To Combine Forecasts or to Combine Information?

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SUPPLEMENTARY APPENDIX

- The three tables (Table 1, Table 2, Table 3) that are attached to the paper present the MSFE ratios with respect to benchmarks.

- In this supplementary appendix (not intended for publication), we present the same three tables in MSFE values.

- Also presented in this supplementary appendix are the the MSFE ratios with R=20 for Table 1B and Table 2B.

Table 1. Monte Carlo Simulation (When CI model is the DGP)

This set of tables presents the performance of each forecasting schemes for predicting y_{t+1} out-of-sample where y_t is by DGP:

$$y_{t+1} = x_t\theta + \eta_{t+1},\ \eta_t \sim N(0, \sigma_\eta^2);\ x_{it} = \phi_i x_{it-1} + v_{it},\ v_t \sim N(0, \Omega),\ i=1,2.$$

We report the out-of-sample MSFE of each forecasting scheme where **bolded** term indicates smaller-than-CI case and the smallest number among them is highlighted.

Panel A. No correlation: $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

R=100, P=100

	MSFE						
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	0.3212	0.5065	1.2779	4.3236	16.5485	**65.2474**	**260.9580**
$\hat{y}^{(2)}$	0.3175	0.5109	1.2780	4.3292	16.5535	**65.2571**	**261.0214**
CI	0.0645	0.2570	1.0303	4.1203	16.4661	65.6717	263.5869
CF-RA1	0.0726	0.2809	1.1199	4.4542	17.4272	67.9914	271.8757
CF-RA2	0.1943	0.3874	1.1665	4.2673	16.6933	66.1483	265.0394
CF-RA3(κ=0)	0.0758	0.2828	1.1084	4.3523	16.9332	66.2953	265.2075
CF-RA3(κ=1)	0.0755	0.2818	1.1044	4.3371	16.8824	66.1349	264.5856
CF-RA3(κ=3)	0.0765	0.2811	1.0978	4.3090	16.7875	65.8398	**263.4426**
CF-RA3(κ=5)	0.0792	0.2823	1.0932	4.2835	16.7018	**65.5790**	**262.4345**
CF-RA3(κ=7)	0.0838	0.2853	1.0905	4.2609	16.6251	**65.3525**	**261.5611**
CF-RA3(κ=9)	0.0902	0.2901	1.0898	4.2410	16.5574	**65.1602**	**260.8224**
CF-RA3(κ=11)	0.0984	0.2967	1.0910	4.2238	16.4988	**65.0021**	**260.2185**
CF-RA3(κ=13)	0.1084	0.3052	1.0942	4.2094	**16.4493**	**64.8784**	**259.7493**
CF-RA3(κ=15)	0.1202	0.3154	1.0993	4.1978	**16.4088**	**64.7889**	**259.4148**
CF-RA3(κ=17)	0.1338	0.3275	1.1064	4.1889	**16.3774**	**64.7336**	**259.2152**
CF-RA3(κ=19)	0.1493	0.3415	1.1154	4.1828	**16.3551**	**64.7127**	**259.1502**
CF-Mean	0.1906	0.3794	1.1427	4.1790	**16.3380**	**64.7908**	**259.4968**

R=1000, P=100

$\hat{y}^{(1)}$	0.3204	0.5195	1.2839	4.2442	16.1167	65.1842	259.6659
$\hat{y}^{(2)}$	0.3070	0.5046	1.2499	4.2812	16.0670	64.9899	**259.3602**
CI	0.0633	0.2533	1.0134	4.0142	15.8976	64.8558	259.4233
CF-RA1	0.0640	0.2552	1.0211	4.0422	16.0124	65.2443	261.2757
CF-RA2	0.1868	0.3849	1.1407	4.1452	15.9879	65.0286	259.7414
CF-RA3(κ=0)	0.0644	0.2550	1.0214	4.0428	15.9915	65.0977	259.9152
CF-RA3(κ=1)	0.0644	0.2550	1.0214	4.0427	15.9908	65.0963	259.9095
CF-RA3(κ=28)	0.0662	0.2567	1.0232	4.0416	15.9748	65.0588	259.7650
CF-RA3(κ=55)	0.0708	0.2615	1.0277	4.0433	15.9619	65.0258	259.6381
CF-RA3(κ=82)	0.0783	0.2693	1.0348	4.0475	15.9523	64.9972	259.5290
CF-RA3(κ=109)	0.0886	0.2801	1.0447	4.0545	15.9459	64.9732	259.4376
CF-RA3(κ=136)	0.1016	0.2939	1.0572	4.0641	15.9427	64.9536	**259.3639**
CF-RA3(κ=163)	0.1176	0.3107	1.0724	4.0765	15.9427	64.9385	**259.3078**
CF-RA3(κ=190)	0.1363	0.3306	1.0903	4.0914	15.9459	64.9279	**259.2695**
CF-RA3(κ=217)	0.1578	0.3534	1.1109	4.1091	15.9524	64.9217	**259.2490**
CF-RA3(κ=244)	0.1822	0.3793	1.1342	4.1295	15.9620	64.9200	**259.2461**
CF-Mean	0.1865	0.3839	1.1384	4.1331	15.9639	64.9202	**259.2473**

Panel B. High correlation: $\Omega = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

R=100, P=100	MSFE						
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	0.1567	0.3446	1.1135	4.1657	**16.3810**	**65.0883**	**260.8334**
$\hat{y}^{(2)}$	0.1546	0.3481	1.1115	4.1746	**16.3699**	**65.0993**	**260.9117**
CI	0.0645	0.2570	1.0303	4.1203	16.4661	65.6717	263.5869
CF-RA1	0.0683	0.2716	1.0901	4.3554	17.4423	68.3369	272.4264
CF-RA2	0.0746	0.2687	1.0485	4.1650	16.6261	66.2329	265.9007
CF-RA3(κ=0)	0.0671	0.2664	1.0670	4.2615	16.9531	66.4927	265.4442
CF-RA3(κ=1)	0.0668	0.2655	1.0632	4.2468	16.8954	66.3195	264.8255
CF-RA3(κ=3)	0.0665	0.2638	1.0562	4.2192	16.7876	66.0012	263.6945
CF-RA3(κ=5)	0.0663	0.2625	1.0500	4.1942	16.6898	65.7203	**262.7054**
CF-RA3(κ=7)	0.0663	0.2614	1.0445	4.1716	16.6021	**65.4767**	**261.8583**
CF-RA3(κ=9)	0.0665	0.2606	1.0397	4.1515	16.5245	**65.2706**	**261.1533**
CF-RA3(κ=11)	0.0669	0.2601	1.0357	4.1340	**16.4569**	**65.1018**	**260.5901**
CF-RA3(κ=13)	0.0674	0.2598	1.0324	**4.1189**	**16.3993**	**64.9704**	**260.1690**
CF-RA3(κ=15)	0.0681	0.2599	**1.0299**	**4.1064**	**16.3519**	**64.8763**	**259.8899**
CF-RA3(κ=17)	0.0690	0.2602	**1.0282**	**4.0963**	**16.3145**	**64.8197**	**259.7527**
CF-RA3(κ=19)	0.0701	0.2608	**1.0272**	**4.0888**	**16.2871**	**64.8004**	**259.7576**
CF-Mean	0.0731	0.2633	**1.0276**	**4.0810**	**16.2623**	**64.8937**	**260.2875**
R=1000, P=100							
$\hat{y}^{(1)}$	0.1570	0.3511	1.0880	4.0553	15.8496	64.9599	254.3646
$\hat{y}^{(2)}$	0.1506	0.3409	1.0995	4.0564	15.8850	**64.7449**	**253.6977**
CI	0.0633	0.2533	1.0035	3.9744	15.8032	64.8558	254.2158
CF-RA1	0.0637	0.2546	1.0087	3.9966	15.8632	65.1936	255.5728
CF-RA2	0.0717	0.2634	1.0144	3.9852	15.8065	**64.8528**	254.2225
CF-RA3(κ=0)	0.0636	0.2541	1.0073	3.9908	15.8524	65.0059	254.3513
CF-RA3(κ=1)	0.0636	0.2541	1.0073	3.9907	15.8519	65.0040	254.3454
CF-RA3(κ=28)	0.0637	0.2540	1.0066	3.9866	15.8389	64.9566	**254.1977**
CF-RA3(κ=55)	0.0639	0.2541	1.0063	3.9832	15.8273	64.9141	**254.0747**
CF-RA3(κ=82)	0.0644	0.2546	1.0062	3.9803	15.8170	64.8765	**253.9763**
CF-RA3(κ=109)	0.0651	0.2552	1.0063	3.9781	15.8081	**64.8438**	**253.9026**
CF-RA3(κ=136)	0.0659	0.2561	1.0067	3.9765	**15.8005**	**64.8160**	**253.8535**
CF-RA3(κ=163)	0.0670	0.2573	1.0074	3.9755	**15.7943**	**64.7931**	**253.8291**
CF-RA3(κ=190)	0.0682	0.2587	1.0083	3.9751	**15.7894**	**64.7751**	**253.8294**
CF-RA3(κ=217)	0.0697	0.2603	1.0095	3.9753	**15.7859**	**64.7621**	**253.8543**
CF-RA3(κ=244)	0.0713	0.2622	1.0110	3.9761	**15.7838**	**64.7539**	**253.9038**
CF-Mean	0.0716	0.2626	1.0113	3.9763	**15.7836**	**64.7531**	**253.9145**

Table 2. Monte Carlo Simulation (When CI model is not the DGP)

This set of tables presents the performance of each forecasting schemes for predicting y_{t+1} out-of-sample where y_t is by DGP:

$$y_{t+1} = x_t\theta + \eta_{t+1},\ \eta_t \sim N(0, \sigma_\eta^2);\ x_{it} = \phi_i x_{it-1} + v_{it},\ v_t \sim N(0, \Omega),\ i=1,2,3.$$

Variable x_{3t} is omitted in each CF and CI schemes.

Panel A. High positive correlations with the omitted variable: $\Omega = \begin{pmatrix} 1 & 0.6 & 0.7 \\ 0.6 & 1 & 0.7 \\ 0.7 & 0.7 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.6 \end{pmatrix}$

R=100, P=100

	MSFE						
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	0.4155	0.6049	1.3705	4.4409	16.6012	**65.3339**	**261.0863**
$\hat{y}^{(2)}$	0.4142	0.6073	1.3695	4.4218	16.6221	**65.2490**	**261.4212**
CI	0.2096	0.4019	1.1749	4.2677	16.5824	65.7799	263.8096
CF-RA1	0.2227	0.4268	1.2487	4.5303	17.6108	68.6328	272.4172
CF-RA2	0.2550	0.4484	1.2271	4.3439	16.7560	66.2857	265.6074
CF-RA3(κ=0)	0.2192	0.4191	1.2249	4.4365	17.1255	66.7912	265.7039
CF-RA3(κ=1)	0.2184	0.4176	1.2204	4.4208	17.0674	66.6093	265.0811
CF-RA3(κ=3)	0.2175	0.4153	1.2122	4.3916	16.9585	66.2726	263.9394
CF-RA3(κ=5)	0.2173	0.4138	1.2054	4.3653	16.8590	65.9722	**262.9361**
CF-RA3(κ=7)	0.2178	0.4131	1.1997	4.3419	16.7692	**65.7081**	**262.0711**
CF-RA3(κ=9)	0.2190	0.4132	1.1954	4.3213	16.6889	**65.4803**	**261.3446**
CF-RA3(κ=11)	0.2210	0.4142	1.1923	4.3036	16.6181	**65.2887**	**260.7564**
CF-RA3(κ=13)	0.2237	0.4160	1.1905	4.2889	**16.5568**	**65.1335**	**260.3067**
CF-RA3(κ=15)	0.2272	0.4187	1.1900	4.2770	**16.5051**	**65.0145**	**259.9953**
CF-RA3(κ=17)	0.2313	0.4221	1.1907	4.2679	**16.4630**	**64.9318**	**259.8223**
CF-RA3(κ=19)	0.2362	0.4264	1.1927	**4.2618**	**16.4304**	**64.8853**	**259.7877**
CF-Mean	0.2500	0.4392	1.2018	**4.2585**	**16.3919**	**64.9134**	**260.2158**
R=1000, P=100							
$\hat{y}^{(1)}$	0.4106	0.6105	1.3208	4.3493	16.7151	65.1866	**258.5414**
$\hat{y}^{(2)}$	0.3987	0.6074	1.3284	4.3789	16.7404	65.2534	**258.2385**
CI	0.1989	0.3982	1.1293	4.1612	16.5457	65.0273	258.5911
CF-RA1	0.1998	0.4013	1.1341	4.1828	16.6283	65.3929	259.0070
CF-RA2	0.2405	0.4454	1.1638	4.1933	16.5904	65.1692	**258.3221**
CF-RA3(κ=0)	0.1994	0.4000	1.1340	4.1718	16.5957	65.2727	**258.2012**
CF-RA3(κ=1)	0.1994	0.4000	1.1339	4.1717	16.5951	65.2705	**258.1976**
CF-RA3(κ=28)	0.1997	0.4006	1.1325	4.1685	16.5823	65.2147	**258.1107**
CF-RA3(κ=55)	0.2010	0.4022	1.1321	4.1666	16.5717	65.1659	**258.0438**
CF-RA3(κ=82)	0.2034	0.4048	1.1328	4.1661	16.5634	65.1240	**257.9969**
CF-RA3(κ=109)	0.2067	0.4085	1.1346	4.1668	16.5574	65.0891	**257.9702**
CF-RA3(κ=136)	0.2110	0.4133	1.1374	4.1689	16.5537	65.0611	**257.9635**
CF-RA3(κ=163)	0.2164	0.4191	1.1414	4.1722	16.5523	65.0401	**257.9768**
CF-RA3(κ=190)	0.2227	0.4260	1.1463	4.1768	16.5532	**65.0260**	**258.0103**
CF-RA3(κ=217)	0.2300	0.4339	1.1524	4.1828	16.5564	**65.0189**	**258.0638**
CF-RA3(κ=244)	0.2383	0.4429	1.1595	4.1900	16.5619	**65.0187**	**258.1374**
CF-Mean	0.2398	0.4445	1.1608	4.1913	16.5630	**65.0193**	**258.1516**

Panel B. High negative correlations with the omitted variable: $\Omega = \begin{pmatrix} 1 & 0.6 & -0.7 \\ 0.6 & 1 & -0.7 \\ -0.7 & -0.7 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.6 \end{pmatrix}$

R=100, P=100				MSFE			
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	**0.2085**	**0.3985**	**1.1634**	**4.2275**	**16.4115**	**65.1202**	**260.9238**
$\hat{y}^{(2)}$	**0.2084**	**0.3987**	**1.1630**	**4.2249**	**16.4105**	**65.0630**	**261.1915**
CI	0.2096	0.4019	1.1749	4.2677	16.5824	65.7799	263.8096
CF-RA1	0.2203	0.4178	1.2134	4.3926	17.0599	67.5417	271.2208
CF-RA2	0.2113	0.4050	1.1832	4.2983	16.6963	66.2598	265.6216
CF-RA3(κ=0)	0.2136	0.4076	1.1822	4.2870	16.6731	66.0403	264.8367
CF-RA3(κ=1)	0.2130	0.4065	1.1794	4.2776	16.6348	65.8960	264.2556
CF-RA3(κ=3)	0.2119	0.4045	**1.1744**	**4.2602**	**16.5647**	**65.6325**	**263.1955**
CF-RA3(κ=5)	0.2109	0.4027	**1.1699**	**4.2452**	**16.5035**	**65.4028**	**262.2718**
CF-RA3(κ=7)	0.2100	**0.4011**	**1.1661**	**4.2323**	**16.4511**	**65.2068**	**261.4843**
CF-RA3(κ=9)	**0.2092**	**0.3997**	**1.1630**	**4.2217**	**16.4075**	**65.0445**	**260.8331**
CF-RA3(κ=11)	**0.2085**	**0.3986**	**1.1604**	**4.2134**	**16.3727**	**64.9158**	**260.3182**
CF-RA3(κ=13)	**0.2080**	**0.3977**	**1.1585**	**4.2072**	**16.3467**	**64.8208**	**259.9396**
CF-RA3(κ=15)	**0.2075**	**0.3970**	**1.1572**	**4.2034**	**16.3296**	**64.7596**	**259.6972**
CF-RA3(κ=17)	**0.2072**	**0.3965**	**1.1565**	**4.2017**	**16.3213**	**64.7320**	**259.5912**
CF-RA3(κ=19)	**0.2070**	**0.3963**	**1.1564**	**4.2023**	**16.3217**	**64.7381**	**259.6214**
CF-Mean	**0.2070**	**0.3966**	**1.1586**	**4.2118**	**16.3550**	**64.8750**	**260.1877**
R=1000, P=100							
$\hat{y}^{(1)}$	0.2078	0.4014	1.1257	4.1023	**16.3682**	**64.9381**	**256.5352**
$\hat{y}^{(2)}$	0.2075	0.4015	**1.1232**	4.1043	**16.3612**	**64.9238**	**256.4619**
CI	0.2070	0.4009	1.1252	4.1015	16.3741	64.9805	256.7990
CF-RA1	0.2080	0.4033	1.1315	4.1310	16.4196	65.2107	257.5531
CF-RA2	0.2074	0.4015	1.1265	4.1073	16.3930	65.0317	256.8528
CF-RA3(κ=0)	0.2078	0.4025	1.1288	4.1168	**16.3688**	65.0926	257.0924
CF-RA3(κ=1)	0.2078	0.4025	1.1287	4.1167	**16.3685**	65.0909	257.0861
CF-RA3(κ=28)	0.2076	0.4022	1.1276	4.1135	**16.3623**	65.0490	256.9270
CF-RA3(κ=55)	0.2074	0.4019	1.1267	4.1107	**16.3573**	65.0126	**256.7891**
CF-RA3(κ=82)	0.2073	0.4016	1.1258	4.1082	**16.3536**	64.9816	**256.6724**
CF-RA3(κ=109)	0.2072	0.4013	**1.1251**	4.1060	**16.3512**	**64.9560**	**256.5769**
CF-RA3(κ=136)	0.2071	0.4011	**1.1245**	4.1043	**16.3501**	**64.9359**	**256.5025**
CF-RA3(κ=163)	0.2070	0.4010	**1.1240**	4.1029	**16.3502**	**64.9213**	**256.4494**
CF-RA3(κ=190)	**0.2069**	**0.4008**	**1.1237**	4.1018	**16.3516**	**64.9121**	**256.4175**
CF-RA3(κ=217)	**0.2069**	**0.4008**	**1.1234**	**4.1012**	**16.3543**	**64.9084**	**256.4068**
CF-RA3(κ=244)	**0.2069**	**0.4007**	**1.1233**	**4.1009**	**16.3583**	**64.9102**	**256.4172**
CF-Mean	**0.2069**	**0.4007**	**1.1233**	**4.1008**	**16.3591**	**64.9110**	**256.4211**

Panel C. High negative correlations with the omitted variable and relatively small θ_3 :

$$\Omega = \begin{pmatrix} 1 & 0.6 & -0.7 \\ 0.6 & 1 & -0.7 \\ -0.7 & -0.7 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.2 \end{pmatrix}$$

R=100, P=100				MSFE			
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	0.1102	0.3000	1.0663	**4.1313**	**16.3078**	**65.0340**	**260.8373**
$\hat{y}^{(2)}$	0.1095	0.3004	1.0660	**4.1228**	**16.3132**	**64.9694**	**261.1324**
CI	0.0808	0.2734	1.0474	4.1390	16.4541	65.6615	263.7154
CF-RA1	0.0857	0.2901	1.1148	4.3701	17.1044	67.5732	271.3472
CF-RA2	0.0879	0.2816	1.0620	4.1768	16.5772	66.1556	265.5008
CF-RA3(κ=0)	0.0842	0.2845	1.0889	4.2406	16.6664	66.0464	264.8405
CF-RA3(κ=1)	0.0839	0.2835	1.0849	4.2276	16.6227	65.8957	264.2558
CF-RA3(κ=3)	0.0834	0.2816	1.0774	4.2034	16.5419	**65.6198**	**263.1885**
CF-RA3(κ=5)	0.0830	0.2800	1.0706	4.1816	16.4701	**65.3781**	**262.2572**
CF-RA3(κ=7)	0.0828	0.2786	1.0644	4.1621	**16.4073**	**65.1706**	**261.4620**
CF-RA3(κ=9)	0.0828	0.2774	1.0589	4.1450	**16.3535**	**64.9973**	**260.8028**
CF-RA3(κ=11)	0.0828	0.2765	1.0541	**4.1303**	**16.3087**	**64.8582**	**260.2795**
CF-RA3(κ=13)	0.0830	0.2758	1.0500	**4.1180**	**16.2729**	**64.7534**	**259.8924**
CF-RA3(κ=15)	0.0833	0.2753	**1.0465**	**4.1080**	**16.2460**	**64.6827**	**259.6412**
CF-RA3(κ=17)	0.0837	0.2750	**1.0437**	**4.1004**	**16.2282**	**64.6462**	**259.5261**
CF-RA3(κ=19)	0.0843	0.2750	**1.0415**	**4.0951**	**16.2193**	**64.6439**	**259.5470**
CF-Mean	0.0861	0.2758	**1.0391**	**4.0920**	**16.2323**	**64.7639**	**260.0913**
R=1000, P=100							
$\hat{y}^{(1)}$	0.1085	0.2995	1.0481	4.0179	**15.8167**	**62.6219**	**253.9382**
$\hat{y}^{(2)}$	0.1080	0.2996	1.0363	4.0201	15.8338	62.7286	**253.8902**
CI	0.0795	0.2706	1.0130	3.9834	15.8218	62.7086	253.9963
CF-RA1	0.0801	0.2723	1.0167	4.0121	15.8992	62.8682	254.5946
CF-RA2	0.0854	0.2771	1.0202	4.0014	15.8399	62.7460	254.2916
CF-RA3(κ=0)	0.0800	0.2717	1.0154	4.0075	15.8663	**62.7004**	254.1153
CF-RA3(κ=1)	0.0800	0.2717	1.0154	4.0074	15.8658	**62.6992**	254.1103
CF-RA3(κ=28)	0.0800	0.2716	1.0148	4.0037	15.8536	**62.6696**	**253.9863**
CF-RA3(κ=55)	0.0801	0.2716	1.0144	4.0006	15.8426	**62.6455**	**253.8848**
CF-RA3(κ=82)	0.0804	0.2718	1.0142	3.9980	15.8327	**62.6268**	**253.8057**
CF-RA3(κ=109)	0.0808	0.2722	1.0142	3.9958	15.8241	**62.6137**	**253.7492**
CF-RA3(κ=136)	0.0814	0.2727	1.0144	3.9941	**15.8167**	**62.6060**	**253.7152**
CF-RA3(κ=163)	0.0821	0.2734	1.0149	3.9930	**15.8105**	**62.6037**	**253.7037**
CF-RA3(κ=190)	0.0829	0.2742	1.0156	3.9923	**15.8054**	**62.6070**	**253.7147**
CF-RA3(κ=217)	0.0839	0.2751	1.0165	3.9921	**15.8016**	**62.6157**	**253.7482**
CF-RA3(κ=244)	0.0850	0.2763	1.0176	3.9924	**15.7990**	**62.6298**	**253.8041**
CF-Mean	0.0852	0.2765	1.0178	3.9925	**15.7986**	**62.6327**	**253.8157**

Panel D. High negative correlations with the omitted variable and $\theta_1 = 3\theta_2$:

$$\Omega = \begin{pmatrix} 1 & 0.6 & -0.7 \\ 0.6 & 1 & -0.7 \\ -0.7 & -0.7 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.6 \\ 0.2 \\ 0.6 \end{pmatrix}$$

R=100, P=100

	MSFE						
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	0.2099	**0.4003**	**1.1647**	**4.2274**	**16.4162**	**65.1204**	**260.9338**
$\hat{y}^{(2)}$	0.2821	0.4733	1.2370	4.2932	**16.4876**	**65.1252**	**261.2787**
CI	0.2096	0.4019	1.1749	4.2677	16.5824	65.7799	263.8096
CF-RA1	0.2201	0.4226	1.2354	4.4638	17.1673	67.6151	271.4548
CF-RA2	0.2140	0.4087	1.1914	4.3153	16.7314	66.2993	265.6514
CF-RA3(κ=0)	0.2147	0.4118	1.2022	4.3394	16.7527	66.1215	264.9521
CF-RA3(κ=1)	0.2140	0.4105	1.1984	4.3277	16.7114	65.9732	264.3680
CF-RA3(κ=3)	0.2131	0.4082	1.1916	4.3062	16.6356	**65.7022**	**263.3020**
CF-RA3(κ=5)	0.2126	0.4066	1.1858	4.2872	**16.5689**	**65.4653**	**262.3721**
CF-RA3(κ=7)	0.2126	0.4056	1.1811	4.2709	**16.5112**	**65.2625**	**261.5784**
CF-RA3(κ=9)	0.2130	0.4052	1.1774	**4.2570**	**16.4626**	**65.0939**	**260.9207**
CF-RA3(κ=11)	0.2139	0.4054	**1.1748**	**4.2457**	**16.4230**	**64.9594**	**260.3993**
CF-RA3(κ=13)	0.2153	0.4061	**1.1732**	**4.2370**	**16.3925**	**64.8590**	**260.0139**
CF-RA3(κ=15)	0.2172	0.4075	**1.1726**	**4.2308**	**16.3710**	**64.7928**	**259.7647**
CF-RA3(κ=17)	0.2195	0.4095	**1.1730**	**4.2272**	**16.3586**	**64.7607**	**259.6516**
CF-RA3(κ=19)	0.2223	0.4121	**1.1745**	**4.2261**	**16.3552**	**64.7627**	**259.6746**
CF-Mean	0.2302	0.4202	1.1817	**4.2330**	**16.3806**	**64.8921**	**260.2241**

R=1000, P=100

$\hat{y}^{(1)}$	0.2091	0.4033	**1.1243**	**4.1011**	**15.9452**	**62.7506**	**253.9645**
$\hat{y}^{(2)}$	0.2791	0.4755	1.1892	4.1891	16.0205	62.9119	**254.0028**
CI	0.2070	0.4009	1.1252	4.1015	15.9636	62.8475	254.0922
CF-RA1	0.2077	0.4026	1.1290	4.1275	16.0387	62.9950	254.7673
CF-RA2	0.2090	0.4034	1.1281	4.1100	15.9755	62.9239	254.3503
CF-RA3(κ=0)	0.2074	0.4019	1.1279	4.1182	15.9911	**62.8152**	254.1649
CF-RA3(κ=1)	0.2074	0.4019	1.1278	4.1181	15.9907	**62.8142**	254.1598
CF-RA3(κ=28)	0.2075	0.4020	1.1266	4.1144	15.9806	**62.7900**	**254.0342**
CF-RA3(κ=55)	0.2081	0.4027	1.1261	4.1119	15.9723	**62.7713**	**253.9322**
CF-RA3(κ=82)	0.2093	0.4039	1.1262	4.1105	15.9658	**62.7579**	**253.8539**
CF-RA3(κ=109)	0.2110	0.4058	1.1270	4.1103	**15.9610**	**62.7500**	**253.7991**
CF-RA3(κ=136)	0.2134	0.4082	1.1284	4.1113	**15.9580**	**62.7476**	**253.7679**
CF-RA3(κ=163)	0.2163	0.4111	1.1305	4.1134	**15.9567**	**62.7505**	**253.7602**
CF-RA3(κ=190)	0.2198	0.4147	1.1332	4.1167	**15.9573**	**62.7589**	**253.7762**
CF-RA3(κ=217)	0.2238	0.4189	1.1366	4.1211	**15.9595**	**62.7728**	**253.8158**
CF-RA3(κ=244)	0.2285	0.4236	1.1407	4.1267	**15.9636**	**62.7921**	**253.8789**
CF-Mean	0.2293	0.4244	1.1414	4.1278	15.9644	**62.7958**	**253.8917**

Table 3. Equity Premium Prediction

Note: Data range from 1927m1 to 2003m12; "kmax", the maximum hypothesized number of factors, is set at 12; "*h*" is the forecast horizon; MSFE is the raw MSFE amplified by 100; MSFE Ratio is the MSFE of each method over that of the Historical Mean model; "*k*" is the number of factors included in the principal component approaches; "Mean/SD" is the mean and standard deviation of the estimated number of factors over the out-of-sample. The case when Historical Mean benchmark is outperformed is indicated in **bold**, and the smallest number among them is highlighted.

Panel A1. Monthly prediction, forecasts begin 1969m1 (*R*=504 and *P*=420)

	h=1			*h*=3			*h*=6			*h*=12		
	MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio	
Historical Mean	0.0407			0.0407			0.0407			0.0407		
CF-Mean	**0.0400**	**0.9820**		**0.0401**	**0.9860**		**0.0403**	**0.9890**		**0.0403**	**0.9891**	
CF-Median	**0.0402**	**0.9887**		**0.0404**	**0.9915**		**0.0404**	**0.9913**		**0.0404**	**0.9904**	
CF-RA1	0.0431	1.0585		0.0434	1.0660		0.0420	1.0325		0.0471	1.1548	
CF-RA2	0.0447	1.0975		0.0441	1.0847		0.0429	1.0538		0.0457	1.1225	
CF-RA3 (κ=0)	0.0439	1.0795		0.0430	1.0581		0.0419	1.0310		0.0457	1.1240	
CF-RA3 (κ=1)	0.0434	1.0670		0.0427	1.0487		0.0417	1.0250		0.0452	1.1116	
CF-RA3 (κ=3)	0.0425	1.0443		0.0420	1.0317		0.0413	1.0141		0.0443	1.0889	
CF-RA3 (κ=5)	0.0417	1.0248		0.0414	1.0172		0.0409	1.0049		0.0435	1.0684	
CF-RA3 (κ=7)	0.0410	1.0086		0.0409	1.0052		**0.0406**	**0.9974**		0.0427	1.0503	
CF-RA3 (κ=9)	**0.0405**	**0.9956**		**0.0405**	**0.9956**		**0.0403**	**0.9916**		0.0421	1.0346	
CF-RA3 (κ=11)	**0.0401**	**0.9859**		**0.0402**	**0.9884**		**0.0402**	**0.9875**		0.0416	1.0213	
CF-RA3 (κ=13)	**0.0398**	**0.9794**		**0.0400**	**0.9837**		**0.0401**	**0.9851**		0.0411	1.0103	
CF-RA3 (κ=15)	**0.0397**	**0.9762**	Mean/SD	**0.0399**	**0.9815**	Mean/SD	**0.0401**	**0.9844**	Mean/SD	0.0408	1.0017	Mean/SD
CF-PC (AIC)	0.0424	1.0429	9.13/3.26	0.0435	1.0697	8.62/3.45	0.0422	1.0363	4.74/4.23	0.0414	1.0158	1.90/2.45
CF-PC (BIC)	**0.0400**	**0.9828**	1.30/1.06	**0.0405**	**0.9962**	1.14/0.49	0.0408	1.0029	1.18/0.42	**0.0407**	**0.9993**	1.06/0.24
CF-PC (*k*=1)	**0.0401**	**0.9858**		**0.0403**	**0.9903**		**0.0407**	**0.9989**		0.0409	1.0049	
CF-PC (*k*=2)	**0.0399**	**0.9801**		**0.0405**	**0.9953**		0.0407	1.0000		**0.0407**	**0.9995**	
CF-PC (*k*=3)	**0.0403**	**0.9912**		0.0410	1.0076		0.0411	1.0090		0.0410	1.0065	
CI-Unrestricted	0.0411	1.0103		0.0434	1.0661		0.0424	1.0400		0.0436	1.0712	
CI-PC (AIC)	0.0413	1.0142	8.70/2.18	0.0429	1.0537	7.47/2.49	0.0434	1.0655	6.22/2.82	0.0413	1.0147	2.35/0.84
CI-PC (BIC)	0.0428	1.0523	3.29/1.85	0.0434	1.0655	2.48/1.39	0.0427	1.0478	1.92/0.99	0.0410	1.0071	1.38/0.63
CI-PC (*k*=1)	**0.0407**	**0.9998**		0.0407	1.0009		**0.0407**	**0.9996**		**0.0405**	**0.9934**	
CI-PC (*k*=2)	0.0409	1.0060		0.0413	1.0151		0.0413	1.0134		**0.0405**	**0.9944**	
CI-PC (*k*=3)	0.0434	1.0673		0.0440	1.0805		0.0432	1.0612		0.0412	1.0115	

Panel A2. Monthly prediction, forecasts begin 1980m1 (R=636 and P=288)

	h=1			h=3			h=6			h=12		
	MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio	
Historical Mean	0.0398			0.0398			0.0398			0.0398		
CF-Mean	**0.0395**	**0.9938**		**0.0397**	**0.9980**		**0.0397**	**0.9981**		**0.0398**	**0.9995**	
CF-Median	**0.0398**	**0.9993**		0.0399	1.0023		**0.0397**	**0.9986**		0.0399	1.0026	
CF-RA1	0.0422	1.0606		0.0412	1.0361		0.0433	1.0873		0.0424	1.0649	
CF-RA2	0.0421	1.0590		0.0423	1.0637		0.0430	1.0811		0.0436	1.0946	
CF-RA3 (κ=0)	0.0431	1.0821		0.0422	1.0605		0.0442	1.1108		0.0425	1.0690	
CF-RA3 (κ=1)	0.0427	1.0741		0.0420	1.0547		0.0438	1.1008		0.0423	1.0642	
CF-RA3 (κ=4)	0.0419	1.0523		0.0413	1.0389		0.0427	1.0734		0.0418	1.0509	
CF-RA3 (κ=7)	0.0411	1.0338		0.0408	1.0256		0.0418	1.0501		0.0413	1.0391	
CF-RA3 (κ=10)	0.0405	1.0187		0.0404	1.0147		0.0410	1.0310		0.0409	1.0288	
CF-RA3 (κ=13)	0.0401	1.0069		0.0400	1.0063		0.0404	1.0161		0.0406	1.0200	
CF-RA3 (κ=16)	**0.0397**	**0.9985**		0.0398	1.0005		0.0400	1.0053		0.0403	1.0128	
CF-RA3 (κ=19)	**0.0395**	**0.9935**		**0.0397**	**0.9970**		**0.0397**	**0.9986**		0.0401	1.0071	
CF-RA3 (κ=22)	**0.0395**	**0.9917**	Mean/SD	**0.0396**	**0.9961**	Mean/SD	**0.0396**	**0.9961**	Mean/SD	0.0399	1.0029	Mean/SD
CF-PC (AIC)	0.0427	1.0741	10.33/3.27	0.0408	1.0251	8.74/3.98	0.0430	1.0815	9.33/3.95	0.0406	1.0198	4.26/4.55
CF-PC (BIC)	**0.0395**	**0.9937**	1.30/0.77	0.0400	1.0063	1.02/0.14	0.0402	1.0104	1.02/0.13	0.0405	1.0161	1/0
CF-PC (k=1)	**0.0394**	**0.9896**		0.0399	1.0038		0.0402	1.0089		0.0405	1.0161	
CF-PC (k=2)	**0.0395**	**0.9918**		0.0402	1.0091		0.0404	1.0154		0.0404	1.0148	
CF-PC (k=3)	**0.0396**	**0.9960**		0.0401	1.0086		0.0404	1.0150		0.0406	1.0200	
CI-Unrestricted	0.0421	1.0592		0.0451	1.1344		0.0419	1.0525		0.0418	1.0495	
CI-PC (AIC)	0.0419	1.0522	8.63/1.87	0.0449	1.1274	7.68/2.12	0.0422	1.0607	6.95/2.53	0.0406	1.0197	2.68/1.14
CI-PC (BIC)	0.0423	1.0639	3.02/1.72	0.0421	1.0578	2.35/1.31	0.0406	1.0199	1.64/1.08	0.0413	1.0376	1.56/0.72
CI-PC (k=1)	0.0403	1.0131		0.0404	1.0150		0.0406	1.0200		0.0406	1.0194	
CI-PC (k=2)	0.0405	1.0175		0.0408	1.0251		0.0409	1.0274		0.0411	1.0315	
CI-PC (k=3)	0.0422	1.0617		0.0423	1.0623		0.0421	1.0575		0.0413	1.0376	

Panel B. Quarterly prediction

	Forecasts begin 1969q1 (R=168 and P=140)						Forecasts begin 1980q1 (R=212 and P=96)					
	h=1			h=4			h=1			h=4		
	MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio		MSFE	MSFE Ratio	
Historical Mean	0.1518			0.1521			0.1346			0.1347		
CF-Mean	**0.1455**	**0.9589**		**0.1486**	**0.9768**		**0.1332**	**0.9899**		0.1356	1.0071	
CF-Median	**0.1471**	**0.9689**		**0.1495**	**0.9831**		**0.1345**	**0.9992**		0.1370	1.0172	
CF-RA1	0.1888	1.2436		0.2655	1.7457		0.1766	1.3127		0.1692	1.2568	
CF-RA2	0.2116	1.3942		0.2510	1.6537		0.1766	1.3120		0.1814	1.3482	
CF-RA3 (κ=0)	0.1970	1.2981		0.2539	1.6728		0.2005	1.4901		0.1725	1.2819	
CF-RA3 (κ=0.25)	0.1922	1.2660		0.2457	1.6185		0.1958	1.4554		0.1703	1.2656	
CF-RA3 (κ=0.5)	0.1875	1.2354		0.2378	1.5665		0.1913	1.4219		0.1682	1.2499	
CF-RA3 (κ=1)	0.1790	1.1791	Mean/SD	0.2230	1.4690	Mean/SD	0.1828	1.3586	Mean/SD	0.1641	1.2198	Mean/SD
CF-PC (AIC)	0.1994	1.3136	7.08/4.40	0.2051	1.3484	3.31/3.98	0.1645	1.2224	8.69/4.05	0.1476	1.0959	4.17/4.87
CF-PC (BIC)	0.1596	1.0512	1.27/0.66	0.1590	1.0451	1.06/0.23	0.1364	1.0136	1.25/0.78	0.1414	1.0499	1.01/0.10
CF-PC (k=1)	0.1523	1.0036		0.1565	1.0286		**0.1344**	**0.9987**		0.1414	1.0501	
CF-PC (k=2)	**0.1517**	**0.9993**		0.1565	1.0287		0.1369	1.0176		0.1388	1.0306	
CF-PC (k=3)	0.1550	1.0214		0.1592	1.0464		0.1375	1.0216		0.1409	1.0467	
CI-Unrestricted	0.1645	1.0835		0.1853	1.2182		0.1756	1.3046		0.1619	1.2026	
CI-PC (AIC)	0.1744	1.1488	7.66/2.21	0.1689	1.1104	2.56/1.35	0.1741	1.2942	8.73/2.10	0.1442	1.0708	2.97/1.84
CI-PC (BIC)	0.1836	1.2094	2.36/0.95	0.1583	1.0409	1.35/0.78	0.1588	1.1799	2.67/1.60	0.1663	1.2350	2.01/1.49
CI-PC (k=1)	**0.1516**	**0.9991**		**0.1511**	**0.9932**		0.1401	1.0414		0.1420	1.0543	
CI-PC (k=2)	0.1549	1.0207		0.1535	1.0091		0.1459	1.0846		0.1516	1.1257	
CI-PC (k=3)	0.1854	1.2214		0.1654	1.0875		0.1630	1.2112		0.1544	1.1467	

Panel C. Annual prediction

	Forecasts begin 1969 (R=42 and P=35)			Forecasts begin 1980(R=53 and P=24)		
	h=1			h=1		
	MSFE	MSFE Ratio		MSFE	MSFE Ratio	
Historical Mean	0.6948			0.4834		
CF-Mean	**0.6320**	**0.9096**		**0.4751**	**0.9828**	
CF-Median	**0.6524**	**0.9390**		0.4925	1.0188	
CF-RA1	3.6004	5.1820		3.1254	6.4651	
CF-RA2	2.8360	4.0819		1.5782	3.2646	
CF-RA3 (κ=0)	2.9970	4.3141		2.4478	5.0635	
CF-RA3 (κ=0.25)	1.5720	2.2625		1.6297	3.3712	
CF-RA3 (κ=0.5)	0.7930	1.1408		1.0294	2.1293	
CF-RA3 (κ=1)	**0.6320**	**0.9096**	Mean/SD	**0.4817**	**0.9965**	Mean/SD
CF-PC (AIC)	3.2141	4.6260	10.14/2.59	2.8428	5.8805	10.08/3.39
CF-PC (BIC)	2.5105	3.6133	5.29/4.62	1.0841	2.2426	4.46/4.70
CF-PC (k=1)	0.6971	1.0034		0.5323	1.1012	
CF-PC (k=2)	**0.6514**	**0.9376**		0.5420	1.1211	
CF-PC (k=3)	0.7300	1.0507		0.6323	1.3079	
CI-Unrestricted	1.3210	1.9013		0.9659	1.9979	
CI-PC (AIC)	1.3247	1.9067	5.34/3.33	0.92799	1.9196	6.33/3.16
CI-PC (BIC)	1.0590	1.5243	3.03/1.87	0.7438	1.5385	1.88/1.33
CI-PC (k=1)	0.7184	1.0340		0.6044	1.2502	
CI-PC (k=2)	0.7362	1.0596		0.6373	1.3183	
CI-PC (k=3)	0.9556	1.3754		0.6678	1.3814	

Monte Carlo experiment with small sample, when *R*=20

Table 1 - Panel B. High correlation: $\Omega = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

***R=20**, P=100*

	MSFE Ratio						
	σ_η =0.25	σ_η =0.5	σ_η =1	σ_η =2	σ_η =4	σ_η =8	σ_η =16
$\hat{y}^{(1)}$	2.3015	1.2622	1.0241	**0.9597**	**0.9406**	**0.9389**	**0.9374**
$\hat{y}^{(2)}$	2.3109	1.2802	1.0204	**0.9571**	**0.9439**	**0.9392**	**0.9378**
CI	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CF-RA1	1.4711	1.4459	1.4531	1.3969	1.2637	1.2094	1.1849
CF-RA2	1.2301	1.0908	1.0677	1.0662	1.0580	1.0556	1.0522
CF-RA3(κ=0)	1.2974	1.2606	1.2433	1.1738	1.0880	1.0565	1.0399
CF-RA3(κ=0.25)	1.2517	1.2180	1.2023	1.1389	1.0615	1.0333	1.0182
CF-RA3(κ=0.5)	1.2100	1.1784	1.1643	1.1067	1.0373	1.0122	**0.9985**
CF-RA3(κ=0.75)	1.1736	1.1424	1.1292	1.0772	1.0154	**0.9933**	**0.9809**
CF-RA3(κ=1)	1.1413	1.1095	1.0971	1.0503	**0.9958**	**0.9765**	**0.9655**
CF-RA3(κ=2)	1.0633	1.0120	**0.9978**	**0.9695**	**0.9406**	**0.9309**	**0.9247**
CF-RA3(κ=3)	1.0619	**0.9681**	**0.9457**	**0.9316**	**0.9223**	**0.9197**	**0.9175**
CF-Mean	1.0902	**0.9664**	**0.9372**	**0.9287**	**0.9270**	**0.9270**	**0.9266**

Table 2 - Panel B. High negative correlations with the omitted variable:

$\Omega = \begin{pmatrix} 1 & 0.6 & -0.7 \\ 0.6 & 1 & -0.7 \\ -0.7 & -0.7 & 1 \end{pmatrix}; \phi_i = 0; \theta = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.6 \end{pmatrix}$

***R=20**, P=100*

$\hat{y}^{(1)}$	**0.9395**	**0.9403**	**0.9395**	**0.9374**	**0.9381**	**0.9389**	**0.9366**
$\hat{y}^{(2)}$	**0.9412**	**0.9398**	**0.9390**	**0.9375**	**0.9380**	**0.9382**	**0.9362**
CI	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CF-RA1	1.2131	1.1957	1.1855	1.1827	1.1827	1.1795	1.1722
CF-RA2	1.0494	1.0488	1.0478	1.0470	1.0501	1.0497	1.0451
CF-RA3(κ=0)	1.0596	1.0490	1.0436	1.0350	1.0331	1.0445	1.0328
CF-RA3(κ=0.25)	1.0358	1.0260	1.0214	1.0134	1.0114	1.0223	1.0116
CF-RA3(κ=0.5)	1.0144	1.0054	1.0012	**0.9939**	**0.9919**	1.0021	**0.9924**
CF-RA3(κ=0.75)	**0.9947**	**0.9867**	**0.9831**	**0.9763**	**0.9744**	**0.9839**	**0.9752**
CF-RA3(κ=1)	**0.9774**	**0.9699**	**0.9671**	**0.9609**	**0.9590**	**0.9678**	**0.9600**
CF-RA3(κ=2)	**0.9288**	**0.9248**	**0.9233**	**0.9195**	**0.9182**	**0.9240**	**0.9192**
CF-RA3(κ=3)	**0.9140**	**0.9132**	**0.9125**	**0.9108**	**0.9106**	**0.9131**	**0.9105**
CF-Mean	**0.9194**	**0.9201**	**0.9195**	**0.9188**	**0.9193**	**0.9200**	**0.9182**