Macroeconomic Stability under Balanced-Budget Rules and No-Income-Effect Preferences*

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Abstract

It has been shown that under an additively separable preference formulation between consumption and hours worked, indeterminacy and sunspots may arise in a standard one-sector or two-sector real business cycle model when the labor tax rate is endogenously determined by a balanced-budget rule with a pre-specified constant level of government expenditures. Our analysis finds that this indeterminacy result will completely disappear within either setting if the period utility function is postulated to exhibit no income effect on the household's demand for leisure. In particular, the model's low-tax steady state displays saddle-path stability and equilibrium uniqueness; whereas the high-tax steady state is either a source or a saddle point.

Keywords: Income Effect; Balanced-Budget Rules; Indeterminacy; Business Cycles.

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1 Introduction

Under the assumptions of perfect competition and constant returns-to-scale in production, the standard one-sector or two-sector real business cycle (RBC) model exhibits an interior steady state that is a locally determinate/isolated saddle point around which there exists a unique convergent rational expectations equilibrium trajectory. In the one-sector macroeconomy with a Cobb-Douglas production technology as well as an additively separable utility function between consumption and hours worked, Schmitt-Grohé and Uribe (1997, section II) analytically examine the aggregate (in)stability effects of a balanced-budget rule whereby constant government expenditures are financed by distortionary taxation on the household's labor income. Given the postulated fiscal specification, a perfect-foresight Laffer curve-type relationship between the labor tax rate and the resulting tax revenue ensues – the model possesses two interior stationary states when the pre-specified level of public spending is lower than the revenue-maximizing counterpart. In this context, these authors derive the necessary and sufficient condition under which the low-tax steady state is an indeterminate sink that can be exploited to yield cyclical fluctuations driven by agents' animal spirits or sunspots. Subsequently, Huang, Meng and Xue (2017, section 2) numerically obtain the qualitatively identical result of equilibrium indeterminacy at the low-tax steady state within a prototypical two-sector (consumption and investment) real business cycle model under the same household-preference and fiscal-policy formulations.[1] When households become optimistic about the economy's future and decide to work harder and invest more, the government is forced to decrease the labor tax rate as total output rises. Schmitt-Grohé and Uribe's (1997) countercyclical balanced-budget scheme helps fulfill the representative agent's initial optimism, and thus destabilizes the macroeconomy by generating endogenous business cycles.[2]

Recently, Abad et al. (2017) extend Schmitt-Grohé and Uribe's (1997) theoretical analyses with a generalized constant returns-to-scale production function and two classes of non-separable preference specifications in a one-sector RBC environment. While the generality of this study is commendable, it is not straightforward to make a direct comparison between its findings versus those of Schmitt-Grohé and Uribe (1997) in a transparent manner. In this paper, we begin with complementing the work of Abad et al. (2017) by maintaining the Cobb-Douglas production structure, and focusing on a specific functional form for the house-

[1] See Benhabib and Farmer (1999) for other mechanisms that may yield indeterminacy and sunspots within various real business cycle models.

[2] Linnemann (2008) incorporates a non-separable preference formulation *à la* King, Plosser and Rebelo (1988) into Schmitt-Grohé and Uribe's (1997) one-sector RBC model. It is found that the utility complementary between consumption and employment will diminish the likelihood of equilibrium indeterminacy under a balanced fiscal budget.

hold utility that was first adopted by Greenwood, Hercowitz and Huffman (GHH, 1988) in the modern business cycle literature. In particular, the period utility function is postulated to exhibit no income effect associated with the representative agent's labor supply decision. As it turns out, although Abad et al. (2017, Proposition 3) touch on the special case with no-income-effect preferences, they do not offer the underlying economic intuition specifically for this setting.[3] Moreover, unlike Schmitt-Grohé and Uribe (1997), they do not explore the possibility of multiple stationary equilibria caused by the balanced-budget fiscal policy under consideration. By contrast, we examine the equilibrium dynamics of each interior steady state for the sake of analytical completeness, and also use the intertemporal consumption Euler equation (in discrete time for ease of interpretation) to help provide a focused and more detailed intuitive explanation.

Under the GHH formulation of non-separable preferences, we find that the relationship between the fixed amount of government spending and the labor tax rate is characterized by a Laffer curve that may possess two interior steady states. Our analysis shows that in sharp contrast to Schmitt-Grohé and Uribe (1997), equilibrium indeterminacy will completely disappear in a one-sector RBC macroeconomy because neither steady state can be a sink. Specifically, saddle-path stability arises when the steady-state tax rate is (i) lower than that maximizes the tax revenue or (ii) higher than a certain threshold value. Intuitively, in order for stationary sunspot equilibria to occur within a dynamic general equilibrium macroeconomic model, the consumption Euler equation must continue to hold in response to a change in agents' non-fundamental expectations. Therefore, upon the anticipation of a higher rate of return on today's investment, households will raise their consumption expenditures and work harder next period. It turns out that this optimism cannot be self-fulfilled under either circumstance since an increase in labor hours large enough to raise the after-tax marginal product of capital (*the after-tax MPK effect*) will generate an unsustainable decrease in the household's intertemporal marginal rate of substitution between current versus future consumption spending. Furthermore, we find that the economy's high-tax steady state turns into a totally unstable source when the stationary equilibrium labor-tax rate falls within the remaining feasible range.

The second part of this paper incorporates the GHH no-income-effect utility function into Huang, Meng and Xue's (2017, section 2) two-sector real business cycle model with Cobb-Douglas production technologies under different capital intensities and countercyclical labor income taxation as in Schmitt-Grohé and Uribe (1997). Since analytical results are

[3] See section 6 of Abad et al. (2017) for the economic intuition of their results under generalized production and utility functions.

generally not available in this case, we quantitatively examine its local stability properties of a calibrated macroeconomy. Under the benchmark parameterization, we find that the model's low-tax steady state exhibits saddle-path stability and equilibrium uniqueness, whereas the high-tax steady state is either a source or a saddle point; and that these baseline results are qualitatively robust to changes in key model parameters over their respective empirically plausible ranges. In addition to the after-tax MPK effect discussed above, agents' optimistic expectations will cause the relative price of investment goods to rise as more productive capital and labor inputs are moved out of the consumption sector (*the price effect*). Our numerical experiments show that the equality in the consumption Euler equation cannot be maintained upon a belief-driven investment spurt. At the low-tax stationary state, an increase in the next period's labor hours large enough to raise the price-weighted after-tax real interest rate, *i.e.* when the after-tax MPK effect outweighs the price effect, will yield a decrease in the household's intertemporal marginal rate of consumption substitution. When the steady-state tax rate is "sufficiently high", an increase in the next period's labor hours small enough to raise the household's intertemporal marginal rate of consumption substitution may reduce the price-weighted after-tax marginal product of capital because of a quantitatively stronger price effect. As a result, indeterminacy and sunspots will not arise within either environment. In sum, our analyses illustrate that under the postulated balanced-budget fiscal policy rule in a canonical one-sector or two-sector real business cycle model, the previous finding of aggregate instability depends crucially on the presence of income effect associated with the representative agent's labor supply decision.

The remainder of this paper is organized as follows. Section 2 presents the one-sector model economy and analytically examines its equilibrium dynamics under perfect foresight. Section 3 studies the two-sector model economy and quantitatively analyzes its local (in)stability properties. Section 4 concludes.

2 One-Sector Economy

This section incorporates a no-income-effect preference formulation, as in Greenwood, Hercowitz and Huffman (GHH, 1988), into Schmitt-Grohé and Uribe's (1997, section II) one-sector real business cycle model with labor income taxation. For the purpose of a direct comparison, we follow Schmitt-Grohé and Uribe (1997) and postulate that the economy's output is generated by a Cobb-Douglas production technology with constant returns-to-scale. This simplification will streamline our exposition and help articulate our intuitive explanations.

2.1 Firms

The production side of this one-sector macroeconomy consists of a unit measure of identical competitive firms. The representative firm produces output Y_t, using capital and labor as inputs, with a constant returns-to-scale Cobb-Douglas production function

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \ \ 0 < \alpha < 1. \tag{1}$$

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{2}$$

$$w_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{3}$$

where r_t is the rental rate of capital and w_t is the real wage rate of labor.

2.2 Households

The macroeconomy is also populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes

$$\int_0^{\infty} e^{-\rho t} \left[\log \left(C_t - A \frac{H_t^{1+\gamma}}{1+\gamma} \right) \right] dt, \quad A > 0, \tag{4}$$

where C_t and H_t are the individual household's consumption and hours worked, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho \in (0,1)$ is the subjective discount rate. We assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (r_t - \delta) K_t + (1-\tau_t) w_t H_t - C_t, \quad K_0 > 0 \text{ given}, \tag{5}$$

where K_t is the household's capital stock, $\delta \in (0,1)$ is the capital depreciation rate, τ_t is the labor-income tax rate. We require that $\tau_t \geq 0$ to rule out the possibility of income subsidies which could only be financed by lump-sum taxation, and that $\tau_t < 1$ such that households have incentive to provide labor services to firms.

The first-order conditions for this household's dynamic optimization problem under perfect foresight are

$$\left(C_t - A\frac{H_t^{1+\gamma}}{1+\gamma}\right)^{-1} = \Lambda_t, \tag{6}$$

$$AH_t^{\gamma} = (1-\tau_t)\, w_t, \tag{7}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - r_t, \tag{8}$$

$$\lim_{t\to\infty} e^{-\rho t}\Lambda_t K_t = 0, \tag{9}$$

where $\Lambda_t > 0$ is the Lagrange multiplier on the budget constraint (5), (7) equates the slope of the representative household's indifference curve to the after-tax real wage, (8) is the standard consumption Euler equation, and (9) is the transversality condition. Since C_t is missing in equation (7), there is no income effect associated with the household's labor supply decision. It follows that the income elasticity of intertemporal substitution in hours worked (or leisure) is zero.

2.3 Government

As in Schmitt-Grohé and Uribe (1997), the government endogenously sets the distortionary tax rate on labor income τ_t to finance a pre-specified constant amount of public expenditures, and balances its budget at each point in time. Hence, the instantaneous government budget constraint is

$$G = \tau_t w_t H_t, \tag{10}$$

where $G \geq 0$ denotes government spending on goods and services. Finally, the aggregate resource constraint for the economy is given by

$$C_t + \dot{K}_t + \delta K_t + G = Y_t, \tag{11}$$

where Y_t represents total output or GDP.

2.4 Analysis of Dynamics

Under Schmitt-Grohé and Uribe's fiscal policy rule with countercyclical labor income taxation, the number of our model's interior steady state(s) can be zero, one or two. Specifically, it is straightforward to show that the government's tax revenue ($= G$) is equal to zero when the steady-state tax rate $\tau^{ss} = 0$ or 1[4]; and that the Laffer curve-type relationship between $G > 0$ and $\tau^{ss} \in (0,\, 1)$ is given by

$$G = \tau^{ss}(1-\alpha)\left[\frac{\alpha}{\rho+\delta}\right]^{\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1+\gamma}{\gamma}\right)}\left[\frac{(1-\alpha)(1-\tau^{ss})}{A}\right]^{\frac{1}{\gamma}}. \tag{12}$$

Setting $\frac{\partial G}{\partial \tau^{ss}} = 0$ yields a unique steady-state tax rate $\tau^* = \frac{\gamma}{1+\gamma}$ that maximizes the level of public expenditures denoted as G^*.[5] It follows that our model possesses zero (two) interior steady states(s) provided $G > (<)\, G^*$, as shown in Figure 1. Therefore, any small deviation from the revenue-maximizing steady state with τ^* and G^* will lead to its disappearance, or the emergence of dual stationary equilibria. This result implies that the economy undergoes a saddle-node bifurcation which may cause the hard loss of equilibrium stability as the government spending passes through the critical level G^*. Figure 1 also shows that when $G \in (0,\, G^*)$, the resulting steady states in our model are characterized by τ_L^{ss} and τ_H^{ss}, where $\tau_L^{ss} < \tau^* < \tau_H^{ss}$. For a given steady-state labor tax rate, the analytical expressions of all remaining endogenous variables can then be easily derived.

Next, we take log-linear approximations to the model's equilibrium conditions in a neighborhood of each interior steady state to obtain the following dynamical system:

$$\begin{bmatrix} \dot{k}_t \\ \dot{\lambda}_t \end{bmatrix} = \mathbf{J}_1 \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}, \;\; k_0 \text{ given}, \tag{13}$$

where k_t and λ_t denote the logarithmic deviations of K_t and Λ_t from their respective steady-state values, and $\mathbf{J}_1$ is the Jacobian matrix of partial derivatives for the transformed dynamical system of our one-sector model. The trace and the determinant of the Jacobian are given by

$$Tr = \rho + \frac{(1-\alpha)(\rho+\delta)\tau^{ss}}{\alpha - \tau^{ss} + \gamma(1-\tau^{ss})}, \tag{14}$$

and

$$Det = \left[\frac{\tau^{ss} - \gamma(1-\tau^{ss})}{\alpha - \tau^{ss} + \gamma(1-\tau^{ss})}\right] \underbrace{\left\{\frac{\delta(1-\alpha)(\rho+\delta)\left[(1-s_i)(1+\gamma) - (1-\alpha)(1+\gamma\tau^{ss})\right]}{s_i(1+\gamma)}\right\}}_{\equiv \Psi(\tau^{ss}) > 0}, \tag{15}$$

[4]When $G = 0$, our model collapses to a standard one-sector RBC macroeconomy with no-income-effect preferences and constant returns-to-scale in production. As shown in Meng and Yip (2008) and Jaimovich (2008), this laissez-faire formulation always exhibits saddle-path stability and equilibrium uniqueness.

[5]When $\gamma = 0$, the revenue-maximizing steady state becomes degenerate with $\tau^* = G^* = 0$. Accordingly, our subsequent analyses of the model's equilibrium dynamics are restricted to cases under $\gamma > 0$.

where $s_i \left(= \frac{\alpha\delta}{\rho+\delta}\right)$ is the steady-state ratio of investment to output.[6] The local stability properties of our model's interior steady state(s) are determined by comparing the eigenvalues of $\mathbf{J}_1$ that have negative real parts to the number of initial conditions in the dynamical system (13), which is equal to one because k_t is a pre-determined state variable. As a result, the steady state exhibits saddle-path stability and equilibrium uniqueness when the two eigenvalues are of opposite signs ($Det < 0$). If both eigenvalues have negative real parts ($Tr < 0$ and $Det > 0$), then the steady state is an indeterminate sink around which there are a continuum of stationary equilibrium trajectories that display endogenous cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues have positive real parts ($Tr > 0$ and $Det > 0$), the steady state becomes a totally unstable source.

In sharp contrast to Schmitt-Grohé and Uribe (1997) with an additively separable household utility in consumption and labor hours, the following Proposition states that local indeterminacy does not arise within our model under non-separable no-income-effect preferences. That is, neither steady state (with τ_L^{ss} or τ_H^{ss}) can be a sink.

Proposition 1. For a given positive level of $G < G^*$, our one-sector economy's low-tax steady state $(0 < \tau_L^{ss} < \tau^*)$ is always a saddle point, whereas the high-tax steady state is either a source $\left(\tau^* < \tau_H^{ss} < \frac{\alpha+\gamma}{1+\gamma}\right)$ or a saddle point $\left(\frac{\alpha+\gamma}{1+\gamma} < \tau_H^{ss} < 1\right)$.

Proof. See the Appendix.

To understand the intuition behind the above no-indeterminacy result, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

$$\frac{C_{t+1} - A\frac{H_{t+1}^{1+\gamma}}{1+\gamma}}{C_t - A\frac{H_t^{1+\gamma}}{1+\gamma}} = \beta[(1-\tau_{t+1})r_{t+1} + 1 - \delta], \tag{16}$$

where β denotes the discount factor. Start the model from an interior steady state at period t, and suppose that agents become optimistic about the economy's future. Acting upon this change in non-fundamental anticipation, the representative household will consume less and invest more today, thus C_t falls while K_{t+1} rises. Due to the lack of income effect, as seen in (7), H_t remains unchanged in response to the lower level of period-t consumption. In addition, a higher K_{t+1} leads to (i) a decrease in r_{t+1} because of diminishing marginal product of capital; and (ii) an increase in H_{t+1} via firms' labor demand function, which in turn raises the economy's output Y_{t+1} as well as the household's consumption C_{t+1}. Under the postulated balanced-budget constraint (10), the government is forced to reduce the labor tax rate τ_{t+1}

[6] Using $\gamma > 0$ (see footnote 3), $s_i \in (0, \alpha)$ and α, $\tau^{ss} \in (0,1)$, it can be shown that the bracket term in the numerator of $\Psi(\cdot)$, given by $(1-s_i)(1+\gamma) - (1-\alpha)(1+\gamma\tau^{ss}) > \gamma(1-\alpha)(1-\tau^{ss}) > 0$. This result, together with $0 < \delta < 1$ and $\rho > 0$, implies that $\Psi(\tau^{ss}) > 0$.

as total income Y_{t+1} increases, thus $(1 - \tau_{t+1})$ rises. Consequently, the change in H_{t+1} will exert two counteracting effects on the intertemporal Euler equation. First, the smaller (bigger) the increase in H_{t+1}, the bigger (smaller, or a decrease may occur) the increase in the left-hand side of (16). Second, the bigger (smaller) the increase in H_{t+1}, the larger (smaller, or a decrease may occur) the rise in the after-tax equilibrium real interest rate $(1 - \tau_{t+1})r_{t+1}$ – this is dubbed as *the after-tax MPK effect*.

For the aforementioned alternative dynamic path to be justified as a self-fulfilling equilibrium, the household's consumption Euler equation must continue to hold in response to agents' rosy expectations. It turns out that the two offsetting effects, described in the previous paragraph, render the equality of (16) impossible within our model. When the economy begins at the low-tax steady state with $0 < \tau_L^{ss} < \tau^* \left(= \frac{\gamma}{1+\gamma}\right)$, a large increase in H_{t+1} is needed for $(1 - \tau_{t+1})r_{t+1}$ and the right-hand side to rise. With C_t falling and C_{t+1} rising, this would in turn decrease the left-hand side. On the other hand, when the starting steady-state tax rate is high over the interval of $\frac{\alpha+\gamma}{1+\gamma} < \tau_H^{ss} < 1$, together with a small increase in H_{t+1} that raises the left-hand side, the after-tax equilibrium return on capital investment $(1 - \tau_{t+1})r_{t+1}$ and the right-hand side may decrease or do not rise enough. As a result, agents' initial optimism cannot be fulfilled under either tax specification, hence the macroeconomy will exhibit saddle-path stability and equilibrium uniqueness. Finally, we find that the high-tax steady state with $\tau^* < \tau_H^{ss} < \frac{\alpha+\gamma}{1+\gamma}$ is a source, which is surrounded by divergent or explosive trajectories that will eventually violate the transversality condition (9).[7]

As a side-by-side comparison, the consumption Euler equation for Schmitt-Grohé and Uribe's (1997) one-sector RBC model, in which the period utility function is postulated to be logarithmic in consumption and additively separable with hours worked, is given by

$$\frac{C_{t+1}}{C_t} = \beta[(1 - \tau_{t+1})r_{t+1} + 1 - \delta]. \tag{17}$$

In this case, agents' optimistic expectations that lead to higher investment today will unambiguously raise the left-hand side of the preceding equation, and result in a lower before-tax real interest rate r_{t+1} due to diminishing returns to productive inputs. Under countercyclical labor income taxation $\left(\frac{\partial \tau_t}{\partial Y_t} < 0\right)$, these authors show that (i) the low-tax steady state may become an indeterminate sink when the right-hand side of (17) rises sufficiently; and (ii) the high-tax steady state is always a saddle point. Overall, our analysis illustrates that under the postulated balanced-budget rule with countercyclical labor income taxation in a standard one-sector RBC model, Schmitt-Grohé and Uribe's (1997, section II) indeterminacy result de-

[7] As in Schmitt-Grohé and Uribe (1997) and Abad et al. (2017), we focus on the model's local stability properties, and leave its (nonlinear) global dynamics for future research.

pends crucially on the presence of income effect associated with the household's labor supply decision.

3 Two-Sector Economy

This section incorporates the GHH no-income-effect preference formulation into Huang, Meng and Xue's (2017, section 2) two-sector real business cycle model with countercyclical labor income taxation *à la* Schmitt-Grohé and Uribe (1997).[8] The production side of the macroeconomy consists of two distinct sectors, consumption and investment. Similar to section 2, competitive firms in each sector produce their respective output with a Cobb-Douglas technology under constant returns-to-scale.

3.1 Firms

In the consumption sector, output is produced by competitive firms using the following production technology:

$$Y_{ct} = K_{ct}^{a} H_{ct}^{1-a}, \quad 0 < a < 1, \tag{18}$$

where K_{ct} and H_{ct} are the capital and labor inputs utilized in the production of consumption goods. Under the assumption that factor markets are perfectly competitive, the first-order conditions for these firms' profit maximization are given by

$$r_t = \frac{aY_{ct}}{K_{ct}}, \tag{19}$$

$$w_t = \frac{(1-a)\,Y_{ct}}{H_{ct}}. \tag{20}$$

Similarly, investment goods are produced by competitive firms using the production technology

$$Y_{It} = K_{It}^{b} H_{It}^{1-b}, \quad 0 < b < 1, \tag{21}$$

where K_{It} and L_{It} are physical capital and labor hours in the investment sector. The first-order conditions that govern the demand for capital and labor in the investment sector are

$$r_t = p_t \frac{bY_{It}}{K_{It}}, \tag{22}$$

$$w_t = p_t \frac{(1-b)\,Y_{It}}{H_{It}}, \tag{23}$$

[8] See Abad and Venditti (2019) for a more detailed analysis on the requisite condition for local indeterminacy within the closed-economy version of Huang, Meng and Xue's (2017) two-sector RBC model.

where p_t denotes the relative price of investment to consumption goods at time t. Notice that firms in each sector face the same factor prices in equilibrium since capital and labor inputs are postulated to be freely mobile across the two production sectors. When $a = b$, our model collapses to a one-sector setting as in section 2. As a result, we will study the cases with $a \neq b$, *i.e.* different capital intensities for producing consumption versus investment goods, from now on.

3.2 Households

In this two-sector macroeconomy, the representative agent's discounted lifetime utility function is

$$\int_0^\infty e^{-\rho t} \left[\log \left(C_t - B \frac{H_t^{1+\gamma}}{1+\gamma} \right) \right] dt, \quad B > 0, \tag{24}$$

subject to the budget constraint given by

$$C_t + p_t \underbrace{\left(\dot{K}_t + \delta K_t \right)}_{= I_t} = r_t K_t + (1 - \tau_t) w_t H_t, \quad K_0 > 0 \text{ given and } 0 \leq \tau_t < 1, \tag{25}$$

where I_t is gross investment. It is then straightforward to derive that the resulting first-order conditions for this household's dynamic optimization problem are

$$\left(C_t - B \frac{H_t^{1+\gamma}}{1+\gamma} \right)^{-1} = \Phi_t, \tag{26}$$

$$BH_t^\gamma = (1 - \tau_t) w_t, \tag{27}$$

$$\frac{\dot{\Phi}_t}{\Phi} = \rho + \delta - \frac{r_t}{p_t}, \tag{28}$$

$$\lim_{t \to \infty} e_t^{-\rho t} \Phi_t K_t = 0, \tag{29}$$

where $\Phi_t > 0$ is the Lagrange multiplier associated with the budget constraint (25), and equations (27)-(29) have the same economic interpretations as those for their one-sector counterparts given by conditions (7)-(9).

3.3 Government

As in section 2, the government balances the budget at each point in time through paying a time-invariant amount of public expenditures with its tax revenue collected from labor services provided within the consumption and investment sectors. It follows that the instantaneous government budget constraint is

$$G = \tau_t w_t H_{ct} + \tau_t w_t H_{It}. \tag{30}$$

Finally, the aggregate resource constraint for the economy is given by

$$C_t + p_t(\dot{K}_t + \delta K_t) + G = Y_t. \tag{31}$$

3.4 Competitive Equilibrium and Local Dynamics

We focus on symmetric competitive equilibria that consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t, H_t, H_{ct}, H_{It}, K_t, K_{ct}, K_{It}\}_{t=0}^{\infty}$ which satisfy the household's and firms' first-order conditions. The market equilibrium conditions for the consumption and investments goods are given by $C_t + \tau_t w_t H_{ct} = Y_{ct}$ and $p_t I_t + \tau_t w_t H_{It} = p_t Y_{It}$. In addition, both the capital and labor markets will clear: $K_{ct} + K_{It} = K_t$ and $H_{ct} + H_{It} = H_t$.

As in section 2, it can be shown that the analytical relationship between government spending $G > 0$ and the steady-state tax rate $\tau^{ss} \in (0, 1)$ within our two-sector model is

$$G = \tau^{ss}\Omega \left\{ (1-\tau^{ss}) \left(\frac{b}{\rho+\delta} \right)^{\frac{a(1+\gamma)}{1-b}} \left[\left(\frac{a}{b} \right)^{a} \left(\frac{1-a}{1-b} \right)^{1-a} \right]^{\frac{a(1+\gamma)}{a-b}} \right\}^{\frac{1}{\gamma}}, \tag{32}$$

where $\Omega \equiv B^{\frac{-1}{\gamma}} \left\{ (1-a) \left[\left(\frac{a}{b} \right)^{b} \left(\frac{1-a}{1-b} \right)^{1-b} \right]^{\frac{a}{b-a}} \right\}^{\frac{1+\gamma}{\gamma}}$; and that a steady-state Laffer curve (similar to Figure 1) will emerge as a result. It follows that the economy possesses two interior stationary states when the pre-specified constant level of public expenditures is lower than the revenue-maximizing counterpart denoted as G^*.

We then log-linearize the model's equilibrium conditions in a neighborhood of each stationary state to obtain the following dynamical system:

$$\begin{bmatrix} \dot{k}_t \\ \dot{\phi}_t \end{bmatrix} = \mathbf{J}_2 \begin{bmatrix} k_t \\ \phi_t \end{bmatrix}, \quad k_0 \text{ given}, \tag{33}$$

where ϕ_t represents the logarithmic deviation of Φ_t from its steady-state value. Since the local (in)stability conditions associated with the Jacobian matrix $\mathbf{J}_2$ cannot be analytically derived, this subsection will quantitatively examine the equilibrium dynamics for a calibrated version of our two-sector RBC macroeconomy with Schmitt-Grohé and Uribe's (1997) balanced-budget fiscal policy and the GHH no-income-effect preference formulation. As in Huang, Meng and Xue's (2017, p. 95), we set $a = 0.33$ and $b = 0.3$ as the baseline capital intensities in firms' production technologies given by (18) and (21). In addition, the time discount rate, ρ, is chosen to be 0.04; the capital depreciation rate, δ, is fixed at 0.1; and the preference parameter, B, is normalized to 1. We also find that under the aforementioned parameter values together with $\tau^{ss} \in (0,\ 1)$, the model's Jacobian becomes singular when $\gamma \in [0,\ 0.0256)$, *i.e.* the household's labor supply elasticity is higher than 39.06. In light of this result, our benchmark parametrization adopts $\gamma = 0.25$, a number that is used by King, Plosser and Rebelo (1988). Using equation (32), it is then straightforward to show that there exists a unique steady-state tax rate $\tau^* = 0.2014$ which will yield the revenue-maximizing level of government spending $G^* = 0.0839$.

Proposition 2. Under the benchmark calibrations on a, b, ρ, δ and γ described above, together with a given positive level of $G < G^*$, our two-sector economy's low-tax steady state $(0 < \tau_L^{ss} < \tau^*)$ is a saddle point, whereas the high-tax steady state is either a source $(\tau^* < \tau_H^{ss} \leq 0.4579)$ or a saddle point $(0.4579 < \tau_H^{ss} < 1)$.

As in section 2, we consider the following discrete-time version of the consumption Euler equation to help understand the preceding Proposition:

$$\frac{C_{t+1} - B\frac{H_{t+1}^{1+\gamma}}{1+\gamma}}{C_t - B\frac{H_t^{1+\gamma}}{1+\gamma}} = \beta \left[\frac{(1-\tau_{t+1})\, r_{t+1} + (1-\delta) p_{t+1}}{p_t} \right]. \tag{34}$$

In order for equilibrium indeterminacy to arise within our two-sector RBC model, equation (34) must be satisfied upon an anticipated increase in the rate of return of period-t investment expenditures. Per our intuitive discussions of (16) for the one-sector macroeconomy, this optimistic expectation leads to (i) a decrease in C_t and no change in H_t, in conjunction with (ii) increases in I_t, K_{t+1}, H_{t+1}, Y_{t+1} and C_{t+1}. These outcomes in turn reduce the labor-income tax rate τ_{t+1} as well as the real interest rate r_{t+1}. It follows that when the increase in H_{t+1} gets higher (lower), the resulting rise in the after-tax marginal product of capital $(1-\tau_{t+1})\, r_{t+1}$, *i.e.* the after-tax MPK effect, will become larger (smaller, or a decrease may occur)

On the other hand, movements of productive resources also affect the relative price of investment goods within the current two-sector framework. Due to the presence of constant returns-to-scale in (18) and (21) with different capital intensities, the economy's production possibility frontier which traces out the trade-off between consumption and investment spending is concave to the origin. As a result, the relative price of investment goods p_t will increase in response to agents' optimism that shifts more capital and labor inputs into the investment sector – this is dubbed as *the price effect.*

As it turns out, our numerical experiments show that the equality in (34) cannot be maintained upon the household's belief-driven investment spurt. When the two-sector economy starts at the low-tax steady state with $0 < \tau_L^{ss} < \tau^*$ $(= 0.2014)$, a large increase in H_{t+1} is needed for the after-tax MPK effect to outweigh the price effect such that the right-hand side rises. However, this will cause the left-hand side to fall because of decreasing C_t and rising C_{t+1}. In addition, when the beginning steady-state tax rate is high given by $0.4579 < \tau_H^{ss} < 1$, together with a small increase in H_{t+1} that raises the left-hand side, the price-weighted after-tax equilibrium return on capital accumulation and the right-hand side may decrease (due to a quantitatively stronger price effect) or do not rise enough. It follows that in either formulation, saddle-path stability and equilibrium uniqueness will result as agents' initial expectation of a higher return on capital is not validated. We also find that the high-tax steady state over the interval of $\tau^* < \tau_H^{ss} \leq 0.4579$ is a totally unstable source, since both eigenvalues of $\mathbf{J}_2$ have positive real parts within this setting. In terms of the sensitivity analysis, the quantitative results reported in Proposition 2 are found to remain qualitatively robust to changes in $\{a,\, b,\, \rho,\, \delta,\, \gamma\}$ over their respective empirically plausible ranges.

For a side-by-side comparison, the consumption Euler equation for the two-sector RBC model *à la* Huang, Meng and Xue's (2017, section 2), in which the period utility function is characterized by constant relative risk aversion (CRRA) in consumption and additively separable with hours worked, is given by

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[\frac{(1-\tau_{t+1})\, r_{t+1} + (1-\delta)p_{t+1}}{p_t}\right], \quad \sigma > 0 \quad \text{and} \quad \sigma \neq 1, \tag{35}$$

where σ is the inverse of the intertemporal elasticity of substitution in consumption. When households become optimistic about the economy's future, they sacrifice current consumption (C_t falls) for higher investment today (raising K_{t+1}), which in turn raises next period's hours worked via firms' demand for labor, thereby producing more output and higher consumption in period $t+1$ (C_{t+1} rises). It follows that the left-hand side of (35) will definitely rise. For this alternative dynamic trajectory to be justified as a self-fulfilling equilibrium path, the price-weighted after-tax rate of return on K_{t+1} net of depreciation, *i.e.* the right-hand side of (35), needs to increase as well. Using the same baseline parameterization on a, b, ρ, δ and γ as in our two-sector macroeconomy under the GHH preference specification, we numerically find that the model's low-tax steady state with $0.4299 \leq \tau_L^{ss} \leq 0.7746$ is a locally indeterminate sink. In this case, the after-tax MPK effect is found to quantitatively dominate the price effect such that the representative agent's initial rosy anticipation can be validated. This in turn implies that the economy will exhibit equilibrium multiplicity and endogenous business cycle fluctuations. Overall, our analysis demonstrates that under the postulated balanced-budget rule with countercyclical labor income taxation in a standard two-sector RBC model, the indeterminacy result of Huang, Meng and Xue (2017, section 2) depends crucially on the presence of income effect associated with the household's labor supply decision.

4 Conclusion

Schmitt-Grohé and Uribe (1997, section II) analytically show that with an additively separable utility function between consumption and hours worked, a standard one-sector real business cycle model may display equilibrium indeterminacy and sunspot-driven aggregate fluctuations when the labor tax rate is endogenously determined by a balanced-budget rule to finance a pre-specified constant level of government spending. Subsequently, Huang, Meng and Xue (2017, section 2) numerically obtain the qualitatively identical result of macroeconomic instability within a prototypical two-sector real business cycle model under the same household-utility and fiscal-policy formulations. This paper complements these previous studies by considering an alternative preference specification that does not exhibit income effect associated with agents' labor supply decisions. We find that local indeterminacy is no longer possible in a no-income-effect macroeconomy with either one or two production sectors. In particular, the model's low-tax steady state is found to display saddle-path stability and equilibrium uniqueness; whereas the high-tax steady state is either a source or a saddle point. In sum, our analyses illustrate that under the postulated balanced-budget rule with countercyclical labor income taxation, the existence of income effect associated with the household's demand for leisure plays a critically important role for generating indeterminacy and sunspots in a canonical one-sector or two-sector representative-agent model economy.

5 Appendix

Proof of Proposition 1. Using (14), it is straightforward to show that $Tr < 0$ when $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma}$, which is higher than $\tau^* = \frac{\gamma}{1+\gamma}$; and that $Tr > 0$ when $\tau^{ss} < \frac{\alpha+\gamma}{1+\gamma} \in (0,1)$. Next, since $\Psi(\tau^{ss}) > 0$ (see footnote 4), the sign of Det given by (15) depends on whether $\frac{\tau^{ss}-\gamma(1-\tau^{ss})}{\alpha-\tau^{ss}+\gamma(1-\tau^{ss})}$ is positive or negative. In particular, $Det > 0$ if and only if $\tau^{ss} - \gamma(1-\tau^{ss})$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss})$ have the same sign, which can happen when

(a) $\tau^{ss} - \gamma(1-\tau^{ss}) < 0$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss}) < 0$. This implies that $\tau^{ss} < \frac{\gamma}{1+\gamma}$ and $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma} > \frac{\gamma}{1+\gamma}$, thus generating a contradiction.

(b) $\tau^{ss} - \gamma(1-\tau^{ss}) > 0$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss}) > 0$. This implies that $\tau^{ss} > \frac{\gamma}{1+\gamma}$ and $\tau^{ss} < \frac{\alpha+\gamma}{1+\gamma}$.

On the other hand, $Det < 0$ if and only if $\tau^{ss} - \gamma(1-\tau^{ss})$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss})$ have opposite signs, which can happen when

(c) $\tau^{ss} - \gamma(1-\tau^{ss}) > 0$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss}) < 0$. This implies that $\tau^{ss} > \frac{\gamma}{1+\gamma}$ and $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma}$, thus the more binding condition is $\tau^{ss} > \frac{\alpha+\gamma}{1+\gamma}$.

(d) $\tau^{ss} - \gamma(1-\tau^{ss}) < 0$ and $\alpha - \tau^{ss} + \gamma(1-\tau^{ss}) > 0$. This implies that $\tau^{ss} < \frac{\gamma}{1+\gamma}$ and $\tau^{ss} < \frac{\alpha+\gamma}{1+\gamma}$, thus the more binding condition is $\tau^{ss} < \frac{\gamma}{1+\gamma}$.

At the low-tax steady state with $0 < \tau_L^{ss} < \frac{\gamma}{1+\gamma}$, the model's Jacobian matrix has $Det < 0$ as in case (d), hence it is a saddle point. In addition, at the high-tax steady state with $\frac{\gamma}{1+\gamma} < \tau_H^{ss} < \frac{\alpha+\gamma}{1+\gamma}$, we find that $Tr > 0$ and $Det > 0$ as in case (b), hence it is a source. Finally, when $\frac{\alpha+\gamma}{1+\gamma} < \tau_H^{ss} < 1$, the high-tax steady state is a saddle point because of $Det < 0$ as in case (c).[9] ■

[9] Local indeterminacy requires that both eigenvalues have negative real parts ($Tr < 0$ and $Det > 0$). However, under case (b) with $\frac{\gamma}{1+\gamma} < \tau^{ss} < \frac{\alpha+\gamma}{1+\gamma}$ and $Det > 0$, the Jacobian's trace is also positive ($Tr > 0$). As a result, neither steady state can be a sink.

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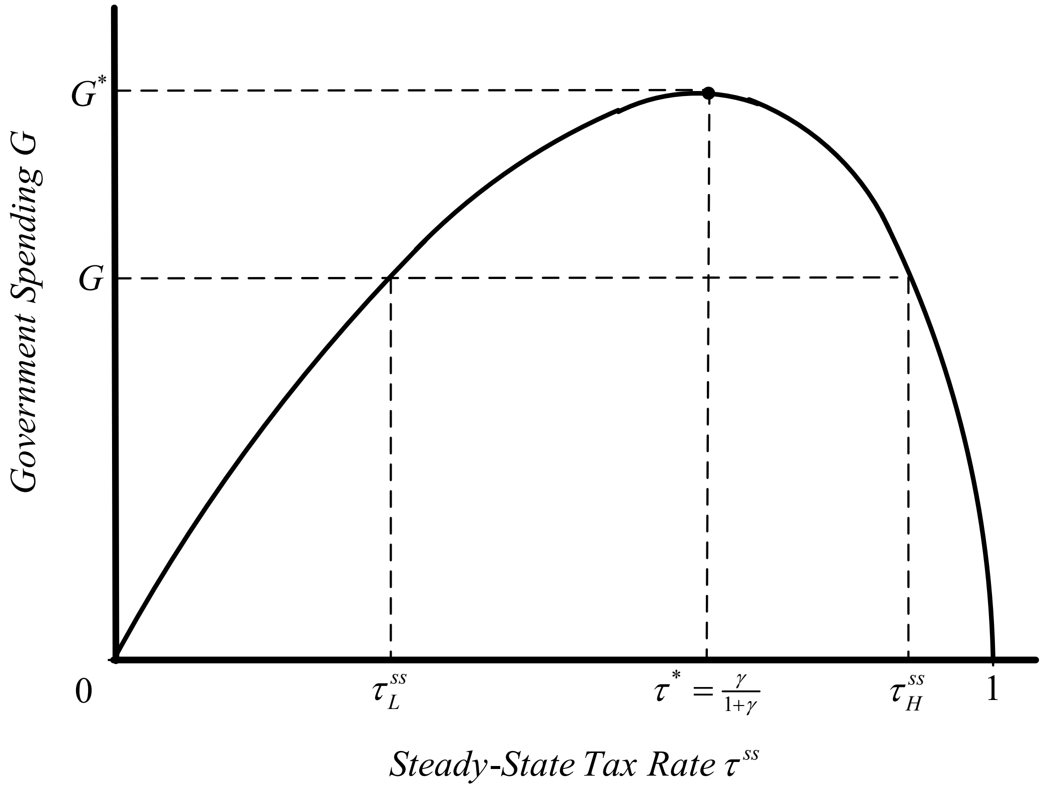

Figure 1. Steady-State Laffer Curve