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Disequilibrium and uncertainty in cointegrated systems: Some empirical evidence

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Abstract

We examine whether uncertainty in predicting cointegrated series depends on degrees of short-run deviation from the long-run cointegrated equilibrium relationship. Predictability of cointegrated series can be improved by including the error correction term in conditional variances just as in conditional means.

Keywords: Cointegration; Error correction; Conditional variance

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1. Introduction

If the first difference of a series can be represented by a stationary and invertible ARMA process, it is said to be integrated. An integrated series contains a long-run dominant component and has a long memory. There may be a set of integrated economic variables whose linear combination can cancel the common long-run component and become stationary. Such variables are cointegrated and move together in the long run. Cointegration links long-run components of a pair or of a group of series. It can be used to discuss some types of equilibrium and to introduce those equilibria into time-series models. If cointegrated series deviate from the equilibrium in the short run due to disturbances to the system, the deviation (which may be called disequilibrium) must be corrected. The short-run deviation has important predictive power for the conditional mean of cointegrated series as the cointegrated series are attracted to each other in the long run, and thus it may be called the error correction term. The dynamic models for this short-run behavior of cointegrated series are error correction models (ECM).

Since the error correction term is such a useful variable in predicting the conditional mean

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so may it be in predicting the conditional variance, since if cointegrated series depart further from each other it may be harder to predict them. Thus we ask if the prediction error variance of cointegrated series can be affected by the degree of the short-run deviations. If disequilibrium may be responsible for the conditional heteroskedasticity it would be modeled accordingly. Our main interest is therefore the potential causality of short-run disequilibrium for uncertainty measured by the conditional variance of the one-step-ahead prediction error in cointegrated systems.

The statistical models are the conditional moments given an information set. Hence any variables that influence either the first or the second moments are likely to affect the other. Forecastability of cointegrated series can thus be improved by including the error correction term in specifying conditional variances just as we do for error correction models in specifying conditional means.

2. The model

Consider a 2×1 vector of cointegrated series X_t which may be generated by the ECM

$$A(B)\,\Delta X_t = \mu + \gamma z_{t-1} + \varepsilon_t\,, \tag{1}$$

where $A(B)$ is a kth order 2×2 matrix polynomial in the backshift operator B, $\Delta = 1 - B$, z_{t-1} is the short-run deviation, and μ and γ are 2×1 vectors of parameters. The error vector $\varepsilon_t = (e_{1t}, e_{2t})'$ is assumed to follow a bivariate conditional normal distribution with mean zero and conditional variance–covariance matrix $H_t \equiv \mathrm{E}(\varepsilon_t \varepsilon_t' \mid \mathcal{F}_{t-1})$, where $\mathcal{F}_{t-1}$ is the σ-field generated by all information available at time $t-1$. Let $h_{ijt} \equiv \mathrm{E}(e_{it} e_{jt} \mid \mathcal{F}_{t-1})$.

The error ε_t is the one-step-ahead forecast error and $\mathrm{E}(\varepsilon_t \mid \mathcal{F}_{t-1}) = 0$ with probability one by construction. However, the error ε_t is not necessarily independent and each element of H_t can be written as a measurable function of the variables in $\mathcal{F}_{t-1}$. A process exhibiting ARCH may be possible and thus we consider the model

$$H_t = C'C + (A'A) \odot (\varepsilon_{t-1}\varepsilon_{t-1}') + B'H_{t-1}B + D'Dz_{t-1}^2\,, \tag{2}$$

where

$$A = \begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 & c_2 \\ 0 & c_3 \end{pmatrix}, \quad D = \begin{pmatrix} d_1 & d_2 \\ 0 & d_3 \end{pmatrix}$$

and $\odot$ indicates element-by-element matrix multiplication. The first element of B and the diagonal elements of A, C, and D are restricted to be positive in order to identify them uniquely. $D \neq 0$ implies that if the short-run deviation is bigger it is harder to predict the series.[1]

[1] The second term in (2) is specified as $(A'A) \odot (\varepsilon_{t-1}\varepsilon_{t-1}')$ following Attanasio (1991). For the third term, just for variety's sake, we choose $B'H_{t-1}B$, following Engle and Kroner (1994), instead of an alternative specification $(B'B) \odot H_{t-1}$, which did not affect our results at all.

Table 1
Unit root tests

	r_t	b_t	p_t	$b_t - r_t$	$p_t - b_t$
PP	−2.21	−2.13	−2.37	−4.57	−3.74
ADF	−1.71 (6)	−1.76 (8)	−1.84 (6)	−5.25 (1)	−3.76 (0)

Note: PP and ADF denote the Phillips–Perron (1988) and the augmented Dickey–Fuller statistics, respectively. Six autocovariances are used to compute PP following Newey and West (1987). The numbers of lag-augmentation in computing ADFs are reported in parentheses, which are chosen using the SIC. Both statistics are computed from regressions with a constant term but without a time trend term.

3. Results

We consider two bivariate cointegrated systems of US monthly interest rate series: (a) the three-month Treasury Bill interest rate (denoted by r_t, Citibase mnemonic is FYGM3) and six-month Treasury Bill interest rate (denoted by b_t, FYGM6) and (b) b_t and interest rates on the six-month Commercial Paper (denoted by p_t, FYCP). The sample period is from January 1959 to December 1992 with 408 observations. The series are monthly averages of daily figures in percent per annum obtained from the Citibase. We thus have the two systems: $X_t = (r_t b_t)'$ and $X_t = (p_t b_t)'$. The lag lengths (k) in Eq. (1) chosen by the use of the Schwarz information criteria (SIC) are 2 and 1 for each system.

We implement the test for the unit root hypothesis. The results in Table 1 indicate that the unit root hypothesis cannot be rejected for r_t, b_t, and p_t, while the two spread series, $z_t = b_t - r_t$ and $z_t = p_t - b_t$, are stationary. Thus $(r_t b_t)'$ and $(p_t b_t)'$ are cointegrated with a known cointegrating vector $(1 \quad -1)'$.

In Table 2 the p-values of residual diagnostics for the estimated models assuming conditional homoskedasticity are presented. As the presence of time-varying higher moments generally induces an incorrect size of the Ljung–Box statistics, we use the robust Lagrange multiplier (LM) test for autocorrelations due to Wooldridge (1990). The robust LM test for autocorrelations of orders 1 and 10 and the LM test for ARCH of orders 5 and 10 are reported. Using the lag lengths (k) chosen by the SIC, the residuals are not serially correlated and there are strong ARCH in both systems.

We assume that ε_t is distributed as conditionally normal. The parameter estimates are then obtained by maximizing the quasi log-likelihood function over the parameter space for all the

Table 2
Diagnostic tests

	Δr_t	Δb_t	Δp_t	Δb_t
AR (1)	0.872	0.706	0.091	0.070
AR (10)	0.998	0.999	0.852	1.000
ARCH (5)	0.000	0.000	0.000	0.000
ARCH (10)	0.000	0.000	0.000	0.000

Note: All of the test statistics follow chi-square distributions with the degrees of freedom being equal to the number in parentheses. All the values reported are asymptotic p-values.

Table 3
Parameter estimates

	$\Delta X_t = (\Delta r_t\ \Delta b_t)'$	$\Delta X_t = (\Delta p_t\ \Delta b_t)'$
c_1	0.106 (6.752)	0.048 (3.211)
c_2	0.124 (7.377)	0.022 (1.129)
c_3	0.009 (1.058)	0.022 (1.571)
a_1	0.601 (11.563)	0.647 (18.243)
a_2	0.537 (9.550)	0.516 (11.916)
a_3	0.195 (8.090)	0.136 (2.796)
b_1	0.670 (34.861)	0.752 (58.563)
b_2	0.680 (19.210)	0.809 (28.563)
d_1	0.510 (5.713)	0.122 (3.991)
d_2	0.462 (4.978)	0.140 (3.712)
d_3	0.063 (0.810)	0.046 (0.816)

Note: The robust asymptotic t-values are in parentheses.

parameters in the conditional mean and variance equations using the method of score with only the first numerical derivatives being used, following Bollerslev and Wooldridge (1992). At the quasi maximum likelihood estimates (QMLE) the robust asymptotic covariance matrix of the estimated parameters and the robust asymptotic t-values are computed à la White (1982) and Bollerslev and Wooldridge (1992).

Table 3 reports the results of the estimated parameters in Eq. (2). To save space the results for the conditional mean equations are not reported. The asymptotic t-values are in parentheses. Several authors have proved that the QMLEs are consistent and asymptotically normal.[2] The estimates of D are significant.

In order to test the validity of the model reported in Table 3, LM tests for ARCH in the residuals standardized by $h_{iit}^{1/2}$ are presented in Table 4. The tests suggest that much of conditional heteroskedasticity is captured by the model.

Table 4
Specification tests for standardized residuals

	Δr_t	Δb_t	Δp_t	Δb_t
ARCH (5)	0.238	0.223	0.887	0.674
ARCH (10)	0.008	0.041	0.944	0.081

Note: All of the test statistics follow chi-square distributions with the degrees of freedom being equal to the number in parentheses. All the values reported are asymptotic p-values.

[2] Weiss (1986), Bollerslev and Wooldridge (1992), Lumsdaine (1991), and Lee and Hansen (1992) provide conditions under which the QMLEs for GARCH models are consistent and asymptotically normal.

4. Concluding remarks

For the system of $X_t = (r_t b_t)'$, the results indicate that when the yield curve is steeper, the interest rates are more volatile and uncertainty increases.

On the system of $X_t = (p_t b_t)'$, recently Friedman and Kuttner (1992) and Bernanke (1990) documented the high information content of the paper-bill spread for future economic activity. Stock and Watson (1989), who examined the information contained in a wide variety of economic variables in an attempt to construct a new index of leading indicators, found that the paper-bill spread outperformed nearly every other variable as a forecaster of the business cycle. Given these results it may be natural to raise a question of how the paper-bill spread itself can be predicted better.

The model seems useful to see how the short-run disequilibrium has an effect on uncertainty in predicting cointegrated series. When one expects increased volatility due to shocks to the system to propagate on both first and second moments, it should be modeled so. We believe that the specification could ensure a better fit and could be consistent empirically with some other examples in economics. The extent to which it is found in other economic series can only be established by further empirical work.

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