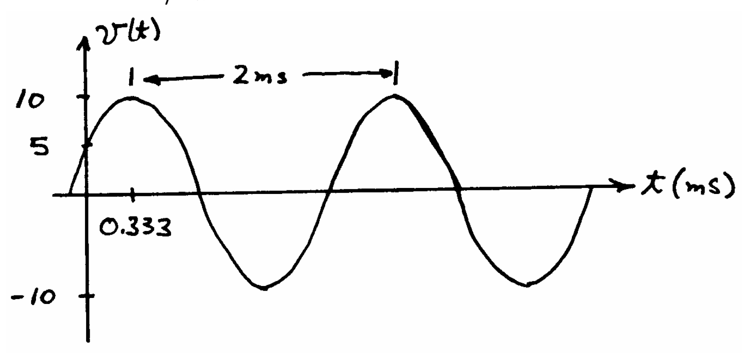
# Homework # 5

**P5.2***

$$v(t) = 10\sin(1000\pi t + 30^\circ) = 10\cos(1000\pi - 60^\circ)$$
$$\omega = 1000\pi \text{ rad/s}$$
$$f = 500 \text{ Hz}$$
$$\text{phase angle} = \theta = -60^\circ = -\pi/3 \text{ radians}$$
$$T = 1/f = 2 \text{ ms}$$
$$V_{rms} = V_m/\sqrt{2} = 10/\sqrt{2} = 7.071 \text{ V}$$
$$P = (V_{rms})^2/R = 1 \text{ W}$$

First positive peak occurs for

$$1000\pi t_{peak} - \pi/3 = 0$$
$$t_{peak} = 0.3333 \text{ ms}$$




**P5.6*** Sinusoidal voltages can be expressed in the form $v(t) = V_m\cos(\omega t + \theta)$. The peak voltage is $V_m = \sqrt{2}V_{rms} = \sqrt{2}20 = 28.28$ V. The frequency is $f = 1/T = 10$ kHz and the angular frequency is $\omega = 2\pi f = 2\pi 10^4$ radians/s. The phase corresponding to a time interval of $\Delta t = 20\ \mu s$ is $\theta = (\Delta t/T)\times 360^\circ = 72^\circ$. Thus we have $v(t) = 28.28\cos(2\pi 10^4 t - 72^\circ)$ V.

**P5.9***

$$V_{rms} = \sqrt{\frac{i}{T}\int_0^T v^2(t)dt}$$
$$= \sqrt{\frac{1}{2}\int_0^1 25dt}$$
$$= 3.536 \text{ V}$$

**P5.11***

$$V_{rms} = \sqrt{\frac{i}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\frac{1}{0.1}\int_0^{0.1}[5 + 10\cos(20\pi t)]^2 dt}$$

$$= \sqrt{\frac{1}{0.1}\int_0^{0.1}[25 + 100\cos(20\pi t) + 100\cos^2(20\pi t)]dt}$$

$$= \sqrt{\frac{1}{0.1}\left[\int_0^{0.1}25dt + \int_0^{0.1}100\cos(20\pi t)dt + \int_0^{0.1}100\cos^2(20\pi t)]dt\right]}$$

$$= \sqrt{\frac{1}{0.1}\left[\int_0^{0.1}25dt + \int_0^{0.1}100\cos(20\pi t)dt + \int_0^{0.1}50dt + \int_0^{0.1}50\cos(40\pi t)]dt\right]}$$

$$= \sqrt{\frac{1}{0.1}[2.5 + 0 + 5 + 0]}$$

$$= 8.660 \quad \text{V}$$

**P5.15***

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

$$= \sqrt{\frac{1}{4}\left(\int_0^2 4dt + \int_2^4 1dt\right)}$$

$$= 1.581 \text{ A}$$

**P5.17***

$$v_1(t) = 100\cos(\omega t)$$

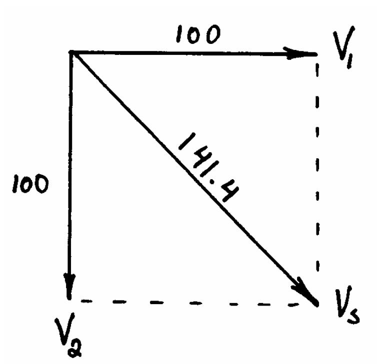
$$v_2(t) = 100\sin(\omega t) = 100\cos(\omega t - 90^\circ)$$

$$\mathbf{V}_1 = 100\angle 0^\circ = 100$$

$$\mathbf{V}_2 = 100\angle -90^\circ = -j100$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2 = 100 - j100 = 141.4\angle -45^\circ$$

$$v_s(t) = 141.4\cos(\omega t - 45^\circ)$$




$\mathbf{V}_2$ lags $\mathbf{V}_1$ by $90^\circ$

$\mathbf{V}_s$ lags $\mathbf{V}_1$ by $45^\circ$

$\mathbf{V}_s$ leads $\mathbf{V}_2$ by $45^\circ$

**P5.21***

$$\omega = 2\pi f = 400\pi$$

$$v_1(t) = 10\cos(400\pi t + 30^\circ)$$
$$v_2(t) = 5\cos(400\pi t + 150^\circ)$$
$$v_3(t) = 10\cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by $120^\circ$
$v_1(t)$ lags $v_3(t)$ by $60^\circ$
$v_2(t)$ leads $v_3(t)$ by $60^\circ$

**P5.23*** We are given the expression

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t)$$

Converting to phasors we obtain

$$5\angle 75^\circ - 3\angle -75^\circ + 4\angle -90^\circ =$$
$$1.2941 + j4.8296 - (0.7765 - j2.8978) - j4 =$$
$$0.5176 + j3.7274 = 3.763\angle 82.09^\circ$$

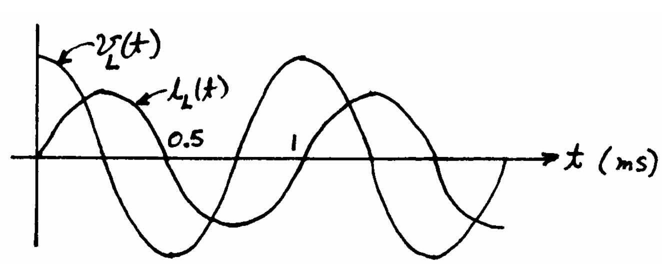
Thus, we have

$$5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t) =$$
$$3.763\cos(\omega t + 82.09^\circ)$$

**P5.28***

$$v_L(t) = 10\cos(2000\pi t)$$
$$\omega = 2000\pi$$
$$Z_L = j\omega L = j200\pi = 200\pi\angle 90^\circ$$
$$\mathbf{V}_L = 10\angle 0^\circ$$
$$\mathbf{I}_L = \mathbf{V}_L / Z_L = (1/20\pi)\angle -90^\circ$$
$$i_L(t) = (1/20\pi)\cos(2000\pi t - 90^\circ) = (1/20\pi)\sin(2000\pi t)$$

$i_L(t)$ lags $v_L(t)$ by $90^\circ$



**P5.29***

$$v_C(t) = 10\cos(2000\pi t)$$

$$\omega = 2000\pi$$

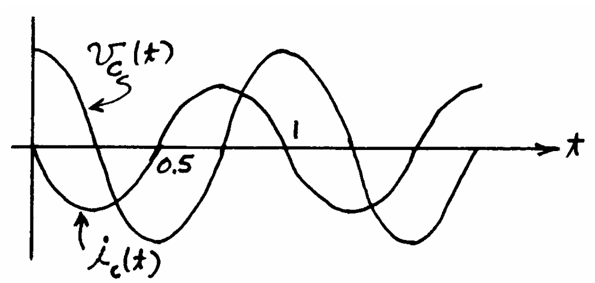
$$Z_C = \frac{-j}{\omega C} = -j15.92 = 15.92\angle -90^\circ\ \Omega$$

$$\mathbf{V}_C = 10\angle 0^\circ$$

$$\mathbf{I}_C = \mathbf{V}_C / Z_C = 0.6283\angle 90^\circ$$

$$i_C(t) = 0.6283\cos(2000\pi t + 90^\circ) = -0.6283\sin(2000\pi t)$$

$i_C(t)$ leads $v_C(t)$ by $90^\circ$




**P5.31***

$$Z = j\omega L + R - j\frac{1}{\omega C}$$

$$\omega = 500: \quad Z = j50 + 50 - j200$$
$$= 50 - j150\ \Omega = 158.1\angle -71.57^\circ$$

$$\omega = 1000: \quad Z = j100 + 50 - j100$$
$$= 50\ \Omega = 50\angle 0^\circ$$

$$\omega = 2000: \quad Z = j200 + 50 - j50$$
$$= 50 + j150\ \Omega = 158.1\angle 71.57^\circ$$

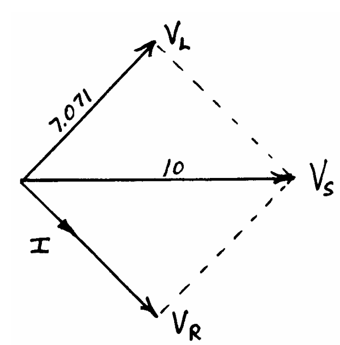
**P5.34***

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L}$$
$$= \frac{10\angle 0^\circ}{100 + j100}$$
$$= 70.71\angle -45^\circ \text{ mA}$$

$$\mathbf{V}_R = R\mathbf{I} = 7.071\angle -45^\circ \text{ V}$$

$$\mathbf{V}_L = j\omega L\mathbf{I} = 7.071\angle 45^\circ \text{ V}$$

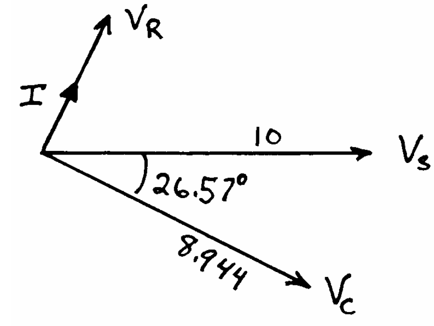
$\mathbf{I}$ lags $\mathbf{V}_s$ by $45^\circ$



**P5.37***

$$\mathbf{I} = \frac{\mathbf{V}_s}{R - j/\omega C}$$
$$= \frac{10\angle 0^\circ}{1000 - j2000}$$
$$= 4.472\angle 63.43^\circ \text{ mA}$$
$$\mathbf{V}_R = R\mathbf{I} = 4.472\angle 63.43^\circ \text{ V}$$
$$\mathbf{V}_C = (-j/\omega C)\mathbf{I} = 8.944\angle -26.57^\circ \text{V}$$

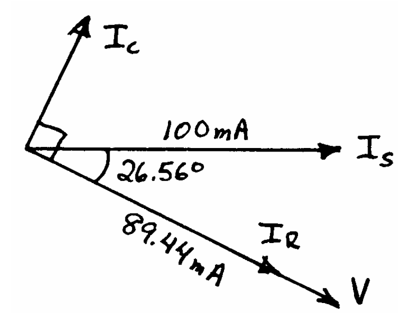
$\mathbf{I}$ leads $\mathbf{V}_s$ by $63.43^\circ$




**P5.39***

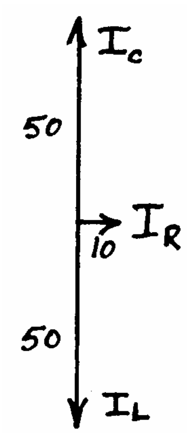
$$\mathbf{I}_s = 100\angle 0^\circ \text{ mA}$$
$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/100 + 1/(-j200)}$$
$$= 8.944\angle -26.56^\circ \text{ V}$$
$$\mathbf{I}_R = \mathbf{V}/R = 89.44\angle -26.56^\circ \text{ mA}$$
$$\mathbf{I}_C = \mathbf{V}(j\omega C) = 44.72\angle 63.44^\circ \text{ mA}$$

$\mathbf{V}$ lags $\mathbf{I}_s$ by $26.56^\circ$




**P5.41***

$$\mathbf{I}_s = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/R + 1/j\omega L + j\omega C}$$
$$= 10^{-2} \frac{1}{1/1000 - j0.005 + j0.005}$$
$$= 10\angle 0^\circ \text{ V}$$
$$\mathbf{I}_R = \mathbf{V}/R = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{I}_L = \mathbf{V}/j\omega L = 50\angle -90^\circ \text{ mA}$$
$$\mathbf{I}_C = \mathbf{V}(j\omega C) = 50\angle 90^\circ \text{ mA}$$




The peak value of $i_L(t)$ is five times larger than the source current! This is possible because current in the capacitance balances the current in the inductance (i.e., $\mathbf{I}_L + \mathbf{I}_C = 0$ ).

**P5.45*** First we write the KVL equation:

$$\mathbf{V}_1 - \mathbf{V}_2 = 10\angle 0^\circ$$

Then we enclose nodes 1 and 2 in a closed surface to form a supernode and write a KCL equation:

$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1}{j20} + \frac{\mathbf{V}_2}{15} + \frac{\mathbf{V}_2}{-j5} = 0$$

The solution to these equations is:

$$\mathbf{V}_1 = 9.402\angle 29.58^\circ$$

$$\mathbf{V}_2 = 4.986\angle 111.45^\circ$$

**P5.46*** Writing KVL equations around the meshes, we obtain

$$5\mathbf{I}_1 + j15(\mathbf{I}_1 - \mathbf{I}_2) = 20$$

$$-j10\mathbf{I}_2 + j15(\mathbf{I}_2 - \mathbf{I}_1) = 10$$

Solving, we obtain:

$$\mathbf{I}_1 = 1.644\angle 80.54^\circ$$

$$\mathbf{I}_2 = 2.977\angle 74.20^\circ$$