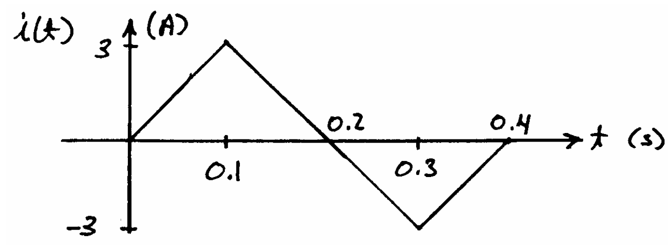
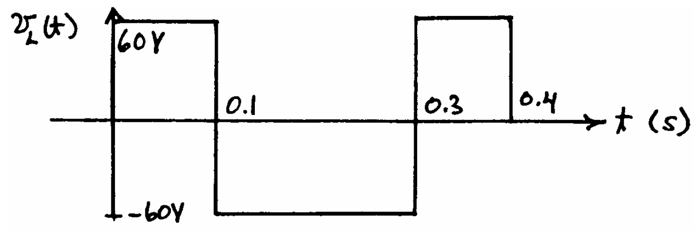
Homework # 5

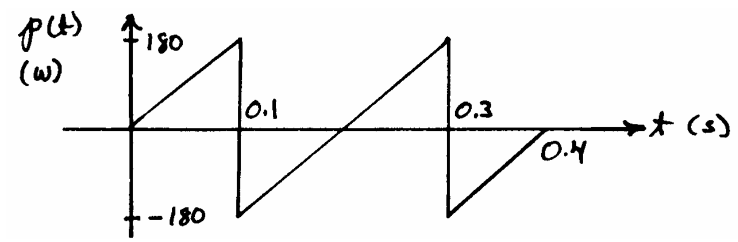
P3.31* $L = 2\,\text{H}$



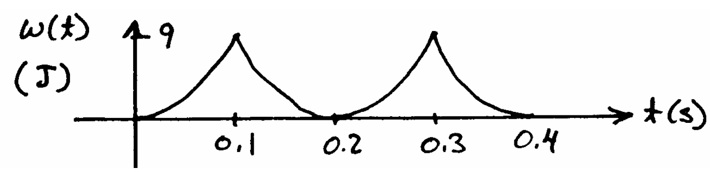
$$v_L(t) = L\frac{di_L(t)}{dt}$$



$$p(t) = v_L(t)i_L(t)$$



$$w(t) = \frac{1}{2}L[i_L(t)]^2$$



P3.36*

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0)$$

$$= 20 \times 10^3 \int_0^t 10dt - 0.1$$

$$= 2 \times 10^5 t - 0.1\,A$$

Solving for the time that the current reaches $+100\,\text{mA}$, we have

$$i_L(t_x) = 0.1 = 2 \times 10^5 t_0 - 0.1$$
$$t_x = 1\ \mu s$$

P3.37* $v_L = L\dfrac{di}{dt} = 0.5\dfrac{4}{0.2} = 10\ \text{V}$

P3.39* (a) $L_{eq} = 1 + \dfrac{1}{1/2 + 1/(1+3)} = 2.333\ \text{H}$

(b) 2 H in parallel with 2 H is equivalent to 1H. Also, 2 H in parallel with 6 H is equivalent to 1.5 H. Finally, we have $L_{eq} = \dfrac{1}{1/4 + 1/(1+1.5)} = 1.538\ \text{H}$

P3.42* $i(t) = \dfrac{1}{L_{eq}}\int_0^t v(t)dt = \dfrac{L_1 + L_2}{L_1 L_2}\int_0^t v(t)dt$

$i_1(t) = \dfrac{1}{L_1}\int_0^t v(t)dt$

Thus, we can write $i_1(t) = \dfrac{L_2}{L_1 + L_2} i(t) = \dfrac{2}{3} i(t)$.

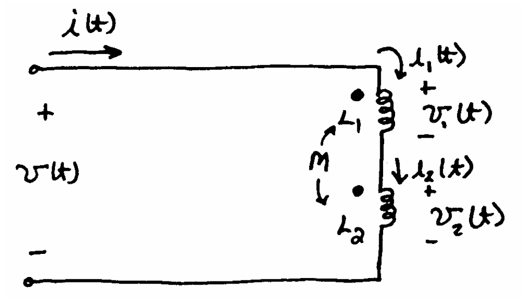
Similarly, we have $i_2(t) = \dfrac{L_1}{L_1 + L_2} i(t) = \dfrac{1}{3} i(t)$.

This is similar to the current-division principle for resistances. Keep in mind that these formulas assume that the initial currents are zero.

P3.47* With the dot moved to the bottom of L_2, we have

$$v_1(t) = L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = 5\cos(10t)$$
$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0$$

P3.48* (a)



As in Figure 3.23a, we can write

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

However, for the circuit at hand, we have $i(t) = i_1(t) = i_2(t)$. Thus,

$$v_1(t) = (L_1 + M)\frac{di(t)}{dt}$$

$$v_2(t) = (L_2 + M)\frac{di(t)}{dt}$$

Also, we have $v(t) = v_1(t) + v_2(t)$.

Substituting, we obtain $v(t) = (L_1 + 2M + L_2)\frac{di(t)}{dt}$.

Thus, we can write $v(t) = L_{eq} \frac{di(t)}{dt}$, in which

$$L_{eq} = L_1 + 2M + L_2.$$

(b) Similarly, for the dot at the bottom end of L_2, we have

$$L_{eq} = L_1 - 2M + L_2$$