Efficiency in the Housing Market with Search Frictions*

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August 26, 2022

Abstract

This paper studies efficiency in the housing market in the presence of search frictions and endogenous entry of buyers and sellers. These two features are essential to explain the housing market stylized facts and to generate an upward-sloping Beveridge Curve in the housing market. Search frictions and endogenous entry create two externalities in the market. First, there is a congestion externality common to markets with search frictions. Sellers do not internalize the effect of listing a house for sale on other sellers' probability of finding a buyer. Second, the endogenous entry of buyers leads to a participation externality, as new entrants in the market raise search costs for all buyers. The equilibrium is inefficient even when the Hosios-Mortensen-Pissarides condition holds. Using a calibration to the US housing market, we quantify the size of these externalities and how far the housing market is from the optimal allocation. We find that the optimal vacancy rate and time-to-sell are about half their equilibrium counterparts, whereas the optimal number of buyers and homeowners are above their decentralized equilibrium values. Given the importance of buyers' entry in the housing market, we also study efficiency in a model with heterogeneous buyers and an alternative entry mechanism. Quantitatively, the efficiency results are very similar to the benchmark model. Finally, we investigate how housing market policies can restore efficiency in the housing market.

JEL Classification: E2, E32, R21, R31.
Keywords: Housing market; Search and matching; Beveridge Curve; Housing liquidity.

*We are grateful to Elliot Anenberg, Richard Arnott, Jan Brueckner, Ed Coulson, Michael Devereux, Gilles Duranton, Jang-Ting Guo, Lu Han, Aaron Hedlund, Ioannis Kospentaris, Pablo Kurlat, Romain Rancière, Guillaume Rocheteau, Stephen Ross, Isaac Sorkin, Carl Walsh, Liang Wang and seminar participants at the 15th North-American Meeting of the Urban Economics Association, the Midwest Macroeconomics Conference Spring 2022, the University of Hawaii Manoa, the University of California Irvine, the University of California Santa Cruz and Virginia Commonwealth University for their helpful comments and suggestions.

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1 Introduction

Any model attempting to describe the housing market must include two essential features: search and matching frictions and entry of both buyers and sellers. Search frictions are necessary to capture that it takes time for buyers to find a suitable house and for sellers to find a buyer, i.e. that search is a costly and time-consuming process. The large fluctuations in time to sell, the co-movement between houses for sale and time-to-sell, and the large dispersion in prices even after controlling for housing observables are all clear indications of the presence of search frictions in the housing market. Less known is the fact that entry of both buyers and sellers is essential to account for the key stylized facts in the housing market: prices are positively correlated with sales and vacancies (i.e. houses for sale), but negatively correlated with time-to-sell. In other words, when house prices are high many houses are listed for sale, there is a high volume of sale transactions, and houses sell fast.[1] As Gabrovski and Ortego-Marti (2019) show, these stylized facts imply that the Beveridge Curve (i.e. the correlation between buyers and vacancies) in the housing market is upward sloping. Search models of the housing market without an endogenous entry of both buyers and sellers generate a counterfactual downward sloping Beveridge Curve and are unable to match the sign of the correlations between prices, sales, time-to-sell and vacancies (Gabrovski and Ortego-Marti, 2019).

These two essential characteristics of the housing market, search and matching frictions and the endogenous entry of buyers, lead to two externalities. The first externality encapsulates both congestion and market thickness externalities that arise in markets with search frictions (Hosios, 1990; Pissarides, 2000). When a seller posts a vacancy, she does not internalize that by doing so she is making it harder for other sellers to find a buyer (congestion externality) while making it easier for buyers to find a home (thick market externality). Absent the endogenous entry of buyers, the efficient allocation is restored if the Hosios-Mortensen-Pissarides (HMP) condition holds, i.e. if the bargaining power of buyers equals the elasticity of the matching rate. Under this condition, the congestion and thick market

[1] More specifically, from Diaz and Jerez (2013): (1) the elasticity of prices with respect to sales is 0.14; (2) the elasticity of prices with respect to time-to-sell is -0.12; (3) the elasticity of prices with respect to vacancies is 0.06. The vacancy quarterly data is from the Housing Vacancy Survey constructed by the US Census Bureau, from 1965:1 to 2010:4. Sales are measured using data from the National Association of Realtors (NAR) from 1968:1 to 2009:4. Time-to-sell data is collected from the American House Survey in the US Census Bureau from 1975:1 to 2010:4. See Diaz and Jerez (2013) for more details. The first two stylized facts (prices are positively correlated with sales but negatively correlated with time-to-sell) have been extensively documented by a number of papers—see Genesove and Mayer (1997, 2001), Glaeser and Gyourko (2006), Krainer (2001); Krainer *et al.* (2008), Ortalo-Magne and Rady (2006), Stein (1995) and the references therein. Ngai and Sheedy (2015) also find that vacancies and prices are positively correlated using vacancies data from the NAR. Using the Case-Shiller Index or FHFA repeat sales prices for house prices give similar correlations. See Gabrovski and Ortego-Marti (2019) for further details.

externalities fully offset each other. However, the equilibrium is not efficient even under the HMP condition because the endogenous entry of buyers leads to an additional participation externality.

This paper studies efficiency in the housing market in the presence of both congestion and participation externalities. In addition, using empirical evidence on the housing market we ask the following question: how far is the housing market from the optimal allocation? To answer these questions, we develop a search and matching model of the housing market with an endogenous entry of both buyers and sellers similar to Gabrovski and Ortego-Marti (2019), in which entry is driven by rising search costs as more buyers enter the market. This leads to an additional externality because buyers do not internalize that by entering the market they raise other buyers' costs.

We begin by characterizing the decentralized equilibrium. The equilibrium is determined by an entry condition for sellers, an entry condition for buyers and equilibrium prices from bargaining. We show that the entry mechanism leads to an upward sloping Beveridge Curve, consistent with the stylized facts in the housing market. We then study the social planner's problem and find the optimal allocation. The equilibrium is inefficient even if the HMP condition holds, because the planner lacks a tool with which to regulate the entry of buyers. Intuitively, in the decentralized economy households only evaluate if, given their utility of owning a home, it is worthwhile to enter the market. They do not, however, internalize their effect on other buyers' search costs. By contrast, the planner internalizes the effect that an additional buyer has on other buyers' costs.[2]

We calibrate the model to the U.S. economy to assess how far the decentralized equilibrium is from the efficient allocation. Given that the housing market is inefficient, a natural follow up question is what is the effect of housing market policies on the equilibrium, and whether they can restore efficiency. This exercise gives us an alternative method to gauge the size of the externalities. We consider four housing policies: (*i*) taxes on new construction; (*ii*) taxes on profits from housing sales; (*iii*) transfer fees to the buyer; (*iv*) property taxes. We study these policies because they affect the entry decision of buyers and sellers and how the trade surplus is split. We find that the optimal vacancy rate and time-to-sell are about half their values in the decentralized economy, and the optimal number of vacancies about 80% of its counterpart in the decentralized economy. By contrast, the optimal number of buyers and homeowners are both above the value in the decentralized economy. Intuitively, the planner finds it optimal to increase homeownership. However, she chooses

[2]In separate derivations not included in this paper (available upon request) we show that the equilibrium is also inefficient when search is directed. The intuition is similar, the planner lacks an additional tool with which to regulate the measure of buyers to its efficient level.

not to achieve this through a faster matching rate for buyers because new home construction is costly. Instead she decreases the number of available housing units for sale and instructs more buyers to enter the market. This ensures that homeownership rates are high and, at the same time, few houses are vacant. We then turn to whether policies can implement the planner's allocation. Since the model features two externalities, two policy tools are required to restore efficiency. We show that a combination of taxes on profits from housing sales and a subsidy to housing construction can implement the constrained-efficient allocation. To provide an alternative measure of the size of the two externalities, we calculate the quantitative magnitude of the policies required to restore efficiency.

Given the importance of entry in the housing market, we investigate efficiency in an alternative model in which participation is driven by households' heterogeneity in their utility of owning a home. In this environment, buyers who value housing the most are the ones who enter the market. The participation externality is due to a compositional effect: homeowners' average value of owning a home decreases as more buyers enter the market. Similarly to our benchmark case, the planner needs at least two policy tools to bring the decentralized allocation to the efficient level. Quantitatively, the model's predictions are strikingly close to those of our benchmark model. The socially optimal time-to-sell and vacancy rate are about half of the value in the decentralized economy, and the planner would like more buyers and homeownership in the market. One advantage of the framework with heterogeneity is that we can use data on house price dispersion to calibrate the entry mechanism. A major disadvantage, however, is that under a normal calibration the Beveridge Curve is downward sloping, so the model cannot account for the stylized facts in the housing market.

Related literature. Since the seminal work in Arnott (1989) and Wheaton (1990), the literature has extensively used search and matching models à la Diamond-Mortensen-Pissarides to study the housing market. This large literature includes Anenberg (2016), Arefeva (2020), Burnside *et al.* (2016), Diaz and Jerez (2013), Gabrovski and Ortego-Marti (2019, 2021a,b,c), Garriga and Hedlund (2020), Genesove and Han (2012), Guren (2018); Guren and McQuade (2018), Han *et al.* (2021), Head *et al.* (2014, 2016), Head *et al.* (2016), Hedlund (2016) Kotova and Zhang (2020), Krainer (2001), Ngai and Tenreyro (2014),Ngai and Sheedy (2020), Novy-Marx (2009), Piazzesi and Schneider (2009), Piazzesi *et al.* (2020) and Smith (2020). Compared to these papers, we study efficiency in the housing market in a framework with search frictions and and an endogenous entry of buyers and sellers mechanism that delivers the observed positive correlation between buyers and vacancies, i.e. an upward-sloping Beveridge Curve. In addition, we characterize the congestion and participation externalities in the housing market and quantify how far the observed equilibrium is from the optimal allocation.

To a lesser extent, our paper is also related to papers that study efficiency in labor markets with search frictions and compositional effects that arise from labor force participation. Papers in this literature include Albrecht *et al.* (2010), Griffy and Masters (2021), Julien and Mangin (2017) and Masters (2015). Although in search models with labor market participation the Beveridge Curve is downward-sloping and consistent with the labor market stylized facts, empirically the Beveridge Curve in the housing market is upward-sloping (Gabrovski and Ortego-Marti, 2019, 2021d). Relative to the above papers in the labor literature, we study efficiency in a framework with a different entry mechanism that is consistent with an upward-sloping Beveridge Curve in the housing market.[3]

2 The housing market with endogenous buyer entry

This section studies efficiency with an endogenous entry mechanism similar to Gabrovski and Ortego-Marti (2019) in which buyers' search costs increase as more buyers enter the market. We begin with a description of the decentralized economy. The main ingredients of the model are the following. First, there are search and matching frictions, which capture that it takes time for buyers to find a house and for sellers to find a buyer for their listed property. Second, there is free entry of both buyers and sellers. As Gabrovski and Ortego-Marti (2019) show, the double entry of buyers and sellers is crucial to match the stylized facts in the housing market, namely that prices are positively correlated with sales and vacancies (i.e. houses for sale) and negatively correlated with time-to-sell (TTS). Combined, these stylized facts imply that buyers and vacancies are positively correlated, i.e. the Beveridge Curve in the housing market is upward sloping. Finally, a key feature of the model is that prices are determined by bargaining.

Next, we derive the efficient allocation. The planner faces two externalities. First, search frictions give rise to the usual congestion and thick market externalities. When sellers list a house for sale, they do not internalize that by posting a house for sale they increase buyers chances of finding a home (thick market externality). At the same time, they also make it more difficult for other sellers to find a buyer (congestion externality). To simplify the exposition, we refer to both thick market and congestion externalities as *congestion externalities*. The second externality arises because buyers do not internalize that by entering the market they raise search costs for other buyers. We denote this externality a *participation externality*. As we show in this section, because of this additional externality, the HMP

[3]Labor force participation models can be viewed as similar to our framework with heterogeneity. In section 4 we characterize the conditions under which the observed positive correlation between buyers and vacancies holds in the environment with heterogeneity and show that quantitatively the model generates a counterfactual downward-sloping Beveridge Curve.

condition does not restore efficiency. Intuitively, the social planner needs an additional tool to first fix the entry of buyers to its efficient level.

2.1 Environment

Time is continuous. Agents are risk-neutral, infinitely lived and discount the future at a rate r. There are three types of agents: households, developers and realtors. Households either own a home, search for a house (i.e. they are buyers), or choose not to participate in the market. Developers may enter the housing market and build a new home at a cost k if not enough existing houses are listed for sale by households through separations. Upon building a house, developers post a vacancy and search for buyers. Houses are identical, regardless of whether they are old or newly built. To capture depreciation in a tractable way, houses are destroyed at an exogenous rate δ.

It takes time for buyers to find a house and for sellers to sell their home. We capture these search frictions in the housing market by assuming a matching function $M(b, v)$, where b is the measure of buyers and v the measure of vacancies or houses for sale—we use both terms interchangeably. The matching function satisfies the usual properties: it is increasing in each of its terms, concave and displays constant returns to scale. Let $\theta \equiv b/v$ denote the housing market tightness. The matching function implies that buyers find homes at a rate $m(\theta) \equiv M(b, v)/b = M(1, \theta^{-1})$ and that sellers find buyers at a rate $\theta m(\theta) = M(b, v)/v = M(\theta, 1)$. As market tightness increases, the home-finding rate for buyers $m(\theta)$ decreases and the finding rate for sellers $\theta m(\theta)$ increases. Intuitively, an increase in market tightness implies that vacancies are relatively more scarce, so it becomes harder for buyers to find a house, but easier for a seller to find a buyer. In addition to these flows, some homeowners become separated from their house at an exogenous rate s. This separation shock captures that the household may need to relocate because of their job, they need to move to a different type of home or area, and so on. Once a separation shock occurs, households list their house for sale.

There is free entry of both buyers and sellers. Given free entry of buyers, households keep entering the market until the value of becoming a buyer equals zero, the value of their outside option. Similarly, free entry of sellers implies that sellers enter the market until the value of a vacancy equals the construction cost k, regardless of whether the house is newly built or existing (all houses are identical). As is common in markets with search frictions, there are rents from matching. We assume that prices are determined by Nash Bargaining (Nash, 1950; Rubinstein, 1982), where β denotes the seller's bargaining strength. Once a buyer and a seller are matched, the buyer pays the price p to the seller and begins enjoying

a utility flow ε from owning a house.

Assume that households must secure the services of a realtor to start searching for a house. Searching for houses is costly for the realtor. Her cost of servicing b buyers is given by $\bar{c}b^{\gamma+1}/(\gamma+1)$, which is consistent with many findings in the real estate literature (Sirmans and Turnbull, 1997). In exchange for her services, the realtor charges a fee c^B, so her revenue is bc^B. Profit maximization implies that the fee is given by $c^B(b) = \bar{c}b^{\gamma}$, which is increasing in the number of buyers. Intuitively, $c^B(b)$ captures search costs such as arranging and scheduling viewings, driving to view houses or locating properties that match buyers' preferences. In reality, buyers incur some of these costs themselves, but for simplicity of the exposition we assume that the realtor bears all the costs and then charges a fee $c^B(b)$. Assuming instead that buyers incur all costs themselves, and that costs are increasing in the number of buyers due to congestion, gives the exact same results. As Gabrovski and Ortego-Marti (2019) show, this endogenous entry mechanism generates an upward-sloping Beveridge Curve, and accounts for the housing market stylized facts qualitatively and quantitatively. Finally, sellers incur search flow cost c^S until they find a buyer.

Similar to many papers in the literature, we do not model the rental market and treat it as a separate market, see for example Diaz and Jerez (2013) and Ngai and Tenreyro (2014), among others. This assumption is well supported empirically by Glaeser and Gyourko (2007), who find that rental and owner occupied homes have very different characteristics, and that there is no arbitrage between both types of homes. In addition, Bachmann and Cooper (2014) find that most flows are within each rental/owner category. In addition, flows from the owner to rental segment are acyclical, and turnover is unrelated to vacancies in the rental market. Overall, the empirical evidence supports the view that the rental market can be treated as a separate market.

2.2 Decentralized equilibrium

Let H, B and V denote the value functions of a homeowner, a buyer and a vacancy. They satisfy the Bellman equations

$$rB = \max\{0, -c^B(b) + m(\theta)(H - B - p))\}, \tag{1}$$

$$rH = \varepsilon - s(H - V - B) - \delta(H - B). \tag{2}$$

Intuitively, (1) captures that buyers can choose whether to enter the housing market. When they search for a house, they incur flow costs $c^B(b)$. At a rate $m(\theta)$, buyers find a house, pay the house price p and become a homeowner, which carries a net gain $H - B - p$. Equation (2) shows that a homeowner derives a utility ε from owning a house. At a rate s a separation

shock occurs, and the household lists her house for sale, which has a value V, and decides whether to become a buyer. At a rate δ the house is destroyed, which implies a net loss of $H - B$. Similarly, the seller's Bellman equation is given by

$$rV = -c^S + \theta m(\theta)(p - V) - \delta V. \tag{3}$$

Sellers incur flow costs c^S while they search. At a rate $\theta m(\theta)$, they meet a buyer and enjoy net capital gains $p - V$. The vacancy is also subject to a destruction shock, which happens at a rate δ.

Because of the frictional nature of the housing market, a match between a buyer and seller generates a positive surplus. As is common in many search models, we assume that the two parties split this surplus according to Nash Bargaining. Let S^B and S^S denote the surplus of the buyer and seller, and let $S \equiv S^B + S^S$ denote the total surplus from matching. Prices solve the following Nash Bargaining problem

$$p = \arg\max_p \left(S^B\right)^{1-\beta} \left(S^S\right)^{\beta}. \tag{4}$$

The first order condition to the above problem gives the Nash sharing rule

$$(1 - \beta)S^S = \beta S^B. \tag{5}$$

In particular, the above sharing rule implies that the buyer extracts a fraction $1 - \beta$ of the surplus and the seller a fraction β, i.e. $S^B = (1 - \beta)S$ and $S^S = \beta S$.

Free entry implies that sellers enter the market until the value of a vacancy covers the construction costs k. On the buyer's side, households participate in the market until the value of being a buyer B equals the value of their outside option, which we normalize to 0. Therefore,

$$V = k, \tag{6}$$

$$B = 0. \tag{7}$$

Intuitively, as we shall see later on, free entry of sellers pins down the market tightness. Given the equilibrium market tightness, free entry of buyers pins down the measure of buyers.

Combining the Bellman equations for the buyer and seller (1) and (2) with the free entry

condition for sellers (6) implies

$$S^B = \frac{\varepsilon + sk}{r + s + \delta} - p, \tag{8}$$

$$S^S = p - k. \tag{9}$$

Combining the above surpluses with the Nash bargaining rule (5) gives the equilibrium price (PP) condition

$$\text{(PP):} \qquad p = k + \beta \left(\frac{\varepsilon + sk}{r + s + \delta} - k \right). \tag{10}$$

Intuitively, due to Nash Bargaining sellers are compensated for their outside option, k, and receive a share β of the surplus.

Free entry of sellers, together with the Bellman equation for a vacancy (3) gives the Housing Entry (HE) condition

$$\text{(HE):} \qquad \frac{(r + \delta)k + c^S}{\theta m(\theta)} = p - k. \tag{11}$$

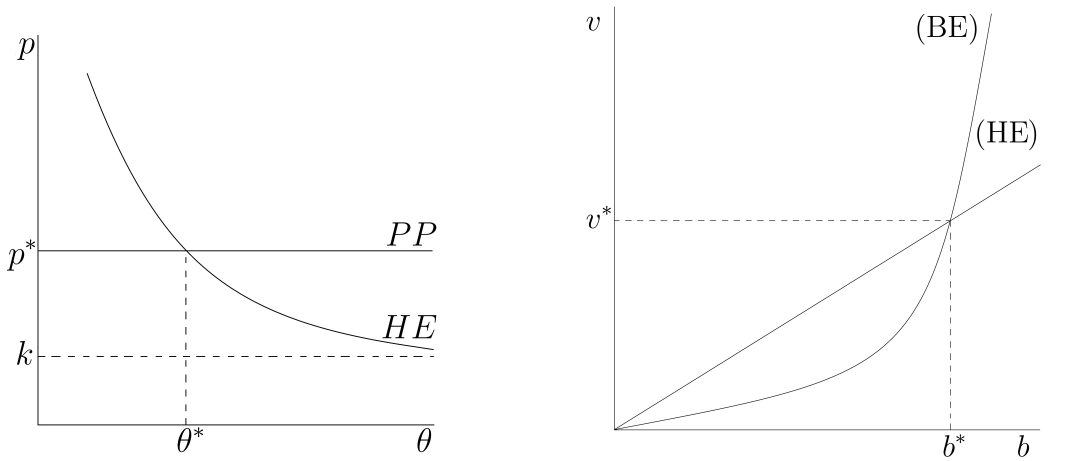
The left hand-side of the above equation captures the seller's expected cost from searching for a buyer: the search cost c^S and the user cost $(r + \delta)k$ for the expected duration of the vacancy $1/(\theta m(\theta))$. The right-hand side corresponds to the seller's surplus. The HE condition shows that developers keep entering the market until the profits from selling the house are just enough to cover the expected cost of finding a buyer.

Free entry of buyers gives the Buyer's Entry (BE) condition

$$\text{(BE):} \qquad \frac{c^B(b)}{m(\theta)} = (1 - \beta) \left(\frac{\varepsilon + (r + \delta)k}{r + \delta + s} \right). \tag{12}$$

The above equation captures that buyers keep entering the market until the marginal buyer's expected cost of finding a home (the left-hand side of (12)) equals the buyer's surplus of being a homeowner, which equals the present discounted value of the return ε net of the user cost $(r + \delta)k$, using the effective discount is $r + s + \delta$.

The PP and HE conditions determine the equilibrium market tightness θ. Given the equilibrium θ, the BE condition yields the equilibrium measure of buyers b. It is straightforward to verify that the equilibrium exists and is unique. This environment yields an upward-sloping Beveridge Curve, which corresponds to the BE curve. Intuitively, as more sellers enter the market, buyers find it more desirable to enter as well because they can find houses more quickly. Hence, buyers and vacancies are positively correlated and the BE curve



(a) Equilibrium price p^* and tightness θ^*

(b) Equilibrium buyers b^* and vacancies v^*

Figure 1a depicts the house entry condition HE from (11) and the price relationship PP from (34), which determine the equilibrium price p^* and market tightness θ^*. From the HE condition, an increase in house prices makes a vacancy more profitable, which lowers market tightness due to the larger entry of sellers. The PP condition is determined by Nash Bargaining between the buyer and the seller. In figure 1b, the HE condition corresponds to a straight line through the origin with a slope given equal to the equilibrium market tightness θ^* from figure 1a. The BE curve depicts the buyers entry condition (37), which captures that an increase in the ratio of vacancies to buyers makes it easier for buyers to find a house and leads to an increase in the entry of buyers. The BE curve corresponds to the Beveridge Curve in the housing market and is upward-sloping.

Figure 1: **Equilibrium in the housing market**

is upward-sloping.[4]

The equilibrium is depicted graphically in figures 1a and 1b. Figure 1a depicts the HE condition and the price relationship PP given by (11) and (10). The HE condition captures that as house prices increase it becomes more profitable to post a vacancy. This leads to more entry of sellers and a lower market tightness. The PP condition comes from Nash Bargaining between buyers and sellers. The equilibrium price p^* and market tightness θ^* are given by the intersection of both curves. The equilibrium market tightness θ^* in figure 1a describes a straight line in the vacancies-buyers space in figure 1b, which we denote as the HE condition. The BE curve in figure 1b corresponds to the BE condition (12). The BE curve captures that as the ratio of vacancies to buyers increases buyers find houses faster, which leads to an increase in the entry of buyers. The BE curve corresponds to the Beveridge Curve in the housing market and is upward-sloping.

[4] Gabrovski and Ortego-Marti (2019) show that a similar environment in discrete time and with business cycle fluctuations can account for the housing market stylized facts qualitatively and quantitatively.

2.3 The social planner's allocation

The social planner faces two externalities. In addition to the usual *congestion* externality in markets with search frictions, the decentralized equilibrium is inefficient because buyers do not internalize the effect their entry has on other buyers' search costs. We refer to this additional externality as the *participation* externality. Let $\tilde{h}$ and c denote the number of homeowners and construction (the number of newly built houses). The planner's problem is given by

$$\max_{\theta,c} \int_0^\infty e^{-rt} \left[x\tilde{h} - v\theta c^B(v\theta) - vc^S - ck \right] dt, \tag{13}$$

subject to

$$\dot{v} = c + sh - \delta v - v\theta m(\theta), \tag{14}$$

$$\dot{\tilde{h}} = v\theta m(\theta) - (s+\delta)h. \tag{15}$$

Setting up the Hamiltonian and using the first order conditions gives the following solution for the planner' allocation at steady state

$$\frac{(r+\delta)k + c^S}{\theta m(\theta)} = \alpha \left(\frac{\varepsilon + sk}{r+s+\delta} - k \right), \tag{16}$$

$$\frac{c^B(b)}{m(\theta)} = \left(\frac{1-\alpha}{1+\gamma} \right) \left(\frac{\varepsilon + sk}{r+s+\delta} - k \right), \tag{17}$$

where $\alpha \equiv -m'(\theta)\theta/m(\theta)$ denotes the elasticity of the matching rate. The appendix provides the derivations and shows that the above conditions are necessary and sufficient. Equations (16) and (17) are the counterpart of the equilibrium conditions (11) and (12) in the decentralized economy. Comparing the decentralized allocation with the optimal one shows that the HMP condition does not restore efficiency. More specifically, restoring efficiency in the decentralized equilibrium requires

$$\beta = \alpha \quad \text{and} \quad 1 - \beta = \frac{1-\alpha}{1+\gamma}. \tag{18}$$

The first condition $\beta = \alpha$ corresponds to the standard HMP condition. If HMP holds, the decentralized equilibrium is efficient if and only if $\gamma = 0$, i.e. there is no entry of buyers. As soon as there is free entry of buyers, the equilibrium is inefficient because the endogenous

entry of buyers generates a participation externality.[5] Intuitively, when there is no buyer entry the only externality is the congestion externality, which can be internalized if the two parties share the surplus in the appropriate fractions, i.e. the seller receives a share α and the buyer receives the rest. With endogenous entry of buyers, however, the participation externality manifests itself through increased search costs for buyers. As a result, the planner finds it optimal to disincentivize entry by giving buyers only a share $(1-\alpha)/(1+\gamma)$ of the surplus and leaving the remaining $\gamma(1-\alpha)/(1+\gamma)$ fraction of the surplus "on the table". Thus, at least two housing market policies are required to eliminate the two externalities and restore efficiency: one policy to set the fraction of surplus that is left "on the table" (i.e. eliminate the participation externality), and one to set the sharing rule for the remainder of the surplus (i.e. eliminate the congestion externality).

3 Quantifying inefficiency in the housing market

In this section we quantify the inefficiency in the housing market implied by the congestion and participation externalities. In other words, we ask: how far is the decentralized equilibrium from the efficient allocation? To answer this question, we calibrate a version of our model which includes housing market policies. We do this for two reasons. First, we are interested in what policies would allow the planner to implement the socially efficient allocation in the decentralized economy. Second, we calibrate the model to the U.S economy, which does feature housing market policies. Upon calibrating the model to the U.S. housing market, we compare the decentralized equilibrium allocation to the planner's efficient allocation. In particular, we focus on three measures of housing market liquidity: time-to-sell, the vacancy rate, and the listing rate. In a second exercise, we examine the housing policies required to restore efficiency.

3.1 The model with housing policies

We consider four policies. First, homeowners pay property taxes τ_p on their home. Second, buyers pay taxes τ_b when they purchase a house. On the seller side, sellers pay a tax τ_s upon selling the house, which we assume applies to the capital gains $p - V$. Finally, construction may be taxed or subsidized at a rate τ_k, where $\tau_k > 0$ corresponds to a tax on construction

[5] A similar result holds when search is directed, efficiency in the decentralized equilibrium is only restored if there is no entry of buyers. A proof for the environment with directed search is available upon request.

and $\tau_k < 0$ to a subsidy. The Bellman equations are given by

$$rB = \max\{0, -c^B(b) + m(\theta)\left[H - B - (1+\tau_b)p\right]\}, \tag{19}$$

$$rH = \varepsilon - \tau_p p - s\left(H - V\right) - \delta H, \tag{20}$$

$$rV = -c^S + \theta m(\theta)(1-\tau_s)\left(p - V\right) - \delta V. \tag{21}$$

The intuition for the above Bellman equations is similar to the intuition for (1)-(3). House prices are determined in a similar way by Nash Bargaining, except that the surplus S is now given by $S = H - B - (1+\tau_b)p + (1-\tau_s)\left(p - V\right)$, where $S^B = H - B - (1+\tau_b)p$ is the buyer surplus and $S^S = (1-\tau_s)\left(p - V\right)$ is the seller's surplus. Nash Bargaining implies the following first order conditions: $S^S = \tilde{\beta} S$ and $S^B = (1-\tilde{\beta})S$, where $\tilde{\beta} \equiv \beta(1-\tau_s)/[\beta(1-\tau_s) + (1-\beta)(1+\tilde{\tau}_b)]$ and $\tilde{\tau}_b \equiv \tau_b + \tau_p/(r+s+\delta)$.[6]

Following the same steps as in section 2.2, we find the following equilibrium conditions

$$p = \frac{\tilde{\beta}}{\tilde{\beta}(1+\tilde{\tau}_b) + (1-\tilde{\beta})(1-\tau_s)}\left[\frac{\varepsilon - (r+\delta)(1+\tau_k)k}{r+s+\delta} - \tilde{\tau}_b(1+\tau_k)k\right] + (1+\tau_k)k, \tag{22}$$

$$\frac{(r+\delta)(1+\tau_k)k + c^S}{\theta m(\theta)} = \tilde{\beta}\left[1 + (1-\beta)\tilde{\tau}_b - \beta\tau_s\right]\left[\frac{\varepsilon + s(1+\tau_k)k}{(r+s+\delta)(1+\tilde{\tau}_b)} - (1+\tau_k)k\right], \tag{23}$$

$$\frac{c^B(b)}{m(\theta)} = (1-\tilde{\beta})\left[1 + (1-\beta)\tilde{\tau}_b - \beta\tau_s\right]\left[\frac{\varepsilon + s(1+\tau_k)k}{(r+s+\delta)(1+\tilde{\tau}_b)} - (1+\tau_k)k\right], \tag{24}$$

which correspond to the PP, HE and BE conditions without policies (10), (11) and (12). We use these conditions to calibrate the model and derive the optimal policies in our numerical exercise below.[7]

3.2 Calibration

Following Gabrovski and Ortego-Marti (2021a), let $r = 0.012$ to match an annual discount factor of 0.953. We set $\delta = 0.004$ to match an annual housing depreciation rate of 1.6% (Van Nieuwerburgh and Weill, 2010). The matching function is assumed to be Cobb-Douglas with $m(\theta) = \mu\theta^{-\alpha}$. The elasticity of the matching function α is equal to 0.16, following the empirical evidence in Genesove and Han (2012). The flow utility of owning a house ε

[6] This is a notable difference compared to the equilibrium without policies. Intuitively, $\tilde{\beta}$ is the effective bargaining weight. Consider for example property taxes. If the buyer pays a higher price, part of the surplus is lost because the homeowner pays higher property taxes. The Nash Bargaining solution takes into account the effect of a change in the price on the surplus, given the housing market policies. The effective weights in the solution capture the effect of the tax rates on the surplus. A similar result emerges with labor market policies (Lagos, 2006; Ortego-Marti, 2020; Pissarides, 2000).

[7] Specifically, we plug in for the socially optimal b and θ from the planner's solution and then solve for the set of τ_k, τ_s, and $\tilde{\tau}_b$ implied by equations (23) and (24).

is normalized to 10.883. This implies a steady state price of 491.2, which is the average empirically observed price (in thousands of dollars) in Kotova and Zhang (2020). We set a profit tax rate for sellers of 3.65%, which corresponds to the average effective tax rate for the Real Estate Developer firms for the years 2014-2019 as reported by Aswath Damodaran.[8] The Tax Foundation in its "Facts and Figures 2018" reports an effective annual property tax rate for the United States of 1.13%. The median real estate transfer tax rate reported by the National Association of Realtors is 0.16%. Following Gabrovski and Ortego-Marti (2019), we assume that $c^B(b) = \bar{c}b^\gamma$. We normalize $b = 1$, which allows us to back out $\bar{c} = 22.327$.

To obtain the rest of the model parameters we target six moments from the data. The time-to-sell is set to 1.76 quarters which is the average of the Median Number of Months on Sales Market reported by the U.S. Bureau of Census for the period of 1987:1–2017:4. We also set the time-to-buy to be equal to the time to sell following Gabrovski and Ortego-Marti (2019), Gabrovski and Ortego-Marti (2021a) and the evidence in Genesove and Han (2012). These two targets yield $\mu = 0.5682$, $k = 451.98$. Ngai and Sheedy (2020) calculate a listing rate, given by the number of new listings on the market divided by the stock of owner-occupied houses not already for sale, equal to 1.667%. This target implies a separation rate $s = 0.0126$. To calibrate c^S, we target expected costs for the seller equal to 8% of the average house price, following Ghent (2012). Following the same reference we also target an average cost for the buyer of 5.1%. These two moments yield $c^S = 14.233$ and $\beta = 0.5234$. Lastly, to calibrate γ we use monthly data on vacancies and sales from the New Residential Sales Release reported by the U.S. Bureau of Census for the period January 1963–December 2019.[9] Given that in our model sales$= bm(\theta)$, the information on sales and vacancies allows us to back out a series for the number of buyers using $b = v[\text{sales}/(\mu v)]^{1/(1-\alpha)}$. Next, we regress the cyclical component of the series for buyers on the cyclical component of the series for vacancies and a constant to arrive at an elasticity of 0.19.[10] Plugging this estimate into equation (14) in Gabrovski and Ortego-Marti (2019) results in $\gamma = 0.68$. Table 1 summarizes the calibration.

3.3 Results

Our calibrated model matches the housing market data well. In particular, vacancy and construction rates, which are not targeted in the calibration, are close to their empirical counterparts. These moments are of particular relevance in our context: the vacancy rate

[8]The data can be found at http://people.stern.nyu.edu/adamodar/New_Home_Page/dataarchived.html. We exclude 2018 for which there is no data on taxes paid.

[9]The series we use for vacancies is Houses for Sale and the series for sales is Houses Sold.

[10]We obtain the cyclical components of the series by first taking natural logs and then detrending using an HP filter with a smoothing parameter of 129,600. The standard error of the estimated coefficient is 0.08.

captures liquidity in the housing market, whereas the construction rate captures the importance of the entry channel, which is at the core of our paper. In the model, the vacancy rate is measured as the ratio of vacancies to the total stock of homes and equals 2.83% in the steady state. In the data, the vacancy rate is 1.9% and is measured as the average of the Homeowner Vacancy Rate reported by the United States Census Bureau for the period 1987:1—2017:4. The model also does a relatively good job at matching the construction rate. In the model, this construction rate is measured by the number of homes built divided by the total housing stock, and is equal to 0.4%. In the data, the average of this ratio for the period 1987:1—2017:4 is 0.27%.[11]

We gauge the distance of the decentralized equilibrium to the planner's social optimum by focusing on three important measures of liquidity in the housing market: the vacancy rate, the listing rate, and the time-to-sell. The values of these liquidity measures at the steady state equilibrium and at the planner's optimum are summarized in Table 2. In general, the planner finds it optimal to reduce the relative amount of vacancies in the market and to have houses match with buyers faster. Intuitively, this is due to the congestion externality. In equilibrium sellers extract too much of the surplus which induces an inefficient over-entry into the market. This leads to expected search and construction costs for sellers that are too high from a socially efficient perspective.

Finally, we quantify the size of the housing policies required to restore efficient. Table 3 reports the optimal tax rates implied by the model and the dollar amount of these policies, in thousands of dollars. The efficient allocation requires fewer houses for sale and more buyers than the decentralized equilibrium. This can be achieved through a combination of taxes on sellers and a subsidy to housing construction.[12] From the HE condition (23), a lower τ_k and a higher τ_s both raise the market tightness, which lowers time-to-sell. From the BE condition, the combination of a lower market tightness, a lower τ_k and a higher τ_s raises the measure of buyers. Quantitatively, raising the profit tax from 3.65% to 69.8% and subsidizing construction at a rate 17.31% brings the housing market to its efficient level. In dollar amounts, these policies correspond to \$44,540 and \$78,250 respectively. To put the dollar amount in perspective, our calibration is such that house prices are $\$491,000$ on average, as in the data.

[11] The data on new construction is taken from the New Privately-Owned Housing Units Completed series by U.S. Census Bureau and U.S. Department of Housing and Urban Development and the data on the stock of homes is from the All Housing Units series reported in the Housing Vacancy Survey of the U.S. Census Bureau.

[12] An examination of equations (24) and (23) reveals that a tax/subsidy on construction is necessary to implement the efficient allocation. The other policy tool the planner can use is either τ_s or $\tilde{\tau}_b$. We focus on the former as this delivers a cleaner comparison with the results from section 4.6.

4 Efficiency with heterogeneity and endogenous participation

In this section we study efficiency in an environment with an alternative entry mechanism. The main departure relative to the framework in section 2 is our assumption that workers are heterogenous in how much they value owning a house. The advantage with this mechanism is that we can use data on price dispersion to calibrate the distribution of households' heterogeneity. The main disadvantage, however, is that the framework with heterogeneity generates a counterfactual Beveridge Curve, as we prove below. Despite this shortcoming, the results on efficiency remain remarkably similar to the environment with rising search costs.

4.1 Environment

The environment maintains the same assumptions as in the model in section 2, except for the following. Households are heterogenous in the utility they derive from owning a home. Let ε denote the household's individual utility from owning a home, and let x denote the utility from homeownership that is common to all households. Overall, households derive a utility εx from owning a home. The parameter x allows us to capture demand shocks, but it is not essential for the study of efficiency. The utility ε is a permanent household characteristic and captures households' heterogeneity in how they value owning a home, i.e. a household with individual utility ε derives a utility εx from all houses. Assume that ε follows a known distribution $F(\varepsilon)$. Relative to the model of section 2, buyers now incur a flow of constant costs c^B while searching for a house.

Given free entry, households keep entering the market until the value of becoming a buyer equals zero, the value of their outside option. Intuitively, this entry condition for buyers pins down the utility of the marginal buyer and determines the endogenous measure of buyers. Similarly, free entry of sellers implies that sellers enter the market until the value of a vacancy equals the construction cost k, regardless of whether the house is newly built or existing (all houses are identical). Once a buyer and a seller are matched, the buyer pays the price $p(\varepsilon)$ to the seller. We maintain our assumption that prices are determined by Nash Bargaining. As we show below, the price depends on the size of the surplus and, therefore, the household's utility from owning a home ε.

4.2 Equilibrium

Let $B(\varepsilon)$ and $H(\varepsilon)$ denote the value functions of a buyer and homeowner with utility ε, and V denote the value function of a vacancy. The value functions satisfy the following Bellman equations

$$rB(\varepsilon) = \max\{0, -c^B + m(\theta)[H(\varepsilon) - B(\varepsilon) - p(\varepsilon)]\}, \tag{25}$$

$$rH(\varepsilon) = \varepsilon x - s[H(\varepsilon) - V - B(\varepsilon)] - \delta[H(\varepsilon) - B(\varepsilon)]. \tag{26}$$

Intuitively, (25) captures that buyers can choose whether to enter the housing market. When they search for a house, they incur flow costs c^B. At a rate $m(\theta)$, buyers find a house, pay the house price $p(\varepsilon)$ and become a homeowner, which carries a net gain $H(\varepsilon) - B(\varepsilon) - p(\varepsilon)$. Equation (26) captures that a homeowner derives a utility εx from owning a house, which depends on her individual utility ε of owning a house. At a rate s a separation shock occurs, and the household lists her house for sale, which has a value V, and decides whether to become a buyer. At a rate δ the house is destroyed, which implies a net loss of $H(\varepsilon) - B(\varepsilon)$. The Bellman equation for a seller listing a house is

$$rV = -c^s + \theta m(\theta) \int_{\varepsilon_R}^{\infty} (p(\varepsilon) - V) \frac{dF(\varepsilon)}{1 - F(\varepsilon_R)} - \delta V. \tag{27}$$

Sellers incur flow costs c^S while they search for buyers. At a rate $\theta m(\theta)$, they meet a buyer from the distribution $F(\varepsilon)/(1 - F(\varepsilon_R))$, where ε_R is the reservation utility. The vacancy is also subject to a destruction shock, which happens at a rate δ.

The buyer and the seller split the surplus from matching according to Nash Bargaining. Let $S^B(\varepsilon)$ and $S^S(\varepsilon)$ denote the surplus of the buyer and seller when the buyer's valuation of homeownership is ε, and let $S(\varepsilon) \equiv S^B(\varepsilon) + S^S(\varepsilon)$ denote the total surplus from matching. Prices solve the following Nash Bargaining problem

$$p(\varepsilon) = \arg\max_{p(\varepsilon)} \left(S^B(\varepsilon)\right)^{1-\beta} \left(S^S(\varepsilon)\right)^{\beta}, \text{ for all } \varepsilon. \tag{28}$$

The first order condition to the above problem gives the Nash sharing rule

$$(1 - \beta) S^S(\varepsilon) = \beta S^B(\varepsilon), \text{ for all } \varepsilon. \tag{29}$$

In particular, the above sharing rule implies that the buyer extracts a fraction β of the surplus and the seller a fraction $1 - \beta$, i.e. $S^B(\varepsilon) = (1 - \beta)S(\varepsilon)$ and $S^S(\varepsilon) = \beta S(\varepsilon)$.

Free entry of sellers implies that sellers enter the market until the value of a vacancy

covers the construction costs k. On the buyer's side, households participate in the market as long as the value of being a buyer $B(\varepsilon)$ is greater than zero. Given that $B(\varepsilon)$ is strictly increasing in the utility ε, free entry of buyers implies that there exists a unique reservation utility ε_R that satisfies $B(\varepsilon_R) = 0$. Buyers with a utility $\varepsilon \geq \varepsilon_R$ participate in the market. In sum, free entry of buyers and sellers imply

$$V = k, \tag{30}$$

$$B(\varepsilon_R) = 0. \tag{31}$$

Intuitively, given a utility of the marginal buyer ε_R, free entry of sellers pins down the market tightness. Given the equilibrium market tightness, free entry of buyers pins down the marginal buyer ε_R and, therefore, the measure of buyers.

Combining the Bellman equations for the buyer and seller (25)—(27) with free entry for sellers $V = k$ implies, for all $\varepsilon \geq \varepsilon_R$

$$S^B(\varepsilon) = \frac{\varepsilon x + c^B + sk + \beta m(\theta)k}{r + s + \delta + \beta m(\theta)} - p(\varepsilon), \tag{32}$$

$$S^S(\varepsilon) = p(\varepsilon) - k. \tag{33}$$

Combining the above surpluses with the Nash bargaining rule (29) gives the equilibrium price (PP) condition

$$\text{(PP):} \qquad p(\varepsilon) = k + \beta \left(\frac{\varepsilon x + c^B + sk + \beta m(\theta)k}{r + s + \delta + \beta m(\theta)} - k \right), \tag{34}$$

i.e. sellers are compensated for their outside option k and receive a share β of the surplus.

Free entry of sellers together with the Bellman equation for a vacancy (27) gives the Housing Entry (HE) condition

$$\text{(HE):} \qquad \frac{(r + \delta)k + c^S}{\theta m(\theta)} = \bar{p} - k, \tag{35}$$

where $\bar{p} \equiv \int_{\varepsilon_R}^{\infty} p(\varepsilon) dF(\varepsilon)/(1 - F(\varepsilon_R))$ is the observed average price. Intuitively, the left hand-side captures the expected costs from finding a buyer, which include the search costs c^S and the user cost $(r + \delta)k$ for the expected duration of the vacancy $1/(\theta m(\theta))$. The right-hand side corresponds to the expected seller's surplus, after taking into account the distribution of buyers' valuation in the market. The HE condition captures that sellers keep entering the market until the expected cost of finding a buyer equals the expected surplus from selling

the house. Integrating prices in (34) and substituting into (35) gives

$$\text{(HE):} \qquad \frac{(r+\delta)k + c^S}{\theta m(\theta)} = (1-\beta)\left[\frac{\bar{\varepsilon}x + c^B - (r+\delta)k}{r+s+\delta+\beta m(\theta)}\right] \tag{36}$$

where $\bar{\varepsilon} \equiv E(\varepsilon|\varepsilon \geq \varepsilon_R) = \int_{\varepsilon_R}^{\infty} \varepsilon dF(\varepsilon)/(1-F(\varepsilon_R))$.

Free entry for buyers yields the Buyer's Entry (BE) condition

$$\text{(BE):} \qquad \frac{c^B}{m(\theta)} = (1-\beta)\left[\frac{\varepsilon_R x - (r+\delta)k}{r+s+\delta}\right]. \tag{37}$$

Intuitively, the above equation captures that buyers keep entering the market until the marginal buyer's expected cost of finding a home (the left-hand side of (37)) equals the buyer's surplus of being a homeowner, which equals the present discounted value of the return ε_R net of the user cost $(r+\delta)k$, using the effective discount is $r+s+\delta$.

4.3 Entry of buyers and the Beveridge Curve

The BE curve defines a relationship between the measure of buyers and vacancies, and corresponds to the Beveridge Curve in the housing market. In this section we show that a main disadvantage of a model with heterogeneity in utility flows and participation is that it generates a downward-sloping Beveridge Curve. The reason is the following. The slope of the BE curve depends on two opposite effects. On the one hand, more vacancies imply a lower market tightness, which makes it easier for buyers to find a house and induces entry of buyers. On the other hand, as buyers find homes more quickly the stock of buyers depletes. This is the usual mechanism in search models of housing without buyer entry, and leads to a counterfactual downward-sloping Beveridge Curve. Whether the BE Curve is upward-sloping depends on which effect dominates. It turns out that, given a standard calibration, the second effect dominates and the BE curve is downward-sloping, as in search models of the labor market with labor force participation.

To see this more clearly, let h denote the homeownership rate, i.e. the fraction of households participating in the market who own a home. The number of buyers is then given by

$$b = N(1-F(\varepsilon_R))(1-h), \tag{38}$$

where N is the large measure of potential buyers and is constant. Increasing vacancies lowers market tightness θ, which lowers the utility of the marginal buyer ε_R, i.e. there is more entry of buyers. This effect leads to a positive relationship between buyers b and vacancies v.

At the same time, however, a lower θ lowers the fraction of market participants who are buyers $1-h=(s+\delta)/(m(\theta)+s+\delta)$, since they find homes more quickly. This effect generates a negative relationship between buyers and vacancies. Whether the BE curve describes a positive of negative relationship between buyers and vacancies depends on which effect dominates. Log-differentiating the above expressions and totally differentiating the BE condition gives the following elasticity $\epsilon_{b,v} \equiv (db/dv)\cdot(v/b)$ of buyers with respect to vacancies

$$\epsilon_{b,v} = -\frac{\Delta(\theta,\varepsilon_R)}{1-\Delta(\theta,\varepsilon_R)}, \tag{39}$$

where $\Delta(\theta,\varepsilon_R)$ is given by

$$\Delta(\theta,\varepsilon_R) = \alpha\left[-\frac{f(\varepsilon_R)\varepsilon_R}{1-F(\varepsilon_R)}\frac{\frac{c^B}{m(\theta)}}{\frac{c^B}{m(\theta)}+\frac{(r+\delta)k}{r+s+\delta}}+\frac{m(\theta)}{m(\theta)+s+\delta}\right], \tag{40}$$

and $f(\varepsilon)$ denotes the pdf of the distribution $F(\varepsilon)$. Using the BE condition (37) to substitute $\varepsilon_R \equiv \varepsilon_R(\theta)$ gives an expression that depends only on θ and parameters. Given a standard calibration $\Delta(\theta,\varepsilon_R(\theta)) \in (0,1)$, which implies a downward-sloping BE curve.

4.4 The social planner's allocation

Similar to the benchmark model with homogenous buyers, the social planner faces two externalities. With heterogenous buyers, however, the participation externality is driven by compositional effects. The marginal households participating in the market are the households who value housing the least. As a result, the average utility of homeownership declines as more buyers enter the market. This leads to a participation externality because buyers do not internalize how their participation affects the distribution of match surpluses. When the planner internalizes both externalities, the HMP condition is again insufficient to restore the efficient allocation. Intuitively, this condition controls for the congestion externality, but again an additional policy is required to restore entry of buyers to the efficient level.

As before c denotes new construction, $\tilde{h}$ denotes the number of homeowners and N is a large measure of the population. We express buyers as a function of the total measure of homeowners $\tilde{h}$ instead of using the fraction of market participants who are homeowners h, as it simplifies the derivations. The planner maximizes

$$\max_{\tilde{h},v,\theta,\varepsilon_R,c}\int_0^\infty e^{-rt}\left\{\left(\int_{\varepsilon_R}^\infty \varepsilon x\frac{dF(\varepsilon)}{1-F(\varepsilon_R)}\right)\tilde{h}-[N(1-F(\varepsilon_R))-\tilde{h}]c^B-vc^S-ck\right\}dt \tag{41}$$

subject to

$$\dot{\tilde{h}} = v\theta m(\theta) - (s+\delta)\tilde{h}, \tag{42}$$

$$\dot{v} = e + sh - \delta v - v\theta m(\theta), \tag{43}$$

$$N(1 - F(\varepsilon_R)) - \tilde{h} = \theta v. \tag{44}$$

The social planner maximizes the present discounted value of the overall utility from homeownership, taking into account households heterogenous' valuations and the amount of entry, net of buyers' search cost, vacancy cost and construction costs. The constraints on the planner's problem include the law of motions for homeownership and vacancies, and the relationship between buyers, vacancies and market tightness.

Solving for the optimal allocation $\{\theta, \varepsilon_R\}$ in steady state yields the two equation system below

$$\frac{(r+\delta)k + c^S}{\theta m(\theta)} = \alpha \left\{ \frac{[(1-h)\bar{\varepsilon} + h\varepsilon_R]x - (r+\delta)k}{r+s+\delta} \right\}, \tag{45}$$

$$\frac{c^B}{m(\theta)} = \left(\frac{1-\alpha}{\alpha}\right) \frac{(r+\delta)k + c^S}{\theta m(\theta)} - \frac{xh(\bar{\varepsilon} - \varepsilon_R)}{m(\theta)}. \tag{46}$$

The appendix includes the derivation. Comparing the planner's first-order conditions with the corresponding HE and BE conditions in the decentralized economy (36) and (37) shows that the HMP condition does not restore efficiency. The optimal allocation reflects that the planner does not only care about the marginal buyer, she is also concerned about the average composition of buyers. By contrast, in the decentralized equilibrium only the marginal buyer matters for entry. When making their decision on whether to enter the housing market, households simply evaluate whether their utility of owning a home yields a positive value of becoming a buyer, i.e. they compare their ε to the marginal value ε_R. If their utility ε is greater than the marginal value ε_R they enter the market. They do not internalize the effect of their entry on the composition of buyers, which affects the overall surplus in matches. Because of this additional participation externality, the decentralized equilibrium is inefficient even when the HMP condition is satisfied.

4.5 Decentralized economy with housing policies

In this section we sketch the equilibrium in the decentralized economy with the same housing market policies as in the baseline framework: (i) homeowners pay property taxes τ_p on their home; (ii) buyers pay taxes τ_b when they purchase a house; (iii) sellers pay a tax τ_s upon selling the house, which we assume applies to the net price $p(\varepsilon) - V$; (iv) developers pay

a construction tax at a rate τ_k, where $\tau_k > 0$ corresponds to a tax on construction and $\tau_k < 0$ to a subsidy.[13] The rest of the environment is the same as in section 4. The Bellman equations are given by

$$rB(\varepsilon) = -c^B + m(\theta)[H(\varepsilon) - B(\varepsilon) - (1+\tau_b)p(\varepsilon)], \tag{47}$$

$$rH(\varepsilon) = \varepsilon x - \tau_p p(\varepsilon) - s(H(\varepsilon) - V - B(\varepsilon)) - \delta(H(\varepsilon) - B(\varepsilon)), \tag{48}$$

$$rV = -c^S + \theta m(\theta)(1-\tau_s)\int_{\varepsilon_R}^{\infty}(p(\varepsilon) - V)\frac{dF(\varepsilon)}{1-F(\varepsilon_R)} - \delta V. \tag{49}$$

The equilibrium is derived following the same procedure as in section 4.2 and is summarized by the following set of equations:

$$p(\varepsilon) = \frac{\tilde{\beta}[\varepsilon + c^B + s(1+\tau_k)k] + (1-\tilde{\beta})(r+s+\delta+m(\theta))(1-\tau_s)(1+\tau_k)k}{r+s+\delta+(1-\tilde{\beta})m(\theta) - \tau_s(1-\tilde{\beta})(r+s+\delta+m(\theta)) + \tilde{\tau} + \tilde{\beta}(r+s+\delta)}, \tag{50}$$

$$\frac{(r+\delta)(1+\tau_k)k + c^S}{\theta m(\theta)(1-\tau_s)} = \bar{p} - (1+\tau_k)k, \tag{51}$$

$$\frac{c^B}{m(\theta)} = (1-\tau_s)(1-\tilde{\beta})\left[\frac{\varepsilon_R x + s(1+\tau_k)k - (1+\tilde{\tau})(1+\tau_k)k}{(r+s+\delta)[(1-\tilde{\beta})(1-\tau_s) + \tilde{\beta}(1+\tilde{\tau})}\right]. \tag{52}$$

4.6 Quantifying inefficiency in the housing market

We keep our calibration strategy as close as possible to the strategy in section 3 to make the quantitative predictions of the two models comparable. In particular, the only moments that are different in the calibration are those that pin down the parameters of the utility distribution $F(\varepsilon)$.[14] As before, set $r = 0.012$ and $\delta = 0.004$. The matching function is still assumed to be Cobb-Douglas with $m(\theta) = \mu\theta^{-\alpha}$ and $\alpha = 0.16$. The development tax τ_k is set to zero, the profit tax rate for the seller to 3.65%, $\tau_p = 0.0028$, and $\tau_b = 0.0016$. Lastly, we normalize the size of the market and set $b = 1$. We assume that the utility distribution follows a Pareto distribution with a shape parameter $\tilde{\alpha}$ and a minimum utility normalized to $\underline{\varepsilon} = 0.1$. We further normalize x to unity.

To calibrate c^B and c^S we target expected costs for the buyer and seller equal to 8% and 5.1% of the average house price, following Ghent (2012). We again target a time-to-sell of 1.76 quarters and set the time-to-buy to be equal to the time to sell. These two targets yield μ and k. As in section 3 we set the listing rate equal to 1.667%. This target implies a separation

[13] As with the homogenous model, the results barely change if we use a lower tax on profits to take into account that some households do not pay profit taxes.

[14] We also drop the moment which pins down γ in the homogeneous case, as this parameter is absent in the heterogeneous buyers model.

rate of $s = 0.0126$. Kotova and Zhang (2020) report housing price dispersion of 16.84%. The authors further attribute 14.67% of the overall dispersion to buyer heterogeneity. Using these estimates we back out an implied mean-min ratio for prices equal to 1.0884. This yields $\tilde{\alpha} = 1.2698$. It turns out that one can normalize ε_R as it acts as a scaling variable. Accordingly we set $\varepsilon_R = 9.934$ so that the average equilibrium price is 491.2, the average price in thousands of dollars reported in Kotova and Zhang (2020). This normalization, together with the buyer entry condition allows us to back out $\beta = 0.2994$. Lastly, the normalization $b = 1$ yields a population parameter $N = 12441$. Table 4 summarizes the calibration.

Given the model's calibration we compute the benchmark equilibrium and the planner's socially optimal allocation. We summarize the results in table 5. The benchmark equilibrium features a vacancy rate of 2.83%. This is very close to the empirically observed vacancy rate for the U.S. of 1.9%. This version of the model also matches relatively well the data on construction: the construction rate in our model is 0.4% whereas the one in the data is 0.27%. Turning our attention to the efficient allocation, the planner finds it optimal to reduce the time-to-sell by almost half. This is achieved by increasing the number of buyers by 13.65% and reducing the number of vacancies by 41.51%. Intuitively, from the planner's perspective the congestion externality induces an over-creation of vacancies in equilibrium. At the same time, the participation externality leads to a sub-optimally low homeownership rate in equilibrium. Therefore, the planner instructs a higher number of households to enter the market. The resulting optimal vacancy rate is 1.64%.

Because our model features two externalities, in general at least two policies are needed to restore the socially optimal allocation. We focus on two possible implementations: (i) a combination of taxes on housing development and profits; (ii) a combination of taxes on profits and taxes on homeowners. If the planner implements the optimal allocation using only taxes on the supply side of the market, then she needs to impose about a 50% profit tax rate and subsidize housing construction at a rate of 4.40%. The dollar amount of these two policies are $\$21,590$ and $\$19,910$ respectively. If the planner implements the social optimum with a combination of τ_s and $\tilde{\tau}_b$, then the optimal tax rate for sales is about 51% and the optimal tax rate for homeowners is about 7.14%. The dollar amount of these two policies are $\$22,440$ and $\$35,430$ respectively. These result suggest that the magnitude of the externalities are comparable to the ones in our baseline model with homogenous buyers.

5 Conclusion

This paper studies efficiency in the housing market with search frictions and endogenous entry of both buyers and sellers. These two features are fundamental to account for the stylized

facts of the housing market, in particular to generate a positive correlation between buyers and vacancies, i.e. an upward sloping Beveridge Curve. We show that in this environment two externalities arise: congestion and participation externalities. We characterize the efficient allocation and show that the decentralized equilibrium is inefficient even when the HMP condition holds. In particular, the decentralized economy features inefficiently low levels of homeownership and inefficiently high vacancy rate and time-to-sell. Finally, this paper studies housing market policies and how they can restore efficiency in the housing market.

Technical Appendix

Social planner's solution in the model with homogeneous buyers

The Hamiltonian for the social planner's problem is given by

$$\begin{aligned} H =& e^{-rt}\{\varepsilon\tilde{h} - vc^S - ck - v\theta c^B(v\theta) + \lambda_{\tilde{h}}[v\theta m(\theta) - (s+\delta)\tilde{h}] && \text{(A1)} \\ &+ \lambda_v[c + s\tilde{h} - \delta v - v\theta m(\theta)]\}. && \text{(A2)} \end{aligned}$$

The first-order conditions are given by

$$\begin{aligned} \frac{\partial H}{\partial c} &= 0 \Rightarrow \lambda_v = k, && \text{(A3)} \\ \frac{\partial H}{\partial \theta} &= 0 \Rightarrow (1+\gamma)c^B(b) = (1-\alpha)m(\theta)(\lambda_{\tilde{h}} - \lambda_v), && \text{(A4)} \\ \frac{\partial H}{\partial \tilde{h}} &= -\dot{\lambda}_{\tilde{h}} + r\lambda_{\tilde{h}} = \varepsilon - (s+\delta)\lambda_{\tilde{h}} + s\lambda_v, && \text{(A5)} \\ \frac{\partial H}{\partial v} &= -\dot{\lambda}_v + r\lambda_v = -(1+\gamma)\theta c^B(b) - c^S + \lambda_{\tilde{h}}\theta m(\theta) - (\delta + \theta m(\theta))\lambda_v. && \text{(A6)} \end{aligned}$$

Combining the above first-order conditions and observing that at steady state $\dot{\lambda h} = \dot{\lambda}_v = 0$ yields the planner's allocation (16) and (17).

To show that the solution to (16) and (17) is indeed the global maximum, re-write the Hamiltonian

$$\begin{aligned} \hat{H}(v,b,c,h) =& \varepsilon\tilde{h} - vc^S - ck - bc^B(b) + \lambda_{\tilde{h}}[M(b,v) - (s+\delta)\tilde{h}] && \text{(A7)} \\ &+ \lambda_v[c + s\tilde{h} - \delta v - M(b,v)] && \text{(A8)} \\ =& \varepsilon\tilde{h} - vc^S - ck - bc^B(b) - \lambda_{\tilde{h}}(s+\delta)\tilde{h} + \lambda_v(c + s\tilde{h} - \delta v) && \text{(A9)} \\ &+ (\lambda_{\tilde{h}} - \lambda_v)M(b,v). && \text{(A10)} \end{aligned}$$

Since $bc^B(b)$ is convex, $M(b,v)$ is concave, and for any admissible tuple $\lambda_{\tilde{h}} - \lambda_v = (\varepsilon - (r+\delta)k)/(r+s+\delta)$, it follows that $\hat{H}(v,b,c,h)$ is concave. Thus, the solution is a global maximum.

Social planner's solution in the model with heterogeneous buyers

The Hamiltonian for the social planner's problem is given by

$$H = e^{-rt}\{x\bar{\varepsilon}\tilde{h} - vc^S - ck - c^B[N(1-F(\varepsilon_R)) - \tilde{h}] + \lambda_{\tilde{h}}[v\theta m(\theta) - (s+\delta)\tilde{h}] \tag{A11}$$

$$+ \lambda_v[c + s\tilde{h} - \delta v - v\theta m(\theta)] + \lambda_\theta[N(1-F(\varepsilon_R)) - \tilde{h} - \theta v]\} \tag{A12}$$

where as before $\bar{\varepsilon} \equiv E(\varepsilon|\varepsilon \geq \varepsilon_R) = \int_{\varepsilon_R}^{\infty} \varepsilon dF(\varepsilon)/(1-F(\varepsilon_R))$. The first-order conditions are given by

$$\frac{\partial H}{\partial c} = 0 \Rightarrow \lambda_v = k, \tag{A13}$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \lambda_\theta = (1-\alpha)m(\theta)(\lambda_{\tilde{h}} - \lambda_v), \tag{A14}$$

$$\frac{\partial H}{\partial \varepsilon_R} = 0 \Rightarrow \lambda_\theta = c^B + xh(\bar{\varepsilon} - \varepsilon_R), \tag{A15}$$

$$\frac{\partial H}{\partial \tilde{h}} = -\dot{\lambda}_{\tilde{h}} + r\lambda_{\tilde{h}} \Rightarrow \lambda_{\tilde{h}} = \frac{x\bar{\varepsilon} + c^B + sk - \lambda_\theta}{r+s+\delta}, \tag{A16}$$

$$\frac{\partial H}{\partial v} = -\dot{\lambda}_v + r\lambda_v \Rightarrow \frac{(r+\delta)k + c^S}{\theta m(\theta)} = \alpha(\lambda_{\tilde{h}} - k). \tag{A17}$$

Combining the above first-order conditions and considering the steady state gives the planner's allocation (45) and (46).

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Preferences/Technology	Parameter	Value	Source/Target
Discount rate	r	0.012	Annual discount factor= 0.953
Utility	ε	10.883	Normalization
Elasticity of Matching Function	α	0.16	Genesove and Han (2012)
Destruction rate	δ	0.004	Van Nieuwerburgh and Weill (2010)
Separation Rate	s	0.0126	Listing rate = 1.67%
Efficiency of Matching Function	μ	0.5682	TTB= 1.76 quarters
Seller cost	c^S	14.233	Average seller cost= 5.1% of price
Bargaining power	β	0.5234	Average seller cost= 8% of price
Construction cost	k	451.98	TTS= 1.76 quarters
Buyer cost parameter	$\bar{c}$	22.327	$b = 1$, normalization
Profit tax rate	τ_s	0.0365	Effective tax rate, real estate firms
Property tax rate	τ_p	0.0028	Tax Foundation Facts and Figures 2018
Transfer rate	τ_b	0.0016	National Association of Realtors
Construction tax	τ_k	0	

Table 1: Baseline Model: Calibration

Moment	Calibrated Benchmark	Optimal
Vacancy rate	2.83	1.55
Listing rate	1.67	1.66
TTS	1.76	0.95
Buyers	1	1.72
Vacancies	1	0.83
Homeowners	34.33	52.65

Table 2: Baseline Model: Moments

Taxes	Calibrated Benchmark	Optimal
τ_s	3.65%	69.8%
	($1.43)	($44.54)
τ_k	0%	−17.31%
	($0)	(−$78.25)

Table 3: Baseline Model: Optimal Policies

Preferences/Technology	Parameter	Value	Source/Target
Discount rate	r	0.012	Annual discount factor= 0.953
Utility scale	x	1	Normalization
Utility distribution support	$\underline{\varepsilon}$	0.1	normalization
Elasticity of Matching Function	α	0.16	Genesove and Han (2012)
Destruction rate	δ	0.004	Van Nieuwerburgh and Weill (2010)
Separation Rate	s	0.0126	Listing rate = 1.67%
Efficiency of Matching Function	μ	0.5682	TTB= 1.76 quarters
Seller cost	c^S	14.233	Average seller cost= 5.1% of price
Buyer cost	c^B	22.326	Average seller cost= 8% of price
Construction cost	k	451.95	TTS= 1.76 quarters
Pareto shape	$\tilde{\alpha}$	1.2698	$\bar{p}/p^R = 1.0884$
Bargaining power	β	0.2994	$\varepsilon_R = 9.934$, normalization
Population	N	12441	$b = 1$, normalization
Profit tax rate	τ_s	0.0365	Effective tax rate, real estate firms
Property tax rate	τ_p	0.0028	Tax Foundation Facts and Figures 2018
Transfer rate	τ_b	0.0016	National Association of Realtors
Construction tax	τ_k	0	

Table 4: Heterogeneous Utility Model: Calibration

Moment	Calibrated Benchmark	Optimal
Vacancy rate	2.83	1.64
Listing rate	1.67	1.66
TTS	1.76	1.01
Buyers	1	1.14
Vacancies	1	0.58
Homeowners	34.33	35.08

Table 5: Heterogeneous Utility Model: Moments

Taxes	Calibrated Benchmark	Optimal
τ_s	3.65%	50.34%
	($1.43)	($21.59)
τ_k	0%	−4.40%
	($0)	(−$19.91)
τ_s	3.65%	50.93%
	($1.43)	($22.44)
$\tilde{\tau}_b$	10%	7.14%
	($49.39)	($35.43)

Table 6: Heterogeneous Utility Model: Optimal Policies