

Homework # 5

P5.2*

$$v(t) = 10 \sin(1000\pi t + 30^\circ) = 10 \cos(1000\pi t - 60^\circ)$$

$$\omega = 1000\pi \text{ rad/s}$$

$$f = 500 \text{ Hz}$$

$$\text{phase angle} = \theta = -60^\circ = -\pi/3 \text{ radians}$$

$$T = 1/f = 2 \text{ ms}$$

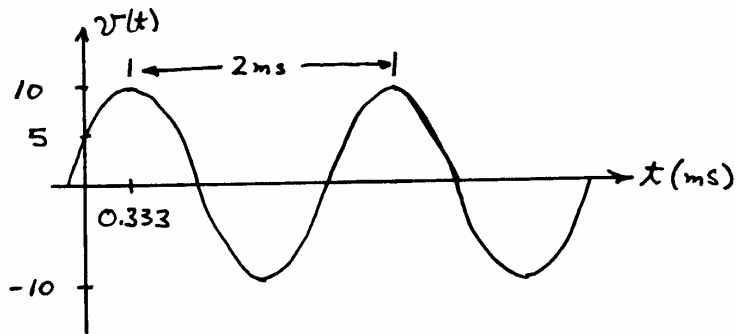
$$V_{rms} = V_m / \sqrt{2} = 10 / \sqrt{2} = 7.071 \text{ V}$$

$$P = (V_{rms})^2 / R = 1 \text{ W}$$

First positive peak occurs for

$$1000\pi t_{peak} - \pi/3 = 0$$

$$t_{peak} = 0.3333 \text{ ms}$$



P5.6*

Sinusoidal voltages can be expressed in the form $v(t) = V_m \cos(\omega t + \theta)$.

The peak voltage is $V_m = \sqrt{2}V_{rms} = \sqrt{2}220 = 28.28 \text{ V}$. The frequency is

$f = 1/T = 10 \text{ kHz}$ and the angular frequency is $\omega = 2\pi f = 2\pi 10^4$

radians/s. The phase corresponding to a time interval of $\Delta t = 20 \mu\text{s}$ is

$\theta = (\Delta t / T) \times 360^\circ = 72^\circ$. Thus we have $v(t) = 28.28 \cos(2\pi 10^4 t - 72^\circ) \text{ V}$.

P5.8*

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

$$= \sqrt{\frac{1}{2} \int_0^1 25 dt}$$

$$= 3.536 \text{ V}$$

P5.12*

$$v_1(t) = 100 \cos(\omega t)$$

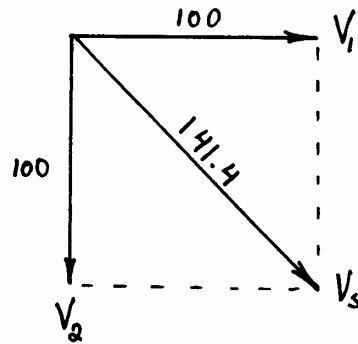
$$v_2(t) = 100 \sin(\omega t) = 100 \cos(\omega t - 90^\circ)$$

$$\mathbf{V}_1 = 100 \angle 0^\circ = 100$$

$$\mathbf{V}_2 = 100 \angle -90^\circ = -j100$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2 = 100 - j100 = 141.4 \angle -45^\circ$$

$$v_s(t) = 141.4 \cos(\omega t - 45^\circ)$$



\mathbf{V}_2 lags \mathbf{V}_1 by 90°

\mathbf{V}_s lags \mathbf{V}_1 by 45°

\mathbf{V}_s leads \mathbf{V}_2 by 45°

P5.14*

$$\omega = 2\pi f = 400\pi$$

$$v_1(t) = 10 \cos(400\pi t + 30^\circ)$$

$$v_2(t) = 5 \cos(400\pi t + 150^\circ)$$

$$v_3(t) = 10 \cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by 120°

$v_1(t)$ lags $v_3(t)$ by 60°

$v_2(t)$ leads $v_3(t)$ by 60°

P5.16* We are given the expression

$$5 \cos(\omega t + 75^\circ) - 3 \cos(\omega t - 75^\circ) + 4 \sin(\omega t)$$

Converting to phasors we obtain

$$5 \angle 75^\circ - 3 \angle -75^\circ + 4 \angle -90^\circ =$$

$$1.2941 + j4.8296 - (0.7765 - j2.8978) - j4 =$$

$$0.5176 + j3.7274 = 3.763 \angle 82.09^\circ$$

Thus, we have

$$5 \cos(\omega t + 75^\circ) - 3 \cos(\omega t - 75^\circ) + 4 \sin(\omega t) =$$

$$3.763 \cos(\omega t + 82.09^\circ)$$

P5.21*

$$v_L(t) = 10 \cos(2000\pi t)$$

$$\omega = 2000\pi$$

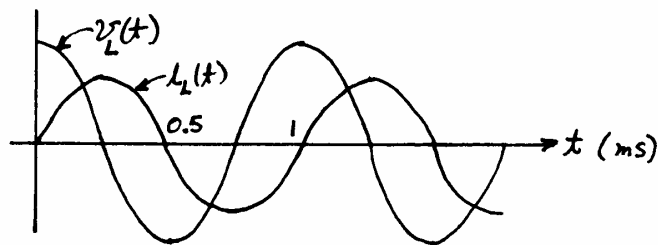
$$Z_L = j\omega L = j200\pi = 200\pi \angle 90^\circ$$

$$\mathbf{V}_L = 10 \angle 0^\circ$$

$$\mathbf{I}_L = \mathbf{V}_L / Z_L = (1/20\pi) \angle -90^\circ$$

$$i_L(t) = (1/20\pi) \cos(2000\pi t - 90^\circ) = (1/20\pi) \sin(2000\pi t)$$

$i_L(t)$ lags $v_L(t)$ by 90°



P5.22*

$$v_C(t) = 10 \cos(2000\pi t)$$

$$\omega = 2000\pi$$

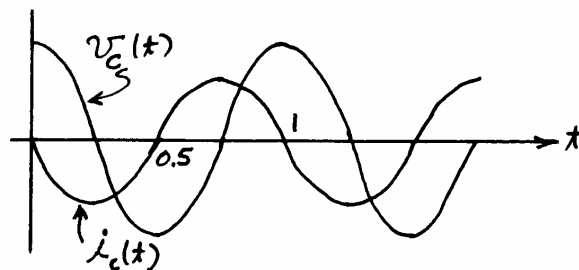
$$Z_C = \frac{-j}{\omega C} = -j15.92 = 15.92 \angle -90^\circ \Omega$$

$$\mathbf{V}_C = 10 \angle 0^\circ$$

$$\mathbf{I}_C = \mathbf{V}_C / Z_C = 0.6283 \angle 90^\circ$$

$$i_C(t) = 0.6283 \cos(2000\pi t + 90^\circ) = -0.6283 \sin(2000\pi t)$$

$i_C(t)$ leads $v_C(t)$ by 90°



P5.23*

$$Z = j\omega L + R - j\frac{1}{\omega C}$$

$$\begin{aligned}\omega = 500: \quad Z &= j50 + 50 - j200 \\ &= 50 - j150 \Omega = 158.1 \angle -71.57^\circ\end{aligned}$$

$$\begin{aligned}\omega = 1000: \quad Z &= j100 + 50 - j100 \\ &= 50 \Omega = 50 \angle 0^\circ\end{aligned}$$

$$\begin{aligned}\omega = 2000: \quad Z &= j200 + 50 - j50 \\ &= 50 + j150 \Omega = 158.1 \angle 71.57^\circ\end{aligned}$$

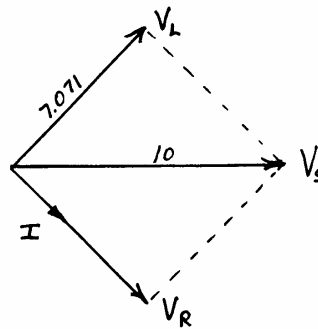
P5.25*

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}_s}{R + j\omega L} \\ &= \frac{10 \angle 0^\circ}{100 + j100} \\ &= 70.71 \angle -45^\circ \text{ mA}\end{aligned}$$

$$\mathbf{V}_R = R\mathbf{I} = 7.071 \angle -45^\circ \text{ V}$$

$$\mathbf{V}_L = j\omega L\mathbf{I} = 7.071 \angle 45^\circ \text{ V}$$

\mathbf{I} lags \mathbf{V}_s by 45°



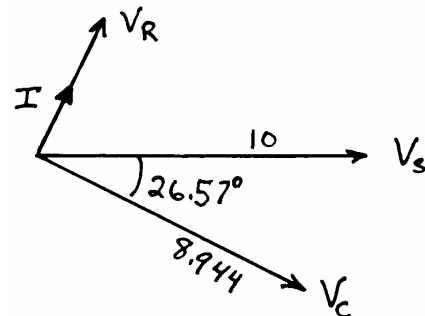
P5.28*

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}_s}{R - j/\omega C} \\ &= \frac{10 \angle 0^\circ}{1000 - j2000} \\ &= 4.472 \angle 63.43^\circ \text{ mA}\end{aligned}$$

$$\mathbf{V}_R = R\mathbf{I} = 4.472 \angle 63.43^\circ \text{ V}$$

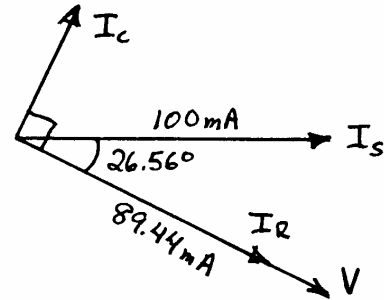
$$\mathbf{V}_C = (-j/\omega C)\mathbf{I} = 8.944 \angle -26.57^\circ \text{ V}$$

\mathbf{I} leads \mathbf{V}_s by 63.43°



P5.30*

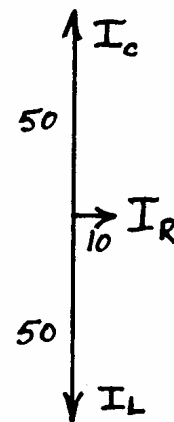
$$\begin{aligned} \mathbf{I}_s &= 100 \angle 0^\circ \text{ mA} \\ \mathbf{V} &= \mathbf{I}_s \frac{1}{1/100 + 1/(-j200)} \\ &= 8.944 \angle -26.56^\circ \text{ V} \\ \mathbf{I}_R &= \mathbf{V}/R = 89.44 \angle -26.56^\circ \text{ mA} \\ \mathbf{I}_C &= \mathbf{V}(j\omega C) = 44.72 \angle 63.44^\circ \text{ mA} \end{aligned}$$



\mathbf{V} lags \mathbf{I}_s by 26.56°

P5.32*

$$\begin{aligned} \mathbf{I}_s &= 10 \angle 0^\circ \text{ mA} \\ \mathbf{V} &= \mathbf{I}_s \frac{1}{1/R + 1/j\omega L + j\omega C} \\ &= 10^{-2} \frac{1}{1/1000 - j0.005 + j0.005} \\ &= 10 \angle 0^\circ \text{ V} \\ \mathbf{I}_R &= \mathbf{V}/R = 10 \angle 0^\circ \text{ mA} \\ \mathbf{I}_L &= \mathbf{V}/j\omega L = 50 \angle -90^\circ \text{ mA} \\ \mathbf{I}_C &= \mathbf{V}(j\omega C) = 50 \angle 90^\circ \text{ mA} \end{aligned}$$



The peak value of $i_L(t)$ is five times larger than the source current! This is possible because current in the capacitance balances the current in the inductance (i.e., $\mathbf{I}_L + \mathbf{I}_C = 0$).

P5.36* First we write the KVL equation:

$$\mathbf{V}_1 - \mathbf{V}_2 = 10 \angle 0^\circ$$

Then we enclose nodes 1 and 2 in a closed surface to form a supernode and write a KCL equation:

$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1}{j20} + \frac{\mathbf{V}_2}{15} + \frac{\mathbf{V}_2}{-j5} = 0$$

The solution to these equations is:

$$\mathbf{V}_1 = 9.402 \angle 29.58^\circ$$

$$\mathbf{V}_2 = 4.986 \angle 111.45^\circ$$

P5.37* Writing KVL equations around the meshes, we obtain

$$5\mathbf{I}_1 + j15(\mathbf{I}_1 - \mathbf{I}_2) = 20$$

$$-j10\mathbf{I}_2 + j15(\mathbf{I}_2 - \mathbf{I}_1) = 10$$

Solving, we obtain:

$$\mathbf{I}_1 = 1.644 \angle 80.54^\circ$$

$$\mathbf{I}_2 = 2.977 \angle 74.20^\circ$$