11 Infinite Horizon Bargaining Games: Theory and Experiments

Eythan Weg
Indiana University

Rami Zwick
Hong Kong University of Science and Technology

This chapter attempts to dismantle the myth that modeling bargaining as an infinite horizon bargaining process is too complex to merit experimentation by providing a simplified approach to the solution of the games underlying this process. In addition, a very rich menu for experimentation with external opportunities and risks is provided. A finite horizon model would be much more cumbersome and *artificial*. The solution of infinite games is approached through a very natural strategic game form built from the original bargaining tree. A distinguished subset of its Nash equilibria are identified for many cases (met in practice) with subgame perfect solutions of the tree. This approach exposes the fact that infinite horizon reasoning relies on a very weak form of backward induction, as opposed to the complete and lengthy induction necessary for finite games.

Although promising, the experimental work surveyed provides only weak and equivocal support for rational behavior. The need arises for further abstraction of the bargaining process for a better fit with the phenomena investigated, a procedure recommended by Rubinstein (1991) in the wider context of game theory in practice.

PURE BARGAINING

How is a valuable object, jointly owned by two parties, shared? This question epitomizes a fundamental question of economics. It encompasses the boundedness of a resource that must be shared if it is to be useful at all.

Two general approaches have been pursued. The axiomatic, with Nash (1950) its most well-known representative, and the procedural, best represented by Rubinstein (1982). The axiomatic method specifies certain desiderata that a bargaining ruling should satisfy. In the best case, these are also sufficient to specify a solution to the problem. This method refrains from promulgating a procedure by which the bargaining outcome is to be attained. Therefore, it is robust regarding changes in the bargaining conditions. The procedural approach specifies exactly how the bargaining is to take place. In this way, it is less dependent on axioms that lack unanimous endorsement. Ideally, as Nash hoped for, a bargaining procedure could be found to justify an axiomatic approach to bargaining. Although this idea has been pursued nothing convincing seems to appear.[1]

It was Rubinstein (1982), equipped with the then new concept of the *subgame*[2] *perfect equilibrium* (SPE) rationality developed by Selten (1975), who was able to exploit what seemed to many to be a natural bargaining scenario. It is this bargaining approach that is explored in this chapter.

Consider a unit (the pie) jointly owned by two people who can enjoy any part of it as described by their respective continuously increasing utility function u_i. Rubinstein considers an atomic building block of bargaining and iterates it as follows. One player suggests a demand x of the pie with the rest offered to the other player. If the latter agrees, the bargaining terminates with the demand accepted. Otherwise, the discrete bargaining clock ticks one period forward and the players swap their roles and start all over again. This continues until the white smoke of agreement is detected.

This description fits perfectly into the standard paradigm of games in extensive form, which, after Selten's innovation, could be solved in a more refined manner than provided by the approach endorsed by Von-Neumann, who saw such games in their folded normal (or strategic) form. Thus, instead of looking for Nash equilibria, which are guaranteed to exist, one looks for a subset of these equilibria that survives after eliminating all those that fail to remain in equilibrium for at least one subgame.[3] The principal beauty of Rubinstein's result is that this set is not empty under a very mild requirement regarding the effects of time: the utility of x consumed at time t is given by $u_i(x)\delta_i^t$, where $0 \leq \delta_i \leq 1$.[4]

To introduce SPE in a very transparent approach (good at least for the periodic structure of bargaining we have in mind), we consider the *fold* of the given bargaining game. This is a two-person strategic game defined as follows. Let

$$X_i(x_j) = \{0 \leq x \leq 1 | u_j(1-x) \geq u_j(x_j)\delta_j\} \qquad (1)$$

be player i's strategy set depending on player j's strategy x_j. The strategy set of each player depends on the strategy chosen by the other. Thus a player's choice of strategy is restricted by the other's choice, with the simple interpretation that in the extensive bargaining game what one offers the other $(1-x)$, must not be lower in utility than the expected utility from the latter's planned action. This interpretation might be characterized as mutual individual rationality. But it is not sufficient to specify any action. For example, the strategy 0 is mutually rational with every strategy of the other player. It is clear that these sets are nonempty, closed, and convex. We define formally the utility of a strategy combination (x_1, x_2) to player i as $u_i(x_i)$. A Nash equilibrium of the fold game is a point $(x_1, x_2) \in X_1(x_2) \times X_2(x_1)$ such that for all $x \in X_i(x_j)$ we have $u_i(x) \leq u_i(x_i)$.

It is clear that the Nash equilibria of the fold game are given by the solutions of the set

$$\begin{aligned} u_1(1-x_2) &\geq \delta_1 u_1(x_1) \\ u_2(1-x_1) &\geq \delta_2 u_2(x_2) \end{aligned} \tag{2}$$

where at least one of these inequalities is satisfied as an equality. It should be intuitively clear that not all Nash equilibria of the fold game are supported in the extensive game by a subgame Nash equilibrium. This distinguished set is characterized as the solutions of Eq. 2 satisfied as equalities. One can view the choice of strategy sets as satisfying a necessary condition for subgame perfectness.

For the immediate experimental applications we assume that $u(x) = x$ or $u(x) = \exp(x)$. One can easily see that for the first case

$$x_1 = \frac{1-\delta_2}{1-\delta_1\delta_2} \tag{3}$$

and for the second

$$x_1 = \begin{cases} 1 & \text{if } \delta_1 > \delta_2 \\ -\log\delta_2 & \text{if } \delta_2 > \delta_1 \\ [-\log\delta_1, 1] & \text{otherwise.} \end{cases} \tag{4}$$

The value of x_2 is then calculated by substitution.

We refer to the first type of time utility interaction as *geometric depreciation* and to the second as *arithmetic depreciation*.[5] This distinction is not mutually exclusive. It has more to do with the framing of the space-time preference structure than with anything else. Geometric depreciation with risk-neutral utilities, however, is directly comparable with the arithmetic case.

Duality

There are situations where the need arises to share a painful or an aversive joint object. Examples are the joint losses of a failed partnership or the division of a shared investment. In such cases, each of the parties attempts to *minimize* his or her share in the joint aversive property that we call, for simplicity, *deficit*. In contrast, the thing desired would be termed joint *surplus*. It will be efficient if the same theory could be applied to both cases. The following *duality* principle shows that this is indeed the case. It serves as a dictionary for the translation of bargaining problems over deficit into bargaining problems over surplus, and vice-versa.

Assume that a timed loss can be measured by the product of an increasing nonnegative function l and a power of $\gamma > 1$. Thus the total loss of a share x at time t is $l(x)\gamma^t$.[6] Then

$$l(x)\gamma^t \leq l(y)\gamma^s \tag{5}$$

iff

$$\frac{1}{l(1-x')}(\frac{1}{\gamma})^t \geq \frac{1}{l(1-y')}(\frac{1}{\gamma})^s \tag{6}$$

where $x = 1 - x'$ and $y = 1 - y'$. If we set $\frac{1}{l(1-x')} = u(x')$, and $\frac{1}{\gamma} = \delta < 1$, we see that to solve a deficit problem amounts to solving a surplus problem and applying the appropriate transformation. Thus we have the following duality:

Theorem (The Dictionary). *Every deficit bargaining situation* $D = (l_1, \gamma_1, l_2, \gamma_2)$ *has a dual surplus problem* $S = (u_1 = \frac{1}{l_1(1-(\cdot))}, \delta_1 = \frac{1}{\gamma_1}, u_2 = \frac{1}{(l_2(1-(\cdot))}, \delta_2 = \frac{1}{\gamma_2})$ *and vice-versa. Moreover, x is a rational share to party 1 in D if and only if* $1 - x$ *is a share to party 1 in S.*

Note that the proof has nothing to do with the bargaining procedure from which the particular solution is derived. Thus, the duality principle can be applied to any bargaining solution where time affects utility in the manner assumed by the theorem.

Corollary. *When* $l_i(x) = x$*, party 1's share is*

$$\frac{1 - \gamma_1}{1 - \gamma_1\gamma_2}. \tag{7}$$

The proof is immediate by invoking the dictionary with Eq. 3.

Example. For the risk-neutral (identity) loss functions with $\gamma_i = 10/9$, party 1's share is $9/19$.

We consider another special case:

Corollary. *Let the loss functions be* $l_i(x) = \exp(x)$*. Then party 1's cost share is*

$$x_1 = \begin{cases} \{0\} & \text{if } \gamma_1 < \gamma_2 \\ \{1 - \log(\gamma_2)\} & \text{if } \gamma_1 > \gamma_2 \\ [0, 1 - \log(\gamma_1)] & \text{if } \gamma_1 = \gamma_2. \end{cases} \tag{8}$$

Proof. This follows immediately from the dictionary and Eq. 4.

Example. Consider an arithmetic depreciation bargaining on a deficit, with bargaining costs of $\log(\gamma_1) = c_1 = 0.2$ and $\log(\gamma_2) = c_2 = 0.1$. The last corollary shows that Player 1's share in the deficit is 0.9.

Again, although the duality principle depends only on the separability of the time and space components of the preference structure, the formulas derived by these corollaries *do* depend on the bargaining procedure and on the rationality concept prescribed — SPE.

SHARING A PIE IN PRACTICE

Several experiments were carried out to test a variety of the hypotheses implied by the solution of the bargaining problem. Some employed arithmetic depreciation (also referred to as fixed cost), some geometric depreciation (also referred to as discount rate) and some even proposed alternative rejection procedures. This section reviews the outcomes of these experiments.

Arithmetic Depreciation

The very first experiment implementing the Rubinstein procedure with a fixed cost ($c = -\log(\delta)$) per period was conducted by Rapoport, Weg, and Felsenthal (1990). This experiment is divided into two studies that differ only by the levels of the fixed costs of the bargaining periods. In each step of this experiment, an initial group of students is subdivided randomly into pairs and each pair is engaged in a division of a pie of 30 Israeli Shekels according to the Rubinstein regime. In order to end each bargaining within reason, bargaining is allowed to continue for at least eight periods but no more than 13 periods. The exact cutoff is randomly chosen within this interval but subjects were not privy to this rule. The logistics of running concurrent bargaining games and stepping through the several games that each member of the group of subjects is engaged in during a given experimental session are governed by a computer program running under a time sharing system.

In both parts of the experiment, three power relationships between the players were conceived: S $(c_1 < c_2)$, W $(c_1 > c_2)$, and E $(c_1 = c_2)$. The c_i are members of $\{0.1, 2.5\}$ for the first study and of $\{0.2, 3.0\}$ for the second. A maximal cost within these respective sets is taken for condition E.

Figure 11.1 presents the data as reflected by last period demands by Player 1 (first mover) in the last iteration of each play.[7] Under condition *S*, Player 1 is expected to demand (in the first period) 30 Shekels in either of the studies and under condition *W*, 2.5 Shekels in Study 1 and 3.0 Shekels in Study 2. This assumes that the bargaining terminates immediately, but if not, it repeatedly starts a new subgame of type *S* or type *W* and because 2.5 or 3.0 are small compared to 30.00 we do not expect to find much of a difference if we look at last period demands.[8] The raw data is particularly impressive as extreme demands by and offers to the strong players are not buried in the averages. Averages, in this case, are not particularly appropriate because boundary outcomes are predicted whereas statistics based on typical values normally assume deviations on *either* side, that is, demands are expected to fall beyond the boundary.

The second study (Weg & Zwick, 1991), which followed a similar design to the Rapoport et al.'s studies investigated the robustness of the rational solution under an isomorphic transformation provided by the duality principle discussed earlier. Specifically, it defines an apparently different arithmetic depreciation bargaining where a better outcome is measured by how small it is. That is, a bargainer minimizes losses instead of maximizes gains. Formally, the disutility of x at time t is measured by $x + tc$ where $c > 0$. In a previous section we have shown that the problem is identical to the maximization of share under the utility $1 - x - tc$. Thus the experiment provides for a comparison between playing dual surplus and deficit games. Weg and Zwick (1991) compared bargaining over losses to bargaining over gains by the same subjects (within subject design) with pies of \$15 and costs set of $\{0.05, 1.25\}$. Condition E studied by Rapoport et al. was not investigated due to the expected uninformative outcomes of such a condition. Fig-

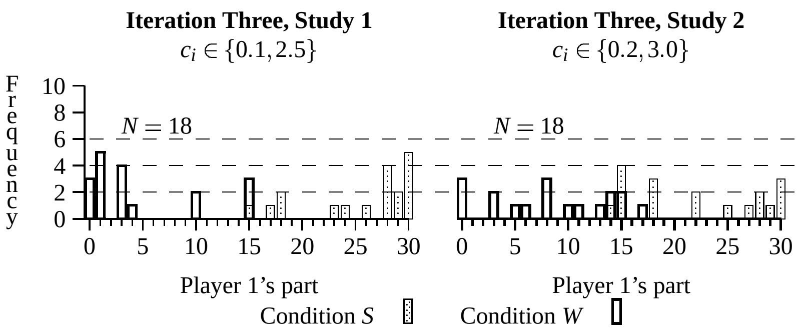

Fig. 11.1. Last period demands.

ure 11.2 presents the raw data for the second (and last) iteration, where payoffs for deficit bargaining are normalized by the duality transformation $1 - x$ which allows comparisons on equal footing with surplus games.

Three conclusions can be drawn from these experiments:

1. Strong parties obtain in general what is predicted by subgame perfect rationality.
2. It is hard to tell the difference between bargaining over losses and bargaining over gains.
3. Weak Player 1s (first movers) cannot in general improve their lot by decree: The game moves on to a subgame where the strong player usually sets the "price". The outcomes are invariant with regard to the position of the strong party except that a strong second party tends to get his or her share later than expected.

What makes the arithmetic depreciation preference structure attractive is the clear-cut extreme predictions it implies. We explore some interesting applications of this structure and now turn to a relatively more complex prediction derived from the Rubinstein paradigm.

Geometric Depreciation

According to Eq. 3, Player 1's demand is a continuous and nonconstant function of the discounting parameters. Can one expect human subjects to attend to the type of influence these parameters have on the predicted (subgame perfect) demands? Equation 4, which is applicable to arithmetic devaluation, is immensely simpler although discontinuous. It is (two-valued) constant almost everywhere. And, moreover, the domain of constancy is a union of two connected regions. Therefore, assuming only rough sensitivity to parameter values by subjects, it stands to reason that rational but perhaps cognitively limited players would ad-

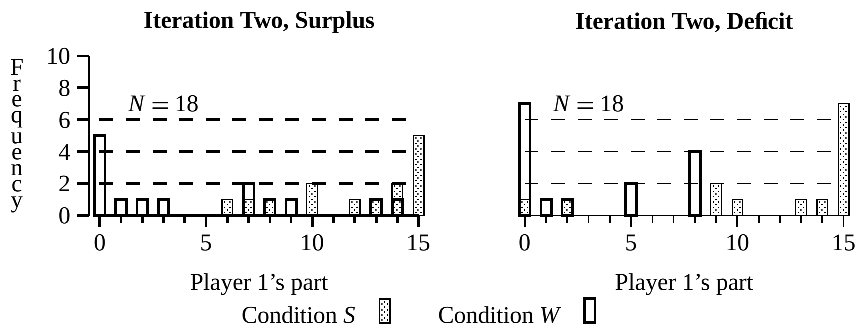

Fig. 11.2. Final demands by Player 1. Seven data points (of plays ended prematurely) are not shown.

---

here more closely to subgame perfect rationality in an arithmetic depreciating environment than in a geometric one.[9]

Much research reports on experiments where the identity utility is depreciated geometrically over time. Weg, Rapoport, and Felsenthal (1990) attempted to test rationality in a purely Rubinstein, alternating offer bargaining. Bargainers in this paradigm have no other option but to come to an agreement in finite time even though time is not formally limited, though no infinite paths are superior to any other path. Thus, bargainers have a strong incentive not to pursue such paths.

Two independent studies, which are replications of each other in every way except for the quotient values (discount rates), are reported. In either study the pie is 60.00 Shekels. In the first study the discounts are relatively mild and specify three conditions: (δ_1, δ_2) is either $(0.90, 0.17)$, or $(0.50, 0.90)$, or $(0.67, 0.67)$. The second study experiments with steeper rates — $(0.50, 0.17)$, $(0.17, 0.50)$, or $(0.17, 0.17)$. Table 11.1 shows the general trends.

The pattern seems to support the conclusion that strong players 1 are only able to extract *at best* half pies, and weak players 1 signal their intention to compensate for their steeper devaluation.[10] Note that these findings are in distinct contradiction to rationality because they fail to account for the cost of time in many ways:

1. A breakdown of monotonicity (in the right direction) with quotient values. For example, if we denote by $x(\alpha, \beta)$ the payoff to player 1 for game parameters (α, β), then for $\delta_1 < \delta_2$ we must have $x(\delta_1, \delta_1) > x(\delta_2, \delta_2)$. In fact, for these cases bargainers split pies in half, which is approximately correct only for extremely high quotients, that is, when time is negligible.
2. Symmetrization of players' positions. In theory, time is valuable and therefore $x(\delta_1, \delta_2) \neq 1 - x(\delta_2, \delta_1)$, or putting it differently, in general bargaining is not symmetric with respect to time. Unfortunately, this is not reflected in the data. Regardless of position, players' payoffs depend solely on their time

Table 11.1
Mean First and Final Shares to Player 1 in Studies 1 and 2

Iteration	$\delta_1 > \delta_2$	$\delta_1 = \delta_2$	$\delta_1 < \delta_2$	$\delta_1 > \delta_2$	$\delta_1 = \delta_2$	$\delta_1 < \delta_2$
	Study 1					
	First Offers			**Final Offers**		
1	29.4	31.6	36.4	27.1	30.5	36.0
2	30.6	31.1	36.9	27.6	30.7	36.4
3	30.1	31.0	36.0	24.4	31.0	36.5
	Study 2					
1	29.5	33.0	42.8	22.4	31.8	40.3
2	28.6	34.3	43.3	23.2	32.5	42.1
3	19.2	36.1	46.1	23.1	34.3	43.6

devaluation.

A Prelude to Optional Game Termination

One of the more interesting applications of arithmetic depreciation bargaining is the assessment of the prevalence of fairness considerations in economic situations (Kahneman, Knetsch, & Thaler, 1986). Güth, Schmittberger, and Schwarze (1982) initiated a long line of ultimatum studies where one person proposes a “take it or leave it” offer regarding the division of a pie. Thus, one player proposes x and the other either accepts it, in which case the game terminates with the proposed outcome, or rejects it where the status quo (normalized to 0) is obtained. This study showed that most proposers would share the pie evenly, in apparent support of fairness considerations, and further, positive offers are rejected in contradiction to rational behavior. Rejection of any part of the pie is an admission that obtaining nothing, in this game, is preferred to consuming a positive part of it, which is normally untrue in the context of individual choice. The problem with the ultimatum game as a tool in the investigation of fairness in economic settings is that it does not have an appropriate control game. Thus, it is not a priori clear whether the proposer is mitigated by a fear of rejection or by a consideration of fairness.[11] We shall now show how the arithmetic cost structure lends itself to several types of tests of the fairness issue. For this we need to dwell a little more on the theory of division problems with so-called outside options.

BASIC OUTSIDE OPTIONS

Rubinstein's alternating offer paradigm is considered as atomic or pure bargaining. There are several directions one can take with the aim of expanding the atomic form. Here we consider one such extension. In a later section, we provide a broader generalization of this route.

In atomic bargaining, players find satisfaction only through an agreement. But this is possible only in a fully deterministic world. Because bargaining proceeds through steps in time, it is conceivable that the intentions of the players may not be fulfilled and/or some new opportunities may arise.

We introduce a new move that results in the realization of the status quo. Thus a player may opt to quit the bargaining on receipt of any offer. Of course, the termination of bargaining without an agreement should lead to some payoffs. If bargaining is to take place, we shall impose some restrictions on these payoffs. A natural rough requirement is that the sum of these payoffs in pie units is no greater than the pie. For otherwise, one party may not have enough incentive to bargain at all.

Sutton (1986) suggested a generalization of this idea. Consider a random event E that may follow a rejection of an offer with a given, and commonly known probability, p. Sutton suggests two interpretations for the occurrence of E:

V. In the *voluntary* interpretation the rejecting player (i) has the option to consume an outside value of s_i. In this case the other player consumes an outside value and the bargaining terminates. If the rejecting player chooses not to consume the outside value, the bargaining clock moves one unit forward and a new demand (normally by the rejecting player) is considered.[12]

F. In the *forced* interpretation, the occurrence of E signifies the necessity to terminate the bargaining with each player consuming his or her outside values s_i. The clock does not tick.

If E does not occur, the bargaining clock simply ticks a unit. We refer to a player as an F-player or a V-player depending on the interpretation of the event E that may follow his or her rejection of an offer.

Note that the notion of outside options is orthogonal to the bargaining procedure and that, in fact, only the Rubinstein procedure has been given experimental treatment in non-cooperative bargaining.

Forced Termination

Consider two F-players characterized by the probability p of being terminated with outside status-quo payoff s_i. How should they play? We imitate the approach we have taken earlier by defining an appropriate fold game. We derive subgame perfectness from the following principles:

Interperiod Rationality. An offer will not be accepted if it dictates less utility than what one offers oneself. This will be translated into a specific operationalization, depending on the bargaining environment. Equation 2 is specific to pure Rubinstein bargaining. For an FF bargaining environment, this is translated to $u_i(1-x_j) \geq (1-p)\delta_i u_i(x_i) + pu_i(s_i)$ for $i \neq j$. The right-hand term in this inequality is the *expected prospect* of a rejection to the rejecting player. This assumes, as is currently all too common, that players obey expected utility. In principle, other utility theories can be grafted as long as they are made to be common knowledge. Thus we set the strategy set for player j,

$$X_j(x_i) = \{0 \leq x \leq 1 | u_i(1-x) \geq (1-p)\delta_i u_i(x_i) + pu_i(s_i)\} \tag{9}$$

and the formal utility for the strategy combination (x_1, x_2) to player i is $u_i(x_i)$.

Interperson Rationality. Again we look for the Nash equilibria for the fold game.

To make the fold game playable, the strategy sets need to be nonempty for all values of their argument. One sees by inspection of the definitions that this condition depends on the probability p and the outside values s_i. We define the *present value* to player i of a promise to share x one period later to be

$$PV_i(x,p) = u_i^{-1}((1-p)\delta_i u_i(x) + pu_i(s_i)) \tag{10}$$

where for an increasing function f, $f^{-1}(x) = \inf\{y | f(y) \geq x\}$.[13] We say "promise" because its fulfillment depends on the occurrence of a random event E whose probability is p. Now we assume that s_i is such that $PV_i(x,p) \leq 1$, which makes the strategy sets non-empty. This is true automatically, for example, when $s_i \leq 1$.

As above, the Nash equilibria of the fold game are of interest. They exist because the equations

$$\begin{aligned} 1 - x_2 &= PV_1(x_1, p) \\ 1 - x_1 &= PV_2(x_2, p) \end{aligned} \tag{11}$$

are equivalent to the equation

$$x_1 = 1 - PV_2(1 - PV_1(x_1, p), p). \tag{12}$$

Because the PV_i maps the unit interval continuously into itself, this equation must have a solution by elementary considerations.

Now (x_1, x_2) is supported by SPE if and only if two conditions are satisfied:

$$PV_1(1-x_2, p) + PV_2(x_2) \leq 1 \tag{13}$$

and

$$PV_2(1-x_1, p) + PV_1(x_1) \leq 1. \tag{14}$$

For example, suppose player 1 demands more then x_1. Then it is rejected by player 2's plan, and the utility of a promise of $1-x_2$ one period later is $u_1(PV_1(1-x_2,p)) \leq u_1(1-PV_2(x_2,p)) = u_1(x_1)$ (we use both mutual consistency (Eq. 13) and the fact that (x_1,x_2) is a solution of Eq. 11). Hence player 1's deviation is not profitable in the extensive game! If Player 2 accepts $x > x_1$ then he does not gain because: $u_2(1-x) < u_1(1-x_2) = u_2(PV_2(x_2))$ by Eq. 11.

A sufficient condition for Eq. 13 and 14 is $s_i \leq x_i$. This follows from

Lemma (PV_i). $PV_i(x,p) \leq \max(s_i,x)$.

The proof is a simple observation.

Corollary. $PV_1(1-x_2,p) \leq \max\{s_1, 1-x_2\} = \max\{s_1, PV_1(x_1,p)\} \leq \max\{s_1,x_1\} = x_1 = 1-PV_2(x_2,p)$.

This chain of inequalities is justified by appeal to Eq. 11 and the lemma which results in Eq. 13.

Example. Here is a case where Eq. 13 and 14 are not satisfied. Suppose the utilities are identities, $s_i = 0.9$, $p = 0.9$ and $\delta_i = 0.1$. Then Eq. 11 reduces to:

$$\begin{aligned} 1-x_2 &= 0.01x_1 + 0.81 \\ 1-x_1 &= 0.01x_2 + 0.81. \end{aligned} \tag{15}$$

Then $x_1 = x_2 = 0.188119$ but when player 1 demands, say $0.20 > x_1$, it is rejected and he or she is "promised" a little-valued future amount of $1-0.188119$ but its present value is much higher (due to the relatively high probability of receiving immediately 0.9), so a deviation is profitable. Hence (x_1,x_2) cannot be subgame perfect. This is the intuition. Plugging in the proper values in Eq. 13 and 14 shows that the conditions are not fulfilled.

Voluntary Termination

The same logic we applied to forced termination can be applied to voluntary termination with $PV_i(x,p) = PV_i^V(x) = u_i^{-1}((1-p)\delta_i u_i(x) + p\max(\delta_i u_i(x), u_i(s_i)))$. If we denote the definition of present value for F-player by PV_i^F we see immediately that $PV_i^V \geq PV_i^F$. Therefore, the intuition that *ceteris paribus*, being under a voluntary regime is advantageous to being under a forced regime is justified by inspecting the equation determining the partitions. In making this statement we imply that the logic can be applied to heterogenous players (F and V).

Note that lemma PV_i with its immediate consequence still holds and for future use we also record the following

Theorem (VV inequalities). *Suppose the players are risk averse and zero at zero (u_i are concave and $u_i(0) = 0$). Then for a VV bargaining where*

$$\delta_2 u_2(1-s_1) \geq u_2(s_2) \tag{16}$$

and

$$\delta_1 u_1(1-s_2) \geq u_1(s_1) \tag{17}$$

we have:

$$\begin{aligned} s_1 \leq 1-\delta_2(1-s_1) \leq & x_1 \leq 1-s_2 \\ s_2 \leq 1-\delta_1(1-s_2) \leq & x_2 \leq 1-s_1. \end{aligned} \tag{18}$$

Note that the conclusion implies that (x_1, x_2) is supported by an SPE strategy combination.

Proof. Note that

$$\begin{aligned} 1-x_2 \geq PV_1(x_1,p) = & u_1^{-1}((1-p)\max(\delta_1 u(x_1), u_1(s_1)) + pu_1(s_1)) \geq \\ & u_1^{-1}(u_1(s_1)) = s_1. \end{aligned} \tag{19}$$

Hence we obtain the second inequality of the second assertion of the theorem. Similarly, we obtain the corresponding inequality of the first assertion.

It follows from what we have just proved and the monotonicity of PV_i with respect to the first variable that $PV_2(x_2,p) \leq PV_2(1-s_1,p)$.

Now

$$\begin{aligned} PV_2(1-s_1,p) = & u_2^{-1}((1-p)\max(\delta_2 u_2(1-s_1), u_2(s_2)) + pu_2(s_2)) \leq \\ u_2^{-1}(\delta_2 u_2(1-s_1)) \leq & u_2^{-1}(u_2(\delta_2(1-s_1))) \leq \delta_2(1-s_1), \end{aligned} \tag{20}$$

which follows because of risk aversion and Eq. 16.

Therefore, by Eq. 11

$$x_1 = 1 - PV_2(x_2,p) \geq 1 - PV_2(1-s_1,p) \geq 1-\delta_2(1-s_1). \tag{21}$$

Note: The conditions on the utilities required by the theorem are satisfied for the identity utilities, which make the theorem useful in experimental work.

Arithmetic Depreciation with Outside Options

We return now to the experimental setting. Weg and Zwick (1994) and Zwick and Weg (1996) experimented with exponential utilities and side options. Their general setup is especially simple. The probability of access to an outside value is always 1 and it is voluntary for each of the players. Thus outside options are always present when an offer is considered by a recipient. What makes the predicted outcomes especially simple is the choice of fundamental utilities for money — $u_i(x) = \exp(x)$. The solution for the fixed points is simple.

We use Eq. 11 with the proper understanding that $PV_i(x,1) = PV_i^V(x,1)$. We see that Eq. 11 is equivalent to

$$x_2 = 1 - \max(x_1 - c_1, s_1) \tag{22}$$

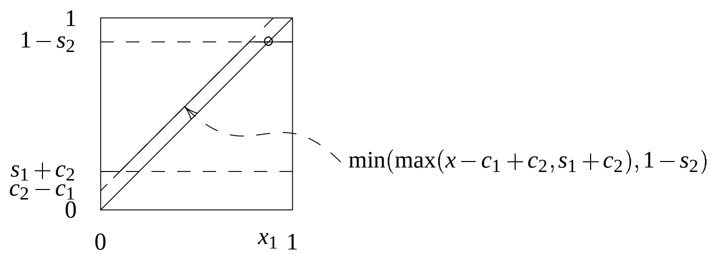

Fig. 11.3. Arithmetic depreciation, $c_1 < c_2$.

and

$$x_1 = 1 - \max(x_2 - c_2, s_2), \tag{23}$$

where $c_i = -\log(\delta_i) \geq 0$. It is only in the special case of $p_i = 1$ that we can represent the utility in logarithmic form, thereby the arithmetic depreciation of the identity utility by the fixed cost c_i. This is in fact the terminology used by Weg and Zwick (1994).

The fixed-point equation is easily shown to be

$$\min(\max(x - c_1 + c_2, s_1 + c_2), 1 - s_2) = x. \tag{24}$$

To make the solution of the bargaining problem a little more interesting, we assume that $s_1 + s_2 + \max(c_1, c_2) < 1$. This condition implies mutual consistency of the solution, as required. Now the predicted payoff to Player 1 is

$$x_1 \begin{cases} 1 - s_2 & \text{if } c_1 < c_2 \\ s_1 + c_2 & \text{if } c_1 > c_2 \\ [s_1 + c_2, 1 - s_2] & \text{if } c_1 = c_2. \end{cases} \tag{25}$$

Figure 11.3 shows the graphic solution. A simple way to conceive this result is given by this rule: *Every bargainer claims his or her outside option and bargains over what's left* (without outside options). This rule has a significant heuristic value. It fact, it corresponds to individual rationality, which is well known in the theory of games in characteristic function form. Every solution concept suggested for that domain satisfies this requirement.

The main purpose of using exponential utilities with fixed discount rates (or what amount to the same thing, identity utilities with fixed costs of depreciation) is to present fairness considerations in a different light. It is particularly suitable for experimentation because as we have seen, the bargaining scene is very simple and must be understood that way, given the "good" behavior under the nonoutside option regime (Rapoport et al., 1990; Weg & Zwick, 1991).

Although the standard economic method employs the ultimatum game that was popularized by Güth (see Güth & Tietz, 1990), Weg and Zwick (1994) opted for an alternative. Recall that an ultimatum is a single period game and therefore using the standard temporal accounting where the present is never discounted, discounts are irrelevant and bargaining is reduced to a single offer that Player 2 is entitled to either accept or reject. In the latter case, the game ends with some predefined status quo. Normally the ultimatum is normalized to have outside options equal to zero. It is obvious that rational offers should amount to nothing (assuming a continuous pie). What Güth et al. (1982) found is that most offers settle on the midpoint of the pie. Their interpretation is that Player 1 normally shows a taste for fairness. This interpretation was given additional support by Kahneman et al. (1986) who, for that purpose, invented the dictator game, an ultimatum where Player 2 is relegated to merely an observer. But naturally, the fairness interpretation attributed to Player 1 was contested with the alternative, which attributes asymmetric fairness component to Player 2's utility. This is postulated to be evident to Player 1 who is as greedy as can be and merely optimizes payoffs by reducing the risk that a small offer might be rejected. Supporters of this alternative hypothesis explain Kahneman, Knetsch, & Thaler's (1986) generous dictators as merely desirous of the experimenter's good will, and therefore imply that dictator experiments are likely to produce artifactual results.[14]

Weg and Zwick (1994) considered arithmetic depreciation a complementary if not better arena to test for fairness. Consider first the analog to the standard dictator given by a bargaining game without outside options and where $c_1 < c_2$. The predicted payoff allocates the whole pie to Player 1 in the first period. Thus the outcome is the same as predicted for the dictator game (as well as the ultimatum game), but without allowing for early termination. Player 1 is the omnipotent player who can, if he or she so chooses, deviate from the rational dictum and offer symmetric allocations. Next, consider the analog to the ultimatum game. Here we take the arithmetic depreciation again but with the provision of outside options of *zero* to each of the players. Again the predicted allocation is the same as in the dictator analog, except that now Player 2 has the same option as Player 2 in the ultimatum game — refuse a small offer by opting out to obtain even a smaller payoff of zero. Because Player 1 in the dictator analog was found to be a highly greedy player, unaffected by his or her appearance to the experimenters (Rapoport et al., 1990; Weg & Zwick, 1991), any mitigation of demands in the ultimatum analogs is attributed to the fear that Player 2 will act on a threat of quitting. This expected behavior was in fact tested by Weg and Zwick (1994). Figure 11.4 is a conceptual schema of the various games and their interrelationships.

This experiment is concerned with the division of pies of \$20.00, with a cost set of $\{0.1, 2.00\}$. Some bargaining games allow for opting out with zero payoffs and some do not. Although the bargaining is conceived as unlimited in time, in practice, a game is terminated if the negotiation reaches the fourteenth period, which in fact occurred only twice in 216 plays. The experiment has a $2 \times 2 \times 3$

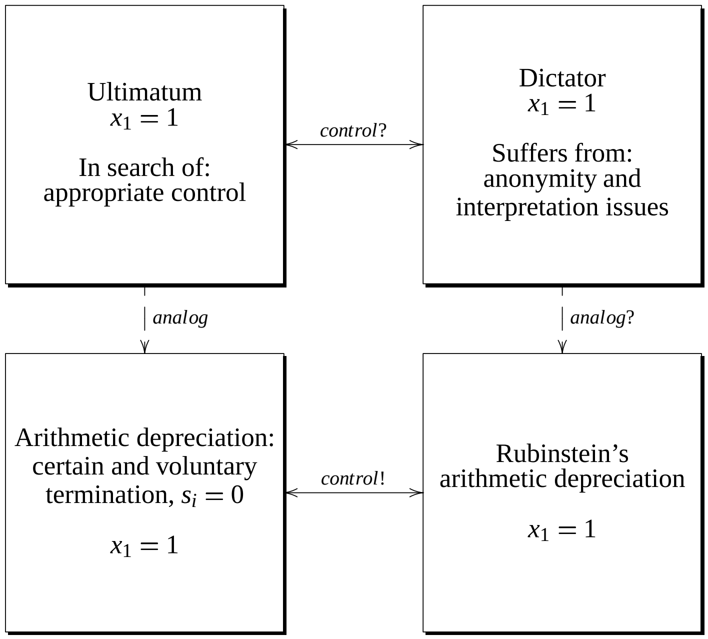

Fig. 11.4. Schematic relationships among games in the search for fairness.

factorial structure consisting of game type (with or without outside option), costs pattern ($c_1 < c_2$ or $c_1 > c_2$), and iteration (whether the first mover holds this role for the first, second, or third time). The last two factors are of the within-subject type, which means that during a single session, subjects play only one of the game types. For our purposes here, our main interest lies with the condition $c_1 < c_2$.

Figure 11.5 presents the frequency distribution of the last offers to the strong player (the one with the smaller cost) in the first and third (last) iteration by game type (with access to null outside options and without). Some summary statistics for this experiment are shown in Table 11.2.

Although the cost-based weaker player seldom exercises the option to opt out (3 times out of 108 games), the mere availability of this option is sufficient to deter the cost-based strong player from high demands. Thus, the main hypothesis that sharing in competitive environments is less affected by fairness considerations than by the threat of lost opportunities is supported, but the extreme greed shown by Rapoport et al. (1990) and Weg and Zwick (1991) failed to materialize in this experiment. A possible explanation for this discrepancy is suggested later. Nonetheless, the principal contribution of this setup is in providing an economic test bed to the hypothesis. In particular, any behavior in a dictator setup is inherently confounded. It is analogous to a boxing match with one contestant fighting

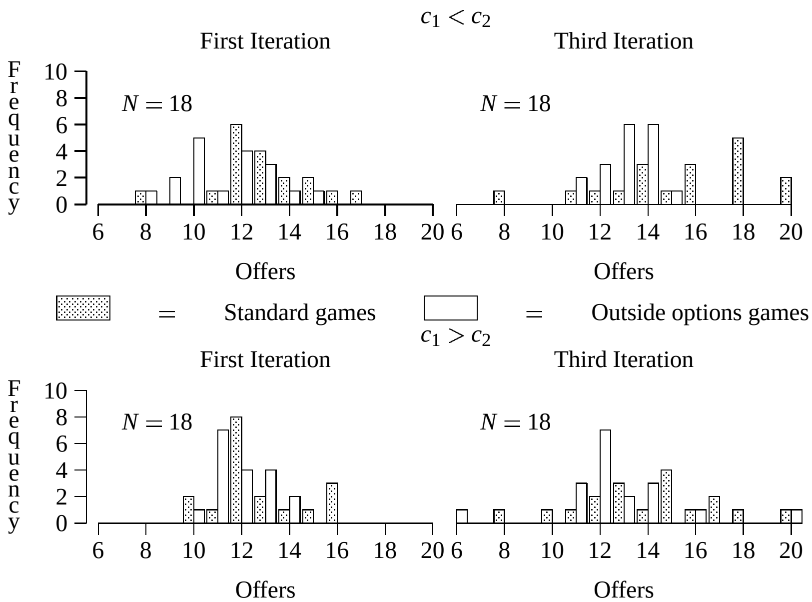

Fig. 11.5. Last period offers to the strong player.

with his arms tied down. [15]

Geometric Depreciation With Outside Options

Several studies were conducted under a richer format than the original Rubinstein's paradigm with several intentions in mind. Some like Binmore, Shaked, and Sutton (1989) and Binmore, Morgan, Shaked, and Sutton (1991) were motivated perhaps to demonstrate the sensitivity of subjects to the structure of bargaining. Bargaining outcomes in these studies depend on the precise specification of the available moves. Thus, Nash's (1950) bargaining solution, which is derived from axioms, would not be applicable predictor for the bargaining procedures employed in these studies.

On the other hand, Zwick, Rapoport, & Howard's (1992) experiment was guided by the formal similarity between discounting and the probability of continuation of the bargaining. And finally, Weg, Zwick, and Rapoport (1996) and Kahn and Murnighan (1993) explored the applicability of rational bargaining with outside options under somewhat less focal predictions. In fact, their games, especially Kahn and Murnighan's, might be described as games in a reasonably general position. Nonetheless, as we shall see, they all carry a significant heuristic as to the relative power of the players, and thus provide clues to reasonable behavior.

Table 11.2
Upper Quartiles of First and Final Offers to the Strong Player (Proportions)

Iteration	Costs Pattern	Game Type	First Period	Final Period
1	$c_1 < c_2$	No Quit	0.80	0.70
		Quit	0.70	0.62
	$c_1 > c_2$	No Quit	0.60	0.70
		Quit	0.60	0.61
2	$c_1 < c_2$	No Quit	0.80	0.75
		Quit	0.70	0.67
	$c_1 > c_2$	No Quit	0.70	0.70
		Quit	0.60	0.65
3	$c_1 < c_2$	No Quit	0.90	0.90
		Quit	0.70	0.70
	$c_1 > c_2$	No Quit	0.75	0.80
		Quit	0.60	0.70

Note. pie 20, $c_i \in \{0.1, 2.0\}$

Deal Me Out (DMO). Binmore et al. (1989) set out to show that the conventional wisdom of evenly sharing the leftover after accounting for outside options (obtained if bargaining fails to reach an agreement) is not always a reliable predictor of behavior. For this they exploit the *VV* bargaining procedure with $s_1 = 0$, $s_2 = \{0, 2/7, 4/7\}$ with identities as utilities. Note that the conditions for theorem *VV inequalities* are satisfied in this case and therefore solutions for Eq. 11 are subgame perfect. They hold regardless of probabilities and discounting quotients!

Conventional wisdom would lead us to predict that player 1's share will be $\frac{1+s_1-s_2}{2}$. This is reasonable, perhaps, if one does not specify the negotiation procedure. But applying Eq. 11 we see that the Nash equation is reduced to

$$x_2 = 1 - \max(\delta_1 x_1, s_1) \tag{26}$$

and

$$x_1 = 1 - \max(\delta_2 x_2, s_2). \tag{27}$$

Hence, by substitutions we need to solve the fixed points for

$$1 - \max(\delta_2(1 - \max(\delta_1 x, s_1)), s_2). \tag{28}$$

Simplification shows that we need to solve

$$\min(\max(1 - \delta_2 + \delta_2\delta_1 x, 1 - \delta_2(1 - s_1)), 1 - s_2) = x. \tag{29}$$

We assume that $\delta_i(1 - s_j) \geq s_i$. From this we conclude that bargaining would continue as usual if and only if

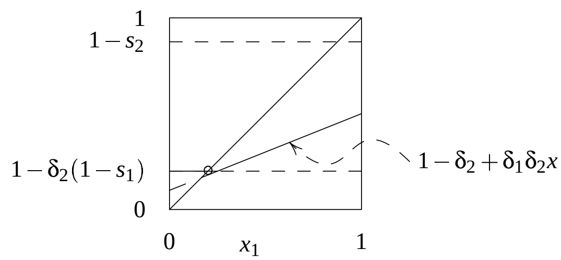

Fig. 11.6. Extreme case of VV solution.

$$1-\delta_2(1-s_1) \leq \frac{1-\delta_2}{1-\delta_1\delta_2} \leq 1-s_2 \qquad \text{(See Fig. 11.6).} \tag{30}$$

Consequently, player 2 does not bargain if bargaining results in an offer of less then s_2 and player 1 demands $1-\delta_2(1-s_1)$ if bargaining results in a smaller payoff. Bargaining here means that players act as if outside options are not available (or they are zero). Otherwise, outside options are too small to make an effect. Note that we have met this situation in a different guise: arithmetic depreciation with outside options.

By choosing a common discount quotient of 0.9 and $p = 1$, Binmore et al. (1989) made the expected bargaining outcome (without outside options) be $0.526 = \frac{1}{1+0.9} \approx 0.5$. They provide experimental evidence that for the two smaller values of s_2, there is a noticeable concentration of offers at about this point, whereas when s_2 is larger than half, the concentration is shifted to about $3/7$ (see Fig. 11.7). This is exactly what is expected when the bargaining procedure, which proceeds in a very well-specified manner, is taken into consideration. Note that the conventional Nash solution is predicted to result in significant increases in payoff to Player 2 (for the nonnull outside options), which is not the empirical case. We come back to this explanation later.

Forced Termination. In an attempt to partially replicate Weg et al. (1990) without the possible drawback of finite implementation of an infinite game, Zwick et al. (1992) substituted probabilities of termination (or rather of continuation) for discount quotients. Their design can be viewed as an FF bargaining with no time devaluation but with probability $p > 0$ and $s_1 = s_2 = 0$. Thus, the problem of termination is built into the design. Using Eq. 11 we see that

$$1-x_i = (1-p)x_j. \tag{31}$$

By setting $\delta = 1-p$ we see that in the bargaining problem with forced termination

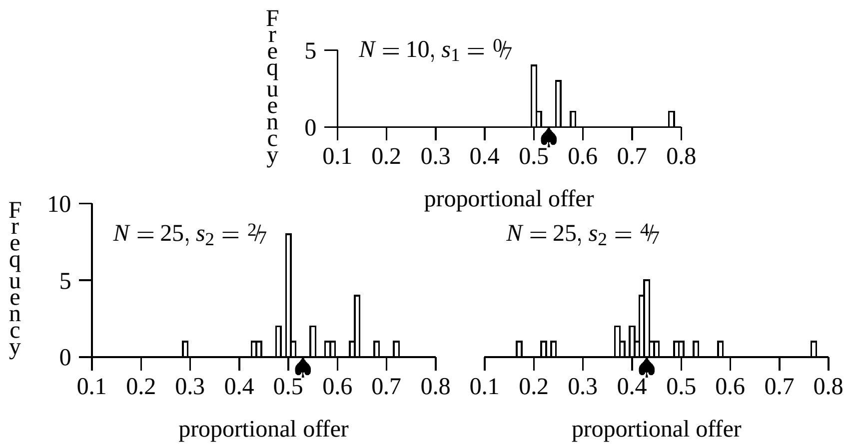

Fig. 11.7. First period demand by player 1. A ♠ marks rational demand.

the probability of continuation can formally be seen as the discount quotient, and therefore rational behavior is predicted by the same formulas as given by the corresponding discount version (Eq. 3). Zwick et al. (1992) experimented with these values of p: $5/6$, $1/3$, and $1/10$ and \$30 pies. Their results are presented in Fig. 11.8 and can be simply summarized by a dictum: equal split (plus a correction favoring the first mover). They follow a similar pattern found in Weg et al. (1990). Note that for high values of p we have an approximate ultimatum, and in this light, fairness considerations might be in force. We discuss possible interpretations of this deviation later.

Split the Difference (STD) vs. DMO — Procedural Implementation. Binmore et al. (1991) can be viewed as an extension of both Binmore et al. (1989) and Zwick et al. (1992). First recall that DMO can be stated as "allocate your opponent a side option unless he or she can do better by bargaining." Binmore et al. (1989) showed that the bargaining part can be given a precise meaning, as in Rubinstein bargaining with having always accessible outside options that are not taken. But the outside option idea is more versatile.

Note that with the Rubinstein's bargaining paradigm, players can be made symmetric only at the limit point, when time is irrelevant. (Of course, at that point the rational outcome of the equal split fails to be unique.) Now, the next step is to ask whether one can obtain by bargaining (in the limit) an equal split, after status quo values (s_1 and s_2) are paid; that is, Player i is paid $\frac{1+s_i-s_j}{2}$. We recall that formally we can take discounts to be interpreted as probability of continuation with $s_i = 0$. Therefore, when $p_i \to 0$ the players share the pie equally. Zwick et al. (1992) experimented with this framework except that their probabilities are never

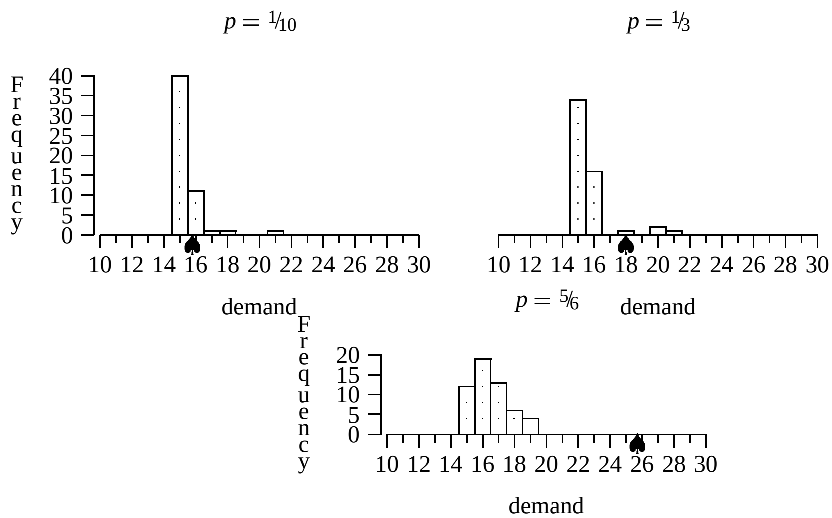

Fig. 11.8. First period demand during last. A ♠ marks rational partition.

close enough to zero. This intuition leads to the prediction that, quite generally, rational payoffs for a nonzero outside option game under an FF regime are split the difference, when p is small enough. This is shown more formally in the next two paragraphs.

Assume that $s_1 + s_2 < 1$, $\delta_i = 1$, and utilities are identities. According to Eq. 11

$$x_2 = 1 - ((1-p)x_1 + ps_1) \tag{32}$$

and

$$x_1 = 1 - ((1-p)x_2 + ps_2). \tag{33}$$

Therefore

$$x_1 = \frac{p(1-s_2) + (1-p)ps_1}{1-(1-p)(1-p)}. \tag{34}$$

Multiplying the above by $1 = \frac{(1/p)}{(1/p)}$ for $p \neq 0$ we see

$$x_1 = \frac{1 + s_1(1-p) - s_2}{2-p}. \tag{35}$$

Taking the limit $p \to 0$ we obtain STD payoffs: each player gets an outside option plus half the remaining interval. Thus, STD as conventional wisdom can be approximated by procedural bargaining. Note that $x_i \geq s_i$ for a small enough p. Thus according to the corollary to lemma PV_i, the solution to Eq. 11 is in fact supported by SPE.

Table 11.3
The Design of Binmore et. al (1991) Study

	High — $s_2 = 0.64$			Low — $s_2 = 0.36$	
	δ	p		δ	p
VV	0.9	1	VV	0.9	1
FF	1	0.1	FF	1	0.1

Does it work in practice? An affirmative answer is the essential claim of Binmore, Morgan, Shaked, & Sutton's (1991) research. They arrange for the play of four types of games, all sharing a common impatience coefficient (discount quotients interpreted as risk values or vice versa), which are classified as shown in Table 11.3 (outside values are normalized to unit pies).

The outside option to Player 1, s_1, is negligible and set to 0.04 for all games. Pies are £5 sterling apiece. As common with equal parameter games, pies and outside options shrink over time whenever impatience is a discount quotient. The main findings can be detected in Fig. 11.9. Referring to Player 2's high outside option condition, Binmore et al. (1991) wrote:

> It is not surprising that 50 : 50 does not do well when player 2 can get 64% without the consent of his partner, but it is instructive that S-T-D predicts *very much better* than D-M-O in forced breakdown games, while D-M-O predicts *better* than S-T-D in optional breakdown games. (p. 304; italics added)

Unfortunately, this beautiful result is not replicated so well when Player 2's outside option is low.[16] Note also that STD, when outside options are zero, is also equal split. Zwick et al. (1992) showed that equal splits are typical *regardless* of the probability of access to the outside option.[17] Thus, the finding of splitting differences in the limiting case of FF games might reflect a general tendency to ignore the effects of time. The fact that DMO is not seen in FF games is obviously due to the very meaning of voluntary exit (which is not available), and the fact that the forced probability of exiting is behaviorally irrelevant! But theoretically, STD depends on low probabilities and Binmore, Morgan, Shaked, & Sutton's (1991) satisfaction over their subjects' good behavior is perhaps not completely justified.

Outside Options — Middle Range Cases. Two other studies followed the path pioneered by Binmore and his associates in the studies just reported.

The research by Weg et al. (1996) is a direct descendant of Binmore et al. (1991). It compares playing VV games to playing FF games where probabilities of access to outside options are *not* boundary values — $p \in \{0.2, 0.8\}$. These events are realized by the spin of an actual wheel of fortune shown to the bargainers. It allows, therefore, the testing of the prevalence of STD where the normative

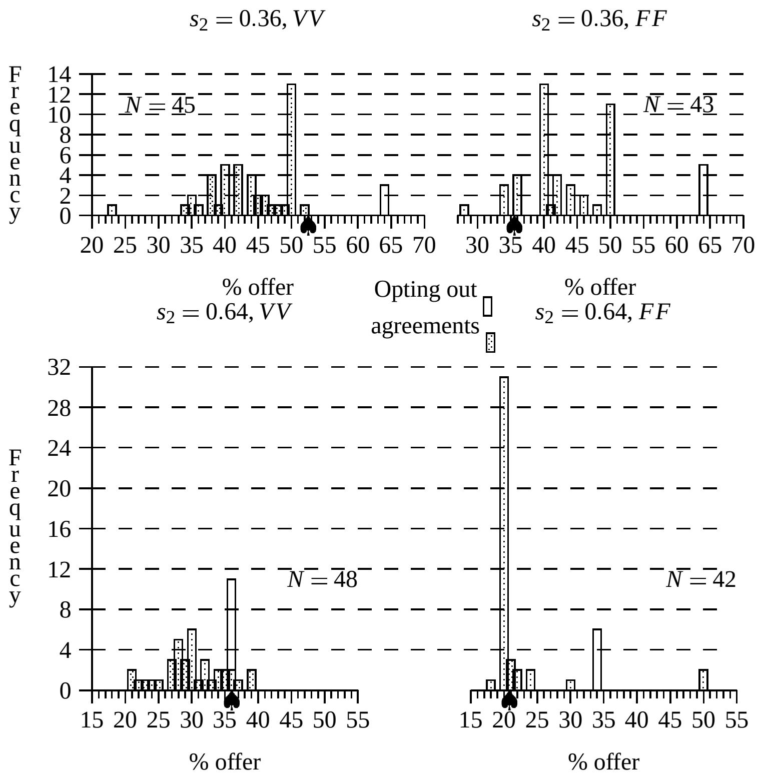

Fig. 11.9. Final demands for Player 1. A ♠ marks rational demand.

point of view forbids it. Subjects paticipated in both types of games in the the same session. The paradigm is a shrinking pie where all pies start with a relatively large sum of \$30.00. Because the shrinking rate is uniformly set to 0.9, games might extend slightly but meaningfully longer in time without dealing with negligible pies. Only Player 1 has a nonzero outside option — one of $\{3, 12, 24\}$ in dollars.[18] Probablities and outside option values do not vary within an experimental session.

Figure 11.10 presents the frequency distribution of first period demand by the main parameters of the experiment. The fact that, for the more interesting parameters, average splits do not support theoretical predictions is perhaps understandable. The best one can say about the results is that first period demands are in general monotonic with the rational demands. Also, general qualitative predictions that players 2 in FF games are worse off than their counterparts in *VV* is borne out. But again, rational outcomes are almost never attained and actual mean

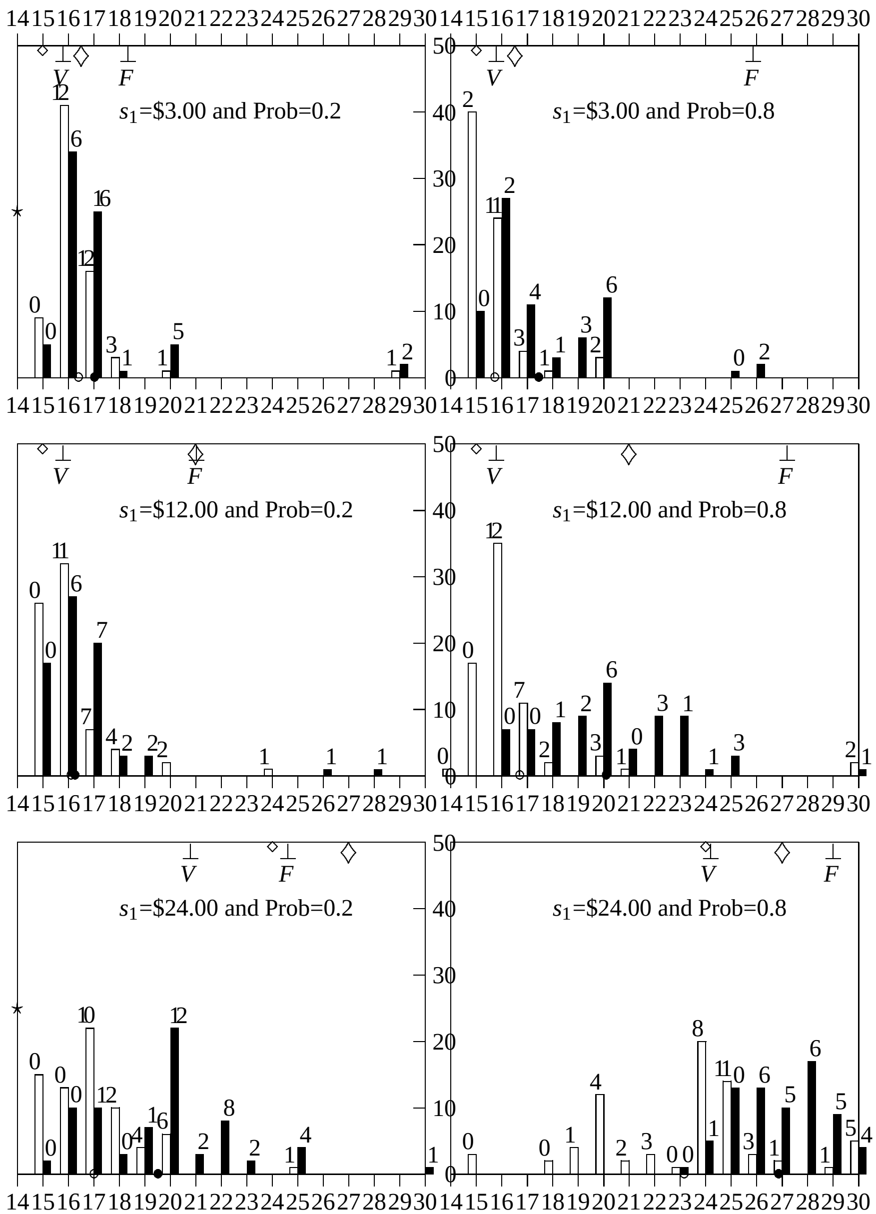

Fig. 11.10. Frequency distribution of first period demands by side value, probability and game type. $\underset{V}{\perp} \Rightarrow SPE$ for VV, $\underset{F}{\perp} \Rightarrow SPE$ for FF, $\Diamond \Rightarrow$ S-T-D, and $\diamond \Rightarrow$ D-M-O. $\blacksquare \Rightarrow FF$, $\square \Rightarrow VV$.Numbers are rejection counts. A $\star \Rightarrow$ uncounted point of less then 14 in a VV game. $\bullet \Rightarrow FF$ mean demand. $\circ \Rightarrow VV$ mean demand.

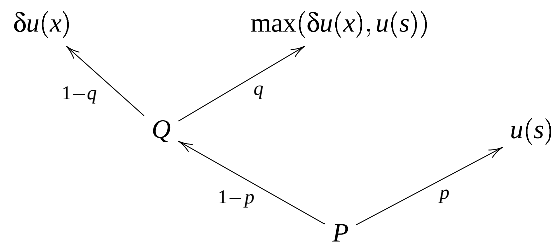

Fig. 11.11. A tree diagram of bargaining termination rules.

behavior is very conservative. The effects of time (or probabilities of termination) are underestimated.

Mixed Options. The research by Kahn and Murnighan (1993) presented an experiment with mixed features of voluntary and forced termination in a single game. In order to provide an appropriate framework for this experiment, we present a general framework that encompasses all bargaining procedures we have discussed so far under a single paradigm. In fact, there are two such (inconsistent) extensions that happen to coincide in the special case of Kahn and Murnighan.

Extension A. When a player receives an offer it may be accepted or rejected. In the former case, the game ends as usual. Otherwise, the game continues as follows. There are two stochastically independent events P and Q. If P occurs, the game terminates immediately with each player consuming an outside option. Otherwise, if Q occurs then the player may announce the immediate termination of bargaining with each of the players consuming an outside option or opt to counterpropose in the next bargaining period. If Q fails to occur, the player counterproposes in the next period.

There are two extreme cases that we have already treated. Assume that P is the impossible event. Then, obviously, we have a voluntary option depending on the occurrence of Q. If Q is the impossible event then we have a forced termination depending on the occurrence of Q. Figure 11.11 depicts the situation with labels attached to the terminal nodes describing the utility of reaching these nodes.

Consider now the standard Rubinstein bargaining tree. To each rejection branch one attaches the tree diagram to obtain the bargaining tree for a general bargaining game with outside options.

In this manner we have unified the existing bargaining schemes into a cube whose dimensions are F, V, and D. A point in the cube and a point in the unit interval[19] is a choice of parameters p, q, δ, and s, respectively, which with the

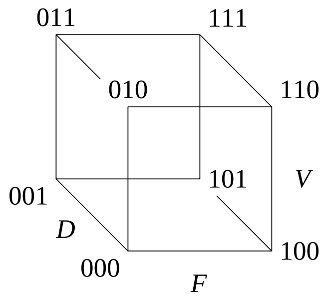

Fig. 11.12. The bargaining regime cube.

addition of a utility u defines a bargaining player. A choice of two such quintets, $P^i = (\delta_i, p_i, q_i, s_i, u_i)$, and an order between them defines a bargaining game (See Fig. 11.12), denoted by the signature (P^1, P^2). This unification facilitates a comparison of plays of bargaining games characterized, perhaps, by slightly different parameters. This is so because the games generated in this way map continuously into the predicted payoffs to Player 1.[20] In this manner we see that the FF games and VV games are not separate entities, as could have been assumed, but rather members of a larger family of hybrid games characterized by events that are peculiar to each of the bargainers.

To solve this type of game we need our standard technique in which we have to define only the appropriate *present value* functions. But these are simple extensions to the voluntary and forced termination paradigms:

$$PV_i^{P,Q}(x, p_i, q_i) = PV_i(x, p_i, q_i) = u_i^{-1}((1-p_i)(1-q_i)x + (1-p_i)q_i \max(u_i(x), u(s_i)) + pu_i(s_i)) \tag{36}$$

that are continuous when u_i are. Again we fold the extensive game and focus on its Nash equilibria that satisfy Eq. 11. Now the conditions in Eq. 13 and 14 look as follows:

$$PV_1(1-x_2, p_2, q_2) + PV_2(x_2, p_2, q_2) \leq 1 \tag{37}$$

and

$$PV_2(1-x_1, p_1, q_1) + PV_1(x_1, p_1, q_1) \leq 1 \tag{38}$$

for any solution (x_1, x_2) of Eq. 11. And we note the indices. Player i would not demand more than x_i, the solution of Eq. 11, if and only if Eq. 37 and Eq. 38 are satisfied and the expectation from a "promise" of $(1-x_j)$ depends on j's probabilities! Lemma PV as well as its corollary still holds in this extended case. Thus, when $x_i \geq s_i$ for a solution (x_1, x_2) of Eq. 11, the latter must be supported by SPE.

Extension B. There is no reason to arrange the events P and Q in this order. Their reversal results in a gamble with, in general, different expected utility. One case in which the two schemes coincide is the case where the event Q is the sure event.

For then

$$PV(x,p,1) = u^{-1}(\max((1-p)\delta u(x) + p_i u(s), u(s))) \tag{39}$$

under scheme-A, and

$$PV(x,p,q) = u^{-1}(q(\max(u(s),(1-p)\delta u(x)) + pu(s)) + (1-q)((1-p)\delta u(x) + pu(s))) \tag{40}$$

under scheme-B. A substitution for $q = 1$ in the latter gives the result.

Very interesting cases arise by the choice of extreme values for some parameters and among those is the case where $q_i = 1$ and $0 < p_i < 1$, which was explored by Kahn and Murnighan (1993). The only asymmetry allowed among the players is reflected by the outside options ratio the experimenters choose — ∞, which means that only one player has a nonzero outside option. In a sense, their experiment is one of several natural continuations of the other experiments reported. The novelty lies in the mixing of voluntary outside options weighted by a certain risk (high or low) of forced termination.

Kahn and Murnighan (1993) opted for games of scheme-B semantics with common termination probability p, common voluntary exit probability $q = 1$, and a common discount quotient δ. One player has zero outside option and the other may opt for 0.1 or 0.9 of the pie in any given game. These options are available (in two different games) in both orders to Player 1 and 2, respectively. The games are implemented by the shrinking pie method and information exchange between players is by human messengers. Random events are realized by coin tossing or chip drawing. All game pies are \$10 at the start of the negotiation. In addition to these parameters, other game parameters are all elements from the product of these sets: $F = \{0.05, 0.5\}$ for probabilities of forced terminations, and $D = \{1, 0.8\}$ for discount quotients.

Two last remarks regarding the design are in order. First, when Player 1 has a nonzero outside option, he or she may leave the bargaining before even giving an offer. Second, because the zero outside option is not normatively effective, the experimenters have opted to express it by not allowing any voluntary opting to its owner. This makes the framing of bargaining a bit more natural.[21] In the language of this research, the probability q could in fact be any value.

Table 11.4[22] presents the subgame perfect prediction and mean first period offers for the various combination cells of the experiment. Admittedly, the table is complex. But a glance at Fig. 11.13 reveals almost all the reader may need to know. It plots the mean first-period demand on the predicted SPE demand. Note that subgame perfectness is rare, and that subjects overdemand when SPE

Table 11.4
Mean First Period Demand (Slanted) and SPE (Upright)

			$q=0$		$q=1$	
p	δ	s	**Rich**	**Poor**	**Rich**	**Poor**
0.05	1	0.1	*0.6500*	*0.6500*	*0.5600*	*0.6200*
			0.5615	0.4615	0.5615	0.4615
		0.9	*0.8500*	*0.6200*	*0.9200*	*0.2500*
			0.9513	0.0513	0.9513	0.0513
	0.8	0.1	*0.5700*	*0.5500*	*0.5700*	*0.6300*
			0.5772	0.5563	0.5772	0.5563
		0.9	*0.5700*	*0.5100*	*0.8400*	*0.2500*
			0.6491	0.4616	0.9240	0.1000
0.5	1	0.1	*0.5300*	*0.6300*	*0.5600*	*0.6400*
			0.7000	0.6000	0.7000	0.6000
		0.9	*0.8100*	*0.4800*	*0.9300*	*0.3000*
			0.9667	0.0667	0.9667	0.0667
	0.8	0.1	*0.5900*	*0.5900*	*0.6200*	*0.6100*
			0.7381	0.6548	0.7381	0.6548
		0.9	*0.8500*	*0.4900*	*0.9100*	*0.2900*
			0.9286	0.1786	0.9600	0.1000

Note. SPE's are significant to at least three digits (the minimum necessary to distinguish between the cells) and observed means are rounded to two digits. The rich player is the one with $s > 0$. Columns classify first movers.

indicates small values, underdemand on high SPE values, and are insensitive in the middle range.[23] It is also misleading to quote the Pearson correlation coefficient of 0.86 here. With the range of predicted values as it is, very approximate behavior is sufficient to induce this artifact.

Abstract Vs. Potential Outside Options. "To accept or not to accept" is the fundamental question for one of the parties at any given period and, of course, by implication, "what an is acceptable demand" to the other. Naturally, it depends on the available alternatives. But what are they? In Rubinstein's formulation, the single option is to initialize the bargaining at the next period. The outside options

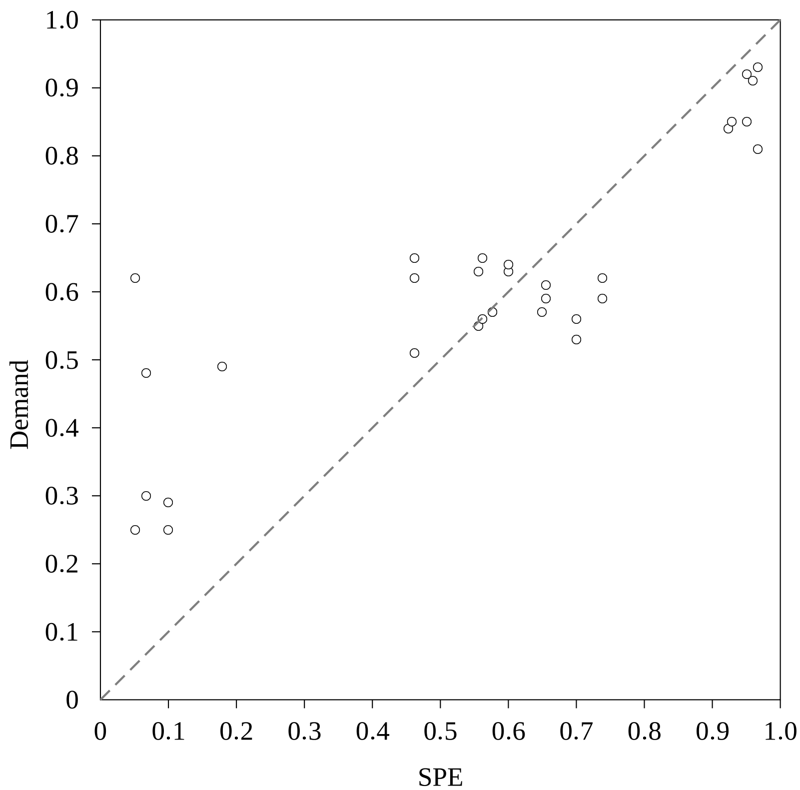

Fig. 11.13. Observed mean first period demand on subgame perfect predictions.

---

add certain other alternatives to the list. These alternatives should be looked at as encapsulations of certain situations that are expected to net a present utility.

We have seen some more or less reasonable bargaining behavior when such outside options are present (Binmore et al., 1989, 1991; Kahn & Murnighan, 1993; Weg et al., 1996). Zwick, Rapoport, and Weg (1996) tested the hypothesis originally made in Weg et al. (1996) that "correct" behavior does not necessarily reflect the centrality of SPE rationality in players' considerations, but is rather due to marginal cues provided in these studies, which in turn narrow the acceptable outcomes significantly. For example, consider Kahn & Murnighan's (1993) games where one player's outside option nets 90% of the pie. Any demand, under many experimental conditions, cannot be too far from the rational demand.

For another example, consider the bargaining problem with signature (P^1, P^2)

where $P^1 = (1, 0.1, 0, 0.64, 1)$ and $P^2 = (1, 0.1, 0, 0, 1)$ investigated by Binmore et al. (1991). This is an FF game with 0.64 outside option to Player 2 and a relatively small probability of the game terminating after any given rejection. Thus, the cost of rejection to the receiving party is rather small. Therefore, the players are made almost *symmetric* with respect to the remaining pie after Player 2 is immediately paid the outside option. Therefore, STD is expected. Of course, for the normative theory to result in a unique prediction, some uncertainty with respect to continuation is necessary and this happens to coincide with the idea that in case of insufficient reason, one tends to argue for the equalizing of treatments. However, when one stays away from the limit cases, either because the discount quotients are steep or because the probability of termination is high, bargaining is greatly affected by time.

We know from Zwick et al. (1992), Weg et al. (1990), and Kahn and Murnighan (1993) that people are highly conservative and lack an appreciation for these effects. Weg et al. (1996) conjectured that when outside options are given in their unencapsulated form, for example, as outside bargaining options, their effects would be washed away. To test this hypothesis, Zwick et al. (1996) considered a game Λ defined recursively as $\Lambda = (P^1, P^2)$ where $P^1 = (\delta, 0, 1, \Lambda, 1)$ and $P^2 = (\delta, 0, 1, 0, 1)$.[24] That is, although Player 2 has a voluntary outside option valued at 0, Player 1 is entitled to opt out to play a bargaining game of the same type with another player. The semantics given to this game are as follows. There are three players, one seller and two buyers. The seller sells a product valued by the seller at \$0 and to each of the buyers at \$10. The selling price and price saving are discounted by δ for every participant. The bargaining starts with the seller offering a price to a buyer of choice. The buyer may accept and the bargaining terminates, opt out and receive nothing while the seller starts the same game with the other buyer (the bargaining clock *does* tick), or reject the offer in order to make a counteroffer to the seller in the next period. The seller then can accept immediately, opt to restart the bargaining with the other buyer at the next period by making him or her a price offer, or reject and make a counteroffer to the same buyer in the next period.

What should reasonable bargainers do? Of course, the outside options are irrelevant! For the buyers an outside option is certainly a nongaining advantage and therefore might as well be ignored. And the seller must be indifferent between either of the buyers and therefore expects to obtain the same as when playing against a single buyer. Therefore, reasonable people are not impressed by the enriched situation. The price is set by the logic of a 2-person Rubinstein paradigm — $\frac{1}{1+\delta}$ to the seller and the rest to one of the buyers.[25]

But the data tell a different, nonetheless familiar story (recall Weg & Zwick, 1994). We do not cover the full design of Zwick et al. (1996) here. They compared standard Rubinstein's bargaining to bargaining under the bargaining rule defined by Λ with two between-subject discount quotients: $1/6$ and $2/3$. The observed first period prices when the seller has the option to switch buyers are sig-

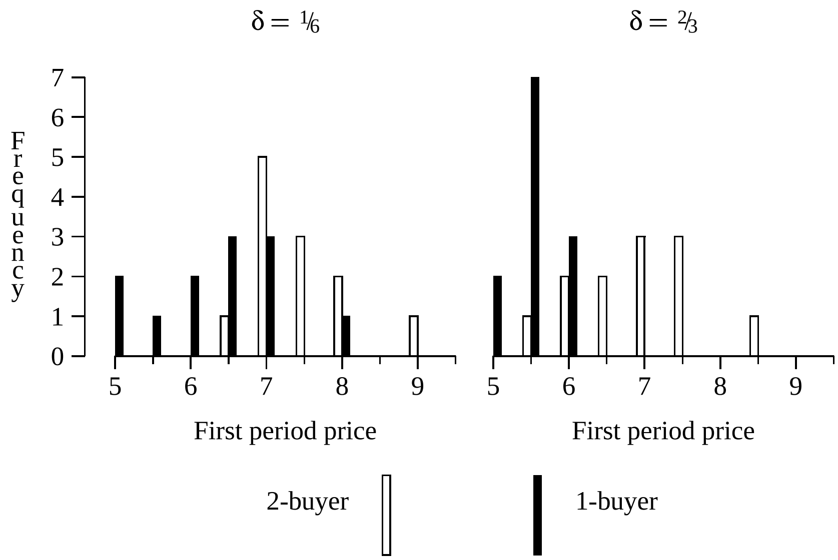

Fig. 11.14. Mean first period prices.

nificantly higher than when he or she does not, contrary to normative expectations (Fig. 11.14). Further, demands in the 1-buyer game are not predicted by rational behavior (note the distinctly modest demands for $\delta = 1/6$). This aspect is a replication of Weg, Rapoport, & Felsenthal's (1990) results.

DISCUSSION

Our conclusions are rather simple, although yet very tentative: People are reasonable within their cognitive limitations and moral constraints. This does not mean that they behave rationally according to point specifications. No one expects them to. Rather, people respond to changing bargaining conditions, in general, in the right directions. This is clear, for example, by recalling that:

i. Voluntarily terminated Player 2 is in general more powerful than a forcibly terminated Player 2 with the same time and utility preferences (Weg et al., 1996).
ii. Under well-chosen parameters, DMO is observed under voluntary termination and STD under forced termination (Binmore et al., 1989, 1991). We also observe the limitation of those rules (Kahn & Murnighan, 1993). The mental accounting for this is the same as for the previous item.
iii. Arithmetic depreciation is, sometimes, understood and acted on very accu-

rately (Rapoport et al., 1990; Weg & Zwick, 1991).

But these rules are easily foiled by well-chosen red herrings. Thus, ultimatum-like game situations attenuate demands (Weg & Zwick, 1994) and implementing outside options as actual bargaining bolsters them (Zwick et al., 1996). In both cases, some seemingly strategic options take center stage in human bargaining behavior.

And finally, there is a cue whose effects can increase or diminish strategic accuracy depending on well-specified circumstances as follows. Nothing, of course, changes in the preference structure of a Rubinstein bargaining game when time is taken into account from the very first period.[26] This rule was adopted by Rapoport et al. (1990) and Weg and Zwick (1991), but the customary rule was followed by Weg and Zwick (1994) in an arithmetic depreciation bargaining game. More extreme demands were found with the immediate depreciation rule. It appears that the advantage of the strong player (particularly when moving first) is more easily seen when the effects of delays in coming into an agreement are immediate. On the other hand, Weg et al. (1990) showed that this immediate depreciation has quite a different effect when implemented geometrically. Subjects seem to solve the equation $x_1\delta_1 = (1 - x_1)\delta_2$ and thus appropriate the pie in proportion to their counterpart's discount quotient. This implies that when discounts are equal, equal split prevails regardless of time effects. We conclude further that the idyllic prescription given by the DMO rule is behaviorally borne out only when the discount quotients are high (when rational and behavioral focal predictions are approximately the same), and can be totally fallacious otherwise. We just have to consider equal but steep time effects. In this case, when Player 2's outside options are high but less than half, he or she may still obtain half the pie unjustifiably. This is because the Rubinstein bargaining partition, which predicts a low share to player 2, is invariably missed. The neglect of strategic advantage on the part of subjects also can be seen when it is derived from risk (Kahn & Murnighan, 1993; Zwick et al., 1992). But in these cases, the argument is slightly weaker due to unaccounted (by experimenters) effects of risk nonneutrality.

What story can we whip up (for the data is rather scarce) from these successes and failures of rationality? Bargaining is a group problem-solving activity with well-specified rules of communication and message content. And, like any other problem solving, it relies on meaningful and reliable clues. Out of the clues people create the story and even set the goals (such as wealth, wealth mitigated by fairness) that they want to achieve. Of course, bargainers are not clones of each other, and therefore the scenarios they build independently of each other are not necessarily compatible, although they often are.

Bargaining Time

The essence of the theoretical success of the noncooperative approach to bargaining rests not on utility, linear or not, but on the diminishing usefulness of any gain over time.[27] This is clear intuitively, and can be seen by two unknowns when the discount quotients are 1, and therefore provides no clue for adequate action. When time of agreement makes a difference, players need to realize what an adequate proposal is.

In the case of a finite horizon, it is very tempting to try to reach it. This leads to the following calculation, which is shown only for the simplest case: the discounts are the same and the utilities are identities. The proper demand for a game of order 0 (which ends after a single demand) is $a_0 = 1$, and the proper demand for a game of order n is $a_n = 1 - \delta a_{n-1}$. The problem boils down to the solution of this recursive definition. Reaching for the horizon means backward induction.[28] Everyone knows what a_0 is, unless perhaps fairness is at issue. But, very quickly, the recursive step goes out of behavioral tune (Neelin, Sonnenschein, & Spiegel, 1988; Ochs & Roth, 1989). In fact, this approach, which may seem attractive, is very cumbersome and seems cognitively intractable for subjects. Learning is expected to be difficult (in fact, a never-ending problem) because of its linear dependence on game order.

The infinite horizon case has two immediate benefits. First, because there is no definite end to the bargaining, the fairness issue derived from last mover advantage, as is known from ultimatum plays, vanishes. This can be seen by imitating the procedure taken in Endnote 28. Assuming that $0 \leq a_0 = b \leq 1$, the payoff to player 1 for a game of order n is $\sum_{0 \leq i < n-1} (-\delta)^i + b(-\delta)^n$. Taking the limit as $n \to \infty$ shows that fairness considerations due to end game effect do not appear to be an issue.

Second, the infinite horizon invites a different approach,[29] which often works but can easily be made to fail. If the present is *not* discounted, there is a strong incentive to ignore differential effects of bargaining delays. If no delays are wanted and the infinite regress is unpalatable, what seems more likely than an equal share? This is the common case, except for a small commission taken by the bargainer who is the proposer. It must be noted that this is the behavior of naïve subjects, who, after a few practice plays, are made to play a few games for real. We have no knowledge at all of what the effect of saturated behavior is.

What if the pie is depreciated immediately and perhaps even at different rates? Again, subjects are guided by manageable tools. If the frame is discount quotients, then equal net share seems appealing. But even this solution may tax communication channels. It relies on both players realizing the reason for such an odd offer, especially when a strong player is to receive it.[30] Failure to coordinate in this manner is the reason for the prevalence of the focal equal split.

The arithmetic frame has a different effect in this case. Recall that the rescaling of the exponential utility structure also provides for a different and much more meaningful consequence of delay in that utility may be reflected in actual loss, penetrating out the pie limits. For this reason, perhaps, many subjects tend to settle on extreme demands very quickly, but only when loss takes its toll immediately.

Outside Options

The introduction of outside options has resulted in some of the best support that subgame perfectness has received in the area of bargaining under geometric depreciation. This might seem surprising given the counterrational behavior under pure bargaining. Nonetheless, we give our interpretation — an interpretation that underscores the need to understand the precise mechanisms governing *pure* bargaining.

The relative success in playing outside option games lies in the clear understanding of the immediate impact of the access to those options.

It is cognitively trivial to realize (because it lies at the very core of what an outside option is and is thus understood at the instructional level) that the larger the outside option one has access to, the larger the piece of the pie it is reasonable to expect under any given conditions. In a sense, the introduction of an immediate outside option brings the game close to a single period game.

When a player is in a forced termination state *(F)*, the higher the probability of termination the closer the payoff to the outside option. This may not be the case if the outside option is low enough to raise fairness restraints.

When a player has the option to terminate the game voluntarily, the outside option provides a certain protection against the otherwise inferior bargaining position. Obviously, the larger the probability of access, the better this protection.

We think that this type of simple deduction, which follows directly from the instructions, drives the results obtained in bargaining experiments with side options. This will be true to a large degree regardless of the precise bargaining procedure. But there is more to the bargaining that can be derived only from the specific nature of the discounted alternating procedure. For example, consider the nature of the protection provided by the voluntary position. In order for this to be compatible with rational behavior, it is required that the players are bargaining rational. For example, under pure bargaining with discounts $(\delta_1, \delta_2) = (0.7, 0.9)$, Player 2 is offered about 73% of the pie through bargaining. If the voluntary outside option is 20%, he or she rejects anything smaller than this value (73%) because of the bargaining advantage. However, if the equal share rule prevails, then even a 50–50 split is reasonable and even attractive. In fact, this behavior seems pretty likely, albeit irrational. Consider now the same parameters except that the game is made into a single period game with Player 2 having an outside option of 50%. In this case, the subgame perfect demand is 50%, which deals Player 2 out. These examples serve to show that estimating the usefulness of an outside option depends on the underlying prevailing bargaining rationality. DMO behavior is thus fortuitous. Its beauty relies more on its mathematical elegance than on its behavioral reality.

Final Remarks

In the abstract to his paper on the interpretation of game theory, Rubinstein (1991) wrote:

> It is argued that a good model in game theory has to be realistic in the sense that it provides a model for the *perception* of real life social phenomena. It should incorporate a description of the relevant factors involved, as perceived by the decision makers. These need not necessarily represent the physical rules of the world. It is not meant to be isomorphic with respect to "reality" but rather with respect to our perception of regular phenomena in reality. (p. 909)

In experimental economics, we often create the "realities" to be tested. It seems that our subjects insist on telling a story different from SPE. Do they perceive a different reality than intended? As we have already pointed out, one step in the abstraction process in the area of bargaining was taken by Rubinstein (1982) himself in his research. The essential step was the freeing of bargaining models from an irrelevant restriction — the finite horizon. Nonetheless, an inherent asymmetry between the players derived from the discrete and ordered procedure is still left — a vestige of technical requirements. These might be overcome by considering the discount quotient — δ, as cumulative over infinitely small subintervals of the unit of time. This is another method of abstraction, although it is not in the formal rule of the game. In that case, the payoff to player 1 in pure bargaining becomes

$$\lim_{x\to 0} \frac{1-\delta_2^x}{1-(\delta_1\delta_2)^x} = \frac{\log\delta_2}{\log\delta_1+\log\delta_2}. \qquad (44)$$

It follows that the order in the alternating procedure loses its significance, for the partition is independent of it. Similarly, one may operate on Eq. 35 in the same manner[31] and obtain the STD rule, regardless of the probability of access per unit of time. In this way, one may argue for the Zwick et al. (1992) results of equal split when the probability of access is far from 0.

The main difficulty, of course, is that the limit process has no implementation.[32] How would subjects be forced to consider smaller intervals of time if their actions are unitized by the experimenter? One may think of the limit process taking the role of an axiom. But then we fall back to square one, started with the Nash axiomatic method. We are inclined to believe that another step in the process of abstraction starting with Rubinstein is needed. As we have heuristically indicated, a procedural symmetrization of players is required to achieve predictions that better accommodate subjects' behavior.

ACKNOWLEDGMENTS

We are grateful to Gary Bolton, Ido Erev, and Jack Ochs for their useful suggestions on an earlier version of this chapter.

ENDNOTES

1. Harsanyi (1977), for example shows that a certain Zeuthen's process for bargaining does lead to Nash's bargaining solution. But it seems that the required procedure is quite cumbersome for practical application. Nash provided his own suggestion, which rests on a technical selection criterion among the many Nash equilibria of a certain demand game.
2. For our needs, a subgame is to a game as a subtree is to a tree.
3. There are at least two general problems in applying Nash equilibria to our case. First, the Nash equilibrium lacks predictive value. There are simply too many of them. In fact every partition is supported by a Nash strategy. Second, it ignores the depth that is inherent in a tree structure, which is related to the first problem. The fact that acting out a strategy in a tree game unfolds over time is responsible for eliminating incredible moves.
4. Consider a general representation of preference over space-time given by the utility $U(x,t)$, which is continuous, increasing in the first variable and decreasing in the second. Assume that $U(x,t) - U(x,s) = k(t-s)$ for $k < 0$. Then $U(x,t) = U(x,0) + kt$. Thus the preference relation is represented by $u(x)\delta^t$ where $u(x) = \exp(U(x,0))$ and $\delta = \exp(k)$. The assumptions permitting this representation are acceptable to us and therefore we limit our discussion to the bargaining procedure and its impact on behavior.
5. This is because the logarithmic transformation that allows for a simple linear representation of utilities over time, because $\exp(x)\delta^s \geq \exp(y)\delta^t$ is equivalent to $x - cs \geq y - ct$ where $c = -log(\delta)$. Although the original representation is useful for a more compact theoretical treatment, the logarithmic representation is particularly convenient for experimentation.
6. Similar to the way compounded interest is treated.
7. Condition *E* is not reported here due to the fairly consistent equal split behavior found.
8. But quite generally, unbounded horizon bargaining games are isomorphic to infinitely many of their subgames. Raw data of last offers show convergent behavior that quite often reveals how closely subjects play the original game.
9. We do not expect, however, that all fixed-cost predictions would be verified. Note, for example, that the expected shares are contingent only on the ordinal relationship between delay costs rather than on actual quantitative levels. Rapoport et al. (1990) and Weg and Zwick (1991) experimented with highly separable parametric values. This is understandable given that an initial test of a theory tends to be done under rather "promising" conditions. However, Zwick and Chen (1997) demonstrated that cost values do indeed affect agreements in a significant way.
10. The experiment discounts from the very first period!
11. This is not exactly the case. One comparison was indeed proposed: the dictator game (Kahneman et al., 1986), which is discussed later.
12. Exactly when the clock ticks makes a difference computationally, but conceptually it has little significance.
13. The function $f^{-1}$is everywhere defined and non-decreasing. Note that f can in general obtain infinite values, but it is irrelevant in our application. It is continuous if it is not infinite (regardless of whether f is).
14. For an ardent attempt to differentiate fairness from greed, and thus to disprove Kah-

neman, Knetsch, & Thaler's (1986) conclusions, see Hoffman, McCabe, Shachat, and Smith (1994) who by careful experimentation argued for selfish dictators. A later attempt by Hoffman, McCabe, and Smith (1996) to replicate their earlier results under improved experimental control has not been completely successful.

15. Another, perhaps more successful, control game to the classical ultimatum, may be found in Bolton and Zwick (1995) where single period bargaining is suggested. Player 1 may choose between a 50/50 split and, without risk, a more extreme partition. A rejection by Player 2 does not affect Player 1's share. The main difference between this game and the dictator analog is that Player 2's acquiescence is not required to implement Player 1's desire. This could be desirable, depending on one's point of view.
16. Binmore, Morgan, Shaked, & Sutton's (1991) attempt to clarify this point through the removal of attractive focal points by changing the denomination of the pie is less than a complete success.
17. Recall that STD is expected only at the limit.
18. Having Player 1 possess the meaningful outside option makes the games more complex than Binmore et al. (1989) and Binmore et al. (1991) due to the need to account for Player 1's opportunities being devalued one period after the commencement of play.
19. We separate the dimensions in this way for the lack of visible four dimensional boxes.
20. For simplicity, this portrayal assumes identity utilities, denoted by 1.
21. Binmore et al. (1989) opted for a similar framing when a player has a voluntary outside option of zero. That this simplification may be behaviorally unwarranted was noted by Weg and Zwick (1994).
22. The authors are thankful to Lawrence Kahn and Keith Murnighan for furnishing the data for this table.
23. By looking at the table one sees that split-the-difference fails for extreme predicted allocations. For $q = 1$, very weak bargainers ignore the realities altogether.
24. For this special case we shall assume that outside options are consumed a period later than the usual convention.
25. In reality, neither buyer has an option to leave the bargaining. In this way, the apparent disparity between the buyers and the seller is made larger.
26. Thus, the utility of x at time t is $u(x)\delta^{t+1}$ for $t = 0, 1, \ldots$, instead of the usual, $u(x)\delta^t$.
27. The stationarity requirement is reviewing Equation 2, which is reduced to a single equation with more technical.
28. Another way is the paper and pencil approach. Let

$$\phi(x) = \sum_{n\geq 0} a_n x^n \tag{41}$$

be a power series with coefficients, a_n, being the payoffs to Player 1 in Ståhl bargaining games of order n and

$$\psi(x) = \sum_{n\geq 0} x^n. \tag{42}$$

Then we see that $(\psi(x) - \delta\phi(x))x + 1 = \phi(x)$ and therefore

$$\phi(x) = \frac{x\psi(x)+1}{1+\delta x} = \frac{\psi(x)}{1+\delta x}. \tag{43}$$

It follows that $a_n = \sum_{n\geq i\geq 0}(-\delta)^i$.

29. Entertaining delays just induces games of the same type, and the "horizon" remains as far away as it normally is.
30. Therefore, this rule is often adopted by weak proposers.
31. That is, by considering

$$\frac{1+(1-p)^x s_1 - s_2}{2-\omega(p,x)} \tag{45}$$

as $x \to 0$ where the function $\omega(p,x) \to 0$ as $x \to 0$.
32. Limit process is a common manner of *definition* in certain areas of mathematics. Take, for example, the concept of area. But this is precisely what it is: a definition, which is not appropriate in the case we are discussing.

REFERENCES

Binmore, K., Shaked, A., & Sutton, J. (1989). An outside option experiment. *Quarterly Journal of Economics*, *104*, 753–770.

Binmore, K., Morgan, P., Shaked, A., & Sutton, J. (1991). Do people exploit their bargaining power? An experimental study. *Games and Economic Behavior*, *3*, 295–322.

Bolton, G. E. & Zwick, R. (1995). Anonymity versus punishment in ultimatum bargaining. *Games and Economic Behavior*, *10*, 95–121.

Güth, W. & Tietz, R. (1990). Ultimatum bargaining behavior: A survey and comparison of experimental results. *Journal of Economic Psychology*, *11*, 417–449.

Güth, W., Schmittberger, R., & Schwarze, B. (1982). An experimental analysis of ultimatum games. *Journal of Economic Behavior and Organization*, *3*, 367–388.

Harsanyi, J. C. (1977). *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge: Cambridge University Press.

Hoffman, E., McCabe, K. K., Shachat, K., & Smith, V. (1994). Preference, property rights, and anonymity in bargaining games. *Games and Economic Behavior*, *7*, 346–380.

Hoffman, E., McCabe, K. K., & Smith, V. (1996). Social distant and other-regarding behavior in dictator games. *American Economic Review*, *96*, 653–660.

Kahn, L. M. & Murnighan, J. K. (1993). A general experiment on bargaining in demand games with outside options. *American Economic Review*, *83*, 1260–1280.

Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1986). Fairness and the assumptions of economics. *Journal of Business*, *59*, S285 – S300.

Nash, J. F. (1950). The bargaining problem. *Econometrica*, *18*, 155–162.

Neelin, J., Sonnenschein, H., & Spiegel, M. (1988). A further test of noncooperative bargaining theory. *American Economic Review*, *78*, 824–836.

Ochs, J. & Roth, A. E. (1989). An experimental study of sequential bargaining. *American Economic Review*, *79*, 355–384.

Rapoport, A., Weg, E., & Felsenthal, D. (1990). Effects of fixed costs in two-person sequential bargaining. *Theory and Decision*, *28*, 47–71.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, *50*, 97–109.

Rubinstein, A. (1991). Comments on the interpretation of game theory. *Econometrica*, *59*, 909–924.

Selten, R. (1975). Reexamination of the perfectness concept for equilibrium points in extensive games. *Econometrica*, *4*, 25–55.

Sutton, J. (1986). Non-cooperative bargaining theory: An introduction. *Review of Economic Studies*, *53*, 709–724.

Weg, E. & Zwick, R. (1991). On the robustness of perfect equilibrium in fixed cost sequential bargaining under isomorphic transformation. *Economics Letters*, *36*, 21–24.

Weg, E. & Zwick, R. (1994). Toward a settlement of the fairness question in sequential bargaining games. *Journal of Economic Behavior and Organization*, *24*, 19–34.

Weg, E., Rapoport, A., & Felsenthal, D. (1990). Two-person bargaining behavior in fixed discounting factors games with infinite horizon. *Games and Economic Behavior*, *2*, 76–95.

Weg, E., Zwick, R., & Rapoport, A. (1996). Bargaining in uncertain environments: A systematic distortion of perfect equilibrium demands. *Games and Economic Behavior*, *14*, 260–286.

Zwick, R. & Chen, X.-P. (1997). *What price for fairness? A bargaining study*. Working Paper 97.088, The Hong Kong University of Science and Technology, Department of Marketing.

Zwick, R. & Weg, E. (1996). *An experimental study of buyer-seller negotiation: self interest versus other regarding behavior*. Working Paper 96.070, The Hong Kong University of Science and Technology, Department of Marketing.

Zwick, R., Rapoport, A., & Howard, J. C. (1992). Two-person sequential bargaining behavior with exogenous breakdown. *Theory and Decision*, *32*, 241–268.

Zwick, R., Rapoport, A., & Weg, E. (1996). *A breakdown of Invariance: the case of two vs. three-person sequential bargaining.* Working Paper 96.070, The Hong Kong University of Science and Technology, Department of Marketing.

Index