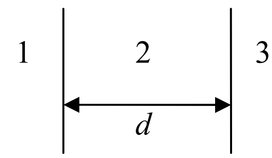
Reflections and transmissions at the interfaces separating 3 media

$$\mathbf{E}_1 = (E_{m1}^{+}\, e^{-\gamma_1 z} + E_{m1}^{-}\, e^{\gamma_1 z})\,\mathbf{a}_x$$
$$\mathbf{E}_2 = (E_{m2}^{+}\, e^{-\gamma_2 z} + E_{m2}^{-}\, e^{\gamma_2 z})\,\mathbf{a}_x$$
$$\mathbf{E}_3 = E_{m3}^{+}\, e^{-\gamma_3 z}\,\mathbf{a}_x$$



Reflection at the first interface: $$\frac{E_{m1}^{-}}{E_{m1}^{+}} = -\frac{(\eta_1+\eta_2)(\eta_2-\eta_3)+e^{2\gamma_2 d}(\eta_1-\eta_2)(\eta_2+\eta_3)}{(\eta_1-\eta_2)(\eta_2-\eta_3)+e^{2\gamma_2 d}(\eta_1+\eta_2)(\eta_2+\eta_3)}$$

Transmission at the first interface: $$\frac{E_{m2}^{+}}{E_{m1}^{+}} = \frac{2e^{2\gamma_2 d}\eta_2(\eta_2+\eta_3)}{(\eta_1-\eta_2)(\eta_2-\eta_3)+e^{2\gamma_2 d}(\eta_1+\eta_2)(\eta_2+\eta_3)}$$

Reflection at the second interface: $$\frac{E_{m2}^{-}}{E_{m1}^{+}} = -\frac{2\eta_2(\eta_2-\eta_3)}{(\eta_1-\eta_2)(\eta_2-\eta_3)+e^{2\gamma_2 d}(\eta_1+\eta_2)(\eta_2+\eta_3)}$$

Transmission at the second interface: $$\frac{E_{m3}^{+}}{E_{m1}^{+}} = \frac{4e^{(\gamma_2+\gamma_3)d}\eta_2\eta_3}{(\eta_1-\eta_2)(\eta_2-\eta_3)+e^{2\gamma_2 d}(\eta_1+\eta_2)(\eta_2+\eta_3)}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{-1/4} e^{j\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$ (wave impedance)

$\gamma = \alpha + j\beta$ (propagation constant)

$$\alpha = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1} \approx \begin{cases} \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}} & \text{if } \dfrac{\sigma}{\omega\varepsilon} \ll 1 \\ \sqrt{\dfrac{\omega\mu\sigma}{2}} & \text{if } \dfrac{\sigma}{\omega\varepsilon} \gg 1 \end{cases}$$

$$\beta = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1} \approx \begin{cases} \omega\sqrt{\mu\varepsilon} & \text{if } \dfrac{\sigma}{\omega\varepsilon} \ll 1 \\ \sqrt{\dfrac{\omega\mu\sigma}{2}} & \text{if } \dfrac{\sigma}{\omega\varepsilon} \gg 1 \end{cases}$$

$\lambda = \dfrac{2\pi}{\beta}$ (wavelength)

$\varepsilon = \varepsilon_r\varepsilon_0$ ($\varepsilon_0 = \dfrac{1}{36\pi}\times 10^{-9}$ F/m)

$\mu = \mu_r\mu_0$ ($\mu_0 = 4\pi\times 10^{-7}$ H/m)