# Homework # 6

**P5.53*** This is a capacitive load because the reactance is negative.

$$P = I_{rms}^2 R = (15)^2 100 = 22.5 \text{ kW}$$

$$Q = I_{rms}^2 X = (15)^2(-50) = -11.25 \text{ kVAR}$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}(-0.5) = 26.57^\circ$$

power factor $= \cos(\theta) = 89.44\%$

**P5.59***

$$\mathbf{I} = \frac{1000\sqrt{2}\angle 0^\circ}{100} + \frac{1000\sqrt{2}\angle 0^\circ}{-j265.3} = 14.14 + j5.331$$

$$= 15.11\angle 20.66^\circ$$

$$P = V_{rms} I_{rms} \cos\theta = 10 \text{ kW}$$

$$Q = V_{rms} I_{rms} \sin\theta = -3.770 \text{ kVAR}$$

$$\text{Apparent power} = V_{rms} I_{rms} = 10.68 \text{ kVA}$$

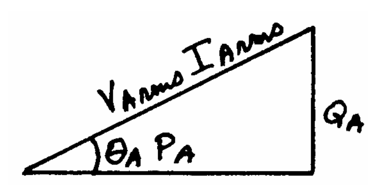
$$\text{Power factor} = \cos(20.66^\circ) = 0.9357 = 93.57\% \text{ leading}$$

**P5.62*** Load $A$:

$$P_A = 10 \text{ kW}$$

$$\theta_A = \cos^{-1}(0.9) = 25.84^\circ$$

$$Q_A = P_A \tan\theta_A = 4.843 \text{ kVAR}$$



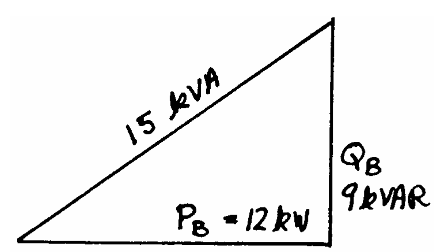

Load $B$:

$$V_{rms} I_{Brms} = 15 \text{ kVA}$$

$$\theta_B = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q_B = V_{rms} I_{Brms} \sin(\theta_B) = 9 \text{ kVAR}$$

$$P_B = V_{rms} I_{Brms} \cos(\theta_B) = 12 \text{ kW}$$




Source:

$$P_s = P_A + P_B = 22 \text{ kW}$$

$$Q_s = Q_A + Q_B = 13.84 \text{ kVAR}$$

$$\text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 26 \text{ kVA}$$

$$\text{Power factor} = \frac{P_s}{\text{Apparent power}} = 0.8462 = 84.62\% \text{ lagging}$$

**P5.64*** (a)

$$\cos\theta = 0.25$$
$$\theta = 75.52^\circ$$
$$P = V_{rms} I_{rms} \cos(\theta)$$
$$I_{rms} = \frac{P}{V_{rms}\cos(\theta)} = \frac{100\text{ kW}}{1\text{ kV}(0.25)} = 400\text{ A}$$
$$\mathbf{I} = 400\sqrt{2}\angle -75.52^\circ$$

(b)

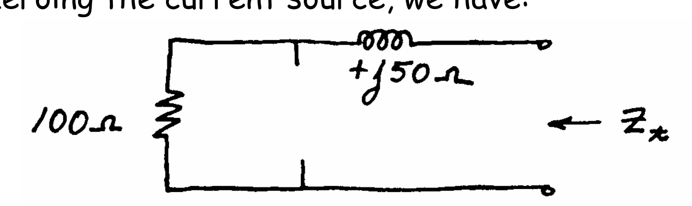
$$Q_{load} = V_{rms} I_{rms} \sin\theta = 387.3\text{ kVAR}$$
$$Q_{total} = 0 = Q_{load} + Q_C$$
$$Q_C = -387.3 \times 10^3 = \frac{(V_{rms})^2}{X_C}$$
$$X_C = -2.582 = -\frac{1}{\omega C}$$
$$C = 1027\,\mu\text{F}$$

The capacitor must be rated for at least 387.3 kVAR. With the capacitor in place, we have:

$$P = 100\text{ kW} = V_{rms} I_{rms}$$
$$I_{rms} = 100\text{ A}$$
$$\mathbf{I} = 100\angle 0^\circ$$

(c) The line current is smaller by a factor of 4 with the capacitor in place, reducing $I^2R$ losses in the line by a factor of 16.

**P5.67*** (a) Zeroing the current source, we have:




Thus, the Thévenin impedance is

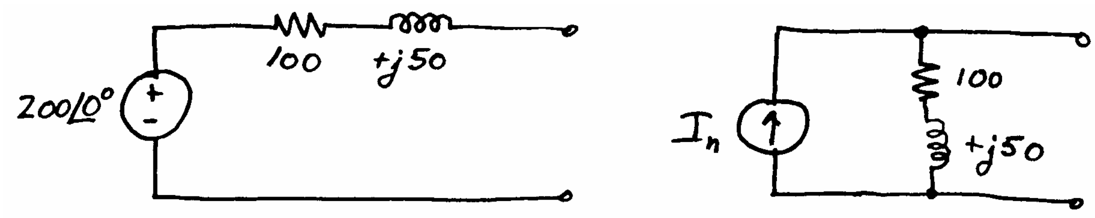
$$Z_t = 100 + j50 = 111.8\angle 26.57^\circ\ \Omega$$

Under open circuit conditions, there is zero voltage across the inductance, the current flows through the resistance, and the Thévenin voltage is

$$\mathbf{V}_t = \mathbf{V}_{oc} = 200\angle 0^\circ$$

$$\mathbf{I}_n = \mathbf{V}_t / Z_t = 1.789\angle -26.57^\circ$$

Thus, the Thévenin and Norton equivalent circuits are:




(b) For maximum power transfer, the load impedance is

$$Z_{load} = 100 - j50$$

$$\mathbf{I}_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 100 - j50} = 1$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 100(1/\sqrt{2})^2 = 50 \text{ W}$$

(c) In the case for which the load must be pure resistance, the load for maximum power transfer is

$$Z_{load} = |Z_t| = 111.8$$

$$I_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{200}{100 + j50 + 111.8} = 0.9190\angle -13.28^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 47.21 \text{ W}$$

**P5.70*** For maximum power transfer, the impedance of the load should be the complex conjugate of the Thévenin impedance:

$$Z_{load} = 10 - j5$$

$$Y_{load} = 1/Z_{load} = 0.08 + j0.04$$

$$Y_{load} = 1/R_{load} + j\omega C_{load} = 0.08 + j0.04$$

Setting real parts equal:

$$1/R_{load} = 0.08$$

$$R_{load} = 12.5\ \Omega$$

Setting imaginary parts equal:

$$\omega C_{load} = 0.04$$

$$C_{load} = 106.1\ \mu\text{F}$$

**P5.73***

$$V_L = \sqrt{3} \times V_Y = \sqrt{3} \times 440 = 762.1 \text{ V rms}$$

$$I_L = \frac{V_Y}{R} = \frac{440}{30} = 14.67 \text{ A rms}$$

$$P = 3 V_Y I_L \cos(\theta) = 3 \times 440 \times 14.67 \times \cos(0)$$

$$= 19.36 \text{ kW}$$

**P5.74***

$$Z_Y = \frac{1}{1/R + j\omega C}$$

$$= \frac{1}{1/50 + j377 \times 10^{-4}}$$

$$= 10.98 - j20.70$$

$$= 23.43\angle -62.05^\circ$$

$$Z_\Delta = 3Z_Y$$

$$= 70.29\angle -62.05^\circ \ \ \Omega$$

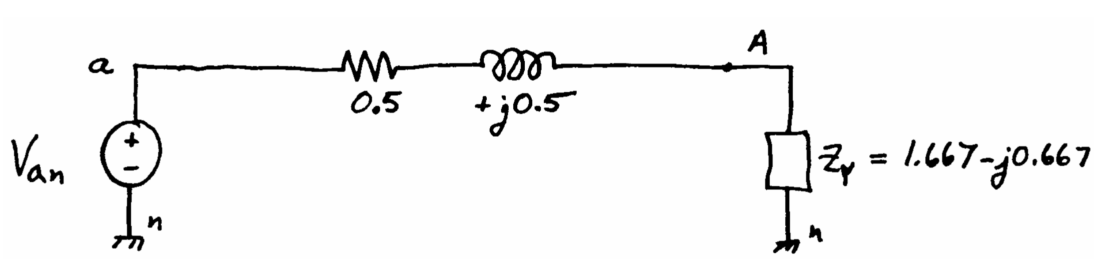
**P5.78*** This is a positive sequence source. The phasor diagram is shown in Figure 5.40 in the book. Thus, we have:

$$\mathbf{V}_{an} = \frac{440\sqrt{2}}{\sqrt{3}}\angle 0^\circ$$

The impedance of a equivalent wye-connected load is

$$Z_Y = \frac{Z_\Delta}{3} = 1.667 - j0.6667 \ \Omega$$

The equivalent circuit for the a-phase of an equivalent wye-wye circuit is:




Thus, the line current is:

$$
\begin{aligned}
\mathbf{I}_{aA} &= \frac{\mathbf{V}_{an}}{0.5 + j0.5 + Z_Y} \\
&= 116.9\sqrt{2}\angle 4.40^\circ \\
\mathbf{V}_{An} &= \mathbf{V}_{an} - \mathbf{I}_{aA}(0.5 + j0.5) \\
&= 209.8\sqrt{2}\angle -17.40^\circ \\
\mathbf{V}_{AB} &= 363.4\sqrt{2}\angle 12.60^\circ \\
\mathbf{I}_{AB} &= \frac{\mathbf{V}_{AB}}{Z_\Delta} \\
&= 67.49\sqrt{2}\angle 34.40^\circ \\
P_{load} &= 3(I_{ABrms})^2 \times 5 = 68.32\ \text{kW} \\
P_{line} &= 3(I_{aArms})^2 \times 0.5 = 20.50\ \text{kW}
\end{aligned}
$$