Nonparametric Bootstrap Specification Testing in Econometric Models*

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ABSTRACT

We consider three nonparametric tests for functional form, varying parameters, and omitted variables in regression models both of time series data and of cross-sectional data. The first test is to compare the sums of squared residuals from the null and the alternative models and the second test is to compare the fitted values of the null and alternative models. The third test is the nonparametric conditional moment test, which is to see if the residuals from the null model is related to the conditioning variables in the alternative models. Bootstrap procedures are used for these tests and their performance is examined via monte carlo experiments.

Key Words: nonparametric test, nonlinearity, variable selection, lag selection, functional-coefficient model, conditional moment test, naive bootstrap, wild bootstrap, monte carlo.

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1 Introduction

Since the path-breaking work of Karl Pearson (1900) the 20th century saw the significant advances in the parametric statistical and econometric hypothesis testing procedures, see Bera (2000) for an excellent survey. A problem with the parametric testing procedures is that the tests may not be consistent under the misspecified alternative hypotheses. In the last two decades a rich literature has developed on constructing consistent model specification tests using nonparametric estimation techniques. Bierens (1982) was first to provide a consistent conditional moment test for model misspecification. Ullah (1985) first suggested the construction of model specification test using nonparametric estimation technique. Nonparametric specification test for time series data was first proposed by Robinson (1989).

Since the publication of these works various test statistics have been proposed for consistently testing parametric regression functional form, e.g. Andrews (1997), Azzalini, Bowman, and Härdle (1989), Bierens (1982, 1990), Bierens and Ploberger (1997), Cai, Fan, and Yao (2000), De Jong (1996), Eubank and Spiegelman (1990), Eubank and Hart (1992), Fan and Li (1996), Fan, Zhang, and Zhang (2001), Gozalo (1993), Härdle and Mammen (1993), Hart (1997), Hong and White (1995), Horowitz and Härdle (1994), Horowitz and Spokoiny (2000), Li and Wang (1996), Robinson (1991), Ullah (1985), Whang (2000), Wooldridge (1992), Yatchew (1992), and Zheng (1996), among others. Similarly several papers have appeared on testing the significance of omitted or excluded variables from the model, e.g. Aït-Sahalia, Bickel, and Stoker (1994), Fan and Li (1996), Härdle and Mammen (1993), Li (1999), Linton and Gozalo (1997), Racine (1997), Ullah and Vinod (1993), and Whang and Andrews (1993), among others. Delgado and Stengos (1994), Lavergne and Vuong (1996), and Ullah and Singh (1989) explore non-nested hypothesis testing problems. In addition to omitted variables and functional forms, there are many papers which look into the nonparametric approach to general hypothesis testing problems encountered in econometrics, e.g. Cai, Fan, and Yao (2000), Fan, Zhang, and Zhang (2001), Hart (1997), Lewbel (1993, 1995), Robinson (1989), and Ullah and Singh (1989), among others. For details, see Pagan and Ullah (1999).

While most of the early developments in the nonparametric hypothesis testing appeared for the independent data, except e.g. Robinson (1989), in recent years the problem of hypothesis testing with the dependent time series data has been addressed by many authors. For example, Berg and Li (1998), Fan and Li (1997), Hjellvik and Tjøstheim (1995, 1996), Hjellvik, Yao, and Tjøstheim (1998), Kreiss, Neumann, and Yao (1998), Lee (2001), and Lee and Ullah (2001), among others, have considered the tests for functional form and omitted variables. In particular, for time series, testing for omitted variables is often to identify the number of lags. Nonparametric lag selection in nonlinear time series models are studied by Auestad and Tjøstheim (1990), Cheng and Tong (1992), Fan and Li (1999a), Granger and Lin (1994), Granger, Maasoumi, and

Racine (2000), Hong and White (2001), Tjøstheim and Auestad (1994), Tschernig and Yang (2000), and Yao and Tong (1994). Chen and Fan (1999) provided consistent tests for time series models, but the asymptotic distributions of their tests are nonstandard. In a major development, Fan and Li (1999b) developed the central limit theorems for the degenerate U-statistics for the weakly dependent data. This has provided a significant breakthrough and important contribution in Li (1999) who shows the asymptotic normality of Li and Wang (1998) type tests for wide range of hypotheses testing problems, with dependent data, e.g. parametric functional forms in regression, single index models, semiparametric regressions, variable selection, and mean-variance ratio hypothesis in finance.

We note here that the test statistics for most of the testing problems described above are based on the following alternative procedures: (1) Ullah (1985) type F or likelihood ratio procedure comparing the residual sum of squares under the null and alternative hypotheses, also see Azzalini, Bowman, and Härdle (1989), Cai, Fan, and Yao (2000) and Fan, Zhang, and Zhang (2001), (2) the procedure comparing the sum of squares of the differences in the fitted values of the models under the null and alternative specifications, e.g. Härdle and Mammen (1993), Ullah and Vinod (1993), Aït-Sahalia, Bickel, and Stoker (1994), and (3) the conditional moment procedure looking into the covariance between the residual under the null and the model specified under the alternative (e.g., Zheng (1996), Fan and Li (1996), and Li and Wang (1998)). These three alternative procedures are equivalent in the sense that they conform to the same population value of the null hypothesis of no difference between the null and alternative specifications. However, sample statistics based on them are different and may give different results. The purpose here is not to introduce a new procedure of nonparametric testing, instead the modest aim is to explore the bootstrap simulation comparison of these three procedures with respect to size and power properties in small as well as large samples. In the earlier simulation studies usually the case of testing a parametric specification is considered by only one of the three procedures. This is perhaps the first study which considers all the three procedures and looks into not only the testing of a parametric specification but the testing of varying parameters and omitted variables. We also consider both the independent cross-section and dependent time-series data models. Both the naive bootstrap and wild bootstrap procedures are used for our analysis.

The plan of the paper is as follows. In Section 2, we present the nonparametric kernel regression estimator. Section 3 presents the three procedures of nonparametric hypothesis testing. Then in Section 4 we give our simulation results. Finally, Section 5 gives conclusions.

2 Nonparametric Regression

Let us consider the regression model

$$y_t = m(x_t) + u_t = E(y_t|x_t) + u_t \tag{1}$$

where, $t = 1, \ldots, n$, y_t is a scalar dependent variable, $x_t = (x_{t1}, \ldots, x_{tk})$ is an $1 \times k$ vector, $m(x_t) = E(y_t|x_t)$ is the true but unknown regression function and u_t is the error term such that $E(u_t|x_t) = 0$. The model in (1) includes the autoregressive model as a special case in which x_t consists of lagged values of y_t. For the time series case we assume that $\{y_t, x_t\}$ is a strictly stationary discrete-time stochastic process.

A parametric approach to estimate $m(x_t)$ in (1) may begin by fitting a linear parametric regression model through the data as

$$\begin{aligned} y_t &= \alpha + \beta x_t + u_t \\ &= X_t \delta + u_t \end{aligned} \tag{2}$$

or more generally a nonlinear parametric model $y_t = m(x_t, \delta) + u_t$, where $X_t = (1\ x_t)$ and $\delta = (\alpha\ \beta)'$. One can obtain a least squares (LS) estimator of $\hat{m}(x_t)$ by $m(x_t, \hat{\delta})$ where $\hat{\delta}$ is the LS estimator of δ obtained by minimizing the global LS objective function

$$\sum_{t=1} u_t^2 = \sum_{t=1} (y - m(x_{t,}\delta))^2. \tag{3}$$

However, this global parametric LS estimator, based on the global modelling, is inconsistent and biased at least in the regions of data where the *a priori* specified regression is not correctly specified.

An alternative improved approach is to use the nonparametric kernel regression estimation of the unknown $m(x_t)$. Essentially the idea behind the kernel regression is to model the regression function $m(x_t)$ locally. For example, to obtain the regression function at a given point x, we apply the standard linear regression technique to the data in the interval of length h around x. That is, for the data in the interval of length h, we consider the linear model

$$\begin{aligned} y_t &= \alpha(x) + x_t \beta(x) + u_t, \\ &= X_t \delta(x) + u_t, \end{aligned} \tag{4}$$

and then estimate $\delta(x)$ by minimizing the local LS or weighted LS errors

$$\sum_{t=1} u_t^2 K_{tx} = \sum_{t=1} (y_t - X_t \delta(x))^2 K_{tx} \tag{5}$$

with respect to $\delta(x)$, where $K_{tx} = K\left(\frac{x_t - x}{h}\right)$ is called a kernel (weight) function and $h \to 0$ as $n \to \infty$ is usually called window width (smoothing parameter). Generally the kernel function can be any probability density function having a finite second moment. The estimator so obtained is

$$\hat{\delta}(x) = \left(\mathsf{X}'\mathsf{K}(x)\mathsf{X}\right)^{-1}\mathsf{X}'\mathsf{K}(x)\mathsf{y} \tag{6}$$

where $\mathsf{K}(x)$ is the $n \times n$ diagonal matrix with diagonal elements K_{tx} $(t = 1, ..., n)$, X is an $n \times (k+1)$ matrix with the t-th row X_t, and y is an $n \times 1$ vector. The estimator of $m(x_t)$ is then given by $m(x_t) = X_t\delta(x_t)$. The approach in (4) is called local linear regression and the estimator in (6) is known as the local linear LS (LLLS) estimator. For details on the kernel regression estimators and the choices of $K(\cdot)$ and h, see Wand and Jones (1995), Fan and Gijbels (1996), and Pagan and Ullah (1999).

It is interesting to note the special cases and generalization of (4) and (6). When $h = \infty$, the local linear regression modelling in (4) becomes global modelling in (2) and the LLLS estimator $\hat{\delta}(x)$ in (6) becomes the global LS estimator of $\hat{\delta}$. This is because when $h = \infty$, $K_{tx} = K(0)$ and the minimization of $\sum (y_t - X_t\delta(x))^2$ becomes the minimization of $K(0)\sum (y_t - X_t\delta(x))^2$ = minimization of $\sum (y_t - X_t\delta)^2$. Also note that when $X_t = 1$, the LLLS estimator $\hat{\delta}(x)$ reduces to

$$\begin{aligned} \hat{\delta}(x) &= \hat{\alpha}(x) = \left(\mathsf{i}'\mathsf{K}(x)\mathsf{i}\right)^{-1}\mathsf{i}'\mathsf{K}(x)\mathsf{y} \\ &= \frac{\sum_{t=1}^{n} y_t K_{tx}}{\sum_{t=1}^{n} K_{tx}} \end{aligned} \tag{7}$$

which is the Nadaraya (1964) and Watson (1964) kernel regression estimator, where i is an $n \times 1$ vector of unit elements. The LLLS can be extended to the p-th order local polynomial LS estimator where, for $k = 1$, $X_t = [1, x_t, \ldots, x_t^p]$ and δ is a $(p+1) \times 1$ vector, see Fan and Gijbels (1996).

One advantage of the local estimators is that they can be viewed as the varying coefficient (functional coefficient) estimators. This is because $\hat{\delta}(x)$ may have varying values at different data points x_t. In this sense the local linear model $y_t = X_t\delta(x) + u_t$ is a varying coefficient model $y_t = m(x_t) + u_t$ where $m(x_t) = X_t\delta(x_t) \simeq X_t\delta(x)$ where $\delta(x_t) \simeq \delta(x)$ is the first term of the Taylor's approximation around x. This is in contrast to the global estimator $\hat{\delta}$ which is the estimator of δ in the constant coefficient model.

The above idea of varying coefficients model can be extended to the situations where the coefficients are varying with respect to z_t which may be a subset of x_t or something else, i.e.,

$$E(y_t|x_t) = m(x_t) = X_t\delta(z_t). \tag{8}$$

Examples of these include functional coefficient autoregressive model (Chen and Tsay 1993, Cai, Fan, and Yao 2000), smooth coefficient model (Li, Huang, and Fu 1997), random coefficient model (Raj and Ullah 1981), smooth transition autoregressive model (Granger and Teräsvirta 1993), exponential autoregressive model (Haggan and Ozaki 1981), and threshold autoregressive model (Tong 1990). Also see Section 4. To estimate $\delta(z_t)$ we can again do a local approximation $\delta(z_t) \simeq \delta(z)$ and then minimize $\sum [y_t - X_t\delta(z)]^2 K_{tz}$

with respect to $\delta(z)$, where $K_{tz} = K(\frac{z_t - z}{h})$. This gives the varying coefficient estimator

$$\tilde{\delta}(z) = (\mathsf{X}'\mathsf{K}(z)\mathsf{X})^{-1}\mathsf{X}'\mathsf{K}(z)\mathsf{y}$$

where $\mathsf{K}(z)$ is a diagonal matrix of $K_{tz}, t = 1, \ldots, n$. When $z_t = x_t$, this reduces to the LLLS estimator $\hat{\delta}(x)$ in (6).

Cai, Fan, and Yao (2000) consider a local linear approximation $\delta(z_t) \simeq \delta(z) + D(z)(z_t - z)'$ where $D(z) = \frac{\partial \delta(z_t)}{\partial z_t^0}$ evaluated at $z_t = z$. The LL varying coefficient (LLVC) estimator of Cai, Fan, and Yao (2000) is then obtained by minimizing

$$\begin{aligned} \sum_{t=1} [y_t - X_t\delta(z_t)]^2 K_{tz} &= \sum_{t=1} [y_t - X_t\delta(z) - [(z_t - z) \otimes X_t] vecD(z)]^2 K_{tz} \\ &= \sum_{t=1} [y_t - X_t^z \delta^z(z)]^2 K_{tz} \end{aligned}$$

with respect to $\delta^z(z) = [\delta(z)' \;\; (vecD(z))']'$ where $X_t^z = [X_t \;\; (z_t - z) \otimes X_t]$. This gives

$$\ddot{\delta}^z(z) = (\mathsf{X}^{z\prime}\mathsf{K}(z)\mathsf{X}^z)^{-1}\mathsf{X}^{z\prime}\mathsf{K}(z)\mathsf{y}, \tag{9}$$

and $\ddot{\delta}(z) = (\mathsf{I}\ 0)\ddot{\delta}^z(z)$. Hence

$$\tilde{m}(x) = (1\ x\ 0)\ddot{\delta}^z(z) = (1\ x)\ddot{\delta}(z). \tag{10}$$

For the asymptotic properties of these varying coefficient estimators, see Cai, Fan, and Yao (2000).

3 Nonparametric Bootstrap Tests

We consider here two types of null hypotheses on $m(\cdot)$:

$$H_0 \;\; : \;\; m(x_t) = m(x_t, \delta) \tag{11}$$

$$H_0 \;\; : \;\; m(x_t) = m(x_{t1}) \tag{12}$$

where $x_t = (x_{t1}, x_{t2})$; x_{t1} is $1 \times k_1$, x_{t2} is $1 \times k_2$, and $k = k_1 + k_2$. The alternative hypothesis in each case is the unspecified nonparametric regression:

$$H_1 : m(x_t) = E(y_t | x_t). \tag{13}$$

The null hypothesis in (11) can be used as the null hypothesis for testing both for the functional form as well as the varying coefficient models. As a simple example of (11), we may consider testing for the functional form to be a linear regression, namely $H_0 : m(x_t, \delta) = X_t\delta$ against H_1 in (13). However, if H_1 in (13) is specified to be local linear models $m(x_t) = X_t\delta(x_t)$ or $m(x_t) = X_t\delta(z_t)$ then the testing for the null $H_0 : m(x_t, \delta) = X_t\delta$ against $H_1 : m(x_t) = X_t\delta(z_t)$ is testing for the constant regression model against the varying coefficient regression, see Section 4 for more example.

The null hypothesis in (12) is for testing the significance of the omitted variables x_{t2}, that is to test for selection of variables or lags. In this case both the null and alternative models are nonparametric. Though not considered here the test statistics considered below can also be used for testing situations where the null hypothesis is the partially linear model, $m(x_t) = x_{t1}\beta + m(x_{t2})$, single index models $m(x_t) = m(x_t, \delta)$, among others.

We will consider three main approaches for the above testing problems.

The first test procedure we consider is, as suggested in Ullah (1985), to compare the residual sum of squares RSS_0 under the null with the nonparametric residual sum of squares under the alternative, RSS_1. The test statistic is

$$T = \frac{(\text{RSS}_0 - \text{RSS}_1)}{\text{RSS}_1} \tag{14}$$

where for the null (11) $\text{RSS}_0 = \sum \hat{u}_t^2$, $\hat{u}_t = y_t - m(x_t, \hat{\delta})$, and for the null (12) $\text{RSS}_0 = \sum \tilde{u}_t^2$, $\tilde{u}_t = y_t - \tilde{m}(x_{t1})$, $\tilde{m}(x_{t1})$ is given by (10) with $x_t = x_{t1}$. Further $\text{RSS}_1 = \sum \tilde{u}_t^2$, $\tilde{u}_t = y_t - \tilde{m}(x_t)$. We reject the null hypothesis when T is large.

Fan and Li (1992) show the asymptotic normality of $nh^{k/2}T$, see also Fan, Zhang, and Zhang (2001, Theorem 5), and Cai, Fan, and Yao (2000).

Fan, Zhang, and Zhang (2001) further show that a suitably normalized T will have its asymptotic null distribution that is independent of nuisance parameters. They call this property the Wilks (1938) phenomenon. An important consequence of this result is that one does not have to derive theoretically the normalizing factors in order to be able to use the test. As long as the Wilks phenomenon holds, one can simply simulate the null distribution of the test statistic T. This is in stark contrast with some other tests whose asymptotic null distributions depend on nuisance parameters. Based on these Wilks results of Fan, Zhang, and Zhang (2001), Cai, Fan, and Yao (2000) suggest to use the bootstrap method which allows the implementation of (14). It involves the following steps to evaluate p-values of T to test the null hypotheses in (11) and (12).

1. Generate the bootstrap residuals $\{\tilde{u}_t^*\}$ from the centered residuals from the nonparametric (NP) alternative model, $(\tilde{u}_t - \bar{u})$ where $\bar{u} = n^{-1}\sum \tilde{u}_t$ and $\tilde{u}_t = y_t - \tilde{m}(x_t)$.

    (a) For naive bootstrap, $\{\tilde{u}_t^*\}$ is obtained from randomly resampling $\{\tilde{u}_t - \bar{u}\}$ with replacement.

    (b) For wild bootstrap, $\tilde{u}_t^* = a(\tilde{u}_t - \bar{u})$ with probability $r = (\sqrt{5}+1)/2\sqrt{5}$ and $\tilde{u}_t^* = b(\tilde{u}_t - \bar{u})$ with probability $1 - r$ $(t = 1, \ldots, n)$, where $a = -(\sqrt{5}-1)/2$ and $b = (\sqrt{5}+1)/2$. See Li and Wang (1998, pp. 150-151).

2. Generate the bootstrap sample $\{y_t^*\}_{t=1}^n$ from the null model; from $y_t^* \equiv m(x_t, \hat{\delta}) + \tilde{u}_t^*$ $(t = 1, \ldots, n)$ for the null in (11) to test for parametric functional form, and from $y_t^* = \tilde{m}(x_{t1}) + \tilde{u}_t^*$ for the null in (12) to test for omitted variables x_{t2}.

3. Using the bootstrap sample $\{y_t^*, x_t, z_t\}_{t=1}^n$, calculate the bootstrap test statistic T^* using, for the sake of simplicity, the same h used in estimation with the original sample as done in Cai, Fan, and Yao (2000).

4. Repeat the above steps B times and use the empirical distribution of T^* as the conditional null distribution of T given $\{y_t^*, x_t, z_t\}_{t=1}^n$. We use $B = 500$. The bootstrap p-value of the test T is simply the relative frequency of the event $\{T^* \geq T\}$ in the bootstrap resamples.

We use both naive bootstrap (Efron 1979) and wild bootstrap (Wu 1986, Liu 1988). The wild bootstrap method preserves the conditional heteroskedasticity in the original residuals. For wild bootstrap, see also Shao and Tu (1995, p. 292), Härdle (1990, p. 247), or Li and Wang (1998, p. 150).

Two more versions of the T-test in (14) can be considered

$$S = \frac{\text{RSS}_0 - \text{RSS}_1}{\text{RSS}_0}, \tag{15}$$

$$R = \frac{1}{n}\left(\text{RSS}_0 - \text{RSS}_1\right), \tag{16}$$

where S is the same as T with RSS_1 in the denominator replaced by RSS_0 in the spirit of Rao's score test, and R is essentially the numerator of T. In our Monte Carlo study in Section 4 the statistics T, S and R are compared and calculated on the basis of weighted (trimmed) RSS to control the tail behavior of the nonparametric estimator. For example, weighted $\text{RSS}_1 = \sum \tilde{u}_t^2 w(z_t, a)$, and for the null in (11) weighted $\text{RSS}_0 = \sum \hat{u}_t^2 w(z_t, a)$, where $w(z_t, a) = \mathbf{1}(|z_t/\hat{\sigma}_z| < a)$, $\hat{\sigma}_z$ is the sample standard deviation, $a = \infty,\ 2,\ 1.5$, and $\mathbf{1}(\cdot)$ is the indicator function. The test statistics with weighted RSS's will be denoted as R_a, S_a, and T_a. Note that $w(z_t, \infty) = 1$ and thus $R_\infty, S_\infty, T_\infty$ are R, S, T in (14)-(16) without weights. When the weight $w(\cdot, \cdot)$ is a known function so that it is not estimated, the Wilks phenomenon continues to hold as shown by Fan, Zhang, and Zhang (2001, Remark 4.2 and Theorem 9) and thus the Cai, Fan, and Yao (2000) bootstrap procedure can be applied to the statistics R_a, S_a and T_a with weighted RSS's.

The second test procedure we consider is to compare the fitted values from the null and alternative models as suggested in Härdle and Mammen (1993), Ullah and Vinod (1993), and Aït-Sahalia, Bickel, and Stoker (1994). For the null in (11), this is given by

$$Q_a = \frac{1}{n}\sum_{t=1}^{n}(m(x_t, \hat{\delta}) - \tilde{m}(x_t))^2 w(z_t, a). \tag{17}$$

For the omitted variable testing in (12), $m(x_t, \hat{\delta})$ in (17) is replaced by the NP estimator $\tilde{m}(x_{t1})$ with x_{t2} omitted. The bootstrap procedure described above for R_a, S_a, T_a may also be applied to Q_a.

The third test procedure we consider is the conditional moment test for $E(u_t|x_t) = 0$, which is identical to testing

$$E[u_t E(u_t|x_t) f(x_t)] = 0, \tag{18}$$

where $f(x_t)$ is the density of x_t. A sample estimator of the left hand side of (18) is

$$\begin{aligned} L' &= \frac{1}{n} \sum_{t=1} \hat{u}_t E(\hat{u}_t|x_t) \hat{f}(x_t) \\ &= \frac{1}{n(n-1)h^k} \sum_{t=1} \sum_{t'=1, t' \neq t} \hat{u}_t \hat{u}_{t'} K_{t't} \end{aligned} \tag{19}$$

where $\hat{u}_t = y_t - m(x_t, \hat{\delta}) = y_t - X_t\hat{\delta}$ to test for the null hypothesis in (11) or $\hat{u}_t = y_t - \tilde{m}(x_{t1})$ to test for the null hypothesis in (12), $E(\hat{u}_t|x_t) = \sum_{t' \neq t} \hat{u}_{t'} K_{t't} / \sum_{t' \neq t} K_{t't}$ from (7), and $\hat{f}(x_t) = [(n-1)h^k]^{-1} \sum_{t' \neq t} K_{t't}$ is the kernel density estimator; $K_{t't} = K(\frac{x_{t'} - x_t}{h})$. Note that we estimate the auxiliary regression function $E(\hat{u}_t|x_t)$ from the local constant LS estimator (7) of Nadaraya and Watson, not from the LLVC estimator of Cai, Fan, and Yao (2000) in (9) just to maintain the original formula of Li and Wang (1998) and Zheng (1996).

The asymptotic test statistic is then given by

$$L = nh^{k/2} \frac{L'}{\sqrt{\hat{\sigma}}} \xrightarrow{d} N(0,1) \tag{20}$$

where $\hat{\sigma} = 2(n(n-1)h^k)^{-1} \sum_t \sum_{t' \neq t} \hat{u}_t^2 \hat{u}_{t'}^2 K_{t't}^2$ is a consistent estimator of the asymptotic variance of $nh^{k/2}L'$, see Zheng (1996), Fan and Li (1996), Li and Wang (1998), Fan and Ullah (1999), and Rahman and Ullah (1999), for details. Also, see Pagan and Ullah (1999, Ch. 3) and Ullah (2001) for the relationship between R, Q and L test statistics. Based on the asymptotic results of Fan and Li (1996, 1997, 1999b) and Li (1999) for dependent data, Berg and Li (1998) establish the asymptotic validity of using the wild bootstrap method for L for time-series. The bootstrap p-values for L to test for the adequacy of the linear parametric model, $m(x_t, \delta) = X_t\delta$ in (11), can be computed as follows.

1. Generate the bootstrap residuals $\{\hat{u}_t^*\}$ from the residual from the null model $\hat{u}_t = y_t - X_t\hat{\delta}$:

    (a) For naive bootstrap, $\{\hat{u}_t^*\}$ is obtained from randomly resampling $\{\hat{u}_t\}$ with replacement.

    (b) For wild bootstrap, $\hat{u}_t^* = a\hat{u}_t$ with probability r and $\hat{u}_t^* = b\hat{u}_t$ with probability $1 - r$ as discussed above.

2. Generate the bootstrap sample $\{y_t^*\}_{t=1}^n$ from the null model $y_t^* \equiv m(x_t, \hat{\delta}) + \hat{u}_t^* = X_t\hat{\delta} + \hat{u}_t^*$ $(t = 1, \ldots, n)$ for the null in (11) to test for neglected nonlinearity.

3. Using the bootstrap sample $\{y_t^*\}_{t=1}^n$, calculate the bootstrap test statistic L^*.

4. Repeat the above steps B times and use the empirical distribution of L^* as the null distribution of L. We use $B = 500$. The bootstrap p-value of the test L is the relative frequency of the event $\{L^* \geq L\}$ in the bootstrap resamples.

For testing the omitted variables in the null (12), we replace $X_t\hat{\delta}$ in the steps 1 and 2 above by the NP regression estimator $\tilde{m}(x_{t1})$ in (10) with $x_t = x_{t1}$ and with x_{t2} omitted, and use centered NP residuals. That is,

1. Generate the bootstrap residuals $\{\hat{u}_t^*\}$ from the centered residuals from the NP alternative model, $(\hat{u}_t - \bar{u})$ where $\bar{u} = n^{-1}\sum \hat{u}_t$ and $\hat{u}_t = y_t - \tilde{m}(x_{t1})$.

2. Generate the bootstrap sample $\{y_t^*\}_{t=1}^n$ from the null model $y_t^* = \tilde{m}(x_{t1}) + \hat{u}_t^*$.

For parametric models, Davidson and MacKinnon (1999) show that the size distortion of a bootstrap test is at least of the order $n^{-1/2}$ smaller than that of the corresponding asymptotic test. For nonparametric models, h also enters in the order of refinement. Li and Wang (1998) show that if the distribution of L^j ($j = A$ for asymptotic, B for naive bootstrap, and W for wild bootstrap) admit an Edgeworth expansion then the bootstrap distribution approximates the null distribution of L with an error of order $n^{-1/2}h^{k/2}$ improving over the normal approximation. Since L is asymptotically normal under the null, the bootstrap tests L^B and L^W are more accurate than the asymptotic test L^A, as confirmed in the simulation of the next section. See Hall (1992) for further discussion on Edgeworth expansions and the extent of the refinements in various contexts.

The above three different testing approaches are related. Under the null in (11) $H_0 : m(x_t) = m(x_t, \delta)$, the RSS based test statistics R, S, T will be expected to be zero as $E(y_t - m(x_t, \delta))^2 = E(y_t - m(x_t))^2$. Also, by construction $E(u_t|x_t) = 0$ under the null, which implies $E[u_tE(u_t|x_t)f(x_t)] = 0$ for the L test. From this, we get the relationship used for the Q test because $E[u_tE(u_t|x_t)f(x_t)] = E[E(u_t|x_t)^2 f(x_t)] = E[\{E(y_t|x_t) - m(x_t, \delta)\}^2 f(x_t)] = 0$.

4 Monte Carlo

In this section we examine the finite sample properties of the test statistics T, Q, and L especially with the empirical null distributions being generated by the bootstrap method. Asymptotic critical values are also used for the L test. We consider four cases as indicated in 'blocks' below. All of the error terms ε_t below are $i.i.d.$ $N(0, 1)$.

BLOCK 1

This block is to study the size and power of the tests for functional form or varying coefficients in time series models. Let $x_t = y_{t-1}$. The following two models are taken from Lee, White, and Granger (1993).

DGP 1 Linear AR(1)

$$y_t = 0.6y_{t-1} + \varepsilon_t$$

DGP 2 Threshold Autoregressive (TAR(1)) (Nonlinear AR)

$$\begin{aligned} y_t &= 0.9y_{t-1} + \varepsilon_t \qquad |y_{t-1}| \leq 1 \\ &= -0.3y_{t-1} + \varepsilon_t \qquad |y_{t-1}| > 1 \end{aligned}$$

Note that DGP 1 is a constant parameter model whereas the alternative DGP 2 is a varying parameter model $y_t = y_{t-1}\delta\left(y_{t-1}\right) + \varepsilon_t$, where $\delta(y_{t-1}) = 0.9$ or -0.3 depending on $x_t = z_t = y_{t-1}$. In this sense testing for DGP 1 against DGP 2 is also a test for varying parameters.

BLOCK 2

This block is to study the size and power of the tests for functional form in cross-sectional models. Let v_{t1} and v_{t2} be drawn from $IN(0,1)$. Two regressors x_{t1} and x_{t2} are defined as $x_{t1} = v_{t1}$ and $x_{t2} = (v_{t1}+v_{t2})/\sqrt{2}$. Let $x_t = (x_{t1}\ x_{t2})$. The following two models are taken from Zheng (1996).

DGP 3

$$y_t = 1 + x_{t1} + x_{t2} + \varepsilon_t$$

DGP 4

$$y_t = |1 + x_{t1} + x_{t2}|^{5/3} + \varepsilon_t$$

BLOCK 3

This block is to study the size and power of the tests for lag selection in time series models. Let $x_t = (y_{t-1}\ y_{t-2})$. The alternative model DGP 6 is taken from Cai, Fan, and Yao (2000).

DGP 5 Exponential AR(1)

$$\begin{aligned} y_t &= a_1(y_{t-1})y_{t-1} + 0.2\varepsilon_t \\ a_1(y_{t-1}) &= 0.138 + (0.316 + 0.982y_{t-1})\exp(-3.89y_{t-1}^2) \end{aligned}$$

DGP 6 Exponential AR(2)

$$y_t = a_1(y_{t-1})y_{t-1} + a_2(y_{t-1})y_{t-2} + 0.2\varepsilon_t$$

$$a_1(y_{t-1}) = 0.138 + (0.316 + 0.982 y_{t-1}) \exp(-3.89 y_{t-1}^2)$$

$$a_2(y_{t-1}) = -0.437 - (0.659 + 1.260 y_{t-1}) \exp(-3.89 y_{t-1}^2)$$

BLOCK 4

This block is to study the size and power of the tests for variable selection in cross-sectional models. Let v_{t1} and v_{t2} be drawn from $IN(0,1)$. Two regressors x_{t1} and x_{t2} are defined as $x_{t1} = v_{t1}$ and $x_{t2} = (v_{t1}+v_{t2})/\sqrt{2}$. Let $x_t = (x_{t1}\ x_{t2})$. The alternative model DGP 8 is taken from Zheng (1996).

DGP 7

$$y_t = |1 + x_{t1}|^{5/3} + \varepsilon_t$$

DGP 8

$$y_t = |1 + x_{t1} + x_{t2}|^{5/3} + \varepsilon_t$$

To estimate $\hat{u}_t$ for the null model and $\tilde{u}_t$ for the alternative model, the information set used are $x_t = y_{t-1}$ for Block 1, $x_t = (y_{t-1}\ y_{t-2})$ for Block 3, and $x_t = (x_{t1}\ x_{t2})$ for Blocks 2, 4. The omitted variable is y_{t-2} for Block 3 and x_{t2} for Block 4.

We use a scalar 'threshold variable' z_t for all models: $z_t = y_{t-1}$ for Blocks 1 and 3, and $z_t = x_{t1}$ for Blocks 2 and 4.

For the Q_a, R_a, S_a, and T_a tests, as suggested by Cai, Fan, and Yao (2000), we select h using out-of-sample cross-validation. Let m and Q be two positive integers such that $n > mQ$. The basic idea is first to use Q sub-series of lengths $n - qm$ $(q = 1, \ldots, Q)$ to estimate the coefficient functions $\delta_q(z_t)$ and then to compute the one-step forecast errors of the next segment of the time series of length m based on the estimated models. That is to choose h minimizing the average of the mean square forecast errors

$$AMS(h) = \sum_{q=1}^{Q} AMS_q(h) \tag{21}$$

where

$$AMS_q(h) = \frac{1}{m} \sum_{t=n-qm+1}^{n-qm+m} [y_t - X_t^z \ddot{\delta}_q^z(z)]^2 \tag{22}$$

and $\ddot{\delta}_q^z(\cdot)$ are computed from the sample $\{y_t\ x_t\}_{t=1}^{n-qm}$. We use $m = [0.1n]$, $Q = 4$, and the Epanechinikov kernel $K(z) = \frac{3}{4}(1 - z^2)1(|z| < 1)$.

For the L test, as in Li and Wang (1998, p. 154), we use a standard normal kernel. Note that x_t is an $1 \times k$ vector, and $k = 1$ for Block 1 and $k = 2$ for Blocks 2, 3, 4. Thus the smoothing parameter h is chosen as $h_i = c\hat{\sigma}_i n^{-1/5}$ $(i = 1)$ for the cases with $k = 1$, and $h_i = c\hat{\sigma}_i n^{-1/6}$ $(i = 1, 2)$ for the cases with $k = 2$, where $\hat{\sigma}_i$ is the sample standard deviation of i-th element of x_t. The four values of $c = 0.1$, 0.5, 1, and 2 are used, and the corresponding estimated rejection probability will be denoted as L_c. In computing L, h^k shown in (19) and (20) is replaced with $\prod_{i=1}^{k} h_i$.

Test statistics are denoted as $Q_a^j, R_a^j, S_a^j, T_a^j$ and L_c^j, with the superscripts $j = A, B, W$ referring to the methods of obtaining the null distributions of the test statistics; asymptotics ($j = A$), naive bootstrap ($j = B$), and wild bootstrap ($j = W$).

Monte Carlo experiments are conducted with 500 bootstrap resamples and 1000 monte carlo replications. The amount of computing time needed to get the size results (Panels A and B) of the 36 statistics shown in tables for both 5% and 10% levels was as follows. It took a 600MHz-256MB Pentium III PC approximately a day for $n = 50$, 2-3 days for $n = 100$, and 5-7 days for $n = 200$. So, it took the PC roughly 7-10 days for size results of each table. It took another 7-10 days for power results (Panels C and D) of each table. It took less for Tables 1 and 2 where the null hypothesis is (11) than for Tables 3 and 4 where the null hypothesis is (12) and thus both the null and alternative models are nonparametric. For the whole results of the paper, it took the PC about 2 months. A GAUSS code for computing all the tests is available from the authors.

Table 1 presents the empirical size (DGP 1) and power (DGP 2) of testing for neglected nonlinearity in time series models in Block 1. We observe the following.

1. The L test using bootstrap (L^B and L^W) exhibits excellent size behavior and is better than all the other tests (L^A, Q^j, R^j, S^j, and T^j, $j = A, B, W$).

2. R is better than S and T. S and T are identical. Q behaves similarly to R. The tests of Q, R, S, T tend to be over-sized for $n = 50$, and under-sized with $n = 100, 200$ which is more apparent with larger sample size.

3. Trimming for R_a, S_a, T_a and Q_a is useful when n is small. For example, for $n = 50$, T_2 works better than T_∞. However, the trimming makes the size worse when n is large (say, $n = 200$).

4. The asymptotic test L^A works better with smaller c. L_c^A is not reliable for $c > 0.1$ and getting worse as c gets larger. The bootstrap tests L^B and L^W work very well with all four values of c. The asymptotic L^A is sensitive to c while the bootstrap tests L^B and L^W are not sensitive to c. This is different from what is found Lee and Ullah (2001) where the bootstrap tests L^B and L^W are also sensitive to c. This is because $\{y_t^*\}$ in this paper is not recursively generated (as described above) while Lee and Ullah (2001) generated $\{y_t^*\}$ recursively for time series data. Note that in this paper we generated the bootstrap data $\{y_t^*\}$ conditional on $\{x_t\}$ for both the cases when x_t is exogenous (Blocks 2, 4) and the cases when x_t is lagged dependent variables (Blocks 1, 3). The bootstrap method used in Lee and

Ullah (2001) may be called the "recursive" bootstrap, while the bootstrap method used in this paper may be called the "conditional" bootstrap. As discussed in Lee (2001), the bootstrap method treating x_t as given and generating $\{y_t^*\}$ conditional on x_t gives more robust size behavior than the recursive bootstrap even for the time series data.

5. Turning to the power behavior, although the size of L^B and L^W are quite robust to c, the power of these tests can vary with c and is generally best with larger c. The tests Q, R, S, T have a similar power pattern but these are slightly worse than L.

Table 2 presents the size (DGP 3) and power (DGP 4) of testing for neglected nonlinearity in cross-sectional models in Block 2. The following observations are made. All the size results in Table 1 for time series (summarized above) hold here for Table 2 with cross-sectional data. While the size of L^B and L^W are quite robust to c, the power of these tests can vary with c and is higher with larger c.

Table 3 presents the size (DGP 5) and power (DGP 6) of testing for lag selection in time series models in Block 3. The results are very similar to those in Tables 1 and 2. The tests L^B and L^W have good size and power in testing for lag selection in nonparametric time series models as well as to test for parametric functional forms.

Table 4 presents the size (DGP 7) and power (DGP 8) of testing for variable selection in cross-sectional models in Block 4. The results are again very similar to those in Tables 1, 2, and 3. The tests L^B and L^W have very good size and power in testing for omitted variables in cross-sectional models.

5 Conclusions

We consider three nonparametric tests for functional form, varying parameters, and omitted variables in regression models both of time series data and of cross-sectional data. The first approach (R, S, T) is to compare the sums of squared residuals from the null and the alternative models and the second test (Q) is to compare the fitted values of the null and alternative models. The third test (L) is the nonparametric conditional moment test, which is to see if the residuals from the null model is related to the conditioning variables in the alternative models. We find that the bootstrap tests of Li and Wang (1998) and Zheng (1996) L^B and L^W have very good size and power properties in all situations we considered.

One of the reasons for the better performance of these L tests compared to R, S, T and Q tests may be due to the fact that the asymptotic distribution of L is asymptotically normal with the mean zero under the null hypothesis, whereas this is not the case for R, S and T tests. Therefore it will be an interesting future study to compare L test with the bias-adjusted R, S, T tests as described in Fan and Li (2001). It will also be useful to develop the theoretical power properties of the tests under various local alternatives, as studied in Hong and Lee (2001) and Tripathi and Kitamura (2000) in different but related contexts. Moreover, this paper has considered the tests based on the kernel smoothing procedure only. It will be useful to study how our study compares with other specification testing procedures, especially using other smoothing procedures such as neural network, spline regression, and Fuzzy c-Means algorithm in Giles and Draeseke (2001). Finally the issue of optimal choice of window-width for the tests considered here needs further future investigations.

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TABLE 1. Testing for Linearity in Time Seris Models (Block 1)

A. Size of Tests at 5% level with DGP 1

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.089	0.125	0.089	0.099	0.081	0.075	0.102	0.135	0.101	0.117	0.097	0.089
100	0.041	0.072	0.029	0.036	0.030	0.024	0.049	0.087	0.039	0.050	0.036	0.036
200	0.033	0.059	0.024	0.026	0.021	0.019	0.037	0.066	0.033	0.036	0.027	0.024

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.022	0.092	0.025	0.073	0.020	0.055	0.022	0.092	0.025	0.073	0.020	0.055
100	0.018	0.062	0.015	0.031	0.019	0.020	0.018	0.062	0.015	0.031	0.019	0.020
200	0.021	0.057	0.024	0.028	0.015	0.018	0.021	0.057	0.024	0.028	0.015	0.018

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.047	0.059	0.055	0.013	0.049	0.048	0.001	0.046	0.042	0.000	0.039	0.039
100	0.031	0.041	0.042	0.015	0.044	0.047	0.006	0.049	0.046	0.000	0.043	0.047
200	0.038	0.045	0.044	0.027	0.052	0.057	0.008	0.047	0.048	0.002	0.050	0.041

B. Size of Tests at 10% level with DGP 1

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.109	0.164	0.112	0.127	0.104	0.096	0.126	0.170	0.128	0.147	0.122	0.113
100	0.057	0.102	0.049	0.051	0.041	0.039	0.080	0.113	0.064	0.069	0.067	0.061
200	0.048	0.097	0.038	0.036	0.033	0.031	0.064	0.121	0.054	0.056	0.050	0.045

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.048	0.127	0.048	0.105	0.045	0.079	0.048	0.127	0.048	0.105	0.045	0.079
100	0.041	0.093	0.032	0.048	0.036	0.043	0.041	0.093	0.032	0.048	0.036	0.043
200	0.048	0.102	0.041	0.044	0.033	0.037	0.048	0.102	0.041	0.044	0.033	0.037

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.089	0.121	0.117	0.033	0.103	0.094	0.005	0.102	0.101	0.000	0.077	0.094
100	0.060	0.096	0.086	0.029	0.093	0.083	0.011	0.092	0.096	0.000	0.090	0.097
200	0.065	0.092	0.103	0.050	0.099	0.095	0.020	0.103	0.100	0.003	0.094	0.090

C. Power of Tests at 5% level with DGP 2

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.422	0.505	0.422	0.452	0.432	0.414	0.457	0.533	0.473	0.488	0.489	0.461
100	0.649	0.731	0.677	0.674	0.684	0.663	0.717	0.776	0.732	0.741	0.744	0.727
200	0.922	0.946	0.908	0.907	0.906	0.905	0.954	0.965	0.947	0.948	0.959	0.956

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.232	0.454	0.250	0.397	0.271	0.362	0.232	0.454	0.250	0.397	0.271	0.362
100	0.628	0.739	0.658	0.694	0.675	0.688	0.628	0.739	0.658	0.694	0.675	0.688
200	0.942	0.962	0.945	0.947	0.950	0.956	0.942	0.962	0.945	0.947	0.950	0.956

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.304	0.331	0.325	0.408	0.567	0.549	0.228	0.575	0.569	0.001	0.335	0.388
100	0.656	0.696	0.706	0.857	0.928	0.922	0.773	0.943	0.943	0.086	0.825	0.840
200	0.967	0.973	0.972	0.997	0.999	0.999	0.996	0.998	0.998	0.842	0.998	0.999

D. Power of Tests at 10% level with DGP 2

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.474	0.578	0.489	0.501	0.492	0.473	0.514	0.606	0.529	0.539	0.551	0.531
100	0.714	0.792	0.724	0.733	0.734	0.727	0.776	0.823	0.783	0.776	0.787	0.782
200	0.953	0.964	0.911	0.911	0.910	0.909	0.970	0.979	0.970	0.966	0.978	0.976

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.329	0.536	0.355	0.472	0.372	0.456	0.329	0.536	0.355	0.472	0.372	0.456
100	0.714	0.807	0.736	0.763	0.741	0.755	0.714	0.807	0.736	0.763	0.741	0.755
200	0.968	0.976	0.961	0.964	0.975	0.975	0.968	0.976	0.961	0.964	0.975	0.975

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.404	0.464	0.450	0.482	0.695	0.702	0.304	0.715	0.724	0.003	0.516	0.573
100	0.763	0.817	0.816	0.902	0.968	0.963	0.836	0.972	0.973	0.163	0.909	0.922
200	0.983	0.989	0.989	0.998	0.999	0.999	0.998	1.000	0.999	0.912	1.000	1.000

Notes: Test statistics are denoted as $Q_a^j, R_a^j, S_a^j, T_a^j$,and L_c^j, with the superscripts $j = A, B, W$ refer to the methods of obtaining the null distributions of the test statistics; using the asymptotics (A), naive bootstrap (B), and wild bootstrap (W). The number of bootstrap resamples = 500 and number of monte carlo replications = 1000. The 95% asymptotic confidence interval of the estimated size is (0.036, 0.064) at 5% nominal level of significance and (0.081, 0.119) at 10% nominal level of significance.

TABLE 2. Testing for Linearity in Cross-Sectional Models (Block 2)

A. Size of Tests at 5% level with DGP 3

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.056	0.109	0.054	0.071	0.040	0.045	0.064	0.122	0.061	0.082	0.047	0.058
100	0.028	0.080	0.020	0.033	0.010	0.013	0.040	0.099	0.035	0.043	0.026	0.028
200	0.020	0.054	0.007	0.012	0.003	0.002	0.024	0.068	0.014	0.018	0.009	0.008

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.014	0.078	0.016	0.052	0.013	0.027	0.014	0.078	0.016	0.052	0.013	0.027
100	0.015	0.061	0.010	0.021	0.007	0.011	0.015	0.061	0.010	0.021	0.007	0.011
200	0.012	0.052	0.005	0.014	0.005	0.007	0.012	0.052	0.005	0.014	0.005	0.007

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.026	0.050	0.030	0.022	0.048	0.049	0.007	0.041	0.041	0.000	0.048	0.050
100	0.038	0.042	0.038	0.016	0.052	0.051	0.009	0.054	0.051	0.001	0.054	0.053
200	0.040	0.046	0.054	0.022	0.040	0.042	0.015	0.053	0.054	0.001	0.057	0.062

B. Size of Tests at 10% level with DGP 3

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.076	0.139	0.077	0.093	0.054	0.058	0.097	0.154	0.095	0.116	0.076	0.085
100	0.054	0.116	0.038	0.051	0.022	0.024	0.075	0.132	0.056	0.075	0.042	0.049
200	0.035	0.089	0.016	0.023	0.008	0.008	0.052	0.103	0.030	0.032	0.021	0.023

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.030	0.114	0.033	0.080	0.027	0.044	0.030	0.114	0.033	0.080	0.027	0.044
100	0.032	0.106	0.026	0.047	0.022	0.034	0.032	0.106	0.026	0.047	0.022	0.034
200	0.024	0.086	0.017	0.025	0.016	0.014	0.024	0.086	0.017	0.025	0.016	0.014

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.098	0.105	0.084	0.038	0.098	0.093	0.012	0.098	0.101	0.000	0.100	0.105
100	0.087	0.093	0.082	0.049	0.109	0.101	0.020	0.107	0.108	0.002	0.114	0.108
200	0.091	0.103	0.104	0.046	0.086	0.088	0.021	0.094	0.096	0.003	0.112	0.112

C. Power of Tests at 5% level with DGP 4

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.522	0.628	0.478	1.000	1.000	0.993	1.000	1.000	0.995	1.000	1.000	1.000
100	0.946	0.961	0.934	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

D. Power of Tests at 10% level with DGP 4

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.792	0.791	0.672	1.000	1.000	0.998	1.000	1.000	0.999	1.000	1.000	1.000
100	0.993	0.994	0.986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 3. Lag Selection in Time Series Models (Block 3)

A. Size of Tests at 5% level with DGP 5

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.073	0.106	0.077	0.099	0.087	0.094	0.078	0.108	0.084	0.099	0.088	0.096
100	0.013	0.034	0.015	0.022	0.017	0.019	0.013	0.037	0.019	0.023	0.021	0.020
200	0.008	0.025	0.012	0.016	0.013	0.014	0.011	0.028	0.016	0.020	0.017	0.017

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.006	0.072	0.010	0.063	0.009	0.055	0.006	0.072	0.010	0.063	0.009	0.055
100	0.002	0.025	0.004	0.014	0.005	0.009	0.002	0.025	0.004	0.014	0.005	0.009
200	0.002	0.021	0.007	0.018	0.011	0.009	0.002	0.021	0.007	0.018	0.011	0.009

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.011	0.037	0.034	0.014	0.051	0.045	0.006	0.040	0.038	0.000	0.045	0.052
100	0.029	0.046	0.043	0.015	0.048	0.048	0.004	0.053	0.056	0.001	0.052	0.054
200	0.040	0.055	0.054	0.018	0.044	0.049	0.008	0.042	0.043	0.003	0.060	0.067

B. Size of Tests at 10% level with DGP 5

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.085	0.124	0.098	0.121	0.106	0.119	0.090	0.123	0.100	0.121	0.108	0.114
100	0.022	0.051	0.030	0.044	0.031	0.031	0.023	0.061	0.032	0.043	0.035	0.034
200	0.014	0.034	0.021	0.028	0.023	0.023	0.018	0.039	0.028	0.033	0.036	0.032

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.018	0.090	0.022	0.080	0.026	0.072	0.018	0.090	0.022	0.080	0.026	0.072
100	0.006	0.035	0.013	0.028	0.013	0.027	0.006	0.035	0.013	0.028	0.013	0.027
200	0.011	0.030	0.017	0.028	0.023	0.027	0.011	0.030	0.017	0.028	0.023	0.027

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.069	0.093	0.075	0.037	0.088	0.092	0.011	0.090	0.084	0.001	0.094	0.092
100	0.090	0.101	0.100	0.031	0.101	0.101	0.010	0.104	0.102	0.003	0.102	0.105
200	0.086	0.096	0.096	0.038	0.098	0.091	0.017	0.096	0.089	0.004	0.110	0.100

C. Power of Tests at 5% level with DGP 6

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.983	0.988	0.985	0.988	0.989	0.990	0.999	0.998	0.999	0.998	1.000	0.998
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.995	0.998	0.995	0.998	0.997	0.998	0.995	0.998	0.995	0.998	0.997	0.998
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.282	0.496	0.336	0.965	0.976	0.967	0.979	0.985	0.982	0.980	0.986	0.994
100	0.828	0.899	0.836	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

D. Power of Tests at 10% level with DGP 6

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.986	0.990	0.987	0.991	0.991	0.991	1.000	0.999	1.000	0.998	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.997	0.998	0.997	0.998	0.998	0.999	0.997	0.998	0.997	0.998	0.998	0.999
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.571	0.680	0.550	0.979	0.978	0.978	0.985	0.987	0.990	0.984	0.991	0.996
100	0.946	0.961	0.931	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 4. Variable Selection in Cross-Sectional Models (Block 4)

A. Size of Tests at 5% level with DGP 7

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.025	0.055	0.032	0.043	0.032	0.036	0.029	0.048	0.035	0.042	0.036	0.036
100	0.011	0.022	0.014	0.021	0.011	0.013	0.013	0.028	0.019	0.024	0.021	0.018
200	0.003	0.014	0.011	0.015	0.008	0.010	0.004	0.013	0.007	0.014	0.008	0.009

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.003	0.024	0.007	0.020	0.010	0.022	0.003	0.024	0.007	0.020	0.010	0.022
100	0.006	0.016	0.007	0.013	0.008	0.011	0.006	0.016	0.007	0.013	0.008	0.011
200	0.002	0.010	0.005	0.011	0.005	0.007	0.002	0.010	0.005	0.011	0.005	0.007

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.027	0.057	0.041	0.018	0.051	0.050	0.005	0.058	0.055	0.000	0.045	0.054
100	0.032	0.041	0.038	0.009	0.039	0.041	0.003	0.043	0.042	0.000	0.039	0.040
200	0.041	0.049	0.053	0.020	0.047	0.047	0.004	0.042	0.041	0.000	0.035	0.035

B. Size of Tests at 10% level with DGP 7

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.045	0.073	0.051	0.067	0.049	0.050	0.039	0.077	0.047	0.070	0.048	0.052
100	0.017	0.037	0.024	0.030	0.024	0.026	0.021	0.049	0.031	0.041	0.036	0.038
200	0.014	0.028	0.018	0.022	0.016	0.018	0.008	0.030	0.025	0.027	0.022	0.020

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.009	0.038	0.016	0.033	0.021	0.035	0.009	0.038	0.016	0.033	0.021	0.035
100	0.008	0.038	0.018	0.030	0.025	0.025	0.008	0.038	0.018	0.030	0.025	0.025
200	0.006	0.023	0.017	0.022	0.013	0.015	0.006	0.023	0.017	0.022	0.013	0.015

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.091	0.098	0.081	0.034	0.105	0.107	0.009	0.114	0.116	0.000	0.097	0.102
100	0.076	0.085	0.079	0.029	0.088	0.084	0.010	0.097	0.100	0.000	0.087	0.090
200	0.085	0.096	0.098	0.038	0.094	0.092	0.011	0.090	0.088	0.000	0.085	0.090

C. Power of Tests at 5% level with DGP 8

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.993	0.993	0.992	0.992	0.992	0.991	0.995	0.995	0.994	0.994	0.996	0.995
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.993	0.994	0.993	0.993	0.994	0.995	0.993	0.994	0.993	0.993	0.994	0.995
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.154	0.259	0.182	0.907	0.964	0.913	0.952	0.994	0.982	0.777	0.993	0.994
100	0.527	0.589	0.526	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	0.941	0.962	0.948	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

D. Power of Tests at 10% level with DGP 8

n	Q^B_∞	Q^W_∞	Q^B_2	Q^W_2	$Q^B_{1.5}$	$Q^W_{1.5}$	R^B_∞	R^W_∞	R^B_2	R^W_2	$R^B_{1.5}$	$R^W_{1.5}$
50	0.993	0.995	0.993	0.994	0.992	0.992	0.996	0.996	0.997	0.996	0.996	0.996
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	S^B_∞	S^W_∞	S^B_2	S^W_2	$S^B_{1.5}$	$S^W_{1.5}$	T^B_∞	T^W_∞	T^B_2	T^W_2	$T^B_{1.5}$	$T^W_{1.5}$
50	0.995	0.996	0.995	0.996	0.995	0.996	0.995	0.996	0.995	0.996	0.995	0.996
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	$L^A_{0.1}$	$L^B_{0.1}$	$L^W_{0.1}$	$L^A_{0.5}$	$L^B_{0.5}$	$L^W_{0.5}$	L^A_1	L^B_1	L^W_1	L^A_2	L^B_2	L^W_2
50	0.369	0.414	0.328	0.942	0.982	0.955	0.968	0.995	0.993	0.860	0.997	0.995
100	0.726	0.751	0.688	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	0.981	0.988	0.977	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000