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Stochastic Trends and Fluctuations in National Income, Wages, and Profits*

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I. Introduction

Macroeconomics and especially the theory of business cycles went through very important changes during the last ten years. During the seventies most macroeconomists believed that economic activity evolved around a deterministic trend. Cyclical components and unanticipated changes in policy were believed to be the source of economic fluctuations. The real business cycle (RBC) models which began to emerge in the early eighties cast doubt on this belief.

According to the RBC theory, the cumulative effect of permanent shocks to productivity explains economic fluctuations. The proponents of the RBC theory assume that productivity shocks are exogenous and are not affected by aggregate demand shocks. Therefore, monetary shocks have no role in explaining economic fluctuations. In fact, according to the RBC theories, money responds positively to fluctuations in production induced by technological shocks. Therefore, the positive correlation between output and money is one of reverse causation.

Critics of the RBC theories argue that productivity shocks cannot be treated strictly as exogenous. Evans [5] provides evidence that a significant portion of the variance of productivity impulse can be attributed to aggregate demand shocks. Another line of research by Christiano and Eichenbaum [4] shows that monetary-policy shocks have persistent liquidity effects as well as persistent increases in output.

In this paper we investigate the role of monetary factors in explaining fluctuations in both

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the level and the (functional) distribution of income.¹ For this purpose, we first consider a simple RBC model with permanent productivity shocks. Within this theoretical framework, we show that income, wages, and profits follow unit root processes; are cointegrated pairwise; and share only one common stochastic trend which is related to productivity.

Employing U.S. data for the period 1959:1–1992:2, we also find empirical evidence that the U.S. national income, wages, and profits are cointegrated and share only one common stochastic trend. Following King et al. [9], the common stochastic trend is estimated and the forecast error variance of income, wages, and profits attributed to innovations in the common stochastic trend are computed from a vector error correction model (VECM). The evidence suggests that innovations in the permanent component explain a substantial variation of the forecast error variance of national income, wages, and profits. The cumulative impulse response functions (CIRFs) indicate that, in response to a shock to the common stochastic trend, wages, profits, and national income respond positively and converge to their steady-state levels in the long-run. All this evidence suggests that real shocks have substantial effects on both the level and the distribution of income.

When the three-variable VECM, consisting of national income, wages, and profits, is extended to include nominal variables, such as, the money supply and interest rates, we find three stochastic trends and identify three shocks. In this extended model consisting of five variables, the sum of permanent components still explains a large portion of the fluctuations in income, wages, and profits but the explanatory power of permanent components is highly reduced relative to that of the three-variable system. Furthermore, the inclusion of nominal variables reduces the importance attributed to real shocks substantially and nominal shocks emerge as important factors in explaining fluctuations in income and its individual components, i.e., wages and profits.

The remainder of the paper is organized as follows. Section II derives the properties of a simple RBC model used to identify the structural disturbances. Section III outlines the identification issues. Section IV describes the data and examines their integration and cointegration properties. Section V presents the empirical findings for the three-variable system consisting of only real variables. Sections VI and VII consider a five-variable system which includes both real and nominal variables, discuss the identification of real and nominal shocks, and analyze the effects of these shocks on income and functional distribution of income. Section VIII presents some concluding remarks.

II. A Simple RBC Model

Consider an economy inhabited by N identical agents with infinite horizon. Each agent seeks to maximize her lifetime utility $E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i})$, where $0 < \beta < 1$ denotes the discount factor, t is a time index and $u(\cdot)$ represents the one-period utility function, which depends on per capita real consumption, C . Application of E_t yields the mathematical expectation of a random variable conditional upon the information set in period t .

The production technology is described by $Y_t = f(A_t, K_t)$, where Y and K denote, respectively, (real) output and capital both in per capita terms and A captures the state of the technology.

1. This is of particular importance if, as in models of steady investment-driven growth, the savings rate as well as the growth rate of the economy depend on the composition of income, that is labor vs. capital income (see the discussion in Bertola [1]). Furthermore, as it has been documented in Lee, Liu, and Wang [10], an increase in labor share generates a more equitable (personal) distribution of income.

Moreover, the function $f(\cdot)$ is assumed to be increasing, strictly concave, linearly homogeneous, satisfying the Inada conditions. The resource constraint for this economy can then be written as

$$K_{t+1} = Y_t - C_t + (1 - \delta)K_t, \tag{1}$$

where $0 \leq \delta \leq 1$ is the depreciation rate. For simplicity, we assume that there is no population growth.

Next, we employ the equivalency between the social optimal and the competitive equilibrium allocations, that exists in an environment like this [12], to derive the necessary conditions for this program. If we let $V(\cdot)$ denote the value function, then by Bellman's principle of optimality, we have $V(K_t) = \max_{C_t} \{u(C_t) + \beta E_t[V(K_{t+1})]\}$ subject to (1). Simple differentiation yields the first-order condition $u_C(C_t) = \beta E_t[V_K(K_{t+1})]$, and the Benveniste-Scheinkman equation for the evolution of the state variable $V_K(K_t) = \beta r_t E_t[V_K(K_{t+1})]$, where subscripts, other than t , denote partial derivatives, and $r_t \equiv (1 - \delta) + f_K$ denotes the gross marginal product of capital (real interest rate). Combining these two equations, one can obtain the Euler equation

$$u_C(C_t) = \beta E_t[u_C(C_{t+1})r_{t+1}], \tag{2}$$

which describes the intertemporal trade-off in consumption.

Next, we show that, within a deterministic setting, output, profits and wages have the same long-run growth rate; analogously, in a stochastic setting the three variables are cointegrated pairwise.

Steady-State Growth

Consider first the case where A_t grows at a constant (gross) rate g , that is, $g \equiv A_{t+1}/A_t$. Assume also that the utility function takes the constant elasticity of intertemporal substitution form, that is, $u(C_t) = (C_t^{1-\sigma} - 1)/(1 - \sigma)$. Using the resource constraint, (1), and the Euler equation, (2), it is straightforward to show that technology, capital, output, and consumption all grow at a common rate, g , while the interest rate is constant over time, $r_t = r$ for all t . Furthermore, total profits, Π_t , defined as $\Pi_t \equiv [r_t - (1 - \delta)]K_t$, and total wages $W_t \equiv Y_t - \Pi_t$ grow also at the rate g .²

Stochastic Growth

Consider next the case where A_t is a random variable. To account for perpetual growth we assume, following, among others, Prescott [16], Christiano [3], and King et al. [9], that technology follows a logarithmic random walk, i.e., $a_t = g + a_{t-1} + \zeta_t$, where $a_t \equiv \ln(A_t)$ and the productivity shocks $\{\zeta_t\}$ are i.i.d. with zero mean.³ In general, an exact analytical solution of the model cannot be obtained and one needs to employ an approximate solution method (see Taylor and Uhlig [20] for a review and a comparison of the methods that are available). Instead, following Long and Plosser [11], we employ a Cobb-Douglas production function and adopt specific parameter values. This enables us to derive an exact analytical solution and to demonstrate the properties

2. In fact, these properties are also shared by several endogenous growth models, such as those found, for example, in King and Rebelo [8] and Rebelo [17].

3. Variables in lower case letters, e.g., a_t, y_t, w_t, π_t , and k_t , denote the natural logarithms of the corresponding variables in upper case letters.

that are important for the empirical implementation of the model. More specifically, we assume that $Y_t = f(A_t, K_t) = A_t^\alpha K_t^{1-\alpha}$, $\delta = 1$, i.e., capital depreciates fully in one period, and $\sigma = 1$, so that the utility function takes the log-linear form, $u(C_t) = \ln(C_t)$. In this case, the Euler equation (2) becomes $1/C_t = \beta E_t[r_{t+1}/C_{t+1}]$, where $r_{t+1} = (1 - \delta) + (1 - \alpha)(A_{t+1}/K_{t+1})^\alpha$. It seems plausible to guess as a solution to this functional equation a function of the form $C_t = \theta A_t^\alpha K_t^{1-\alpha}$ and hence, by using the resource constraint, to obtain $K_{t+1} = (1 - \theta)A_t^\alpha K_t^{1-\alpha}$. By substituting the last two equations for C_t and K_{t+1} into (2), we obtain $\theta = 1 - \beta(1 - \alpha)$. Thus, $k_{t+1} = \ln[\beta(1 - \alpha)] + \alpha a_t + (1 - \alpha)k_t$, or $\Delta k_t = \alpha g + \alpha \zeta_{t-1} + (1 - \alpha)\Delta k_{t-1}$, where Δ is the difference operator. Using the production function, we also obtain

$$\Delta y_t = \alpha g + \alpha \zeta_t + (1 - \alpha)\Delta y_{t-1}. \quad (3)$$

Thus, the logarithms of both capital and output follow unit root processes. Nevertheless, the two variables are cointegrated since their difference

$$y_t - k_t = -\alpha \ln[\beta(1 - \alpha)] + \alpha \Delta a_t + (1 - \alpha)(y_{t-1} - k_{t-1})$$

is stationary. Moreover, $\Pi_t = (1 - \alpha)A_t^\alpha K_t^{1-\alpha}$, or $\pi_t = \ln(1 - \alpha) + \alpha a_t + (1 - \alpha)k_t$, or

$$\Delta \pi_t = \alpha g + \alpha \zeta_t + (1 - \alpha)\Delta \pi_{t-1}, \quad (4)$$

that is, the logarithm of profits is an I(1) series. Similarly, the logarithm of wages is also an I(1) series which can be expressed as

$$\Delta w_t = \alpha g + \alpha \zeta_t + (1 - \alpha)\Delta w_{t-1}. \quad (5)$$

Finally, notice that although profits and wages follow unit root process they are cointegrated with each other and with output. These properties hold in more general cases as well. To verify this, one can follow Campbell [2] and approximate analytically the solution of the model by log-linearizing the resource constraint (equation (1)) and the Euler equation (equation (2)).

III. Identification of the Permanent Real Shock

Employing equation (3) one can show that $\Delta y_t = [1 - (1 - \alpha)B]^{-1}(\alpha g + \alpha \zeta_t) \equiv g + \omega_{1t}$, where $B \equiv 1 - \Delta$ is the backshift operator and $\omega_{1t} = \alpha [1 - (1 - \alpha)B]^{-1} \zeta_t = \alpha \sum_{j=0}^{\infty} (1 - \alpha)^j \zeta_{t-j}$ is the normalized moving average of all the past real shocks ζ_{t-j} with geometrically declining weights, and is thus an integrated process of order zero [I(0)] (In the sequel, we refer to ω_{1t} as the “permanent real shock”). Therefore, $y_t = y_0 + gt + h_t$, where $h_t \equiv \sum_{i=0}^{\infty} \omega_{1,t-i}$. The series is thus decomposed into the initial value y_0 , a linear deterministic trend gt , and an I(1) stochastic trend h_t . Moreover, considering possible stationary measurement errors or some unmodelled idiosyncratic transitory shocks to the series (denoted by \tilde{y}_t), the actual series can be represented as

$$y_t = y_0 + gt + h_t + \tilde{y}_t.$$

Similarly, from (5) and (4),

$$w_t = w_0 + gt + h_t + \tilde{w}_t,$$

and

$$\pi_t = \pi_0 + gt + h_t + \tilde{\pi}_t,$$

where \tilde{w}_t and $\tilde{\pi}_t$ denote I(0) transitory components. Let $X_t \equiv (y_t \ w_t \ \pi_t)'$. Then

$$X_t = X_0 + \mu t + Jh_t + \tilde{X}_t \tag{6}$$

where $\mu = (g \ g \ g)'$, $J = (1 \ 1 \ 1)'$, h_t is the scalar I(1) common permanent component, and $\tilde{X}_t \equiv (\tilde{y}_t \ \tilde{w}_t \ \tilde{\pi}_t)'$ consists of I(0) idiosyncratic transitory components.

The series $X_t = (y_t \ w_t \ \pi_t)'$ can be generated from the common factor representation (6) or from a vector error correction model (VECM):

$$\Delta X_t = \mu + A_1 \Delta X_{t-1} + \dots + A_{k-1} \Delta X_{t-k+1} + A_k X_{t-1} + \varepsilon_t \tag{7}$$

where A_i , $i = 1, \dots, k$, is a 3×3 matrix of parameters and ε_t is a 3×1 vector white noise. As there exists only one common factor h_t , the cointegrating rank (r) is equal to 2 and thus A_k is of rank 2.

Since the VECM can be used for forecasting, we compute the fractions of the forecast error variance of ΔX_t due to the innovations ($\omega_{1t} = \Delta h_t$) to h_t . This can yield information about the relative importance of the common stochastic trend in each series. We estimate the VECM given by (7), and then transform it to a vector moving average (VMA) model:⁴

$$\Delta X_t = \mu + C(B)\varepsilon_t \tag{8}$$

where $C(B)$ is a 3×3 matrix polynomial in B . Moreover, $C(1)$ is of rank 1 and hence there exist 3×1 vectors K and D such that $C(1) = KD'$.

To identify the common factor h_t we impose some identifying restrictions. First, we rewrite (8) as

$$\Delta X_t = \mu + \Gamma(B)\omega_t \tag{9}$$

where $\Gamma(B)$ is a 3×3 matrix polynomial in B , $\Gamma(1)$ is of rank 1, and $\omega_t = (\omega_{1t} \ \omega_{2t} \ \omega_{3t})'$ is a 3×1 vector white noise. The imposed identifying restrictions then are: $\Gamma(1) = KE'$, $K = J$, and $E = (1 \ 0 \ 0)'$. Under these restrictions, (9) implies

$$\begin{aligned} X_t - X_0 - \mu t &= \Delta^{-1}\Gamma(B)\omega_t = \Delta^{-1}[\Gamma(1) + \Delta\Gamma^*(B)]\omega_t \\ &= J\Delta^{-1}\omega_{1t} + \Gamma^*(B)\omega_t = Jh_t + \Gamma^*(B)\omega_t, \end{aligned}$$

which is the same as (6) with $\tilde{X}_t = \Gamma^*(B)\omega_t$. Hence, the common factor is identified as $h_t = \Delta^{-1}\omega_{1t}$, with $\omega_{1t} = D'\varepsilon_t$.

4. Although the order of the estimated vector moving average model that we use is 24, we have also experimented with VMA models of different order, e.g., 16, 20, and 36, and found almost identical results.

IV. The Time Series Properties of the Data

The data consist of quarterly U.S. observations from 1959:1 to 1992:2 (134 observations) on real national income (Citibase mnemonic GYQ), nominal national income (GY), compensation on employees ($GCOMP$), corporate profits ($GPJVA$), proprietors' income ($GPROJ$), rental income ($GPRENJ$), and net interest ($GNINT$). The ratio GY/GYQ is used as a price deflator. Furthermore, the monthly U.S. observations from 1959:1 to 1992:2 on total civilian population ($PI6$), M2 ($FM2$), and the three-month U.S. Treasury bill rate ($FYGM3$) series are converted to quarterly series by taking quarterly averages.

The labor income is calculated as the sum of $GCOMP_t$ and $(weight_t \times GPROJ_t)$, where $weight_t = GCOMP_t \div (GY_t - GPROJ_t)$. The capital income, on the other hand, is taken as the remainder of the national income and is equal to $GPJVA_t + (1 - weight_t) \times GPROJ_t + GPRENJ_t + GNINT_t$. Notice, in particular, that we split the proprietors' income proportionately into labor and capital income using the variable $weight_t$.⁵

In our empirical study we employ the logarithms of per capita real national income (y_t), per capita real labor income (w_t), per capita real capital income (π_t), per capita nominal money supply (m_t), and the price series (p_t). The per capita real money supply is defined as $(m_t - p_t)$ and the three-month Treasury bill rate (R_t) is expressed in a percentage form. Most series display upward trends (The actual series minus the first observations ($X_t - X_1$) are plotted in Figures 1 and 3 in bold lines).

Furthermore, the series y_t , w_t , π_t , m_t , $m_t - p_t$, and R_t can be characterized as I(1) processes according to the augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests [15, Table 1]. The inflation series Δp_t , on the other hand, is I(0) according to the PP test, though the ADF test does not reject the null hypothesis of a unit root in Δp_t . Inflation rate series are often found to be I(1). But as we use the national income deflator, $p_t = \ln(GY_t/GYQ_t)$, to deflate all nominal series, and since the inflation rate, Δp_t , obtained from this series is I(0), we do not include it in our model.

The results of the Johansen cointegration tests [7] for various systems are reported in Table I. We test for cointegration among the following systems: $X_{1t} = (y_t \ w_t \ \pi_t)'$, $X_{2t} = (m_t \ R_t \ y_t \ w_t \ \pi_t)'$, $X_{3t} = (y_t \ w_t)'$, $X_{4t} = (y_t \ \pi_t)'$, $X_{5t} = (y_t \ m_t)'$, $X_{6t} = (y_t \ m_t - p_t)'$, $X_{7t} = (y_t \ R_t)'$, and $X_{8t} = (m_t \ R_t)'$ (Our benchmark systems are X_{1t} and X_{2t}). We find cointegration in X_{1t} , X_{2t} , X_{3t} , X_{4t} , and X_{6t} . The series in X_{1t} share only one common stochastic trend while the ones in X_{2t} share three stochastic trends. Furthermore, $X_{1t} = (y_t \ w_t \ \pi_t)'$ is cointegrated with the real money supply but not with the nominal money supply. These cointegration results are consistent with economic theory. Hence, $X_{1t} = (y_t \ w_t \ \pi_t)'$, m_t , and R_t are three sets that do not share the same common stochastic trends in the full system of $X_{2t} = (y_t \ w_t \ \pi_t \ m_t \ R_t)'$.

V. Empirical Results for the Three-Variable Model

We estimate the VECM given by equation (7) using the Johansen method [7] for $X_t = X_{1t}$. The estimated permanent component plus the deterministic trend ($gt + h_t$) is plotted along with the

5. The sample mean of $weight_t$ is 0.799. We have also used the fixed value of 2/3 as the weight, following Summers [19], and found the results to be very similar.

Table I. Tests for Unit Root and Cointegration

Unit Root Tests										
	y_t	w_t	π_t	m_t	$m_t - p_t$	R_t	Δp_t			
<i>ADF1</i>	-2.23 (2)	-1.86 (1)	-2.34 (2)	-0.65 (1)	-2.43 (1)	-2.63 (5)	-2.42 (2)			
<i>ADF2</i>	-2.57 (2)	-2.03 (1)	-2.55 (2)	-1.33 (1)	-2.42 (1)	-2.73 (5)	-2.24 (2)			
<i>PP1</i>	-1.94	-2.07	-2.26	-0.35	-2.36	-2.10	-5.28			
<i>PP2</i>	-2.17	-1.90	-2.62	-1.26	-2.06	-1.91	-5.55			

Johansen Tests for Cointegration										
critical values										
	90%	95%	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
<i>Trace</i>										
$m = 1$	2.69	3.76	5.11	2.90	4.67	4.64	0.08	4.65	1.10	2.06
$m = 2$	13.33	15.41	14.76	7.62	23.04	24.00	8.30	28.39	9.57	11.91
$m = 3$	26.79	29.68	41.10	17.80						
$m = 4$	43.95	47.21		54.13						
$m = 5$	64.84	68.52		92.97						
λ max										
$m = 1$	2.69	3.76	5.11	2.90	4.67	4.64	0.08	4.65	1.10	2.06
$m = 2$	12.07	14.07	9.65	4.73	18.37	19.36	8.22	23.75	8.48	9.85
$m = 3$	18.60	20.97	26.34	10.18						
$m = 4$	24.73	27.07		36.33						
$m = 5$	30.90	33.46		38.84						

Notes for Unit Root Tests: ADF and PP denote the augmented Dickey-Fuller test and the Phillips-Perron test statistics, respectively. *ADF1*, *PP1* are computed with a constant term, and *ADF2*, *PP2* are with a constant and a linear trend. The numbers in parentheses for ADF are the number of lag-augmentation, chosen using the SIC. The results do not change when the AIC is used. We report PP's with four non-zero autocovariances in Newey-West correction [13]. The critical values for both statistics, which are asymptotically equivalent, may be obtained from Fuller [6, 373].

Notes for Cointegration Tests: If p is the number of variables in a system, then $m \equiv p - r$ is the number of common stochastic trends in the system. The critical values in the second and third columns are obtained from Osterwald-Lenum [14]. $X_{1t} = (y_t, w_t, \pi_t)'$, $X_{2t} = (m_t, R_t, y_t, w_t, \pi_t)'$, $X_{3t} = (y_t, w_t)'$, $X_{4t} = (y_t, \pi_t)'$, $X_{5t} = (y_t, m_t)'$, $X_{6t} = (y_t, m_t - p_t)'$, $X_{7t} = (y_t, R_t)'$, and $X_{8t} = (m_t, R_t)'$. $k = 2$ for all X_{it} but X_{7t} , for which $k = 3$.

actual series ($X_t - X_1$) in Figure 1. The plots suggest that a large part of the fluctuations in income, wages, and profits can be explained by movements in the common stochastic trend.

We first choose the lag length k using the Akaike and Schwarz information criteria (AIC and SIC). The value chosen for X_{1t} is $k = 2$. The fraction of the forecast-error variances of ΔX_t attributed to innovations ω_{1t} in the common stochastic trend along with their simulated standard errors (obtained by normal approximation) are presented in Table II. In computing these, as in King et al. [9], we imposed the assumption that the permanent shock is uncorrelated with transitory shocks, that is, $E\omega_{1t}\omega_{2t} = 0$ and $E\omega_{1t}\omega_{3t} = 0$. The point estimates suggest that at the end of a 24 quarter horizon, 94% of the fluctuations in income, 65% of the fluctuations in wages, and 76% of the fluctuations in profits can be attributed to innovations in the common stochastic trend.

The impulse responses of X_t to an innovation of one standard deviation in the common stochastic trend are plotted in Figure 2, along with the one-standard-deviation confidence intervals, computed by Monte Carlo simulation using 1000 replications (dotted lines). In response to the permanent real shock, π_t increases in the short-run and then gradually declines, while y_t and w_t complete the adjustment process much more slowly. As may be expected from (6) and since

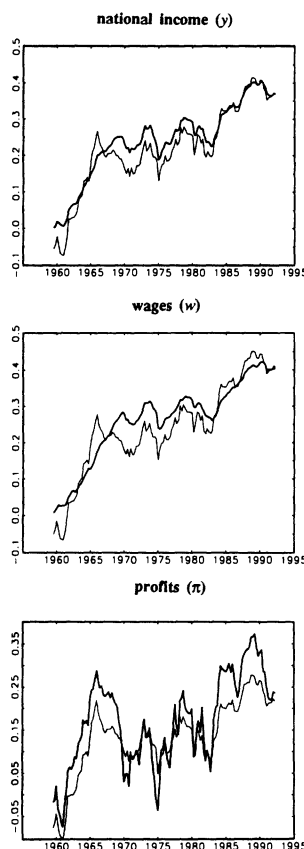


Figure 1. Common stochastic trend in $X_{1t} = (y_t \ w_t \ \pi_t)'$

Note: Actual series in bold lines and the estimated common trend in thin lines.

$K = J$, the long-run multiplier of the permanent shock is unity, that is, $\lim_{h \rightarrow \infty} y_t / \partial \omega_{1t-h} = \lim_{h \rightarrow \infty} \partial w_t / \partial \omega_{1t-h} = \lim_{h \rightarrow \infty} \partial \pi_t / \partial \omega_{1t-h} = 1$. This can also be seen in Figure 2.

These results indicate that a large portion of the fluctuations in national income, wages, and profits can be explained by permanent real shocks.

VI. Extensions with Nominal Variables: Identification

In this section we investigate the possibility of additional permanent shocks by considering other cointegrating relations which include nominal variables. The model employed is $X_{2t} = (m_t \ R_t \ y_t \ w_t \ \pi_t)'$.

Let X_t denote X_{2t} and have the Wold representation of the form equation (8) with $C(B)$ being a 5×5 matrix polynomial in B . Recall, from the Johansen cointegration test for X_{2t} , presented in Table I, that X_t is cointegrated with $r = 2$ and $m = 3$. Thus, $C(1)$ has rank 3. To identify the 3×1 common factor F_t , consider the model presented in equation (9) with $\Gamma(B)$ being a 5×5 matrix polynomial in B and $\omega_t = (\omega_{1t} \ \omega_{2t} \ \omega_{3t} \ \omega_{4t} \ \omega_{5t})'$ being a 5×1 vector white noise. As before we have $X_t - X_0 - \mu t = \Delta^{-1} \Gamma(B) \omega_t = \Delta^{-1} [\Gamma(1) + \Delta \Gamma^*(B)] \omega_t = \Gamma(1) \Delta^{-1} \omega_t + \Gamma^*(B) \omega_t$.

Table II. Forecast Error Variance Decomposition: $X_{1t} = (y_t w_t \pi_t)'$

Horizon	Δy_t	Δw_t	$\Delta \pi_t$
1	.968	.568	.778
4	.941	.640	.764
8	.938	.649	.758
12	.937	.651	.758
24	.936	.652	.758

Note: Based on an estimated VECM for X_{1t} , with $p = 3$, $k = 2$, $r = 2$, and $m = p - r = 1$.

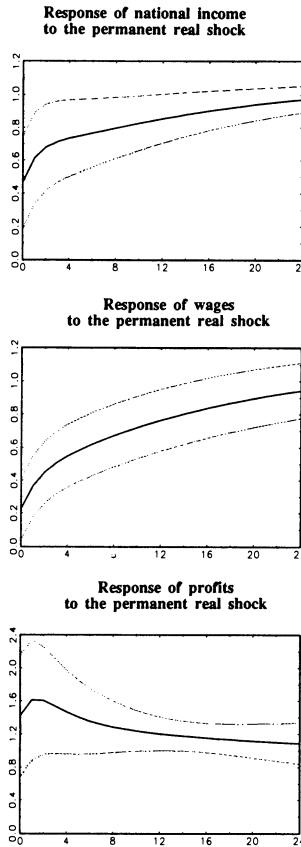


Figure 2. Response of $X_{1t} = (y_t w_t \pi_t)'$ to shock ω_{1t}

Since $\Gamma(1)$ is of rank 3, it can be written as $\Gamma(1) = [A \ 0]$ where A is a known 5×3 matrix of rank 3 and 0 is a 5×2 null matrix. Then,

$$X_t = X_0 + \mu t + AH_t + \Gamma^*(B)\omega_t \tag{10}$$

where $H_t = (h_{1t} h_{2t} h_{3t})' = (\Delta^{-1}\omega_{1t} \Delta^{-1}\omega_{2t} \Delta^{-1}\omega_{3t})'$ is a 3×1 vector of $I(1)$ common stochastic trends in the sense of Stock and Watson [18].

Let a_{ij} be the (i, j) element of A . Assuming that the cointegrating vector of both $(y_t w_t)'$ and $(y_t \pi_t)'$ is $(1 \ -1)'$, we can set $a_{33} = a_{43} = a_{53} = 1$, $a_{32} = a_{42} = a_{52} = a$, and

$a_{31} = a_{41} = a_{51} = b$. Since $(y_t, w_t, \pi_t)'$ and $(m_t, R_t)'$ are not cointegrated, we also set $a_{13} = a_{23} = 0$. Moreover, since m_t is not cointegrated with R_t , we set $a_{12} = 0$. Finally, we normalize the system by setting $a_{11} = a_{22} = 1$ and let $a_{21} = c$ to simplify the notation. Then

$$AH_t = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{pmatrix} \begin{pmatrix} h_{1t} \\ h_{2t} \\ h_{3t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ c & 1 & 0 \\ b & a & 1 \\ b & a & 1 \\ b & a & 1 \end{pmatrix} \begin{pmatrix} h_{1t} \\ h_{2t} \\ h_{3t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ c & 1 & 0 \\ b & a & 1 \end{pmatrix} \begin{pmatrix} h_{1t} \\ h_{2t} \\ h_{3t} \end{pmatrix} \equiv \tilde{A}\Pi H_t. \quad (11)$$

Thus, $A = \tilde{A}\Pi$. If $F_t = (f_{1t}, f_{2t}, f_{3t})' \equiv \Pi H_t$ is the 3×1 permanent stochastic component of X_t , then $AH_t = \tilde{A}F_t = (f_{1t}, f_{2t}, f_{3t}, f_{3t}, f_{3t})'$ and

$$X_t = X_0 + \mu t + (f_{1t}, f_{2t}, f_{3t}, f_{3t}, f_{3t})' + \Gamma^*(B)\omega_t, \quad (12)$$

where f_{1t} , f_{2t} , and f_{3t} are the common stochastic trends in $(y_t, w_t, \pi_t)'$, m_t , and R_t , respectively. It should be noted that A is the matrix of the CIRF's in the infinite horizon, i.e.,

$$a_{ij} = \lim_{h \rightarrow \infty} CIRF_{ijh}$$

where $CIRF_{ijh} = \partial x_i / \partial \omega_{j,t-h}$ denotes the cumulative impulse response of x_i to the j th shock ω_j , $j = 1, 2, 3$, for the previous h periods and x_i is the i th variable in the system X_t .

We compute the cumulative impulse response functions (CIRF's) and the fractions of the forecast error variance of X_t attributed to the permanent shocks $(\omega_{1t}, \omega_{2t}, \omega_{3t})'$ as follows. First note that, since $F_t = \Pi H_t$, $f_{1t} = h_{1t}$, $f_{2t} = ch_{1t} + h_{2t}$, and $f_{3t} = bh_{1t} + ah_{2t} + h_{3t}$. As $\Delta h_{it} = \omega_{it}$, the short-run causal-chain of the shocks can be described as

$$\begin{aligned} \omega_{1t} &\Rightarrow h_{1t} \Rightarrow (f_{1t}, f_{2t}, f_{3t}) \Rightarrow m_t, R_t, (y_t, w_t, \pi_t)' \\ \omega_{2t} &\Rightarrow h_{2t} \Rightarrow (f_{2t}, f_{3t}) \Rightarrow R_t, (y_t, w_t, \pi_t)' \\ \omega_{3t} &\Rightarrow h_{3t} \Rightarrow f_{3t} \Rightarrow (y_t, w_t, \pi_t)'. \end{aligned}$$

Since $f_{1t} = h_{1t}$ and f_{1t} is the dominant I(1) component in m_t , $\omega_{1t} = \Delta h_{1t}$ can be considered as the monetary shock. Therefore, by selecting the ordering $(m_t, R_t, y_t, w_t, \pi_t)'$ we design the model so that the monetary shock ω_{1t} affects $(y_t, w_t, \pi_t)'$ (R_t) if $b \neq 0$ ($c \neq 0$). Furthermore, as long as $a \neq 0$, the interest rate shock ω_{2t} also affects these real variables $(y_t, w_t, \pi_t)'$. Finally, since $f_{3t} = bh_{1t} + ah_{2t} + h_{3t}$, ω_{3t} is designed to affect only $(y_t, w_t, \pi_t)'$ and hence it is called the (permanent) real shock.⁶

VII. Empirical Results for the Five-Variable System

The five-variable VECM with three stochastic trends is also estimated by the Johansen method [7] employing a lag length $k = 2$, chosen by the AIC and SIC criteria. The estimated permanent components with trend are plotted along with the actual series in Figure 3. The plots suggest that while movements in the sum of permanent components continue to explain a large portion

6. The robustness of the empirical findings with respect to different orderings is investigated in the next section.

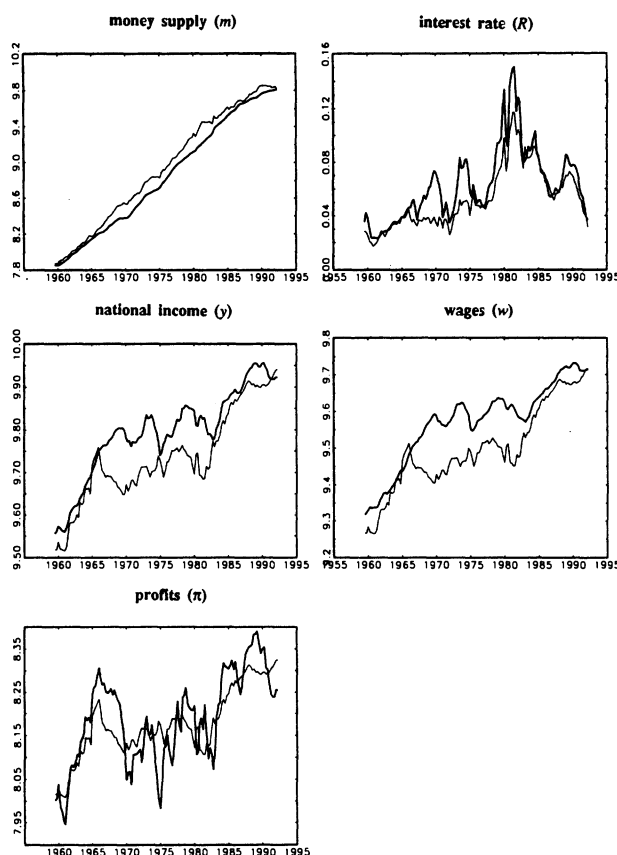


Figure 3. Common stochastic trend in $X_{2t} = (m_t R_t y_t w_t \pi_t)'$

Note: Actual series in bold lines and the estimated common trend in thin lines.

of the fluctuations in the actual series, the explanatory power of permanent shocks is highly reduced compared to that of the three-variable system. Similar evidence is obtained by analyzing the forecast error variance decomposition in Table III. At the end of the 24-quarter horizon, the sum of the three shocks explains only 53% of the variation in income, 50% of the variation in wages, and 40% of the variation in profits.

The fraction of the forecast-error variance of ΔX_t attributed to innovations in the three stochastic trends, characterized as money, interest rate, and real shocks, are presented in Panels A, B, and C of Table III. The point estimates suggest that at the end of the 24-quarter horizon the permanent component that can be attributed to monetary shocks explains 26.1 percent of the fluctuations in income, 31.6 percent of the fluctuations in wages, and 12.6 percent of the fluctuations in profits.

There is a striking difference between the three-variable and the five-variable VECM's when we compare the impact of real shocks. As it can be seen from Table III, the inclusion of monetary and financial variables reduces the importance that can be attributed to real shocks immensely. Moreover, changing the ordering of the variables and placing the real variables ahead of the nominal variables does not change this result.

Further insight is gained by analyzing the CIRF's. The responses of the system variables to

Table III. Forecast Error Variance Decomposition: $X_{2t} = (m_t, R_t, y_t, w_t, \pi_t)'$

Horizon	Δm	ΔR	Δy	Δw	$\Delta \pi$
A. Fraction of the forecast error variance attributed to ω_1					
1	.762	.086	.133	.173	.049
4	.526	.154	.184	.231	.080
8	.501	.153	.229	.250	.123
12	.500	.155	.252	.289	.126
24	.520	.156	.261	.316	.126
B. Fraction of the forecast error variance attributed to ω_2					
1	.078	.818	.079	.000	.166
4	.250	.738	.170	.085	.197
8	.251	.734	.174	.127	.182
12	.252	.731	.167	.123	.182
24	.237	.730	.165	.120	.182
C. Fraction of the forecast error variance attributed to ω_3					
1	.069	.060	.034	.048	.025
4	.125	.056	.086	.023	.086
8	.122	.058	.102	.053	.090
12	.125	.058	.101	.058	.089
24	.124	.058	.101	.060	.090

Notes: Based on an estimated VECM for X_{2t} , with $p = 5$, $k = 2$, $r = 2$, and $m = p - r = 3$. ω_1 , ω_2 , and ω_3 are the money supply, interest rate, and permanent real shocks, respectively.

monetary shocks are plotted in Figure 4. The response of the nominal interest rate is positive for the entire horizon. Income, on the other hand, responds positively for the first seven quarters and negatively thereafter. Likewise, wages (profits) respond positively for the first nine (four) quarters and negatively after that.⁷

Although our results can be interpreted within different frameworks we favor one in which money is introduced via a cash-in-advance constraint. A positive shock to the money growth rate leads to a higher inflation rate which raises in turn the transactions frequency and releases part of the money holdings. In equilibrium, this raises the nominal interest rate, capital, output and factor incomes. Nevertheless, in the long-run the reverse Tobin effect dominates. More specifically, a higher rate of inflation raises also the cost of holding money and thus decreases the net rate of return on capital, given the cash-in-advance constraint. This leads to a permanently lower level of capital stock, output and hence factor incomes.

CIRFs of m_t , R_t , y_t , w_t , and π_t to the interest rate shock are plotted in Figure 5. Profits respond positively to the interest rate shock for the first two quarters. This is not surprising because interest income is included in profits. The fall in profits over time indicates that the initial increase in profits due to an increase in interest income is offset by the depressing effects of an increase in interest rates on economic activity. The negative response of wages and income to the interest rate shock is consistent with the transmission mechanism that is common to a large class of economic models.

Figure 6 plots the CIRF's of m_t , R_t , y_t , w_t , and π_t to a permanent real shock. The response of

7. Some simulated confidence intervals converge to zero since the estimated long-run multipliers in each simulation are almost degenerate around the imposed cointegrating relationships.

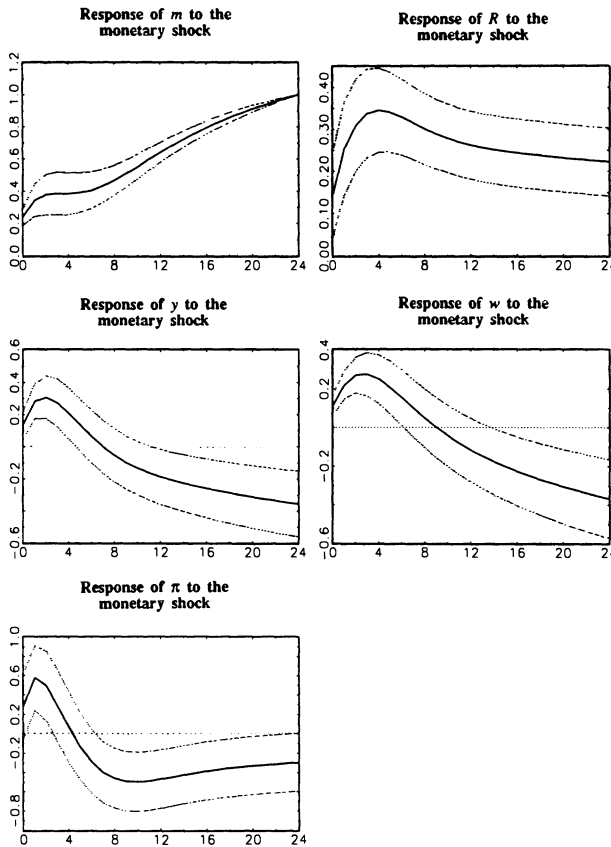


Figure 4. Response of $X_{2t} = (m_t R_t y_t w_t \pi_t)'$ to shock ω_{1t}

y_t , w_t , and π_t to a real shock is positive and in this sense it is similar to that in the three-variable VECM.

The matrix Π in (11) is assumed to be lower triangular. To examine whether the results are sensitive to this assumption we alternatively examine the case where Π is upper triangular. Thus, suppose Π' replaces Π . Then, $F_t = \Pi'H_t$, $f_{1t} = h_{1t} + ch_{2t} + bh_{3t}$, $f_{2t} = h_{2t} + bh_{3t}$, and $f_{3t} = h_{3t}$. The short-run causal-chain of the shocks then can be written as follows: $\omega_{1t} \Rightarrow h_{1t} \Rightarrow f_{1t} \Rightarrow m_t$; $\omega_{2t} \Rightarrow h_{2t} \Rightarrow (f_{1t} f_{2t}) \Rightarrow m_t$ and R_t ; and $\omega_{3t} \Rightarrow h_{3t} \Rightarrow (f_{1t} f_{2t} f_{3t}) \Rightarrow m_t, R_t, (y_t w_t \pi_t)'$. Since $f_{3t} = h_{3t}$, ω_{3t} may now be called the real shock to $(y_t w_t \pi_t)'$. The empirical results, therefore, could be sensitive to the ordering of the variables.

To investigate the robustness of our empirical findings, we also used the orderings $(y_t w_t \pi_t m_t R_t)'$ and $(R_t m_t y_t w_t \pi_t)'$. The forecast error variance decompositions and the cumulative impulse response functions (provided in an appendix available by the authors upon request) indicate that the *main findings* of the paper are robust to different orderings. In all the cases the sum of the three shocks explain 53% of the variation in income, 50% of the variation in wages, and 40% of the variation in profits. In the ordering $(y_t w_t \pi_t m_t R_t)'$, where real variables are ordered first, the forecast error variance that can be attributed to real shocks is greater than the ones obtained when the monetary variables are ordered first. Nonetheless, the monetary shocks are still important sources of fluctuations. Likewise, the CIRF's are very similar to those presented in Figures 4, 5, and 6. In the ordering $(R_t m_t y_t w_t \pi_t)'$, where the interest rate is ordered before

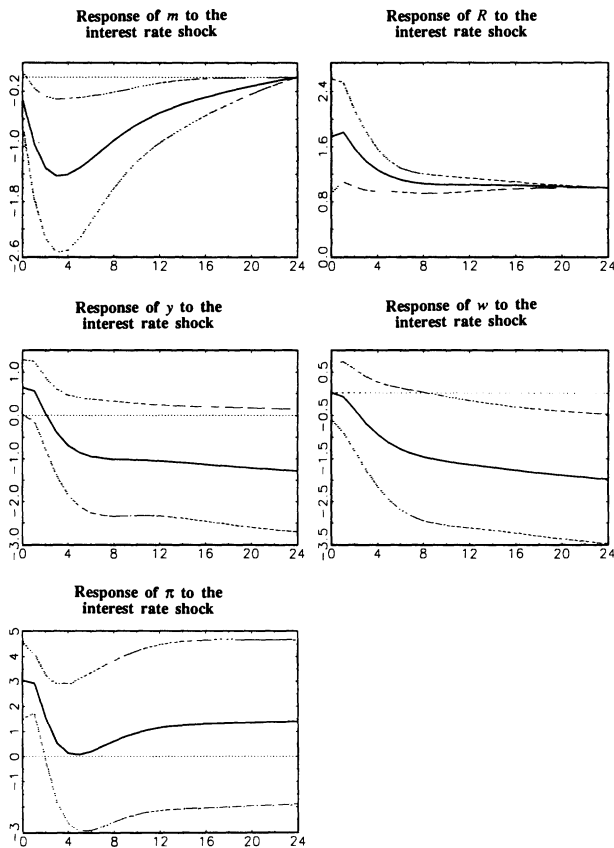


Figure 5. Response of $X_{2t} = (m_t R_t y_t w_t \pi_t)'$ to shock ω_{2t}

the money supply, the forecast error variance that can be attributed to the interest rate shocks is greater than that of the monetary shocks, though the latter are still important in explaining the variation in income and wages at the end of the 24-quarter horizon. Finally, the CIRF's also support the robustness of our findings.

VIII. Concluding Remarks

What determines the fluctuations in income and functional distribution of income? To answer this question we employ a simple real business cycle model. We show that income, wages, and profits per person are cointegrated with each other and share only one common stochastic trend which is related to permanent real shocks. Within the context of this model fluctuations in total income and factor incomes can be explained only by shocks to this common stochastic trend.

Our empirical findings confirm that total income, labor income, and capital income per person are cointegrated with only one common stochastic trend that is related to productivity. We find that substantial amounts of forecast error of the series are due to real shocks, and the estimated long-run responses of the series seem to be consistent with what a RBC model predicts.

When nominal variables are added to the system containing only real variables, however, the

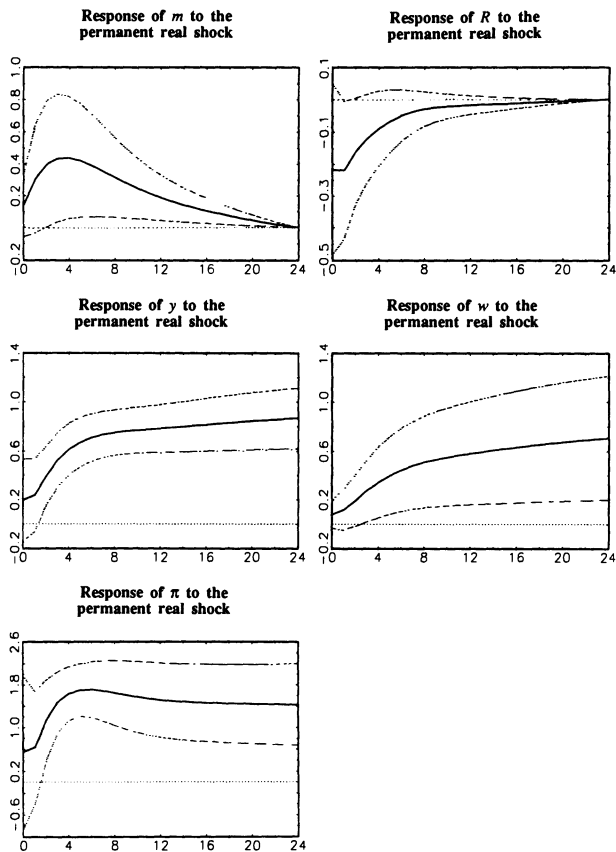


Figure 6. Response of $X_{2t} = (m_t, R_t, y_t, w_t, \pi_t)'$ to shock ω_{3t}

results change dramatically. We find that the system consisting of both real and nominal variables has three stochastic factors. Taking into account nominal shocks reduces the explanatory power of permanent real shocks substantially in explaining fluctuations in income, wages, and profits. The empirical evidence, therefore, presented in this paper suggests that monetary and financial factors cannot be ignored in explaining fluctuations in income and functional distribution of income.

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