ELSEVIER Journal of Econometrics 73 (1996) 401-410

JOURNAL OF Econometrics

Cointegration tests with conditional heteroskedasticity

Tae-Hwy Lee*[,a], Yiuman Tse[b]

[a]Department of Economics, University of California, Riverside, CA 92521, USA
[b]Department of Finance, Insurance, and Real Estate, University of Memphis, Memphis, TN 38152, USA

(Received February 1994; final version received March 1995)

Abstract

We examine the performance of Johansen's (1988) likelihood ratio tests for cointegration in the presence of GARCH and compare with other cointegration tests. The tests tend to overreject the null hypothesis of no cointegration in favor of finding cointegration, but the problem is generally not very serious.

Key words: Cointegration test; GARCH
JEL classification: C15; C32

1. Introduction

Cointegration is an important issue in time series and conditional heteroskedasticity seems ubiquitous in macro and financial time series. In this paper we examine the performance of Johansen's (1988) likelihood ratio tests for cointegration under the presence of conditional heteroskedasticity of the GARCH form (Engle, 1982; Bollerslev, 1986; Bollerslev, Engle, and Nelson, 1995). Comparisons are also conducted with the tests of Dickey and Fuller (1979, DF hereafter) and Sargan and Bhargava (1983).

The main assumption of the Johansen tests is that the disturbances in vector error correction models are i.i.d. Gaussian. Many studies have examined the performance of the Johansen tests under various situations where the assumptions are violated. Cheung and Lai (1993), Gonzalo (1994), Reimers (1991), and Reinsel and Ahn (1992) have examined the effect of dynamic components of the

* Corresponding author.

We would like to thank an anonymous referee for valuable comments. Any errors are ours.

SSDI 0304-4076(95)01745-Y

system on the performance of the Johansen tests. Gonzalo and Pitarakis (1994) examine the effect of the dimension of the system. Gonzalo and Lee (1995) consider situations where variables are not exactly I(1) but are very difficult to tell from I(1) using standard unit root tests. Franses and Haldrup (1994) examine the effect of outliers. Cheung and Lai (1993) and Gonzalo (1994) examine the effect of non-Gaussian error distribution. In many of these studies it is found that the Johansen procedure tends to find spurious cointegration more often than it should.

Cheung and Lai (1993) examine the bias in the test size due to nonnormal innovations, including nonsymmetric and leptokurtic ones. They find that the Johansen tests are reasonably robust to excess kurtosis. A commonly known source of leptokurtic innovations is conditional heteroskedasticity, which leads to a heavy-tailed distribution. In this paper we examine the results of Cheung and Lai (1993) more specifically by considering the case that the error variances follow a GARCH(1,1) model. We examine the empirical size and power of the Johansen trace test (trace, hereafter) and the Johansen maximum eigenvalue test (maxeigen). For comparison, we also consider the DF tests and the cointegrating regression Durbin-Waton test (CRDW) studied by Sargan and Bhargava (1983). The DF τ-statistic (denoted τ) is the usual t-value from the DF OLS regression, and another DF test statistic, $T(\hat{\alpha}-1)$, is obtained using the OLS estimate of the first-order autoregressive coefficient α of the DF regression and the sample size T.

Our experiment is an extension of the work by Kim and Schmidt (1993, KS hereafter) on unit root tests to the cointegration tests. KS examine the DF unit root tests when the errors are conditionally heteroskedastic. They find the DF tests overreject but only moderately. Our results are similar to and confirm the results of Cheung and Lai and KS. These cointegration tests tend to overreject the null hypothesis of no cointegration in favor of finding cointegration in the presence of GARCH errors, but the bias is not very serious. The results of KS for the DF unit root tests with GARCH errors generally carry over to cointegration tests under the presence of GARCH errors.

2. The simulation design

In the experiment for examining the size of the tests, we generate noncointegrated systems with GARCH(1,1) errors. Let $X_t = (x_{1t} \dots x_{Nt})'$ be an $N \times 1$ vector of integrated series with $\Delta X_t = \varepsilon_t$. The error vector $\varepsilon_t = (e_{1t} \dots e_{Nt})'$ is assumed to follow an N-variate conditional normal distribution with $\mathrm{E}(e_{it}|\mathscr{F}_{t-1}) = 0$ and $\mathrm{E}(e_{it}^2|\mathscr{F}_{t-1}) \equiv \sigma_{it}^2$, where $i = 1, \dots, N$, $\mathscr{F}_{t-1}$ is the σ-field generated by all information available at time $t-1$, and

$$\sigma_{it}^2 = \phi_{i0} + \phi_{i1}e_{i,t-1}^2 + \phi_{i2}\sigma_{i,t-1}^2.$$

Let $z_{it} \equiv e_{it}/\sigma_{it}$ be i.i.d. with $\mathrm{E}(z_{it}) = 0$, $\mathrm{var}(z_{it}) = 1$, and $\mathrm{E}(z_{it}z_{jt}) = 0$, $i \neq j$. Drawings of the pseudo random innovations z_{it}, for $i = 1, \dots, N$ and $t = 1, \dots, T + d$, are performed from the standard normal distribution. The first $d = 500$ observations are discarded.

We consider $T = 100$, 1000 and $N = 2$ with various choices of parameter values of $(\phi_{i0}, \phi_{i1}, \phi_{i2})$, $i = 1,2$. As we use the same parameter values $(\phi_{i0}, \phi_{i1}, \phi_{i2})$ for all i, we use the simpler notation σ_t^2 and (ϕ_0, ϕ_1, ϕ_2), omitting the index i in reporting the results. We use $T = 1000$ to examine the large-sample properties for the case when the asymptotic theory is not available. We generate the critical values for $T = 100$, 1000, based on 10,000 replications. For consistency, we use the same seed for all the simulations so that we work with the same random numbers. All simulations are based on 10,000 replications. All the reported results are at the nominal 5% significance level. The asymptotic 95% confidence interval of the empirical size is (0.0456, 0.0544). In all simulations to compute the residual-based test statistics (such as τ, $T(\hat{\alpha} - 1)$, and CRDW), x_{1t} is used as the dependent variable in the cointegrating regressions.

In the experiment for examining the power of the tests, we generate bivariate cointegrated series with GARCH errors. The system generated is $\Delta x_{1t} = -0.2(x_{1,t-1} - x_{2,t-1}) + e_{1t}$ and $\Delta x_{2t} = e_{2t}$, where e_{1t} and e_{2t} have the conditional variances of the GARCH(1,1) form discussed above. As we consider only a particular data generating process (DGP) the power of the test is dependent on the specification of the DGP, the results may only be illustrative. For a general study, see the method used in Johansen (1989) who investigates the power function using the theory of near-integrated processes developed in Phillips (1988).

Following KS, the DF τ-static with White's (1980) correction for heteroskedasticity is also examined. In computing the size and power of the τ-statistic with the White correction, we use two different critical values. One critical value is obtained from the 5% low tail percentile of the DF τ-statistic with the White correction. The other critical value is obtained from the 5% low tail percentile of the DF τ-static without the correction (the same one used for τ). The size and power using the first critical value (with the White correction) are reported under the notation 'τ-White', and the results using the second critical value are reported under the notation 'τ'-White'.

The paper is organized as follows. Tables 1, 3, 4, 5, and 6 show the empirical size of tests, the frequency that the null hypothesis stating that truly noncointegrated processes are not cointegrated is rejected in 10,000 trials. Table 2 shows the empirical power of tests, the frequency that the null hypothesis stating that truly cointegrated processes are not cointegrated is rejected in 10,000 trials. $\{z_{it}\}$ are generated from the standard normal distribution, except for Table 5 where $\{z_{it}\}$ are drawn from the Student-t distribution. Asymmetric conditional heteroskedasticity of the exponential GARCH (EGARCH) form and the time-varying conditional covariance are also considered in Table 6.

3. Results

Table 1 contains the results on the size of the tests. The size distortion is larger when $\phi_1 + \phi_2$ is larger. The condition for the existence of the unconditional fourth moment is $3\phi_1^2 + 2\phi_1\phi_2 + \phi_2^2 < 1$ (Bollerslev, 1986); accordingly, the condition is $\phi_2 < 0.606$ if $\phi_1 = 0.3$ and $\phi_2 < 0.890$ if $\phi_1 = 0.1$. In many applications with high-frequency financial data the estimate for $\phi_1 + \phi_2$ turns out to be very close to one and ϕ_0 is almost zero. For example, French et al. (1987) obtain the sum of the GARCH parameters equal to 0.996 and a very small (but significant) estimate $\phi_0 = 6e\text{-}7$ for a daily stock market return process. For the same value of $\phi_1 + \phi_2$, the size distortion is bigger with a higher ϕ_1 (see also Table 4). This is because, fixing $\phi_1 + \phi_2$, the unconditional kurtosis increases as ϕ_1 increases (Bollerslev, 1986). The size bias generally increases with sample size T when $\phi_1 + \phi_2 = 0.999$, while it generally decreases with T when $\phi_1 + \phi_2 = 0.9$ and 0.95.

In Table 2, we investigate the power of the tests. As the system generated is $\Delta x_{1t} = -0.2(x_{1,t-1} - x_{2,t-1}) + e_{1t}$ and $\Delta x_{2t} = e_{2t}$, the error correction term $w_t \equiv x_{1t} - x_{2t}$ follows an AR(1) model with the first autoregressive coefficient equals to 0.8, i.e., $w_t = 0.8w_{t-1} + \zeta_t$, where $\zeta_t \equiv e_{1t} - e_{2t}$ is a white noise. The

Table 1
The size of the tests at 5% level

T	100	1000	100	1000	100	1000
(ϕ_1, ϕ_2)	(0.3, 0.6)		(0.3, 0.65)		(0.3, 0.699)	
trace	0.0721	0.0609	0.0774	0.0648	0.0946	0.0987
maxeigen	0.0716	0.0606	0.0790	0.0670	0.0970	0.1032
CRDW	0.0712	0.0648	0.0753	0.0784	0.0806	0.1217
$T(\hat{\alpha} - 1)$	0.0711	0.0630	0.0756	0.0755	0.0819	0.1188
τ	0.0696	0.0628	0.0748	0.0748	0.0818	0.1195
τ-White	0.0449	0.0413	0.0439	0.0356	0.0428	0.0266
τ'- White	0.0907	0.0650	0.0883	0.0539	0.0845	0.0438
(ϕ_1, ϕ_2)	(0.1, 0.8)		(0.1, 0.85)		(0.1, 0.899)	
trace	0.0571	0.0521	0.0612	0.0548	0.0683	0.0803
maxeigen	0.0565	0.0531	0.0617	0.0566	0.0674	0.0865
CRDW	0.0548	0.0507	0.0545	0.0546	0.0578	0.0867
$T(\hat{\alpha} - 1)$	0.0557	0.0506	0.0568	0.0542	0.0587	0.0882
τ	0.0560	0.0511	0.0572	0.0530	0.0600	0.0885
τ-White	0.0474	0.0475	0.0479	0.0476	0.0470	0.0335
τ'- White	0.0984	0.0747	0.0989	0.0743	0.0969	0.0582

$\phi_0 = \sigma_0^2(1 - \phi_1 - \phi_2)$, $\sigma_0^2 = 1$.

Table 2
The power of the tests at 5% level

T	100	1000	100	1000	100	1000	100	1000
(ϕ_1,ϕ_2)	(0,0)		(0.3,0.6)		(0.3,0.65)		(0.3,0.699)	
trace	0.9856	1.0000	0.9594	1.0000	0.9488	0.9999	0.9278	1.0000
maxeigen	0.9906	1.0000	0.9637	1.0000	0.9537	0.9999	0.9345	1.0000
CRDW	0.7056	1.0000	0.6884	1.0000	0.6828	1.0000	0.6773	1.0000
$T(\hat{\alpha}-1)$	0.6750	1.0000	0.6669	1.0000	0.6635	0.9999	0.6584	0.9999
τ	0.6782	1.0000	0.6680	0.9999	0.6634	0.9999	0.6579	0.9999
τ-White	0.5283	1.0000	0.3504	0.9932	0.3274	0.9733	0.2974	0.9056
τ'- White	0.7335	1.0000	0.5295	0.9946	0.4964	0.9786	0.4584	0.9210
(ϕ_1,ϕ_2)			(0.1,0.8)		(0.1,0.85)		(0.1,0.899)	
trace			0.9797	1.0000	0.9731	1.0000	0.9284	1.0000
maxeigen			0.9834	1.0000	0.9784	1.0000	0.9356	1.0000
CRDW			0.6991	1.0000	0.6975	1.0000	0.6885	1.0000
$T(\hat{\alpha}-1)$			0.6762	1.0000	0.6720	1.0000	0.6676	1.0000
τ			0.6750	1.0000	0.6713	1.0000	0.6684	1.0000
τ-White			0.4719	1.0000	0.4626	1.0000	0.4495	0.9977
τ'- White			0.6746	1.0000	0.6645	1.0000	0.6394	0.9991

$\phi_0 = \sigma_0^2(1-\phi_1-\phi_2), \sigma_0^2 = 1$. The data are generated from $\Delta x_{1t} = -0.2(x_{1,t-1} - x_{2,t-1}) + e_{1t}$ and $\Delta x_{2t} = e_{2t}$.

regression-based tests for cointegration in $(x_{1t}, x_{2t})'$ is to test for a unit root in w_t when ζ_t is conditionally heteroskedastic. A comparable simulation result when $(\phi_1,\phi_2) = (0,0)$ and $T = 100$ may be found in Engle and Granger (1987, Table II), where the power of CRDW and τ are simulated for the case when $w_t = 0.8w_{t-1} + \zeta_t$ without GARCH in ζ_t. The power of the Johansen tests in Table 2 for $T = 100$ is higher than that of the DF and CRDW tests. When the conditional heteroskedasticity is stronger (i.e., when $\phi_1 + \phi_2$ is higher and ϕ_1 is larger if $\phi_1 + \phi_2$ is the same), all of the tests have lower power even when they are oversized.

As KS conclude that the White correction for τ for testing unit roots is generally helpful, we consider it for testing cointegration. The results in Table 1 shows that White's correction, τ-White (but not τ'-White), may improve the size distortion problem, especially when $(\phi_1 + \phi_2)$ is close to one. However, even if this optimistic perspective is taken for τ-White or τ'-White statistics, the story is only half-told, since the power of the tests must come into question. Although KS (p. 293) notice that the empirical size falls significantly below the nominal size when the White correction is made, especially for large T, they did not examine the power performance of τ-White (or τ'-White). It seems that White's heteroskedasticity correction lowers the power of the DF test when the heteroskedasticity is larger (when $\phi_1 + \phi_2$ is larger and ϕ_1 is larger if $\phi_1 + \phi_2$ is the same).

Nelson (1990a) shows that if $\phi_0 = 0$ and $E[\ln(\phi_2 + \phi_1 z_t^2] < 0$, then $\sigma_t^2 \to 0$ with probability one from any starting point. In particular, IGARCH is degenerate if $\phi_0 = 0$. Consequently, if $\phi_0 = 0$, the components in $X_t = (x_{1t}, x_{2t})'$ tends to zero when t gets large and they behave like being cointegrated (trivially), and therefore the tests reject the null hypothesis of no cointegration. But this degenerate case is not empirically interesting, since in the real world volatility does not seem to decline inexorably toward zero over time. Nelson (1990a) also shows that the σ_t^2 process has a strictly stationary and ergodic distribution if and only if $\phi_0 > 0$ and $E[\ln(\phi_2 + \phi_1 z_t^2)] < 0$. In particular, IGARCH with $\phi_0 > 0$ has a strictly and ergodic limit distribution. Therefore there is a fundamental difference between $\phi_0 = 0$ and ϕ_0 small but positive. We thus investigate the cases with $\phi_0 > 0$ in Table 3, where we set $\phi_1 + \phi_2 = 1$ and vary ϕ_0.

When $\phi_0 > 0$, ϕ_0 is simply scale parameter – e.g., doubling ϕ_0 shifts the density of $\ln(\sigma_t^2)$ a distance $\ln(2)$ to the right. The results are invariant to changes in the initial variance σ_0^2, so long as ϕ_0/σ_0^2 is held constant. In our experiment, however, we fix σ_0^2 (at $\sigma_0^2 = 1$) and change ϕ_0. Will ϕ_0 affect the size of the tests? No, if a Monte Carlo is properly designed. If ϕ_0 is very small, $\sigma_0^2 = 1$ is initialized too far in the right tail of its stationary distribution, so σ_t^2 tends to decline as t gets large. Thus, in order for the results not to depend on ϕ_0 for fixed σ_0^2, d should be fairly large. Discarding enough observations makes the empirical sizes in Table 3 very similar for all $\phi_0 > 0$.[1]

To be comparable with KS's results we also consider an experiment in KS's Table 4, following the parameterization of Nelson (1990b), where $\phi_0 = 0.01\gamma$, $\phi_1 = 0.3\gamma^{1/2}$, $\phi_2 = 1 - \phi_1$, $\sigma_0^2 = 1$. The performance of the tests improves as γ declines, since the unconditional kurtosis becomes smaller as ϕ_1 declines. This is also observed in Table 1, where the size distortion is more serious when $\phi_1 = 0.3$ than when $\phi_1 = 0.1$. The problem is worse when T is larger.

So far we have generated $\{z_{it}\}$ from the standard normal distribution. In Table 5 we simulate the series $\{z_{it}\}$ from the Student-t distribution with the degree of freedom (v) being equal to 5 and 8. The values of v are chosen based on the empirically estimated v by Bollerslev (1987) and Baillie and Myers (1991). The kurtosis of the Student-t density is given by $3(v-2)/(v-4)$ for $v > 4$: it is 9 for $v = 5$ and 4.5 for $v = 8$. The results for $(\phi_1, \phi_2) = (0.3, 0.65)$ are reported. When $v = 8$, the sizes of the tests are similar to those reported in Table 1 using

[1] We thank a referee for drawing our attention to this point. We choose $d = 500$ after examining plots. When $\phi_0 = 0.0001$ in Table 3, but if only $d = 50$ observations are discarded, the empirical size is very much different than reported (for example, when $\phi_0 = 0.0001$, the size of the trace test is 0.2277 for $T = 100$ and 0.4382 for $T = 1000$). When $\phi_0 = 0.01$ or larger, we find that either using $d = 50$ or $d = 500$ does not matter, suggesting that $d = 50$ is enough when ϕ_0 is not too small.

Table 3
The size of the tests at 5% level

ϕ_0	0.0001		0.01		1		100	
T	100	1000	100	1000	100	1000	100	1000
trace	0.0933	0.0937	0.0929	0.0997	0.0914	0.1006	0.0912	0.1003
maxeigen	0.0956	0.0990	0.0948	0.1052	0.0922	0.1060	0.0919	0.1060
CRDW	0.0810	0.1228	0.0810	0.1228	0.0810	0.1228	0.0810	0.1228
$T(\hat{\alpha}-1)$	0.0822	0.1204	0.0822	0.1203	0.0822	0.1203	0.0822	0.1203
τ	0.0819	0.1215	0.0819	0.1214	0.0819	0.1214	0.0819	0.1214
τ-White	0.0431	0.0263	0.0431	0.0263	0.0431	0.0263	0.0431	0.0263
τ'- White	0.0845	0.0437	0.0845	0.0437	0.0845	0.0437	0.0845	0.0437

$\phi_1 = 0.3$, $\phi_2 = 0.7$, $\sigma_0^2 = 1$.

Table 4
The size of the tests at 5% level

γ	1		0.09		0.01	
(ϕ_0, ϕ_1, ϕ_2)	(0.01, 0.3, 0.7)		(0.0009, 0.09, 0.91)		(0.0001, 0.03, 0.97)	
T	100	1000	100	1000	100	1000
trace	0.0929	0.0997	0.0661	0.0817	0.0545	0.0677
maxeigen	0.0948	0.1052	0.0657	0.0859	0.0536	0.0753
CRDW	0.0810	0.1228	0.0575	0.0842	0.0514	0.0598
$T(\hat{\alpha}-1)$	0.0822	0.1203	0.0582	0.0876	0.0522	0.0635
τ	0.0819	0.1214	0.0587	0.0887	0.0520	0.0650
τ-White	0.0431	0.0263	0.0480	0.0345	0.0502	0.0480
τ'-White	0.0845	0.0437	0.0982	0.0572	0.1051	0.0766

$\phi_0 = 0.01\gamma$, $\phi_1 = 0.3\gamma^{1/2}$, $\phi_2 = 1 - \phi_1$, $\sigma_0^2 = 1$.

the normal distribution. However, when $\nu = 5$, the size distortion becomes larger.

In an attempt to capture the asymmetric impact of innovation on volatility, Nelson (1991) develops the EGARCH model of the form

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1(|z_{t-1}| - \mathrm{E}|z_{t-1}|) + \theta z_{t-1} + \alpha_2 \ln(\sigma_{t-1}^2).$$

He shows that θ is significantly negative for modeling the stock market index volatility, suggesting that the variance tends to rise (fall) when the past innovation is negative (positive) in accordance with the empirical evidence for stock returns. In the first part of Table 6, we generate series with the conditional

Table 5
The size of the tests at 5% level, t-distribution (ν = degrees of freedom)

ν	8	5	8	5	8	5
T	100	100	100	100	1000	1000
(ϕ_1, ϕ_2)	(0,0)		(0.3,0.65)		(0.3,0.65)	
trace	0.0560	0.0508	0.0776	0.0842	0.0717	0.0972
maxeigen	0.0554	0.0497	0.0794	0.0846	0.0734	0.1032
CRDW	0.0529	0.0560	0.0859	0.0888	0.0847	0.1273
$T(\hat{\alpha} - 1)$	0.0516	0.0544	0.0851	0.0889	0.0805	0.1262
τ	0.0522	0.0544	0.0858	0.0867	0.0815	0.1257
τ-White	0.0457	0.0364	0.0416	0.0382	0.0323	0.0256
τ'-White	0.0921	0.0809	0.0858	0.0767	0.0511	0.0446

$\phi_0 = \sigma_0^2(1 - \phi_1 - \phi_2)$, $\sigma_0^2 = 1$.

variances of EGARCH. We use $\alpha_0 = -0.0082$, $\alpha_1 = 0.19$, and $\alpha_2 = 0.91$, which are the parameter values estimated in French and Sichel (1993), who model quarterly U.S. real GNP for 1947:2 to 1991:1. Their estimated θ is -0.19. The asymmetric volatility with $\theta = -0.19$ does not seem to make much difference from the previous cases with symmetric GARCH models. However, when we increase the symmetric volatility parameter to $\theta = -0.50$, the bias of the empirical size becomes larger.

Although we have reported the results under the assumption that the conditional covariances are zero for all t, $\mathrm{E}(e_{1t}e_{2t} | \mathscr{F}_{t-1}) = 0$, we also experimented with time-varying conditional covariances in the second part of Table 6. The conditional covariance is $\mathrm{E}(e_{1t}e_{2t} | \mathscr{F}_{t-1}) \equiv \rho_t \sigma_{1t} \sigma_{2t}$, where ρ_t is the conditional correlation. Let us assume that the conditional correlation is constant over time, i.e., $\rho_t = \rho$ for all t, as in Bollerslev (1990). We use this specification of the time-constant conditional correlation because we want to control only one parameter ρ. The data z_{1t} is generated from the standard normal distribution, and $z_{2t} = \rho z_{1t} + (1 - \rho^2)^{1/2} z_{3t}$, where z_{3t}, where z_{3t} is generated from the standard normal distribution and independent of z_{1t}. Thus $\mathrm{E}z_{1t} = \mathrm{E}z_{2t} = 0$, $\mathrm{E}z_{1t}^2 = \mathrm{E}z_{2t}^2 = 1$, and $\mathrm{E}(z_{1t}z_{2t}) = \rho$. The results with $\rho = 0$, 0.5, and 0.9 are reported.[2] The GARCH parameters used are $(\phi_1, \phi_2) = (0.3, 0.65)$, $\phi_0 = \sigma_0^2(1 - \phi_1 - \phi_2)$, and $\sigma_0^2 = 1$. It is shown that the size distortion increases moderately with ρ.[3]

[2] A large ρ is very likely in practice. For example, in Kroner and Sultan (1993), estimated ρ is ranged from 0.96 to 0.99 for weekly spot and future foreign currency series.

[3] If $\phi_1 = \phi_2 = 0$, ρ does not affect the tests at all.

Table 6
The size of the tests at 5% level, EGARCH and time-varying conditional covariance, $T = 100$

	EGARCH		Time-varying conditional covariance		
	$\theta = -0.19$	$\theta = -0.50$	$\rho = 0$	$\rho = 0.5$	$\rho = 0.9$
trace	0.0580	0.0803	0.0774	0.0810	0.0920
maxeigen	0.0574	0.0772	0.0790	0.0830	0.0946
CRDW	0.0613	0.1070	0.0753	0.0819	0.0965
$T(\hat{\alpha} - 1)$	0.0629	0.1097	0.0756	0.0836	0.0999
τ	0.0621	0.1105	0.0748	0.0838	0.0965
τ-White	0.0457	0.0301	0.0439	0.0452	0.0445
τ'-White	0.0965	0.0605	0.0883	0.0914	0.0899

1. EGARCH: $\ln(\sigma_t^2) = \alpha_0 + \alpha_1(|z_{t-1}| - E|z_{t-1}|) + \theta z_{t-1} + \alpha_2 \ln(\sigma_{t-1}^2)$, where $\alpha_0 = -0.0082$, $\alpha_1 = 0.19$, $\alpha_2 = 0.91$, and $E|z_{t-1}| = (2/\pi)^{1/2}$.

2. $\rho = E(z_{1t}z_{2t}|\mathscr{F}_{t-1})$, $(\phi_1, \phi_2) = (0.3, 0.65)$, $\phi_0 = \sigma_0^2(1 - \phi_1 - \phi_2)$, $\sigma_0^2 = 1$.

4. Concluding remarks

We examine the performance of various tests for cointegration under the presence of GARCH. These tests are generally oversized, but not very seriously, except when $\phi_1 + \phi_2$ is close or equal to unity and ϕ_1 is large. The White (1980) heteroskedasticity correction on the Dickey–Fuller tests improves the size of the test but lowers power performance. The size distortion under the presence of conditional heteroskedasticity increases when the Student-t distribution, EGARCH, and time-varying conditional covariance are considered.

References

Baillie, R.T. and R.J. Myers, 1991, Bivariate GARCH estimation of the optimal commodity futures hedge, Journal of Applied Econometrics 6, 109–124.

Bollerslev, T., 1986, Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics 31, 307–327.

Bollerslev, T., 1987, A conditional heteroskedastic time series model for speculative prices and rates of return, Review of Economics and Statistics 31, 542–547.

Bollerslev, T., 1990, Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model, Review of Economics and Statistics 72, 498–505.

Bollerslev, T., R.F. Engle, and D.B. Nelson, 1995, ARCH models, in: R.F. Engle and D. McFadden, eds., Handbook of econometrics, Vol. 4 (North-Holland, Amsterdam).

Cheung Y.-W. and K.S. Lai, 1993, Finite sample sizes of Johansen's likelihood ratio tests for cointegration, Oxford Bulletin of Economics and Statistics 55, 313–328.

Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427–31.

Engle, R.F., 1982, Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, Econometrica 50, 987–1007.

Engle, R.F. and C.W.J. Granger, 1987, Cointegration and error correction: Representation, estimation, and testing, Econometrica 55, 251–276.
French, K.R., G.W. Schwert, and R.F. Stambaugh, 1987, Expected stock returns and volatility, Journal of Financial Economics 19, 3–29.
French, M.W. and D.E. Sichel, 1993, Cyclical patterns in the variances of economic activity, Journal of Business and Economic Statistics 11, 113–119.
Franses, P.H. and N. Haldrup, 1994, The effects of additive outliers on tests for unit roots and cointegration, Journal of Business and Economic Statistics 12, 471–478.
Gonzalo, J., 1994, Five alternative methods of estimating long-run equilibrium relationships, Journal of Econometrics 60, 203–233.
Gonzalo, J. and T.H. Lee, 1995, Pitfalls in testing for long run relationships (Boston University, Boston, MA; University of California, Riverside, CA).
Gonzalo, J. and J.Y. Pitarakis, 1994, Comovements in large systems, Ruth Pollak working paper no. 33 (Boston University, Boston, MA).
Johansen, S., 1988, Statistical analysis of cointegration vectors, Journal of Economic Dynamics and Control 12, 231–254.
Johansen, S., 1989, The power function of the likelihood ratio test for cointegration, in: J. Gruber, ed., Econometric decision models, Lecture notes in economics and mathematical systems, Vol. 366 (Springer-Verlag, Berlin) 323–335.
Kim, K. and P. Schmidt, 1993, Unit root tests with conditional heteroskedasticity, Journal of Econometrics 59, 287–300.
Kroner, K.F. and J. Sultan, 1993, Time-varying distributions and dynamic hedging with foreign currency futures, Journal of Financial and Quantitative Analysis 28, 535–551.
Nelson, D.B., 1990a, Stationarity and persistence in the GARCH(1,1) model, Econometric Theory 6, 318–334.
Nelson, D.B., 1990b, ARCH models as diffusion approximations, Journal of Econometrics 45, 7–38.
Nelson, D.B., 1991, Conditional heteroskedasticity in asset returns: A new approach, Econometrica 59, 347–370.
Phillips, P.C.B., 1988, Regression theory for near-integrated time series, Econometrica 56, 1021–1043.
Reimers, H.E., 1991, Comparisons of tests for multivariate cointegration (Deutsche Bunndesbank, Frankfurt).
Reinsel, G.C. and S.K. Ahn, 1992, Vector autoregressive models with unit roots and reduced rank structure: Estimation, likelihood ratio test, and forecasting, Journal of Time Series Analysis 13, 353–375.
Sargan, J.D. and A. Bhargava, 1983, Testing residuals from least squares regression for being generated by the Gaussian random walk, Econometrica 51, 153–174.
White, Halbert, 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, Econometrica 48, 817–838.