.1014<br>[190.77(-)] | 1.0884<br>[117.74(-)] |
| personal consumption | 1.0776<br>[187.96(-)] | 1.0666<br>[92.02(-)] | 1.0648<br>[63.80(-)] | 1.0504<br>[31.11(-)] |
| CPI | 1.0330<br>[45.58(-)] | 1.0275<br>[33.48(+)] | 1.0156<br>[21.27(-)] | 1.0078<br>[15.30(-)] |
| PPI | 1.0369<br>[83.94(-)] | 1.0319<br>[48.51(-)] | 1.0223<br>[32.37(-)] | 1.0125<br>[18.39(-)] |
| CFNAI | 1.1040<br>[135.05(-)] | 1.1043<br>[75.94(-)] | 1.1046<br>[63.20(-)] | 1.1036<br>[34.58(-)] |
| new order | 1.0680<br>[159.72(-)] | 1.0626<br>[102.94(-)] | 1.0547<br>[105.52(-)] | 1.0423<br>[50.02(-)] |
| monetary base | 1.0672<br>[161.94(-)] | 1.0648<br>[81.64(-)] | 1.0558<br>[51.35(-)] | 1.0454<br>[30.21(-)] |
| consumer sentiment | 1.0186<br>[65.33(-)] | 1.0138<br>[31.04(-)] | 1.0053<br>[24.86(-)] | 0.9939<br>[10.16(+)] |
| read GDP volatility | 1.0650<br>[152.46(-)] | 1.0586<br>[77.84(-)] | 1.0521<br>[49.23(-)] | 1.0403<br>[29.02(-)] |
| inflation volatility | 1.0544<br>[141.19(-)] | 1.0428<br>[56.67(-)] | 1.0373<br>[37.62(-)] | 1.0233<br>[17.79(-)] |
| term spread | 1.0316<br>[99.73(-)] | 1.0265<br>[47.13(-)] | 1.0126<br>[32.44(-)] | 1.0001<br>[16.83(-)] |
| default spread | 1.0380<br>[78.89(-)] | 1.0326<br>[74.25(-)] | 1.0253<br>[67.97(-)] | 1.0128<br>[28.57(-)] |
| MKT | 1.0575<br>[126.22(-)] | 1.0531<br>[64.15(-)] | 1.0493<br>[46.47(-)] | 1.0434<br>[31.75(-)] |
| STR | 1.0530<br>[138.56(-)] | 1.0474<br>[60.69(-)] | 1.0397<br>[37.23(-)] | 1.0276<br>[19.30(-)] |
| IV | 1.0746<br>[22.78(-)] | 1.0773<br>[11.09(-)] | 1.0774<br>[9.96(-)] | 1.0616<br>[5.72(-)] |
| RV | 1.0415<br>[78.59(-)] | 1.0345<br>[26.27(-)] | 1.0314<br>[20.25(-)] | 1.0146<br>[18.77(-)] |
| **Model 3** | 1.3895<br>[155.95(-)] | 1.3179<br>[79.79(-)] | 1.2069<br>[48.33(-)] | 1.1932<br>[32.59(-)] |
| **Model 4** | 1.0666<br>[190.37(-)] | 1.0613<br>[123.50(-)] | 1.0513<br>[36.72(-)] | 1.0440<br>[25.42(-)] |

*Note*: This table reports the out-of-sample forecast evaluation. Each model for one forecast horizon has two numbers: the number in the first row is MSFE ratio relative to the benchmark model, and the numbers in square brackets are the GW statistics. A positive sign in parentheses indicates the corresponding model outperforms Model 1, and a negative sign indicates the opposite. The critical value for GW statistics at 1% and 5% significant level is 9.21 and 5.99, respectively.

**TABLE 4** Subsample analysis

| **GARCH parameters and constant estimates** | | | | | |
|---|---|---|---|---|---|
| $\mu$ | $\alpha$ | $\beta$ | $\gamma$ | $m$ | |
| 0.0108*** | 0.0133*** | 0.9174*** | 0.0998*** | -2.3659*** | |
| (0.0036) | (0.0049) | (0.0151) | (0.0211) | (0.2179) | |
| **Parameter estimates for the long-term component** | | | | | |
| $\theta^{UE}$ | $\omega_1^{UE}$ | $\omega_2^{UE}$ | $\theta^{HS}$ | $\omega_1^{HS}$ | $\omega_2^{HS}$ |
| -17.3074*** | 7.6523 | 2.1373* | -13.2804*** | 3.7790* | 7.0482** |
| (4.7181) | (4.9609) | (1.2374) | (3.3679) | (1.9479) | (3.4016) |
| $\theta^{PPI}$ | $\omega_1^{PPI}$ | $\omega_2^{PPI}$ | $\theta^{DEF}$ | $\omega_1^{DEF}$ | $\omega_2^{DEF}$ |
| -12.6481* | -28.9939*** | 9.2574 | 0.6589*** | 252.0353*** | 187.5867*** |
| (7.4460) | (9.9355) | (10.8838) | (0.1616) | (3.4337) | (4.2685) |

*Note*: This table reports the subsample estimation results from 1968Q1-2007Q4. The long-term component is described as in Equation (10). *UE*, *HS*, *PPI*, and *DEF* indicate unemployment level, housing starts, PPI and default spread. The numbers in parentheses are the robust standard errors, and ***, **, * indicate 1%, 5% and 10% significant levels, respectively.

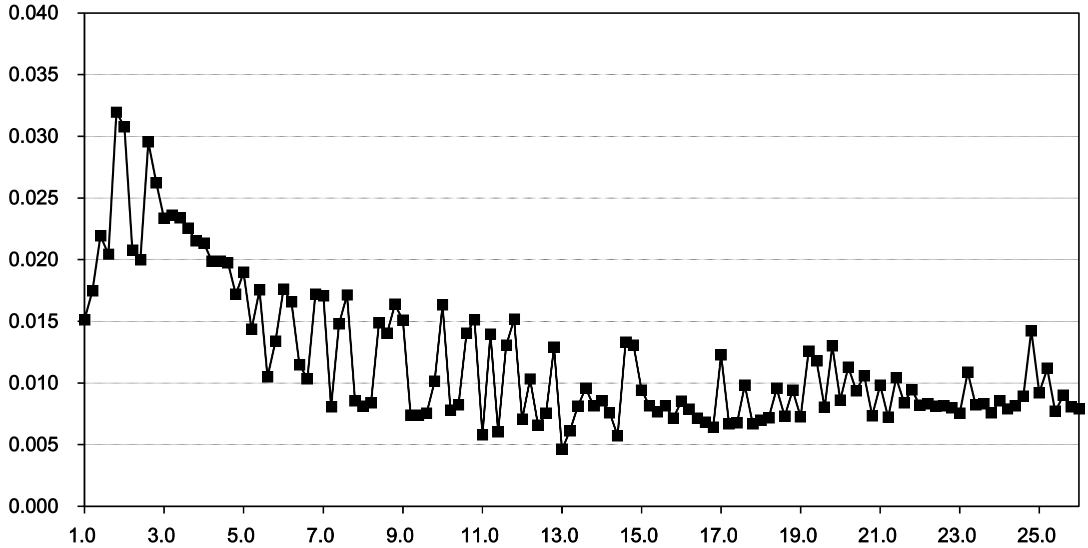


**FIGURE 1** Generalized Information Criteria. *Note*: Figure 1 reports $GIC_{\lambda}$ as a function of the tuning parameter $\lambda$, where $\lambda$ is considered on a 126-point grid on [1, 26] with an increment of 0.2.

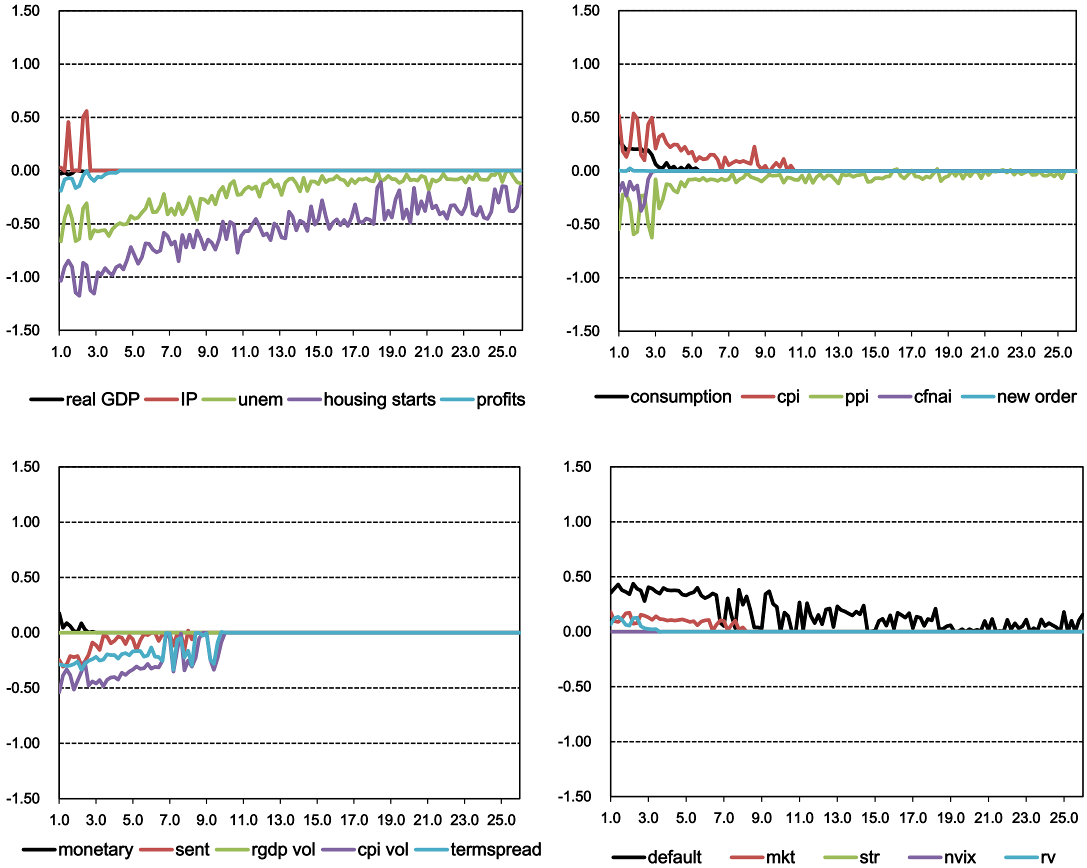


**FIGURE 2** The parameter estimates for each $\lambda$. *Note*: Figure 2 reports the parameter estimates as a function of the turning parameter $\lambda$, where $\lambda$ is considered on a 126-point grid on [1, 26] with an increment of 0.2.

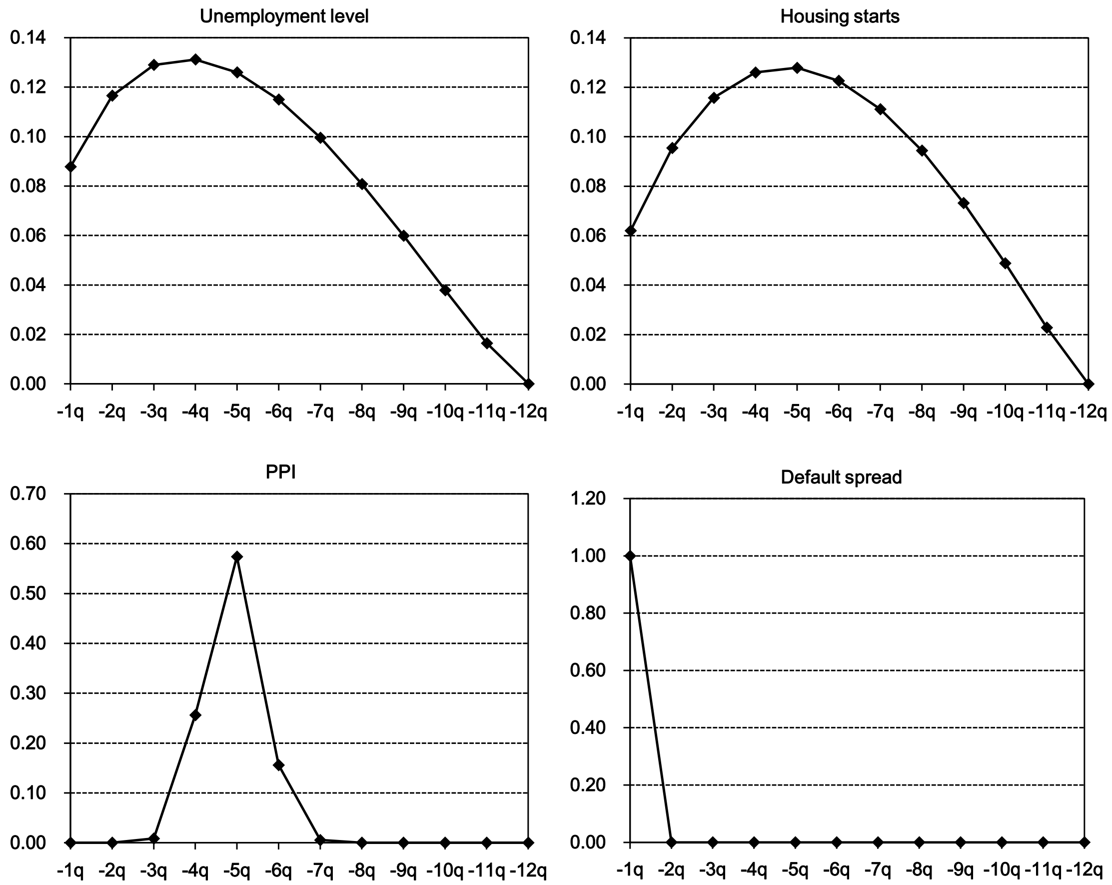


**FIGURE 3** Estimated Beta weights. *Notes*: Figure 3 reports the estimated Beta weights $\varphi_k(\omega_1, \omega_2)$ for the GARCH-MIDAS model with four selected variables, which are unemployment level, housing starts, PPI and default spread.