Homework # 4

P3.5*

$$i = C\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{i}{C} = \frac{100 \times 10^{-6}}{2000 \times 10^{-6}} = 0.05 \text{ V/s}$$

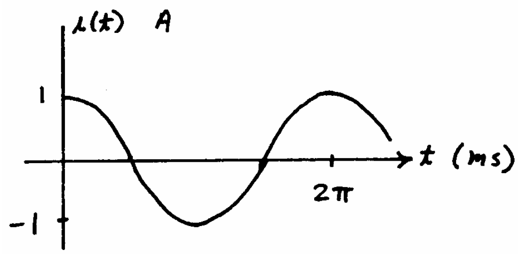
$$\Delta t = \frac{\Delta v}{dv/dt} = \frac{100}{0.05} = 2000 \text{ s}$$

P3.7*

$$i(t) = C\frac{dv}{dt}$$

$$= 10^{-5}\frac{d}{dt}(100\sin 1000t)$$

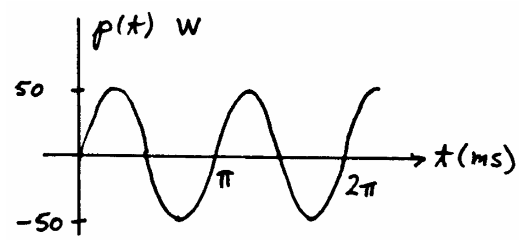
$$= \cos(1000t)$$



$$p(t) = v(t)i(t)$$

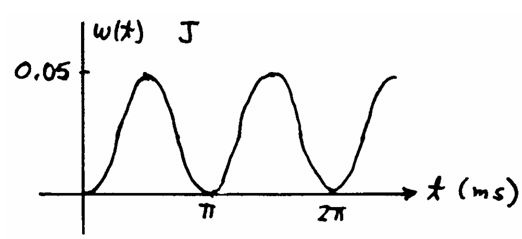
$$= 100\cos(1000t)\sin(1000t)$$

$$= 50\sin(2000t)$$



$$w(t) = \frac{1}{2}C[v(t)]^2$$

$$= 0.05\sin^2(1000t)$$



P3.11* $W = \text{power} \times \text{time} \qquad = 5 \text{ hp} \times 746 \text{ W / hp} \times 3600 \text{ s}$

$$= 13.4 \times 10^6 \text{ J}$$

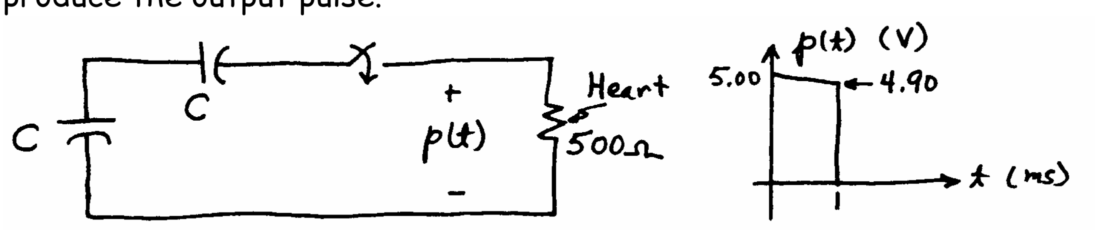
$$V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 13.4 \times 10^6}{0.01}} = 51.8 \text{ kV}$$

It turns out that a 0.01-F capacitor rated for this voltage would be much too large and massive for powering an automobile. Besides, to have reasonable performance, an automobile would need much more than 5 hp for an hour.

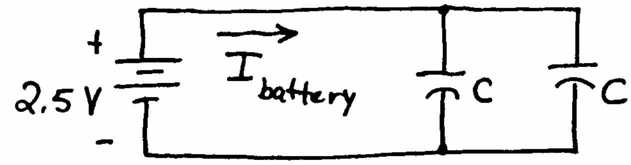
P3.15* (a) $C_{eq} = 1 + \dfrac{1}{1/2 + 1/2} = 2\ \mu\text{F}$

(b) The two 4-μF capacitances are in series and have an equivalent capacitance of $\dfrac{1}{1/4 + 1/4} = 2\ \mu\text{F}$. This combination is a parallel with the 2-μF capacitance, giving an equivalent of 4 μF. Then the 6 μF is in series, giving a capacitance of $\dfrac{1}{1/6 + 1/4} = 2.4\ \mu\text{F}$. Finally, the 3 μF is in parallel, giving an equivalent capacitance of $C_{eq} = 3 + 2.4 = 5.4\ \mu\text{F}$.

P3.19* As shown below, the two capacitors are placed in series with the heart to produce the output pulse.



While the capacitors are connected, the average voltage supplied to the heart is 4.95 V. Thus, the average current is $I_{pulse} = 4.95/500 = 9.9\text{ mA}$. The charge removed from each capacitor during the pulse is $\Delta Q = 9.9\text{ mA} \times 1\text{ ms} = 9.9\ \mu C$. This results in a 0.l V change in voltage, so we have $C_{eq} = \dfrac{C}{2} = \dfrac{\Delta Q}{\Delta V} = \dfrac{9.9 \times 10^{-6}}{0.1} = 99\ \mu\text{F}$. Thus, each capacitor has a capacitance of $C = 198\ \mu\text{F}$. Then as shown below, the capacitors are placed in parallel with the 2.5 V battery to recharge them.



The battery must supply 9.9 μC to each battery. Thus, the average current supplied by the battery is $I_{battery} = \dfrac{2 \times 9.9\ \mu C}{1s} = 19.8\ \mu A$. The ampere-hour rating of the battery is

$$19.8 \times 10^{-6} \times 5 \times 365 \times 24 = 0.867 \text{ Ampere hours}.$$

P3.21* $$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{15 \times 8.85 \times 10^{-12} \times 10 \times 10^{-2} \times 30 \times 10^{-2}}{0.01 \times 10^{-3}} = 0.398\ \mu\text{F}$$

P3.23* The charge Q remains constant because the terminals of the capacitor are open-circuited.

$$Q = C_1 V_1 = 1000 \times 10^{-12} \times 1000 = 1\ \mu C$$

$$W_1 = (1/2) C_1 (V_1)^2 = 500\ \mu\text{J}$$

After the distance between the plates is doubled, the capacitance becomes $C_2 = 500\text{ pF}$.

The voltage increases to $V_2 = \dfrac{Q}{C_2} = \dfrac{10^{-6}}{500 \times 10^{-12}} = 2000\text{ V}$ and the stored energy is $W_2 = (1/2) C_2 (V_2)^2 = 1000\ \mu\text{J}$. The additional energy is supplied by the force needed to pull the plates apart.

P3.26* For square plates, we have $L = W$, the plate area is $A = L^2$, and the volume of the dielectric is

$$\text{Vol} = L^2 d.$$

The minimum thickness of the dielectric is

$$d_{\min} = \frac{V_{\max}}{K} = \frac{1000}{32 \times 10^5} = 0.3125\text{ mm}$$

The required volume is

$$\text{Vol} = Ad = \frac{2W_{\max}}{\varepsilon_0 \varepsilon_r K^2} = \frac{2 \times 10^{-3}}{8.85 \times 10^{-12} (32 \times 10^5)^2} = 22.07 \times 10^{-6}\text{ m}^3$$

and the area is

$$A = \text{Vol}/d = 0.07062\text{ m}^2$$

The length of each side of the square plate is

$$L = \sqrt{A} = 0.2657\text{ m}$$