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INVESTIGATION OF PRODUCTION, SALES AND INVENTORY RELATIONSHIPS USING MULTICOINTEGRATION AND NON-SYMMETRIC ERROR CORRECTION MODELS

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SUMMARY

The accumulated sum of a stationary series is called integrated. If a linear combination of some of the integrated series is stationary, it is said to be cointegrated. This paper presents empirical results on inventories to consider the possibility of a deeper form of cointegration called multicointegration, which is introduced in Granger and Lee (1989). A vector integrated series is multicointegrated if the accumulated sum of its stationary (cointegrated) linear combination is again cointegrated with itself. Inventory, which is the accumulated sum of production minus sales, is probably cointegrated with production and sales. The empirical results generally support the presence of multicointegration of production and sales in many U.S. industries and industrial aggregates. The results also favour the non-symmetric error correction model, providing evidence that the strength of attraction is different on both sides of the attractor. A modified (S, s) rule for inventory control is investigated in the context of a non-symmetric error correction model, and the results generally do not support the (S,s) rule. Sufficient evidence is found to conclude that multicointegration is a useful concept in the area of inventory determination.

1. INTRODUCTION

In an earlier paper the authors introduced a deeper form of cointegration, called multicointegration. An area where this concept may seem to be of relevance is that of inventory series, as inventory may be cointegrated with sales but change of inventory is itself defined by a cointegrating relationship between sales and production. In what follows, a series is said to be I(0) if it is stationary and has spectrum that is finite at all frequencies and is positive at zero frequency.

If x_t, y_t are both I(1) then it is typically true that any linear combination $x_t + by_t$ will also be I(1). However, for some pairs of I(1) series there does exist a linear combination

$$z_t = x_t - Ay_t,$$

that is I(0). When this occurs, x_t, y_t are said to be cointegrated. If x_t, y_t are cointegrated they may be considered to be generated by an error-correcting model of the form

$$\Delta x_t = \rho_1 z_{t-1} + \text{lagged}(\Delta x_t,\ \Delta y_t) + \varepsilon_{xt}$$
$$\Delta y_t = \rho_2 z_{t-1} + \text{lagged}(\Delta x_t,\ \Delta y_t) + \varepsilon_{yt},$$

where at least one of ρ_1, ρ_2 is non-zero and ε_{xt}, ε_{yt} are jointly white noise.

It is generally true that for any vector X_t of N I(1) series, there will be at most r vectors α

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such that $\alpha' X_t$ is I(0), with $r \leqslant N-1$. However, it is also true that any pair of I(1) series may be cointegrated and this does allow the possibility of a deeper form of cointegration. Suppose that x_t, y_t are both I(1), have no trend and are cointegrated, so that $z_t = x_t - Ay_t$ is I(0). It follows that

$$Q_t = \sum_{j=0}^{t} z_{t-j}$$

will be I(1) and x_t, y_t will be said to be multicointegrated if Q_t and x_t are cointegrated. Q_t and y_t will also be cointegrated. It follows that

$$\omega_t = x_t - DQ_t \sim I(0).$$

It should be noted that

$$\omega_t = (1 - D\Delta^{-1}, AD\Delta^{-1})X_t,$$

where $X_t = (x_t, y_t)'$. Thus multicointegration allows two cointegrations at different levels between just two series.

The Cramer representation of the vector I(0) series is

$$\Delta X_t = C(B)\varepsilon_t. \tag{1.1}$$

It was shown in Granger (1983) and Engle and Granger (1987) that for the components of X_t to be cointegrated it is necessary and sufficient that the determinant of $C(B)$ has a root $(1 - B)$. It was shown in Granger and Lee (1989) the requirement for X_t to be multicointegrated is that the determinant of $C(B)$ has a root $(1 - B)^2$. If

$$\det C(B) = (1 - B)^2 d(B)$$

and if $A(B)$ is the adjunct matrix of $C(B)$, (1.1) may then be written

$$A(B)\Delta X_t = (1 - B)^2 d(B)\varepsilon_t. \tag{1.2}$$

Using the notation

$$A(B) = A(1) + \Delta A^*(B)$$
$$A^*(B) = A^*(1) + \Delta A^{**}(B)$$

and

$$A(B) = A(1)B + \Delta \tilde{A}(B)$$
$$\tilde{A}(B) = \tilde{A}(1)B + \Delta \bar{A}(B)$$

where

$$\tilde{A}(B) = A(1) + A^*(B)$$
$$\bar{A}(B) = A(1) + A^*(1) + A^{**}(B),$$

it was proved in Granger and Lee (1989) that

$$\tilde{A}(B)\Delta X_t = -\gamma z_{t-1} + d(B)\Delta \varepsilon_t$$

and

$$\bar{A}(B)\Delta X_t = -\gamma_1 \omega_{t-1} - \gamma_2 z_{t-1} + d(B)\varepsilon_t \tag{1.3}$$

where

$$\gamma = \begin{pmatrix} A_{11}(1) \\ A_{21}(1) \end{pmatrix}$$

$$\gamma_1 = -D^{-1}\gamma$$

$$\gamma_2 = \gamma - A^{-1}\begin{pmatrix} A_{12}^*(1) \\ A_{22}^*(1) \end{pmatrix}.$$

(1.3) is the error-correction model for a pair of multicointegrated series, in which changes of X_t are related to the pair of lagged cointegration errors, $z_t = x_t - Ay_t$ and $\omega_t = x_t - DQ_t$. For multicointegration, ΔX_t is generated by (1.3), with the necessary condition that at least one component of each of γ_1 and γ_2 is non-zero. It should be noted that the integral correction term ω_{t-1} in (1.3) allows the system to be more robust to disturbances because it has a buffer Q_t, and leads to potentially improved forecasts of component of ΔX_t.

The next section investigates an application where x_t is production, y_t sales, z_t change of inventories, and Q_t is level of inventory in various US industries and industrial groupings. Section 3 examines non-symmetric error correction models. In section 4 a modified (S, s) inventory control rule is discussed using non-symmetric error correction model. The empirical results were generally supportive of the presence of multicointegration and the non-symmetric error correction, but not compatible with the (S, s) inventory control rule.

2. TESTS OF MULTICOINTEGRATION FOR INVENTORY DATA

If p_t is a production series and s_t a sales series for an industry, both are likely to be I(1) with a growing economy but $z_t = p_t - s_t$, which is the change in inventory, may be I(0). Thus, sales and production for an industry are very likely to be cointegrated. The level of inventory I_t, being the initial inventory level plus the sum of the changes in inventory, will be I(1) as $\Delta I_t = z_t$ is I(0). If the target level of inventory, I_t^*, is a fixed proportion λ of sales, then $I_t - \lambda s_t = u_t$ is the control error and this again should be expected to be I(0). Thus, in this situation, p_t, s_t will be multicointegrated. The alternative (S, s) rule for inventory control will be discussed at the end of the paper.

To consider the possibility of multicointegration monthly sales and inventory level series were used from the Citibank data base for 27 US industries and industrial aggregates. The data were for the period January 1967 to April 1987, in 1982 constant dollars and seasonally adjusted. The sample size is 244 observations. The change in inventory was estimated directly from the inventory levels and production estimated as sales plus the change in inventory. Thus, $z_t = p_t - s_t$ was not estimated from a cointegrating regression but was available directly from the data. The data are described further in the appendix, and the numbers used for the various industrial groupings are also defined there. Each industry and aggregate was analysed separately, although the series are not independent as disaggregated data are used to form the more aggregate series.

For each pair of sales (s_t) and inventory level (I_t) series, the following steps in the analysis were conducted:

(i) Form the production series

$$p_t = s_t + \Delta I_t.$$

Denote $z_t = \Delta I_t$.

(ii) Test if the series p_t, s_t and I_t are I(1) using the estimated autocorrelations of each series plus the augmented Dickey–Fuller (ADF) test, using 12 lags. Thus for a series z_t, this test forms the regressions

$$\Delta x_t = \rho x_{t-1} + \sum_{j=1}^{12} \gamma_j \Delta x_{t-j} + \text{residual}$$

and uses as the test statistic the usual t-value for the coefficient ρ. The null hypothesis is that $x_t \sim I(1)$, so that $\rho = 0$. The alternative is that x_t is I(0) so that ρ is significantly non-zero and negative. The t-statistic does not have its usual distribution and tables of significance are found in Fuller (1976, page 373). For the sample size used here $-t$ has to be greater than 2·88 for H_0 to be rejected at the 95 per cent level of confidence.

(iii) Test if z_t is I(0) using the same procedure.

(iv) Form the regressions

$$\begin{aligned} I_t &= a_1 + b_1 p_t + u_{1t} \\ I_t &= a_2 + b_2 s_t + u_{2t} \end{aligned} \tag{2.1}$$

and test if the u's are I(0). One method is to use the Durbin–Watson (d) statistics from these regressions. Any value of d greater than 0·08 suggests rejecting the null hypothesis that u is I(1) at the 95 per cent level. A rather better test, according to Engle and Granger (1987), is to use the augmented Dickey–Fuller test as before. However, as some parameters have been estimated in forming the u's the critical values for the test are changed. According to Engle and Yoo (1987) a value of $-t$ greater than 3·25 suggests rejection of the null hypothesis that the series is I(1) at the 95 per cent level. The autocorrelations of the u's are also of interest.

In the complete analysis, the regressions

$$\begin{aligned} p_t &= c_1 + d_1 I_t + w_{1t} \\ s_t &= c_2 + d_2 I_t + w_{2t}, \end{aligned} \tag{2.2}$$

were also formed, but all the results are not reported here as the outcomes were virtually identical.

(v) The following pair of error correction models were estimated:

$$\begin{aligned} \Delta p_t &= \alpha + \beta_1 z_{t-1} + \beta_2 u_{2,t-1} + \gamma_1 \Delta p_{t-1} + \gamma_2 \Delta s_{t-1} + \text{residual} \\ \Delta s_t &= \alpha' + \beta_1' z_{t-1} + \beta_2' u_{2,t-1} + \gamma_1' \Delta p_{t-1} + \gamma_2' \Delta s_{t-1} + \text{residual.} \end{aligned} \tag{2.3}$$

Just single lags for Δp_t, Δs_t were used for ease of computing and reporting of the results, although in some cases further lags may also have been significant. A significant value for the t-statistic on β_2 or β_2' would indicate evidence in favour of multicointegration. In the complete analysis, these error correction models were also estimated with u_{2t} replaced by u_{1t}, w_{1t}, and w_{2t}. Again, the results were very similar and are not reported.

To illustrate the steps of the analysis, values are presented for series 1, which is the most aggregated and is entitled 'Manufacturing and Trade'.

Step (ii) Unit Root Tests

Sample autocorrelations

	r_1	r_5	r_{10}	r_{12}	ADF
p_t	0·983	0·912	0·821	0·787	−0·55
s_t	0·983	0·915	0·826	0·794	−0·41
I_t	0·988	0·937	0·869	0·842	−1·01

$r_k = \text{corr}(x_t, x_{t-k})$

These values are all consistent with p_t, s_t and I_t being I(1).

Step (iii) and (iv) Multicointegration Tests

	r_1	r_5	r_{10}	r_{12}	ADF
z_t	0·432	0·230	0·069	−0·050	−4·28
u_{1t}	0·950	0·725	0·418	0·296	−3·16
u_{2t}	0·938	0·717	0·423	0·308	−3·02

Regression results are

$$I_t = 14\cdot 19 + 1\cdot 52 p_t + u_{1t}$$
$$\bar{R}^2 = 0\cdot 93 \quad \text{DW} = 0\cdot 10$$

$$I_t = 13\cdot 01 + 1\cdot 53 s_t + u_{2t}$$
$$\bar{R}^2 = 0\cdot 94 \quad \text{DW} = 0\cdot 11,$$

(t-values are not shown as the u's are far from the white noises).

The evidence is clear that z_t is I(0) and suggestive that the u's are also I(0).

Step (v) Error Correction Models

$$\Delta p_t = \underset{(5\cdot 06)}{1\cdot 83} - \underset{(3\cdot 94)}{0\cdot 75} z_{t-1} - \underset{(3\cdot 15)}{0\cdot 05} u_{2,t-1} + \underset{(1\cdot 20)}{0\cdot 19} \Delta p_{t-1} - \underset{(2\cdot 15)}{0\cdot 37} \Delta s_{t-1} + \text{residual}$$
$$\bar{R}^2 = 0\cdot 06 \quad \text{DW} = 1\cdot 98$$

$$\Delta s_t = \underset{(2.82)}{1\cdot 00} - \underset{(0.20)}{0\cdot 04} z_{t-1} - \underset{(0.72)}{0\cdot 01} u_{2,t-1} - \underset{(2.51)}{0\cdot 42} \Delta s_{t-1} + \underset{(1.53)}{0\cdot 24} \Delta p_{t-1} + \text{residual}$$
$$\bar{R}^2 = 0\cdot 03 \quad \text{DW} = 2\cdot 01$$

(moduli of t-values are shown).

Taking these results uncritically, the equation for Δp_t suggests multicointegration, as both z_{t-1} and $u_{2,t-1}$ have significant coefficients. Neither of these terms comes in significantly for Δs_t, the only significant term being Δs_{t-1}. Both $\bar{R}^2$ values are low.

Overall, there are several indications that multicointegration is present, the significant

Table I. Augmented Dickey–Fuller statistics

	p_t	s_t	I_t	z_t	w_{1t}	u_{1t}	w_{2t}	u_{2t}
1	−0·55	−0·41	−1·01	−4·28	−3·44	−3·16	−3·32	−3·02
2	−1·36	−1·23	−1·60	−3·95	−3·15	−2·88	−3·06	−2·88
3	−2·03	−1·76	−1·39	−4·21	−3·54	−2·82	−3·25	−2·76
4	−1·68	−1·81	−0·53	−3·30	−2·19	−1·11	−2·35	−1·24
5	−2·27	−1·86	−1·67	−3·73	−2·55	−1·88	−2·10	−1·80
6	−1·53	−1·13	−2·06	−3·82	−2·28	−2·17	−1·55	−1·78
7	−0·07	0·18	−0·04	−3·26	−3·64	−3·59	−3·45	−3·48
8	−2·35	−2·17	−0·17	−3·61	−3·06	−1·56	−2·82	−1·51
9	−1·23	−1·05	−2·29	−3·90	−2·48	−2·46	−2·33	−2·43
10	−1·07	−0·95	−2·14	−4·29	−2·32	−2·40	−2·21	−2·34
11	0·06	−0·06	−1·64	−4·87	−0·17	−1·15	−0·41	−1·30
12	−1·28	−1·16	−2·25	−4·40	−3·11	−3·01	−3·01	−2·97
13	0·10	0·21	−0·44	−4·57	−2·74	−2·64	−2·81	−2·76
14	−1·68	−1·48	−1·77	−4·40	−3·83	−3·59	−3·56	−3·47
15	−1·87	−1·91	−1·55	−4·81	−2·81	−2·38	−2·86	−2·45
16	−1·84	−1·68	−2·46	−3·86	−2·62	−2·43	−2·48	−2·37
17	−0·75	−0·45	−2·17	−4·56	−0·39	−1·38	−0·05	−1·35
18	−0·02	0·11	−0·62	−4·20	−3·06	−3·09	−3·06	−3·12
19	−0·53	−0·48	−1·00	−4·38	−2·78	−2·67	−2·77	−2·73
20	0·57	0·73	0·33	−4·48	−2·48	−2·52	−2·44	−2·50
21	−0·02	0·05	0·67	−5·27	−2·31	−2·25	−2·67	−2·66
22	0·35	0·55	−0·21	−4·02	−2·52	−2·64	−2·28	−2·44
23	−0·47	−0·37	−0·47	−3·50	−4·37	−4·39	−4·18	−4·22
24	−0·11	−0·03	−0·50	−3·92	−2·31	−2·54	−2·21	−2·56
25	−0·64	−0·48	−0·78	−4·66	−2·70	−2·97	−2·39	−2·80
26	−0·87	−1·02	0·31	−3·50	−2·34	−2·05	−2·07	−1·77
27	−1·33	−1·34	1·60	−4·96	−3·10	−2·51	−2·95	−2·34

coefficients in the error correction model, the Durbin–Watson statistic in the regressions that determine the u's, the fairly rapidly declining autocorrelations, and the almost significant ADF statistics for the u's.

Applying these steps of the analysis to all 27 industries and aggregates provides the following results:

(a) All the sales, production and inventory series appears to be I(1), as seen from the ADF statistics in Table I.

(b) All z_t series (change in inventories) are I(0), implying that the sales and production of any industry are cointegrated, which is hardly surprising. The value of the ADF statistics are shown in Table I.

(c) The ADF statistics in Table I and the Durbin–Watson statistics in Table II show some evidence of multicointegration. Although only 10 ADF statistics for w_{1t} are less than $-3 \cdot 0$ it is also true that only two are greater than $-2 \cdot 0$, and all are clearly negative; 44 out of the 54 Durbin–Watson statistics are 0·10 or greater and all but one are 0·8 or more.

Table II also shows the coefficients b's and d's in equations (2.1) and (2.2). As z_t is so clearly I(0) b_1, d_1 are very similar to b_2, d_2, respectively. d_1's range from 0·18 to 1·35 with an average of 0·65.

(d) The error correction models (2.3) using z_{t-1}, $u_{2,t-1}$ and single lags of Δp_t, Δs_t were estimated for each series. The coefficients, plus $|t|$ values, for the z_{t-1} and the $u_{2,t-1}$ series are shown in Table III.

Table II. Regression results of (2.1) and (2.2)

	b_1	d_1	$\bar{R}^2$	DW	b_2	d_2	$\bar{R}^2$	DW
1	1·52	0·62	0·93	0·10	1·53	0·62	0·94	0·12
2	1·65	0·46	0·75	0·08	1·68	0·46	0·77	0·09
3	1·93	0·30	0·57	0·09	2·04	0·30	0·62	0·10
4	0·62	0·19	0·11	0·08	0·64	0·18	0·11	0·02
5	1·10	0·18	0·19	0·17	1·22	0·18	0·22	0·13
6	2·51	0·28	0·70	0·08	2·57	0·28	0·73	0·08
7	2·32	0·40	0·93	0·25	2·37	0·40	0·95	0·29
8	0·87	0·26	0·22	0·21	0·92	0·24	0·22	0·19
9	1·91	0·38	0·73	0·12	1·94	0·39	0·75	0·09
10	1·10	0·81	0·89	0·11	1·09	0·81	0·89	0·12
11	0·73	1·10	0·80	0·14	0·73	1·10	0·79	0·19
12	1·18	0·73	0·86	0·12	1·18	0·73	0·87	0·12
13	1·27	0·64	0·81	0·13	1·29	0·63	0·82	0·13
14	1·25	0·69	0·86	0·18	1·25	0·69	0·87	0·17
15	0·76	0·68	0·51	0·09	0·76	0·68	0·52	0·15
16	1·24	0·56	0·69	0·11	1·26	0·56	0·71	0·08
17	0·81	0·68	0·55	0·14	0·81	0·69	0·55	0·13
18	1·27	0·76	0·97	0·20	1·27	0·76	0·97	0·21
19	1·86	0·49	0·91	0·11	1·88	0·49	0·92	0·10
20	0·72	1·34	0·97	0·30	0·72	1·34	0·97	0·32
21	0·73	1·27	0·93	0·28	0·73	1·27	0·92	0·31
22	0·71	1·35	0·95	0·27	0·71	1·34	0·95	0·31
23	1·52	0·64	0·97	0·32	1·54	0·63	0·97	0·54
24	1·73	0·52	0·90	0·35	1·76	0·52	0·91	0·48
25	1·43	0·56	0·81	0·31	1·45	0·55	0·80	0·58
26	1·21	0·78	0·94	0·11	1·22	0·77	0·94	0·11
27	1·04	0·89	0·92	0·21	1·05	0·88	0·92	0·25

For the Δp_t equations, 23 out of 27 coefficients on z_{t-1} have significant $|t|$ values and everyone of these coefficients is negative, indicating a clear and consistent negative effect of z_{t-1} values on the next changes in production. The average value of these coefficients is $-0 \cdot 676$ and the range is $-0 \cdot 10$ to $-1 \cdot 45$. For the $u_{2,t-1}$ coefficients, eight have significant $|t|$ values but only six of the coefficients are not negative. The range is $0 \cdot 12$ to $-0 \cdot 05$, with an average $-0 \cdot 007$.

For the Δs_t equations there is no consistency of signs of the coefficients of either z_{t-1} or of $u_{2,t-1}$. Twelve of the former have significant $|t|$ values and six of the latter.

Overall, it seems that the error corrections are stronger for Δp_t than for Δs_t, which may be expected as production is a controllable variable, and the control mechanism may well react to the value of the previous z_t. The sales series may be thought of as being largely exogenously determined, unless sales are reduced due to very low inventory levels which are unable to meet a high, unexpected demand. However, the occasional observed relationship between Δs_t and the error correction terms may be due to temporal aggregation, which is well known to induce a weak feedback relationship from a true single causal one (e.g. Weiss, 1984). Presumably, actual production values are determined at a shorter interval than a month.

Multicointegration is found for several industries, using at least one of the criteria available, particularly for the most aggregate series 1 (manufacturing and trade), 2 (manufacturing), 3 (durable goods manufacturing), 18 (merchant wholesalers), and 23 (retail trade). It is rather surprising to find evidence of multicointegration mostly at the aggregate levels rather than at

Table III. Error correction model (2.3)

	Δp_t				Δs_t			
	z_{t-1}		$u_{2,t-1}$		z_{t-1}		$u_{2,t-1}$	
	Coeff.	$\lvert t \rvert$	Coeff.	$\lvert t \rvert$	Coeff.	$\lvert t \rvert$	Coeff.	$\lvert t \rvert$
1	−0·75	3·94	−0·05	3·15	−0·04	0·20	−0·01	0·72
2	−0·96	5·56	−0·04	3·73	−0·47	2·76	−0·02	1·81
3	−0·78	4·63	−0·03	3·11	−0·21	1·31	−0·01	0·90
4	−0·17	1·22	−0·00	0·20	0·28	1·79	0·00	0·41
5	−0·71	5·05	−0·02	1·62	−0·08	0·64	−0·00	0·19
6	−0·10	0·99	−0·01	1·03	0·13	1·40	0·00	0·29
7	−0·20	1·39	−0·01	0·52	0·29	2·30	0·03	1·72
8	−0·12	6·07	−0·00	0·08	−0·02	0·09	0·01	0·69
9	−0·67	4·68	−0·03	2·48	−0·08	0·63	−0·01	0·58
10	−0·93	5·99	−0·03	1·72	−0·32	2·19	−0·00	0·05
11	−0·45	2·84	0·00	0·05	0·37	2·71	0·03	1·34
12	−1·17	7·29	−0·04	2·41	−0·55	3·69	−0·01	0·47
13	−1·45	9·61	−0·04	2·44	−0·48	3·50	−0·00	0·20
14	−1·01	6·58	−0·02	0·96	−0·33	2·40	0·02	1·15
15	−0·90	4·93	−0·00	0·02	−0·13	0·76	0·03	1·24
16	−1·00	5·53	−0·03	1·79	−0·04	0·30	−0·00	0·10
17	−0·83	5·16	−0·02	1·02	−0·06	0·45	0·01	0·35
18	−0·55	2·73	−0·01	0·19	0·30	1·71	0·03	1·47
19	−1·05	6·55	−0·04	2·84	−0·07	0·56	−0·00	0·22
20	−0·78	3·38	0·12	2·19	0·21	1·10	−0·15	3·23
21	−0·99	4·94	0·09	1·78	0·04	0·23	0·13	3·03
22	−0·47	2·00	0·07	1·30	0·51	2·59	0·11	2·51
23	−0·22	2·00	−0·03	1·25	0·48	4·26	0·06	2·29
24	−0·42	3·61	−0·03	1·48	0·51	4·83	0·05	2·63
25	−0·14	1·27	−0·01	0·65	0·55	4·56	0·07	2·64
26	−0·78	6·18	0·00	0·21	0·15	1·61	0·01	1·12
27	−0·65	3·09	0·03	1·02	0·57	3·15	0·05	1·77

the less aggregated ones, as generally cointegration at an aggregate level implies it at disaggregated levels (see Gonzalo (1989) for discussion).

Certain industrial groupings display problems with this analysis, particularly 4 (primary metals manufacturing), 5 (fabricated metals manufacturing) and 8 (transportation equipment manufacturing) which have low $\bar{R}^2$ values when p_t is used to explain I_t, and 4 and 6 which have insignificant $|t|$ values for z_{t-1} in both error correction models. The plots of p_t, s_t and I_t for the primary metal series show a short period, starting in late 1974, in which sales and production decline substantially, but slightly out of phase, so that the inventory level expands rapidly in the same period, explaining the low $\bar{R}^2$ value between production and inventory. If data just for the subperiod 1978:1 to 1987:4 are used in the regression, $\bar{R}^2$ increases to 0·59 with a Durbin–Watson statistic of 0·13, which is nearer with the figures obtained with the other series. Detailed investigation of all the strange series has not been attempted.

An alternative way to judge the relevance of multicointegration is to compare the forecasting ability of error correction models using, or not using, the $u_{2,t-1}$ terms. Thus, the first error correction model (EC1) explained Δp_t by z_{t-1}, Δp_{t-1} and Δs_{t-1} whereas the second model (EC2) used these variable, plus $u_{2,t-1}$. Similarly, two error correction models were estimated for Δs_t. All these models were estimated, having saved 48 terms for post-sample evaluations.

Using the mean squared one-step forecast error as criterion, EC2 outperformed EC1 for 18 out of 27 series (67 per cent) with Δp_t as the dependent variable. Using Δs_t, EC2 beat EC1 on 14 out of 25 occasions (56 per cent), with two ties. The results again give some support to multicointegration being present, but the evidence is certainly not overwhelming.

3. NON-SYMMETRIC ERROR CORRECTION MODELS

The error corrections in the models considered above are symmetric so that the extent of the effect of $|z_{t-1}|$ is the same regardless of the sign of z_{t-1}. However, when choosing the level of production, or its change, it may well matter whether z_{t-1} (previous production – previous sales) was positive or negative or whether $u_{2,t-1}$ (interpreted as inventory level minus its target level of λ times previous sales) was positive or negative.

To investigate these possibilities there further sets of error correction models were conducted, using the notation $w = w^+ + w^-$, $w^+ = \max(w, 0)$ and $w^- = \min(w, 0)$:

(A) Δp_t regressed on z_{t-1}, $u_{2,t-1}^+$, $u_{2,t-1}^-$, Δs_{t-1}
(B) Δp_t regressed on z_{t-1}^+, z_{t-1}^-, $u_{2,t-1}$, Δp_{t-1}, Δs_{t-1}
(C) Δp_t regressed on z_{t-1}^+, z_{t-1}^-, $u_{2,t-1}^+$, $u_{2,t-1}^-$, Δp_{t-1}, Δs_{t-1}

and similar equations for Δs_t.

As an illustration, the results for series 1, the aggregate manufacturing and trade were:

Error correction model (A)

$$\underset{(5\cdot 89)}{\Delta p_t = 3\cdot 16} - \underset{(4\cdot 71)}{0\cdot 91 z_{t-1}} - \underset{(4\cdot 59)}{0\cdot 12 u_{2,t-1}^+} + \underset{(1\cdot 02)}{0\cdot 03 u_{2,t-1}^-} + \underset{(1\cdot 51)}{0\cdot 24 \Delta p_{t-1}} - \underset{(2\cdot 46)}{0\cdot 42 \Delta s_{t-1}} + \text{residual}$$

$$\bar{R}^2 = 0\cdot 10 \quad \mathrm{DW} = 1\cdot 94$$

$$\underset{(3\cdot 73)}{\Delta s_t = 1\cdot 97} - \underset{(0\cdot 80)}{0\cdot 15 z_{t-1}} - \underset{(2\cdot 36)}{0\cdot 06 u_{2,t-1}^+} + \underset{(1\cdot 67)}{0\cdot 05 u_{2,t-2}^-} + \underset{(1\cdot 75)}{0\cdot 27 \Delta p_{t-1}} - \underset{(2\cdot 73)}{0\cdot 46 \Delta s_{t-1}} + \text{residual.}$$

$$\bar{R}^2 = 0\cdot 05 \quad \mathrm{DW} = 1\cdot 98$$

Error correction model (B)

$$\underset{(3\cdot 72)}{\Delta p_t = 1\cdot 89} - \underset{(3\cdot 12)}{0\cdot 78 z_{t-1}^+} - \underset{(1\cdot 84)}{0\cdot 70 z_{t-1}^-} - \underset{(3\cdot 08)}{0\cdot 05 u_{2,t-1}} + \underset{(1\cdot 21)}{0\cdot 19 \Delta p_{t-1}} - \underset{(2\cdot 15)}{0\cdot 37 \Delta s_{t-1}} + \text{residual}$$

$$\bar{R}^2 = 0\cdot 06 \quad \mathrm{DW} = 1\cdot 97$$

$$\underset{(2\cdot 10)}{\Delta s_t = 1\cdot 04} - \underset{(0\cdot 24)}{0\cdot 06 z_{t-1}^+} + \underset{(0\cdot 01)}{0\cdot 003 z_{t-1}^-} - \underset{(0\cdot 69)}{0\cdot 01 u_{2,t-1}} + \underset{(1\cdot 53)}{0\cdot 24 \Delta p_{t-1}} - \underset{(2\cdot 50)}{0\cdot 42 \Delta s_{t-1}} + \text{residual.}$$

$$\bar{R}^2 = 0\cdot 03 \quad \mathrm{DW} = 2\cdot 01$$

Error correction model (C)

$$\underset{(5\cdot 02)}{\Delta p_t = 2\cdot 97} - \underset{(3\cdot 27)}{0\cdot 79 z_{t-1}^+} - \underset{(2\cdot 93)}{1\cdot 17 z_{t-1}^-} - \underset{(4\cdot 60)}{0\cdot 12 u_{2,t-1}^+} + \underset{(1\cdot 12)}{0\cdot 03 u_{2,t-1}^-} + \underset{(1\cdot 48)}{0\cdot 23 \Delta p_{t-1}} - \underset{(2\cdot 44)}{0\cdot 41 \Delta s_{t-1}} + \text{residual}$$

$$\bar{R}^2 = 0\cdot 10 \quad \mathrm{DW} = 1\cdot 96$$

$$\underset{(3\cdot 15)}{\Delta s_t = 1\cdot 84} - \underset{(0\cdot 30)}{0\cdot 07 z_{t-1}^+} - \underset{(0\cdot 86)}{0\cdot 34 z_{t-1}^-} - \underset{(2\cdot 41)}{0\cdot 12 u_{2,t-1}^+} + \underset{(1\cdot 72)}{0\cdot 03 u_{2,t-1}^-} + \underset{(1\cdot 73)}{0\cdot 27 \Delta p_{t-1}} - \underset{(2\cdot 71)}{0\cdot 45 \Delta s_{t-1}} + \text{residual.}$$

$$\bar{R}^2 = 0\cdot 05 \quad \mathrm{DW} = 1\cdot 99$$

One may expect that if production is greater than sales (z_{t-1} is positive) then next production will be reduced, so that z_{t-1}^+ comes in the error correction models for Δp_t with a negative coefficient. If production is smaller than sales then next production will be raised, so that z_{t-1}^-

comes in also with a negative coefficient as $z_{t-1}^{-} < 0$. Similarly, interpreting λs_t as a target inventory level, one would expect $u_{2,t-1}^{+}$ and $u_{2,t-1}^{-}$ to have negative coefficients. There is also seen to be an induction that $u_{2,t-1}^{+}$ has a more significant coefficient in (A), (C) than does $u_{2,t-1}$ in (B).

Tables IV, V and VI summarize the results for these three error correction models, showing the coefficients and $|t|$ values for z_{t-1}, z_{t-1}^{+}, z_{t-1}^{-} and $u_{2,t-1}$, $u_{2,t-1}^{+}$, $u_{2,t-1}^{-}$.

The percentages of significant ($|t| > 1.96$) coefficients in these tables are:

	Δp_t						Δs_t					
	z_{t-1}	z_{t-1}^{+}	z_{t-1}^{-}	$u_{2,t-1}$	$u_{2,t-1}^{+}$	$u_{2,t-1}^{-}$	z_{t-1}	z_{t-1}^{+}	z_{t-1}^{-}	$u_{2,t-1}$	$u_{2,t-1}^{+}$	$u_{2,t-1}^{-}$
A	89	—	—	—	63	15	44	—	—	—	19	30
B	—	78	59	30	—	—	—	22	26	22	—	—
C	—	78	63	—	59	15	—	22	26	—	15	30

It is seen that for Δp_t many more $u_{2,t-1}^{+}$ terms are significant than $u_{2,t-1}$ and that few $u_{2,t-1}^{-}$

Table IV. Error correction model (A)

	Δp_t						Δs_t					
	z_{t-1}		$u_{2,t-1}^{+}$		$u_{2,t-1}^{-}$		z_{t-1}		$u_{2,t-1}^{+}$		$u_{2,t-1}^{-}$	
	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$
1	−0·91	4·71	−0·12	4·59	0·03	1·02	−0·15	0·80	−0·06	2·36	0·05	1·67
2	−1·06	5·90	−0·07	4·39	0·01	0·42	−0·51	2·98	−0·04	2·59	0·02	0·73
3	−0·79	4·71	−0·05	3·15	−0·01	0·29	−0·22	1·37	−0·02	1·47	0·01	0·55
4	−0·23	1·58	−0·05	2·09	0·05	1·98	0·24	1·53	−0·03	1·15	0·05	1·50
5	−0·72	5·12	−0·04	1·74	0·01	0·34	−0·08	0·67	−0·01	0·62	0·01	0·45
6	−0·24	2·24	−0·05	3·52	0·04	2·58	0·04	0·44	−0·02	1·89	0·03	2·17
7	−0·21	1·49	−0·06	2·14	0·06	1·74	0·28	2·22	−0·03	1·20	0·10	3·32
8	−1·14	6·16	0·04	1·21	−0·04	1·29	−0·03	0·17	0·04	1·35	−0·02	0·71
9	−0·78	5·23	−0·07	3·36	0·01	0·72	−0·13	1·00	−0·02	1·44	0·01	0·83
10	−0·94	6·11	−0·09	2·73	0·04	1·03	−0·32	2·23	−0·03	0·87	0·03	0·87
11	−0·45	2·81	0·01	0·19	−0·01	0·14	0·36	2·69	0·03	0·54	0·05	0·84
12	−1·23	7·76	−0·12	4·05	0·07	1·81	−0·60	4·00	−0·06	2·30	0·07	2·01
13	−1·45	9·60	−0·06	2·03	−0·02	0·55	−0·48	3·49	−0·00	0·01	−0·01	0·20
14	−1·04	6·73	−0·08	1·88	0·03	0·80	−0·33	2·43	0·01	0·16	0·03	1·04
15	−0·92	5·01	−0·04	1·10	0·07	1·20	−0·14	0·84	−0·01	0·27	0·08	1·61
16	−1·04	5·72	−0·08	2·59	0·04	1·15	−0·06	0·41	−0·03	1·11	0·03	1·07
17	−0·82	5·13	−0·02	0·56	−0·02	0·42	−0·06	0·45	0·01	0·17	0·01	0·07
18	−0·61	3·05	−0·10	2·20	0·09	1·98	0·27	1·52	−0·02	0·40	0·08	2·13
19	−1·15	7·14	−0·08	4·23	0·02	1·02	−0·12	0·92	−0·03	1·61	0·03	1·50
20	−0·78	3·36	0·24	2·38	0·00	0·03	0·22	1·11	0·21	2·45	0·09	1·08
21	−0·99	4·94	0·13	1·50	0·04	0·40	0·04	0·23	0·11	1·57	0·14	1·71
22	−0·49	2·09	−0·04	0·41	0·16	1·77	0·50	2·50	0·03	0·37	0·18	2·31
23	−0·22	1·98	−0·09	2·25	0·04	0·88	0·48	4·38	−0·03	0·82	0·17	3·73
23	−0·22	1·98	−0·09	2·25	0·04	0·88	0·48	4·38	−0·03	0·82	0·17	3·73
24	−0·46	3·91	−0·09	2·32	0·03	0·70	0·45	4·20	−0·04	1·28	0·15	4·23
25	−0·18	1·68	−0·12	3·31	0·11	2·75	0·49	4·24	−0·08	2·11	0·25	5·44
26	−0·78	6·17	0·00	0·01	0·01	0·21	0·15	1·60	0·01	0·28	0·02	0·85
27	−0·66	3·09	0·04	0·65	0·02	0·41	0·58	3·15	0·04	0·83	0·05	1·01

Table V. Error correction model (B)

	Δp_t						Δs_t					
	z^+_{t-1}		z^-_{t-1}		$u_{2,t-1}$		z^+_{t-1}		z^-_{t-1}		$u_{2,t-1}$	
	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$
1	−0·78	3·12	−0·70	1·84	−0·05	3·08	−0·06	0·24	0·00	0·01	−0·01	0·69
2	−0·93	3·95	−1·02	2·91	−0·04	3·65	−0·43	1·82	−0·57	1·63	−0·02	1·83
3	−0·82	3·55	−0·69	2·17	−0·03	2·99	−0·28	1·24	−0·10	0·33	−0·01	0·80
4	−0·72	3·17	0·39	1·67	−0·00	0·11	−0·36	1·44	0·94	3·69	0·01	0·51
5	−0·58	2·72	−0·86	3·71	−0·02	1·59	0·07	0·38	−0·24	1·23	−0·00	0·16
6	−0·24	1·54	0·09	0·48	−0·01	0·96	−0·07	0·47	0·40	2·25	0·00	0·39
7	−0·12	0·65	−0·37	1·31	−0·01	0·60	0·40	2·51	0·04	0·17	0·02	1·59
8	−1·39	6·19	−0·62	2·07	−0·00	0·07	−0·25	1·21	0·41	1·48	0·01	0·70
9	−0·89	4·41	−0·33	1·23	−0·02	2·17	−0·20	1·19	0·12	0·54	−0·00	0·38
10	−0·93	4·14	−0·91	2·96	−0·03	1·68	−0·31	1·46	−0·33	1·15	−0·00	0·06
11	−0·29	1·33	−0·64	2·59	0·00	0·10	0·39	2·06	0·34	1·60	0·04	1·34
12	−1·37	6·06	−0·82	2·64	−0·04	2·10	−0·76	3·59	−0·21	0·71	−0·00	0·18
13	−1·65	8·99	−1·11	4·70	−0·03	2·25	−0·75	4·60	0·00	0·01	0·00	0·09
14	−0·89	4·16	−1·22	4·25	−0·02	1·07	−0·29	1·54	−0·39	1·52	0·02	1·01
15	−0·87	3·56	−0·95	3·08	−0·00	0·04	−0·02	0·08	−0·31	1·05	0·02	1·15
16	−1·02	4·37	−0·98	3·67	−0·03	1·74	0·08	0·40	−0·20	0·94	−0·00	0·25
17	−1·06	4·15	−0·62	2·59	−0·02	0·87	−0·03	0·14	−0·09	0·43	0·01	0·32
18	−0·64	2·50	−0·38	0·97	−0·00	0·09	0·16	0·75	0·59	1·77	0·04	1·63
19	−1·21	5·98	−0·77	2·79	−0·03	2·25	−0·17	0·99	0·09	0·41	0·00	0·11
20	−0·48	1·57	−1·21	3·25	0·12	2·16	0·45	1·75	−0·13	0·41	0·15	3·21
21	−0·93	3·26	−1·06	3·48	0·09	1·74	0·06	0·27	0·01	0·04	0·12	3·00
22	−0·36	1·21	−0·66	1·70	0·07	1·28	0·59	2·37	0·38	1·16	0·11	2·49
23	−0·54	3·36	0·23	1·14	−0·03	1·09	0·09	0·58	1·03	5·20	0·06	2·55
24	−0·30	1·82	−0·53	3·32	−0·03	1·30	0·33	2·21	0·69	4·69	0·05	2·35
25	−0·52	3·13	0·19	1·24	−0·01	0·52	0·05	0·28	0·97	5·74	0·07	2·87
26	−1·01	6·14	−0·36	1·52	0·00	0·16	0·03	0·28	0·37	2·12	0·01	1·09
27	−0·89	3·34	−0·30	0·93	0·03	1·17	0·44	1·94	0·77	2·77	0·05	1·85

terms are significant. Thus, there is much clearer evidence of multicointegration in the non-symmetric error correction models for Δp_t compared to symmetric models. No such clear change occurs for the Δs_t equations.

Some other generalizations are:

(a) For the Δp_t equations are coefficients on z_{t-1}, z^+_{t-1} and most of the coefficients on z^-_{t-1} are negative.

(b) In (A), (C) for Δp_t, all but six of the coefficients of $u^+_{2,t-1}$ are negative. Only one positive coefficient is significant. In (A) for Δp_t only five of the coefficients on $u^-_{2,t-1}$ are negative, with 6 in (C). None are significant.

(c) $\bar{R}^2$ values for (A), (C) are generally higher than for (B).

(d) A similar forecasting exercise to that reported in the previous section, between (A) and the error correction model (2.3) using z_{t-1} and $u_{2,t-1}$, was conducted. Using Δp_t the non-symmetric error correction model outforecast the symmetric one 56 per cent of the time, for Δs_t it was better 69 per cent of the time.

Overall, the evidence is in favour of non-symmetric multicointegration, but this is property not found for each series. An optimizing process that is consistent with this finding is to choose

Table VI. Error correction model (C)

	Δp_t								Δs_t							
	z^+_{t-1}		z^-_{t-1}		$u^+_{2,t-1}$		$u^-_{2,t-1}$		z^+_{t-1}		z^-_{t-1}		$u^+_{2,t-1}$		$u^-_{2,t-1}$	
	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$	Coeff.	$\|t\|$
1	−0·79	3·27	−1·17	2·93	−0·12	4·60	0·03	1·12	−0·07	0·30	−0·34	0·86	−0·06	2·41	0·05	1·72
2	−0·91	3·88	−1·24	3·48	−0·08	4·39	0·01	0·45	−0·41	1·74	−0·73	2·04	−0·05	2·67	0·20	0·76
3	−0·81	3·49	−0·76	2·35	−0·05	3·04	−0·01	0·29	−0·27	1·19	−0·15	0·47	−0·02	1·37	0·01	0·55
4	−0·89	3·84	0·42	1·83	−0·06	2·68	0·07	2·66	−0·50	1·93	0·96	3·81	−0·05	1·74	0·07	2·18
5	−0·56	2·63	−0·90	3·87	−0·04	1·84	0·01	0·47	0·07	0·42	−0·26	1·33	−0·02	0·75	0·01	0·61
6	−0·29	1·87	−0·16	0·78	−0·05	3·33	0·04	2·46	−0·09	0·65	0·25	1·35	−0·02	1·55	0·03	1·92
7	−0·08	0·43	−0·51	1·77	−0·07	2·35	0·06	1·88	0·45	2·82	−0·11	0·42	−0·04	1·55	0·11	3·51
8	−1·38	6·14	−0·68	2·23	0·03	0·90	−0·03	0·97	−0·24	1·17	0·37	1·30	0·03	1·06	−0·01	0·41
9	−0·94	4·66	−0·50	1·83	−0·06	3·01	0·01	0·67	−0·23	1·31	0·04	0·17	−0·02	1·21	0·01	0·79
10	−0·91	4·04	−0·99	3·21	−0·09	2·71	0·04	1·03	−0·30	1·40	−0·37	1·27	−0·03	0·89	0·03	0·87
11	−0·29	1·32	−0·64	2·57	0·01	0·20	−0·01	0·10	0·39	2·04	0·34	1·58	0·03	0·54	0·05	0·85
12	−1·37	6·14	−0·98	3·17	−0·11	3·75	0·07	1·79	−0·76	3·61	0·32	1·10	−0·06	2·01	0·07	1·99
13	−1·64	8·92	−1·12	4·74	−0·05	1·76	−0·02	0·61	−0·76	4·60	0·01	0·04	0·01	0·41	−0·01	0·31
14	−0·89	4·20	−1·27	4·43	−0·08	2·00	0·03	0·79	−0·29	1·54	−0·40	1·56	0·00	0·11	0·03	1·04
15	−0·87	3·58	−0·99	3·19	−0·05	1·12	0·07	1·20	−0·02	0·09	−0·34	1·16	−0·01	0·37	0·08	1·62
16	−1·02	4·39	−1·06	3·96	−0·08	2·57	0·04	1·15	0·08	0·40	−0·25	1·13	−0·03	1·30	0·04	1·14
17	−1·06	4·14	−0·62	2·58	−0·02	0·49	−0·02	0·36	−0·03	0·14	−0·09	0·43	0·01	0·16	0·01	0·16
18	−0·62	2·48	−0·59	1·50	−0·10	2·11	0·09	1·97	0·17	0·77	0·49	1·42	−0·01	0·19	0·08	2·09
19	−1·20	6·05	−1·04	3·62	−0·08	3·65	0·02	1·00	−0·16	0·98	−0·04	0·19	−0·02	1·29	0·03	1·48
20	−0·50	1·63	−1·17	3·14	0·23	2·25	0·01	0·12	0·44	1·71	−0·11	0·35	0·20	2·33	0·10	1·16
21	−0·95	3·29	−1·04	3·39	0·13	1·43	0·04	0·41	0·07	0·29	0·00	0·01	0·11	1·51	0·14	1·72
22	−0·39	1·32	−0·66	1·70	−0·04	0·39	0·16	1·73	0·57	2·26	0·38	1·16	0·03	0·38	0·18	2·27
23	−0·51	3·13	0·19	0·93	−0·07	1·81	0·03	0·61	0·15	0·92	0·97	4·87	−0·01	0·30	0·16	3·43
24	−0·29	1·76	−0·64	3·83	−0·10	2·49	0·04	1·10	0·35	2·34	0·55	3·64	−0·04	1·16	0·14	3·78
25	−0·44	2·69	0·06	0·04	−0·10	2·69	0·09	2·25	0·16	0·88	0·78	4·61	−0·06	1·46	0·22	4·85
26	−1·02	6·13	−0·33	1·37	0·02	0·45	−0·01	0·30	0·03	0·27	0·37	2·06	0·02	0·57	0·01	0·50
27	−0·89	3·34	−0·30	0·92	0·05	0·81	0·02	0·40	0·45	1·93	0·77	2·76	0·05	0·93	0·05	1·00

production at each instant of time to minimize

$$J = E[\theta_1[(p_{t+1} - p_t^*)^+]^2 + \theta_2[(p_{t+1} - p_t^*)^-]^2 + \theta_3[(I_{t+1} - \lambda s_t)^+]^2 + \theta_4[(I_{t+1} - \lambda s_t)^-]^2 + \theta_5(p_{t+1} - p_t)^2],$$

with all $\theta_j \geqslant 0$. The target for p_{t+1} is $p_t^* = s_t$ and the target for I_{t+1} is λs_t. The system is completed by a generating equation for s_t. The final term represents a cost of changing production. If $\theta_1 \neq \theta_2$ then z^+ and z^- will enter the error correction model separately. This can be interpreted as having $p_t = s_t$ as an attractor (equilibrium) in the phase space (p, s), with the strength of attraction different on both sides of the attractor (Granger, 1987). Similarly for the attractor $I_t = \lambda s_t$ if $\theta_3 \neq \theta_4$.

It should be noted that if z_t is I(0), so will be z_t^+ and z_t^-. However for the attractor intepretation it is important that θ_1 and θ_2 are both non-zero, and similarly for θ_3 and θ_4. If, for example, $\theta_1 > 0$ but $\theta_2 = 0$, attraction would only occur on one side of the line and cointegration would not occur. Thus, in the error correction models (A), (B) and (C) if there is multicointegration both non-symmetric coefficients should be significant, but in practice this rarely occurs. It may be that many of these coefficients are non-zero but small.

The question arises how the series can be actually generated, given non-symmetric multicointegration. Rather than use the variables p_t, s_t it is easier to use the equivalent pair s_t and z_t $(= p_t - s_t)$. Δs_t is generated by an error correction model, such as (A) if relevant, with Δp_t replaced by $\Delta s_t + \Delta z_t$. A generating mechanism for z_t is found by subtracting error correction equations for Δp_t and Δs_t. This will usually be a non-linear ARMAX model, with Δs_t the exogenous variable. However, sometimes a simpler model may occur. An example are the (B) error correction models for series 1, given above. Subtracting gives

$$\begin{aligned}\Delta z_t &= \Delta p_t - \Delta s_t \\ &= 0{\cdot}85 - 0{\cdot}72 z_{t-1}^+ - 0{\cdot}703 z_{t-1}^- - 0{\cdot}04 u_{2,t-1} - 0{\cdot}05 \Delta p_{t-1} + 0{\cdot}05 \Delta s_{t-1} + \text{residual}.\end{aligned}$$

This can be approximately written as

$$\Delta z_t = 0{\cdot}85 - 0{\cdot}7(z_{t-1}^+ + z_{t-1}^-) - 0{\cdot}05\Delta(p_{t-1} - s_{t-1}) + \text{residual},$$

assuming the term in $u_{2,t-1}$ is negligible, gives

$$z_t = 0{\cdot}85 + 0{\cdot}24 z_{t-1} + 0{\cdot}05 z_{t-2} + \text{residual},$$

which is a stationary AR(2) model for z_t. Such simplifications do not always occur.

4. (S, s) INVENTORY CONTROL RULE

It is believed that many companies, particularly in the trade sectors (18 to 27), use an (S, s) inventory control rule. In this rule, if ever the inventory level falls to or below a minimum critical level s, the firm changes production to increase the level to a value S. Such a rule may be optimum for a stationary sales series but is unlikely to hold when both sales and production are I(1). If the inventory level series were bounded it would then appear to be I(0) according to the Dickey–Fuller test as the series would have a bounded variance but this is not observed for the series we investigated. Further, if the (S, s) rule is used, the inventory/sales ratio will often lie in a very narrow band, so that production has to be changed very frequently to add to inventory. A more likely rule is one based on the inventory/sales ratio, i.e. have a pair of values (S', s'), if inventory $\leqslant s' \times$ sales, increase production until inventory $= S' \times$ sales. Such a rule would allow sales, production and inventories all be I(1). Blinder (1981, page 462) used a simple example to show that the original (S, s) rules could aggregate into a stable inventory/sales ratio in the stationary sales case. If different companies use an (S', s') rule with different S', s' values, the aggregates may similarly appear to be multicointegrated.

An indirect test of the (S', s') rule can be derived from the non-symmetric error correction models considered in the previous section. Let $u_t = I_t - \lambda s_t$ and $q_t = I_t - s's_t$. If production were driven only by considerations of inventory then one has

$$q_{t-1} > 0 \rightarrow \Delta p_t = 0$$
$$q_{t-1} \leqslant 0 \rightarrow \Delta p_t > 0.$$

With $\lambda > 0$, if $q_t \leqslant 0$, then $u_t < 0$. Thus in the non-symmetric error correction models, if all companies in an industry were using the same (S', s') rule, and if there is no temporal aggregation, one should find u_{t-1}^- having a significant negative coefficient in the equation for Δp_t. However, this is not what is observed. In Table IV, for instance, $u_{2,t-1}^-$ is significant in only 4 out of 27 industries, none of which are negative. Thus to this extent the evidence is not in favour of the (S', s') rule. The effects of cross-sectional and temporal aggregation on this rule need further study.

5. CONCLUSION

Sufficient evidence has been found for us to conclude that multicointegration is a useful concept in the area of inventory determination. A better test would be on data from a single corporation, but these are not currently available to us.

APPENDIX

Data are taken from the US Department of Commerce, Bureau of Economic Analysis, as available on the Citibank data base. Figures are monthly for the period 1967:1 to 1987:4 for final sales and level of inventory at the end of each month, in 1982 constant dollars and seasonally adjusted. The sample size is 244 observations. The numbers used for the industries and groupings are defined as follows:

1 Manufacturing and Trade (2 + 18 + 23)
2 Manufacturing (3 + 10)
3 Durable Goods Manufacturing (4 + 5 + 6 + 7 + 8 + 9)
4 Primary Metals
5 Fabricated Metals
6 Machinery, Except Electrical
7 Electrical Machinery
8 Total Transportation Equipment
9 Other Durable Goods Manufacturing
10 Non-durable Goods Manufacturing (11 + 12)
11 Food and Kindred products
12 Non-food (13 + 14 + 15 + 16 + 17)
13 Paper and Allied Products
14 Chemicals and Allied Products
15 Petroleum and Coal Products
16 Rubber and Plastic Products
17 Other Non-durable Goods Manufacturing
18 Merchant Wholesalers (19 + 20)

19 Durable Goods Wholesalers
20 Non-durable Goods Wholesalers (21 + 22)
21 Groceries and Farm Products Wholesalers
22 Other Non-durable Goods Wholesalers
23 Retail Trade (24 + 26)
24 Durable Goods Retailers ($25 \subset 24$)
25 Auto Dealers
26 Non-durable Goods Retailers ($27 \subset 26$)
27 Food Stores

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