# Spread and volatility in spot and forward exchange rates

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This paper is concerned with modeling the conditional heteroscedasticity of the prediction error of foreign exchange rates. As spot and forward rates are cointegrated we use a system of error correction models for mean prediction. To predict the variance we use a bivariate generalized autoregressive conditional heteroscedasticity (GARCH) model as a function of the spread. Using daily series for seven currencies, we find that unmodeled conditional heteroscedasticity by GARCH can generally be explained by the squared spread. This indicates that as the spread is bigger the exchange rates are more volatile. (JEL F31, C32).

The characterizations of exchange rate movements have important implications for many issues in international finance and macroeconomics. It is therefore important to carefully model any temporal variations in the volatility process as well as in the process of change in the exchange rate series. A standard way to specify volatility is autoregressive conditional heteroscedasticity (ARCH) due to Engle (1982) and generalized ARCH (GARCH) of Bollerslev (1986). In this paper, we consider an extension of the GARCH model for the error correction models (ECM) of cointegrated series. Cointegration links long-run components of a group of integrated series. It can be used to discuss some types of equilibrium and to introduce those equilibria into time series models. The dynamic models for the short-run behavior of cointegrated series are called error correction models.

An ECM may be considered as a prediction equation of the cointegrated series as it is a conditional expectation of the first difference of the series given an information set. Engle and Yoo (1987) show that the error correction term, which is the short-run deviation from a long-run cointegrated relationship, has important predictive power for conditional mean of the cointegrated series. If the error correction term is an important variable in the conditional mean, so may it be in the conditional variance. This may imply that if the series deviate further from each other they are harder to predict. If disequilibrium (measured by error correction term) is responsible for uncertainty (measured by conditional variance),

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conditional heteroscedasticity may be modeled with a function of several lagged error correction terms. Examining the behavior of the variances over time as a function of short-run deviations is reasonable when one expects increased volatility due to shocks to the system which propagate on both the first and the second moments. We thus use a system of ECMs for the conditional mean, and an extended bivariate GARCH model with error correction term for the conditional variance. The main virtue of the model lies in its capability of pointing to a particular feature of cointegrated series, which is the potential relationship between disequilibrium and uncertainty. The model seems appropriate for testing for causality in variance through the error correction term.

Using this model we investigate the predictive power of the spread between daily spot and forward exchange rates for seven currencies in predicting volatility of the exchange rate changes. We find that unmodeled conditional heteroscedasticity by the GARCH(1, 1) model can be explained by a function of the spread. The conditional variance of the prediction error is positively related with the squared spread. This indicates that as the spread is bigger the exchange rates are more volatile and harder to predict. The short-run error from the cointegrating long-run relationship is therefore a useful variable in modeling conditional variance as well as conditional mean. This may be exploited to obtain more precise time varying confidence intervals for point forecasts of exchange rate changes.

#### I. The model

Much empirical research on the risk premium in forward foreign exchange relies on intertemporal asset pricing models such as that of Lucas (1978). An important relationship in an international environment that can be derived from there is the conditional pricing relation for a forward premium:  $E_tQ_{t+k}z_{t+k}=0$ , where  $E_t$  is the conditional expectation given an information available at time t,  $Q_{t+k} = (U'(C_{t+k})/U'(C_t))(P_t/P_{t+k})$  is the marginal rate of substitution between dates t and t+k,  $U'(C_t)$  is the marginal utility of real per capita consumption  $C_t$ ,  $P_t$  is a price index in the domestic economy at time t,  $z_{t+k} = (S_{t+k}^j - F_{t,k}^j)$  is the forward premium,  $S_t^j$  is the spot exchange rate between the domestic currency j at time t,  $F_{t,k}^t$  is the forward exchange rate which is the domestic currency price of a unit of currency j established at t for payment at t+k. Then we have  $E_tQ_{t+k}E_tz_{t+k} + \rho_t(Q_{t+k}, z_{t+k}) \sigma_t(Q_{t+k}) \sigma_t(z_{t+k})=0$ , where  $\rho_t$  is the conditional correlation and  $\sigma_t$  is the conditional standard deviation. The premium on forward foreign exchange is seen to be related to the conditional second moments.<sup>1</sup>

Although asset pricing theories relates first moments to second moments, most empirical studies (e.g. Giovannini and Jorion, 1989; Kaminsky and Peruga, 1990; McCurdy and Morgan, 1991; Attanasio, 1991) have focused on a specification in which first moments are explained by second moments, especially using GARCH in mean (GARCH-M) model of Engle *et al.* (1987). Exceptions are Cumby and Obstfeld (1984) and Hodrick (1989), who consider the converse. While the GARCH-M specification is useful to explain excess return in terms of conditional second moments, examining the converse is also interesting due to the reason discussed before especially when the system is cointegrated. The purpose of the paper is to examine this converse specification using a multivariate GARCH specification in vector error correction models. Error correction terms TAE-HWY LEE

often turn out to be the most important terms for conditional means, especially in the long horizons. Since most finance theory links conditional means to conditional second moments, it is naturally interesting to see if the most important variables for conditional means do in fact affect conditional variances. This allows us to examine the potential relationship between disequilibrium and uncertainty in the cointegrated system.

Now we turn to our econometric model. Consider a  $2 \times 1$  vector of cointegrated series  $X_t$ . They may be considered to be generated by the following ECM

$$\langle 1 \rangle \qquad \Delta X_{y} = \mu + \gamma z_{t-1} + \Gamma_{1} \Delta X_{t-1} + \ldots + \Gamma_{p} \Delta X_{t-p} + \varepsilon_{t},$$

where  $\Delta = 1 - B$ , B is the backshift operator,  $z_t \equiv \alpha' X_t$  is the magnitude of the short-run deviation at time t, and  $\mu$ ,  $\gamma$ ,  $\alpha$  are all  $2 \times 1$  vectors of parameters and  $\Gamma$ s are  $2 \times 2$  matrices of parameters.  $\alpha$  is the cointegrating vector which determines the long-run equilibrium relationship and  $\gamma$  is the parameter measuring the speed of error corrections. The error vector  $\varepsilon_t = (e_{1t} e_{2t})'$  is assumed to follow a bivariate conditional normal distribution with mean zero and conditional variance-covariance matrix  $H_t \equiv E(\varepsilon_t \varepsilon'_t | \mathscr{F}_{t-1})$ , where  $\mathscr{F}_{t-1}$  is the  $\sigma$ -field generated by all information available at time t-1.

If the ECM  $\langle 1 \rangle$  is correctly specified for the conditional mean  $E(\Delta X_t | \mathscr{F}_{t-1})$ , then  $E(\varepsilon_t | \mathscr{F}_{t-1}) = 0$  with probability one by construction. The error  $\varepsilon_t$  is a martingale difference process but is not necessarily independent. The squared error may be predictable using the information set, and thus the conditional variance may be a function of the variables in the information set.  $H_t$  can be written as a measurable function of the variables in  $\mathscr{F}_{t-1}$ . We may consider a function of  $\Delta X_{t-s}$  such as selective elements of  $\Delta X_{t-s} \Delta X'_{t-s}$  for some  $s \ge 1$ . However, the short-run disequilibrium error  $z_{t-1}$  may seem an especially interesting one. We consider the squared  $z_{t-1}$  in a bivariate GARCH(1, 1) model suggested by Baba *et al.* (1991),

$$\langle 2 \rangle \qquad \qquad H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B + D'Dz_{t-1}^2$$

where A, B, C and D are  $2 \times 2$  matrices, C is an upper triangular matrix with  $c_1$ ,  $c_2$  in its diagonal and  $c_3$  off the diagonal, and D has the same form as C with corresponding elements  $d_1$ ,  $d_2$  and  $d_3$ . The model will be referred to as 'GARCH-X'. The estimates of  $d_i$  may quantify the extent that this model explains the relationship between disequilibrium and conditional volatility not explained by GARCH(1, 1). This parameterization guarantees  $H_t$  to be positive definite and it allows the conditional covariances to change signs over time.<sup>2</sup> In the empirical study in the next section we suppose that A and B are diagonal matrices with  $a_1$ ,  $a_2$  and  $b_1$ ,  $b_2$  in the diagonals, respectively.

#### **II.** Empirical results

In the following analysis we use daily spot and 30-day forward exchange rate data from the New York Foreign Exchange Market. The data begin on March 1, 1980 and end on January 28, 1985, which constitute a total of 1245 observations. The data are those used in Baillie and Bollerslev (1989a, 1989b). They are opening bid prices for British Pound (BP), German Deutche mark (DM), Japanese Yen (JY), Canadian Dollar (CD), French Franc (FF), Italian Lira (IL) and the Swiss Franc (SF). Apart from the BP all the series are in terms of the number of US

Dollars for one unit of foreign currency. The notation  $s_t$  and  $f_t$  is used for the logarithms of the spot and forward series, respectively, so that  $X_t = (s_t f_t)'$ .

Baillie and Bollerslev (1989a) present the results that the unit root hypothesis cannot be rejected for all the series. They also report the results of the cointegration tests of Phillips (1987) between  $s_{t+22}$  and  $f_t$ , which are found strongly cointegrated. However, if  $X_t = (s_{t+22} f_t)'$ , the ECM  $\langle 1 \rangle$  includes variables not available at time t. As we are interested in prediction using the conditional expectations given information at time t, we instead use  $X_t = (s_t f_t)'$  so that the right-hand side of equation  $\langle 1 \rangle$  should not include the variables unknown at time t.  $s_t$  and  $f_t$  are found strongly cointegrated in all currencies at less than one per cent level. We use  $z_t \equiv f_t - s_t$ , which is found stationary. To save space the estimated statistics are not reported.

The lag lengths (p) in ECM  $\langle 1 \rangle$ , chosen by the use of the Schwarz information criterion (SIC), are 4 (BP), 4 (DM), 5 (JY), 5 (CD), 0 (FF), 0 (IL) and 4 (SF).

In Table 1, the asymptotic probability values for various specification tests for the ECM  $\langle 1 \rangle$  estimated by least squares assuming conditional homoscedasticity are presented. These tests are: (a) the Ljung-Box test and the McLeod-Li (1983) test for up to the twentieth order serial correlation in the residuals and in the squared residuals; (b) Wooldridge's (1990) robust regression based LM test for autocorrelations (AR); (c) the LM test for ARCH; (d) White's (1989) neural network test for neglected non-linearity in conditional mean; and (e) the LM test for GARCH-X. Using the lag lengths (p) in ECM  $\langle 1 \rangle$  chosen by the SIC, the residuals are not serially correlated, while the McLeod-Li statistics and the LM test statistics for ARCH are very significant.

We estimate the model with formulations of the conditional mean  $E(\Delta X_t | \mathscr{F}_{t-1})$ as in  $\langle 1 \rangle$  and the conditional variance  $H_t \equiv E(\varepsilon_t \varepsilon'_t | \mathscr{F}_{t-1})$  as in  $\langle 2 \rangle$ . Let  $\theta$  be the vector of all the parameters in the conditional mean and conditional variance. The parameter estimates are obtained by maximizing the quasi (normal) loglikelihood function over  $\theta$  using scoring methods with only first numerical derivatives being used (à la Bollerslev and Wooldridge, 1992). At the maximizing value of  $\theta$  (the QMLE), the asymptotic robust standard errors are obtained (à la White, 1982; White and Domowitz, 1984; Weiss, 1986; Bollerslev and Wooldridge, 1992). In Table 2 the estimated parameters of the conditional variances are reported for five currencies. To save space the results for the conditional means are not reported. The asymptotic robust standard errors are in parentheses. BP and JY are not included as we cannot find a step of increasing likelihood from many different initial values of  $\theta$ . The estimates of D are generally significant.

Nelson (1990) and Bougerol and Picard (1992) establish conditions for the stationarity and ergodicity of the (integrated) GARCH(1, 1) process. Bollerslev and Wooldridge (1992) provide regularity conditions under which the QMLE will be consistent and asymptotically normal, some of which are verified by Lumsdaine (1991). She also proves that the QMLE is consistent and asymptotically normal without assuming a finite fourth moment of errors (which is assumed in Weiss, 1986). Lee and Hansen (1992) show that these may hold when the GARCH(1, 1) process is integrated or even when it is mildly explosive provided that the *conditional* fourth moment of *standardized* error is bounded (which is a fairly weak condition).

Based on the asymptotic normality of the QMLE we test the GARCH-X using Lagrange multiplier (LM) test, Wald tests, and likelihood ratio (LR) tests.<sup>3</sup> Our

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Table

	D	M	U	D	ц	ιF	Ι	L	S	н
	$\Delta S_t$	$\Delta f_t$	$\Delta s_t$	$\Delta f_t$	$\Delta s_t$	$\Delta f_t$	$\Delta s_t$	۵fr	$\Delta s_t$	$\Delta f_t$
Ljung-Box(20)	0.660	0.823	0.108	0.110	0.242	0.109	0.091	0.186	0.351	0.342
McLeod-Li(20)	0.000	0.000	0.000	0.000	0.085	0.000	0.000	0.000	0.000	0.000
Neural(3)	0.139	0.321	0.020	0.273	0.080	0.007	0.483	0.171	0.290	0.341
AR(1)	0.992	0.677	0.464	0.280	0.037	0.040	0.075	0.053	0.848	0.530
AR(2)	0.953	0.970	0.504	0.572	0.999	0.987	0.993	0.865	0.955	0.991
AR(5)	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	0.889	0.874
ARCH(1)	0.003	0.003	0.000	0.001	0.161	0.026	0.000	0.000	0.091	0.163
ARCH(5)	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.013	0.000	0.000	0.000	0.000	0.000
ARCH[0]-X	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000
ARCH[1]-X	0.025	0.027	0.001	0.003	0.143	0.017	0.239	0.929	0.013	0.013
ARCH[5]-X	0.128	0.133	0.016	0.028	0.160	0.057	0.315	0.852	0.088	0.090
ARCH[10]-X	0.106	0.106	0.049	0.071	0.159	0.077	0.379	0.947	0.101	0.108
GARCH[1,1]-X	0.038	0.005	0.004	0.552	0.985	0.027	0.020	0.163	0.003	0.012
Note: The number in pa All the values are	rentheses is the asymptotic <i>p</i> -va	degree of freede alues. For the r	om. The numbe seural network	r in square brates the test we use 10	ackets is the or phantom hide	der of GARCF den units, 3 pri	I fitted under th incipal comport	he null hypothe nents of them,	esis to test for ( and 5 draws of	GARCH-X.

computing the Hochberg Bonferroni based (see Lee *et al.*, 1993). We do not report results for BP and JY as we could not find a step of increasing likelihood for the ECM model with GARCH-X specification from many different initial values of  $\theta$ .

	E	ЭM	C	CD	F	F		1L	Ş	SF
$c_1$	0.003	(3e-4)	5e-4	(5e-5)	0.003	(4e-4)	0.005	(2e-4)	0.001	(6e-4)
$c_2$	9e-5	(6e-5)	4e-5	(1e-5)	<1e-5	(0.715)	6e-4	(1e-4)	6e-5	(8e-5)
$c_3$	0.003	(3e-4)	5e-4	(5e-5)	0.003	(4e-4)	0.005	(2e-4)	0.001	(6e-4)
$a_1 \\ a_2$	0.627	(0.075)	0.441	(0.016)	0.516	(0.039)	0.409	(0.029)	0.483	(0.053)
	0.607	(0.072)	0.447	(0.017)	0.547	(0.044)	0.424	(0.028)	0.473	(0.052)
$b_1 \\ b_2$	0.758	(0.013)	0.870	(0.004)	0.782	(0.017)	0.635	(0.015)	0.880	(0.011)
	0.760	(0.011)	0.870	(0.005)	0.767	(0.019)	0.602	(0.025)	0.882	(0.012)
$d_1$	0.444	(0.077)	0.312	(0.039)	0.206	(0.096)	0.076	(0.067)	0.195	(0.079)
$d_2$	0.030	(0.009)	0.023	(0.005)	0.107	(0.036)	0.092	(0.016)	0.018	(0.005)
$d_3$	0.450	(0.069)	0.301	(0.040)	0.136	(0.099)	0.076	(0.080)	0.202	(0.076)

TABLE 2. Quasi-maximum likelihood estimates of GARCH-X parameters.

*Note:* Asymptotic robust standard errors are in parentheses. For BP and JY we could not find a step of increasing likelihood using many different parameter values.

LM test (Table 1) is based on a consequence of the null hypothesis. Let  $h_{ijt} \equiv E(e_{it}e_{jt} | \mathscr{F}_{t-1})$ . When  $h_{iit}$  is estimated under the null hypothesis that the conditional variance is correctly specified, it should be that  $E(e_{it}^2 - h_{iit} | \mathscr{F}_{t-1}) = 0$ , i = 1, 2 with probability one if the null hypothesis is true. Consequently,  $(e_{it}^2 - h_{iit})$  is uncorrelated with any measurable function of the variables in  $\mathscr{F}_{t-1}$ . We construct an LM test statistic for the null hypothesis that  $(e_{it}^2 - h_{iit})$  is uncorrelated with  $z_{t-1}^2$ . From the standard asymptotic arguments the  $TR^2$  statistic has the  $\chi^2(1)$  distribution asymptotically when the null hypothesis is true, where T is the number of observations and  $R^2$  is the uncentered squared multiple correlation from a standard linear regression of  $(e_{it}^2 - h_{iit})$  on  $z_{t-1}^2$ . We consider the null hypothesis that  $h_{iit}$  is ARCH(q), q = 0, 1, 5, 10. The test with q = 0 provides information on the correlation structure between  $e_{it}^2$  and  $z_{t-1}^2$ . We also consider the null hypothesis that  $h_{iit}$  is GARCH(1, 1). The test generally indicates that  $z_{t-1}^2$  may improve the prediction of  $X_t$  through the conditional second moments.

The GARCH-X effect is also tested by Wald tests (Table 2) and by the LR tests (Table 3) for five currencies. Both tests suggest the presence of the GARCH-X effect for all five currencies. The model selection criteria (Table 3) also indicate that the model with GARCH(1, 1)-X has a better fit than the model with GARCH(1, 1).

These tests may have power in some other directions and other misspecification can possibly be detected. If the results indicate the presence of the GARCH-X effect, they might not provide definitive evidence of neglected GARCH-X effect. The possible presence of higher order GARCH effects or the potential effects of other neglected variables in the information set may lead to the observed results. Much of the model selection literature in which model choice is based on the SIC, the Akaike information criterion (AIC), or other criteria, is concerned with selecting parsimoniously undominated models usually in terms of likelihood and subject to a restriction of correct specification of a model in that the residual processes from the (first and higher) conditional moments be martingale difference sequences. If a model is correctly specified in conditional mean and conditional

	Model	Log-likelihood	LR statistic	AIC	SIC
BP	Model 1 Model 2	11222.65 11244.08	42.85	- 22399.30 - 22434.15	- 22281.38 - 22295.73
DM	Model 1 Model 2 Model 3	11794.28 12026.35 12037.22	464.13 21.74	- 23542.56 - 23998.69 - 24014.43*	-23424.65 -23860.27 -23860.63*
JY	Model 1 Model 2	10349.67 10437.53	175.71	20645.34 20813.05	- 20506.92 - 20654.12
CD	Model 1 Model 2 Model 3	13820.89 14197.91 14225.52	754.03 55.22	27587.79 28333.81 28383.04*	27449.36 28174.88 28208.72*
FF	Model 1 Model 2 Model 3	10020.61 11087.17 11220.71	2133.12 267.09	20027.21 22152.33 22413.42*	
IL	Model 1 Model 2 Model 3	10750.58 10959.91 11000.04	418.65 80.27	21487.16 21897.81 21972.08*	21451.27 21841.41 21900.30*
SF	Model 1 Model 2 Model 3	11187.19 11718.79 11729.96	1063.21 22.32	22328.38 23383.59 23399.91*	22210.46 23245.16 23246.11*

TABLE 3. Log-likelihood, the LR statistics and model selection criteria.

*Note:* \* denotes the minimum of AIC or SIC. Model 1 is the ECM estimated by least squares assuming the conditional homoscedasticity; Model 2 is the ECM with GARCH specification; and Model 3 is the ECM with GARCH-X specification. Model 2 and Model 3 are estimated by maximizing the normal likelihood function (QMLE) using the method of scoring. The number of parameters in Model 1 is 4(p+1)+3 where p is the number of lags in ECM  $\langle 1 \rangle$ . Model 2 has four more parameters than Model 1 and Model 3 has three more parameters than Model 2. For BP and JY we could not find a step of increasing likelihood for Model 3 from many different initial values of parameters.

variance, then we may use the GARCH-X specification as a useful description of the salient aspects of the chosen phenomena.

In order to test the validity of the model, a series of specification tests for the model standardized by  $h_{iit}^{1/2}$  were conducted (but not reported), where  $h_{iit}$  is estimated by GARCH(1, 1)-X. The McLeod-Li tests and LM tests for ARCH become less significant, suggesting that some conditional heteroscedasticity is captured by the model. However, as is well recognized in Baillie and Bollerslev (1989b) and Hsieh (1989) among many others, daily exchange rate series show a considerable amount of leptokurtosis even after accounting for GARCH(1, 1). All of the standardized residuals have mean close to zero and variance close to unity. The LM tests indicate no serial correlation in the standardized residuals. The skewness and the excess kurtosis of the standardized residuals become generally smaller in modulus. From the diagnostics, we think the model using bivariate GARCH(1, 1)-X in the system of the error correction models are reasonably well specified.

We have seen that the conditional variances of prediction errors of the exchange rate changes are positively related to the short-run deviation from the long-run relationship. This means that when the spread is bigger, the exchange rates are more volatile and uncertainty increases. If  $z_{t-1}^2$  has additional predictive power for the changing variances of the spot and forward exchange rate changes, this may be exploited to obtain more precise time varying confidence intervals for point forecasts of exchange rate changes.

### **III.** Concluding remarks

In this paper we investigate a model which seems useful in examining how the short-run disequilibrium has an effect on uncertainty in predicting cointegrated series. Examining the behavior of the variances over time as a function of disequilibrium is reasonable when one expects increased volatility due to shocks to the system which propagate on first and second moments. The model is thus appropriate for testing for causality in variance as well as in mean through the error correction term.

It seems that this specification would ensure a better fit and would be useful empirically in other possible examples in economics. We have conducted the same analysis using monthly short- and long-term interest rate series as well as monthly interest rate series of the commercial paper and the Treasury bill. The results indicate stront GARCH-X for those data. Hence the model seems useful to study the relationship between the short-run deviation from a long-run relationship (disequilibrium) and uncertainty.

#### Notes

- 1. See Hansen and Hodrick (1983) or Mark (1985) for more details. They also derive more explicit relationship between the first and second conditional moments under additional assumptions on the joint distribution of Q and z and consumer preferences.
- 2. In order to allow  $z_{t-1}^2$  to have negative coefficients in  $\langle 2 \rangle$  we also used a symmetric matrix G instead of D'D. As the diagonal elements of G turn out to be positive for our data, we use D'D which guarantees  $H_t$  to be positive definite. The off-diagonal element of D'D may be negative.
- 3. It may be noted that our LM and LR tests are not robust to departures from normality or information equality while our Wald tests are. Robust LM tests are available in literature (Wooldridge, 1990; Bollerslev and Wooldridge, 1992), which we do not pursue here. LR tests do not follow an asymptotic chi-square distribution in the presence of ARCH (White, 1984, p. 76).

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