The Credibility of Commitment and Optimal Nonlinear Savings Taxation*

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Abstract

Previous studies that examine optimal nonlinear taxation of savings/capital have assumed either full-commitment or no-commitment by the government. This raises the question as to whether the results under full-commitment and no-commitment provide upper and lower bounds on the optimal marginal savings tax rates. This paper shows that they do not. Specifically, we consider an infinite-horizon overlapping generations model in which agents attach some probability to whether or not the government can commit. When these probabilistic beliefs differ among high-skill individuals, the optimal steady-state marginal savings tax rates may fall outside those under the polar cases of full-commitment and no-commitment. Our numerical analysis finds that this theoretical possibility can occur under a baseline calibration with empirically plausible values of model parameters, and that it remains qualitatively robust with respect to various parametric changes.

Keywords: Savings Taxation, Commitment, Multi-Dimensional Screening.

JEL Classifications: E60, H21, H24.

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1 Introduction

Previous studies that examine optimal nonlinear taxation of savings/capital have typically assumed full commitment by the government, while a few studies have considered no commitment. Under full-commitment, the government announces its tax policies for the present and future, and then simply implements those policies. Importantly, it is implicit that agents completely believe that the government will implement its announced policies. Under no-commitment, the government re-optimizes its tax policies period by period, irrespective of any previous promises or announcements. In this environment, individuals are aware that the government will re-optimize each period. It follows that under full-commitment, it is common knowledge among agents and the government that the probability of commitment is one, whereas this probability is zero under no-commitment.

In this paper, our aim is to examine a setting that falls between the aforementioned polar cases of full-commitment and no-commitment, and that better reflects realistic behavior and beliefs. To illustrate our thinking, consider for example corporate tax competition among governments. In order to entice a company to locate in its country, a government may promise to levy a low corporate tax rate. However, if the company does undertake operations in its country, the government will be tempted to raise its corporate tax rate. This temptation exits because the company is now a resident, and moving is costly. Moreover, when making its location decision, the company is aware of this temptation and the possibility that it may face higher taxation in the future. The government will then be aware that the company is aware of this temptation, and so on. If one were to assume full-commitment, it is common knowledge that the government will never succumb to this temptation. On the other hand, it is common knowledge that the government will always succumb to this temptation under no-commitment. In our view, a more realistic setting is one we call ‘commitment without credibility’. In this case, the government sets taxes as it would under full-commitment, but it is aware that individuals attach some probability to re-optimization. Therefore, the government’s promise to commit is not completely credible.\footnote{Notice that ‘commitment without credibility’ is conceptually different from ‘partial commitment’. The latter describes a scenario in which the government lacks full commitment and may exploit information collected at previous dates. However, this paper considers the government with full commitment, but its promise to commit is not fully credible \textit{vis-à-vis} agents.}

As in the related literature, we adopt the Mirrlees (1971) information-constrained approach to analyze optimal nonlinear taxation of labor income and savings under commitment-without-credibility in an infinite-horizon overlapping generations (OLG) model that is inhabited by two-ability-type (high-skill and low-skill) workers \textit{à la} Stiglitz’s...
(1982). Each agent is postulated to live for two periods: working in the first period and living-off savings in the second period. These features in turn create a redistributive role for taxation, as well as the possibility that individuals have doubt about the government’s commitment to its savings tax policy. In addition, based on the empirical evidence that citizens who receive higher (lower) education degrees and/or earn higher (lower) levels of income are more (less) likely to participate in public affairs, we first postulate that high-skill agents are sophisticated and can have heterogeneous beliefs about the government’s credibility. Given the redistributive objective associated with the tax policy, low-skill individuals will never want to choose the high-skill type’s allocation, whereas high-skill individuals may want to mimic low-skill workers by choosing their allocation. The government will deter such mimicking behavior by making sure that the allocations it offers satisfy the high-skill type’s incentive-compatibility constraints.

We also postulate that low-skill agents are naive and have a common belief regarding the probability of commitment; but we do not need to specify its notation because the incentive-compatibility constraints for low-skill individuals are always slack and can be omitted. It follows that this commonly-shared belief does not play any role in deriving the optimal tax system.

In the context of a standard Mirrlees-style model in which agents differ only by their abilities or skills, the existing literature has found that zero marginal savings taxation is optimal under full-commitment as in Atkinson and Stiglitz (1976). But savings taxation will be progressive under no-commitment, in that the optimal marginal savings tax rates are increasing with respect to individuals’ skills (see, e.g. Farhi, et al. 2012; Brett and Weymark 2019). As a result, it is tempting to view the results under no-commitment as providing the upper-bound on the size of the optimal marginal savings tax distortions. In the model economy that we consider, it is shown that if high-skill agents have the same beliefs regarding the probability of commitment, then the optimal marginal savings tax rates do always fall between those under full-commitment and no-commitment. Intuitively, high-skill individuals know that if the government re-optimizes the savings tax in the second period of their lives, it will redistribute some of their savings toward low-skill workers. This creates an incentive for high-skill agents to pretend as low-skill individuals. It follows that in order to deter mimicking, the government brings forward consumption by high-skill workers (through taxing their savings) and delays consumption by low-skill individuals (through subsidizing their savings). The intuition

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2 The first paper to examine Mirrlees-style taxation in the context of an OLG setting is Ordover and Phelps (1979). There are now a number of papers that study nonlinear taxation within OLG frameworks, though the specifics of our OLG model are most similar to those in Brett (2012) and Krause (2019).
for the non-zero marginal savings tax rates under commitment-without-credibility is qualitatively identical to that under no-commitment discussed above, but they are closer to zero because there is some chance of commitment.

We also find that when high-skill agents have different beliefs, optimal nonlinear savings tax rates do not simply fall between those under full-commitment and no-commitment – this is our main result. Specifically, some high-skill workers may face larger marginal savings tax distortions than they would under no-commitment, even though all individuals believe that there is some probability of commitment. The explanation for this counter-intuitive finding follows from the possibility that high-skill agents may disagree vis-à-vis the probability of commitment. Under full-commitment and no-commitment, all workers know and agree that the probability of commitment is one and zero, respectively. Therefore, individuals differ only by their skills. Under commitment-without-credibility, agents differ by their skills as well as their beliefs. This makes the optimal tax problem more complicated, as it must now take into account this second source of heterogeneity, i.e. belief heterogeneity itself calls for marginal savings tax distortions. It is this new rationale on taxing savings that makes it theoretically possible for some high-skill individuals to face larger marginal savings tax distortions under commitment-without-credibility than they would under no-commitment. To gain further insights, we numerically show that this theoretical possibility can take place under a baseline calibration with empirically plausible values of model parameters, and that it remains qualitatively robust with respect to various parametric changes. In sum, the general point of our analysis is to illustrate that a minor relaxation of the full-commitment assumption, particularly when agents do not completely believe the government’s credibility to commit, may yield substantive effects on the optimal marginal savings tax rates.

In terms of policy relevance, we have developed an analytical framework in which the commitment-without-credibility government should impose a higher marginal savings tax rate on certain high-skill individuals than that under the no-commitment regime. Moreover, our numerical experiments find that this highest marginal tax rate on savings will become higher when (i) the degree of belief heterogeneity among high-skill workers rises, or (ii) there is an increase in the proportion of high-skill agents who believe that the government’s credibility is strong, or (iii) the ability/productivity gap between high-skill versus low-skill individuals is enlarged. It can be shown that any of the above three scenarios increases the information rent that high-skill workers may receive from their mimicking behavior. In order to induce truth-telling, the government will front-load these agents’ consumption through raising the top marginal tax rate on savings. These results are valuable not only for their theoretical insights to the academic literature,
but also for their broad implications about the design and implementation of optimal nonlinear savings taxation within an infinite-horizon OLG model.

As it turns out, heterogeneous beliefs have been incorporated into some recent research in macroeconomics and finance; see, for example, David (2008) and Cogley, Sargent and Tsyrenniko (2014), among others. However, this feature has not been extensively considered in the field of public finance theory. A notable exception is Blume and Easley (2006) that analyze the asymptotic properties of Pareto optimal allocations when heterogeneous consumers differ in their beliefs about the state of the economy. In addition, our work is closely related to Farhi and Gabaix (2020) who examine nonlinear optimal taxation with behavioral individuals that exhibit heterogeneous beliefs. These authors show that in the presence of agents’ misperception, the properties of optimal nonlinear taxation stand in sharp contrast to the traditional results. Specifically, if poor agents do not fully recognize the future benefits of work, the optimal marginal tax rate of labor income for these individuals should be negative. Since Farhi and Gabaix (2020) study a static Mirrlees-style model, the implications of the intertemporal tax wedge (or the marginal savings tax rates) are not investigated. On the contrary, this paper examines constrained-efficient allocations in a dynamic commitment-without-credibility setting with unobservable skill as well as belief heterogeneities.

The remainder of this paper is organized as follows. Section 2 outlines our modelling framework, Section 3 analytically as well as numerically examines optimal nonlinear taxation of savings under commitment-without-credibility, and Section 4 concludes. A number of mathematical derivations are contained in three appendices.

2 Preliminaries

We examine a simple model that makes the commitment-without-credibility optimal tax problem analytically tractable. In spite of this necessary simplicity, our formulation is sufficiently rich to capture the salient features of the problem. The framework is an infinite-horizon overlapping generations (OLG) model in which each agent lives for two periods. Individuals work in the first period of their lives, and they may be a high-skill or low-skill worker, which creates a redistributive role for taxation. In the second period of their lives agents are retired, and must live-off their savings. This creates a need for savings, as well as the possibility that individuals have doubt about the government’s

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3 The commitment-without-credibility optimal tax problem is a multi-dimensional screening problem. It is well-known that optimal tax problems with two unobserved characteristics can be difficult to solve, even in simple settings. See, for example, Boadway, et al. (2002) and the discussion within. Other recent papers that examine optimal taxation with multi-dimensional types include Jaquet and Lehman (2014) and Rothschild and Scheuer (2015).
commitment to its savings tax policy.

The ability or productivity levels of low-skill and high-skill agents are denoted by $a_1$ and $a_2$ respectively, with $a_2 > a_1 > 0$, such that type 1 individuals are low-skill workers and type 2 individuals are high-skill workers. Several previous studies have found empirical support for a highly-positive correlation between citizen engagement and education/income. According to U.S. Bureau of the Census (2002), the characteristics of participating voters reflect the attributes of people with the biggest stakes in society, such as those who receive higher education degrees and/or earn higher levels of income. On the contrary, citizens who are less educated and/or poor are more unlikely or reluctant to participate in public affairs as documented by Perkins, Brown and Taylor (1996). In light of this evidence, we first postulate that high-skill workers are sophisticated and have skepticism about the government’s credibility on policy commitment. Since the degree of these agents’ suspicion can vary along various dimensions (e.g. age, gender and races, among others), heterogeneous beliefs on the reliability of the tax policy are considered. For the sake of analytical simplicity, each high-skill individual believes that the probability of commitment is either high or low, denoted as $p^H$ and $p^L$ respectively, with $1 > p^H > p^L > 0$. Let $\pi_j$ denote the proportion of high-skill workers who believe that the probability of commitment is $j$, where $j = L$ or $H$ for low or high probability and $\pi_L + \pi_H = 1$.

In accordance with the above-mentioned stylized fact, we also postulate that low-skill individuals are naive and have a common belief regarding the probability of commitment that may take on any value over the closed interval $[0, 1]$; but we do not need to specify its notation because the incentive-compatibility constraints for low-skill agents are always slack and can be omitted. It follows that this commonly-shared belief does not play any role in the design of the optimal tax system. Although our model could be extended to incorporate heterogeneous beliefs among low-skill workers, the resulting optimal tax problem will become intractable and much more complicated, making it less clear as to which incentive-compatibility constraints may bind (see Section 3).

2.1 Consumers

All agents live for two periods, period $t$ and period $t + 1$. An individual born in period $t$ consumes, works, and saves in period $t$, and then lives-off her/his savings in period $t + 1$. Preferences are represented by the quasi-linear utility function:

$$u(c^t_{ij}) - l^t_{ij} + \delta u(x^{t+1}_{ij}), \quad (2.1)$$

where $c^t_{ij}$ is type $ij$’s period $t$ consumption, $l^t_{ij}$ is type $ij$’s labor supply, and $x^{t+1}_{ij}$ is type $ij$’s period $t + 1$ consumption. For high-skill individuals, $i = 2$ and $j = L$ or $H$ for low or high expected probability of commitment. For low-skill individuals the subscript $j$ is
redundant; thus we have $c_i^t$, $l_i^t$, and $x_i^{t+1}$, where $i = 1$. The specification that the utility function is quasi-linear is stronger than the usual additively-separable assumption in the literature. However, quasi-linearity enables us to compare the size (not just sign) of the optimal marginal savings tax rates across different commitment regimes. The function $u(\cdot)$ is increasing and strictly concave, while $\delta \in (0, 1)$ denotes the discount factor. For future reference, we use $y_{ij}^t = w^t a_i l_{ij}^t$ to denote the pre-tax income of a type $ij$ individual in period $t$, where $w^t$ is the wage rate and $a_i l_{ij}^t$ is type $ij$’s effective labor supply.

In the absence of taxation, individuals would choose $c_{ij}^t$, $l_{ij}^t$, $s_{ij}^t$, and $x_{ij}^{t+1}$ to maximize the utility function (2.1) subject to the budget constraints:

$$c_{ij}^t + s_{ij}^t \leq w^t a_i l_{ij}^t,$$
$$x_{ij}^{t+1} \leq (1 + r^{t+1}) s_{ij}^t,$$  

where $s_{ij}^t$ is type $ij$’s savings in period $t$, and $r^{t+1}$ is the rate-of-return on savings in period $t + 1$. It is shown in Appendix A.1 that the solution to program (2.1) – (2.3) yields the marginal condition:

$$\frac{u'(c_{ij}^t)}{\delta (1 + r^{t+1}) u'(x_{ij}^{t+1})} = 1.$$  

However, in the presence of taxation, equation (2.4) might not be satisfied. The resulting marginal distortion is commonly interpreted as a ‘tax wedge’ or ‘implicit marginal tax rate’ on savings. Thus, we define:

$$\tau_{ij}^t := 1 - \frac{u'(c_{ij}^t)}{\delta (1 + r^{t+1}) u'(x_{ij}^{t+1})},$$

as the (implicit) marginal tax rate on savings in period $t$ faced by a type $ij$ individual.

2.2 Producer

The production side of our OLG model is standard. There is a representative profit-maximizing producer with a constant returns-to-scale production function:

$$Q^t = F(K^t, Z^t),$$

where $Q^t$ is total output in period $t$, $K^t$ is the aggregate capital stock in period $t$, and $Z^t$ is total effective labor in period $t$. Total effective labor consists of total low-skill and high-skill effective labor: $Z^t = Z_1^t + Z_2^t$, where $Z_1^t = N_1^t a_1 l_1^t$ and $Z_2^t = N_2^t (\pi_L a_2 l_{2L}^t + \pi_H a_2 l_{2H}^t)$, with $N_i^t$ representing the number of type $i$ workers in period $t$. We make the simplifying assumption that $N_1^t = N_2^t$, and that the populations of both low-skill and high-skill individuals grow at the same (positive) fixed rate of $n$ per period, i.e. $n = \frac{N_i^{t+1} - N_i^t}{N_i^t}$. 

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The preceding production function can be rewritten as:

\[ Q_t = Z_t f(k^t) \]  

(2.7)

where \( k^t = \frac{K_t}{Z_t} \) is the ratio of capital to effective labor, and \( f(\cdot) \) is increasing and strictly concave. Profit maximization implies that:

\[ \frac{\partial Q_t}{\partial Z_t} = f(k^t) - f'(k^t)k^t = w^t, \]  

(2.8)

\[ \frac{\partial Q_t}{\partial K_t} = f'(k^t) = r^t, \]  

(2.9)

which are the standard conditions that the marginal product of (effective) labor equals the wage rate, and the marginal product of capital equals the rental rate.

2.3 Steady-State Equilibrium

We focus on the steady-state equilibrium of our OLG model in which all per capita variables are constant through time. It is shown in Appendix A.2 that the steady-state equilibrium condition is:

\[ f(k) = \frac{w \left[ c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1+n} \right]}{y_1 + \pi_L y_{2L} + \pi_H y_{2H}} + nk, \]  

(2.10)

where the absence of the time superscript \( t \) represents the steady-state value of each variable.

3 Commitment Without Credibility

Under commitment-without-credibility, the government sets taxes as it would under full-commitment, but is aware that agents attach some probability to its re-optimizing the savings tax rates after individuals have made their savings decisions. We examine optimal steady-state nonlinear taxation under Mirrlees-style information constraints that the government cannot observe any worker’s skill type, nor her/his belief \( \text{vis-à-vis} \) the probability of commitment. Accordingly, the government’s choice of an optimal nonlinear tax schedule is equivalent to its choosing a steady-state allocation for each type of individuals, \( \langle c_1, y_1, x_1, c_{2L}, y_{2L}, x_{2L} \rangle, \langle c_{2H}, y_{2H}, x_{2H} \rangle \), and \( k \) to maximize:

\[ u(c_1) - \frac{y_1}{w a_1} + \delta u(x_1) + \sum_{j=L,H} \pi_j \left[ u(c_{2j}) - \frac{y_{2j}}{w a_2} + \delta u(x_{2j}) \right], \]  

(3.1)

subject to:

\[ f(k) \geq \frac{w \left[ c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1+n} \right]}{y_1 + \pi_L y_{2L} + \pi_H y_{2H}} + nk, \]  

(3.2)
\[ u(c_{2H}) - \frac{y_{2H}}{w_2} + \delta \left[ p^H u(x_{2H}) + (1 - p^H)u(x) \right] \geq u(c_{2L}) - \frac{y_{2L}}{w_2} + \delta \left[ p^H u(x_{2L}) + (1 - p^H)u(x) \right], \]
\[ u(c_{2H}) - \frac{y_{2H}}{w_2} + \delta \left[ p^H u(x_{2H}) + (1 - p^H)u(x) \right] \geq u(c_1) - \frac{y_1}{w_2} + \delta \left[ p^H u(x_1) + (1 - p^H)u(x) \right], \]
\[ u(c_{2L}) - \frac{y_{2L}}{w_2} + \delta \left[ p^L u(x_{2L}) + (1 - p^L)u(x) \right] \geq u(c_1) - \frac{y_1}{w_2} + \delta \left[ p^L u(x_1) + (1 - p^L)u(x) \right], \]

where \( w = f(k) - f'(k)k \) by profit maximization (equation 2.8). Equation (3.1) is a utilitarian social welfare function,\(^4\) which is analogous to the aggregate welfare function that the government would maximize under full-commitment. This reflects the government’s promise that it will not re-optimize the savings tax policy. Equation (3.2) is a restatement of the steady-state equilibrium condition (2.10), and it ensures that the chosen allocation is feasible. It also ensures, indirectly, that the government’s budget will be balanced.

Equations (3.3) – (3.5) are incentive-compatibility constraints. At this point, we encounter the usual challenge associated with multi-dimensional screening problems in that there are numerous (in our model, six) incentive-compatibility constraints that need to be considered.\(^5\) However, we will analyze solutions to the optimal tax problem in which only (3.3) – (3.5) may be binding, for the following reasons. Consider incentive-compatibility constraint (3.3), which links the \( 2H \) and \( 2L \) types. Recall that \( 2H \) agents are high-skill individuals who believe that there is a high probability of commitment. These individuals think that there is a good chance that the government will not re-optimize the savings tax rates, therefore their retirement consumption will be \( x_{2H} \). But if the government does re-optimize, it will equalize consumption across all individuals as is optimal under no-commitment (Farhi, et al. 2012; Brett and Weymark 2019). Let \( x \) denote this equalized level of consumption in retirement. In this no-commitment environment, high-skill agents would then have their savings taxed to subsidize the retirement consumption of low-skill workers (see Appendix B). On the other hand, \( 2L \) agents are high-skill individuals who believe that the probability of commitment is low. They think it is relatively more likely that the government will re-optimize the savings tax rates, and their retirement consumption will be only \( x \). Accordingly, \( 2L \) workers are relatively more reluctant to reveal their type, and they must be offered a generous allocation to do so. This implies that \( 2H \) agents will be attracted to the allocation intended for \( 2L \) individuals, but not vice versa. Incentive-compatibility constraint (3.3) prevents such mimicking behavior, by ensuring that the utility a \( 2H \) individual can

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\(^4\)Recall that there are equal numbers of low-skill and high-skill individuals.

\(^5\)If low-skill individuals were also distinguished by their beliefs regarding the probability of commitment, there would be twelve incentive-compatibility constraints to consider.
expect from choosing \((c_{2H}, y_{2H}, x_{2H})\) is greater than or equal to what s/he could obtain by choosing \((c_{2L}, y_{2L}, x_{2L})\). Specifically, \(2H\) workers believe that under the probability of commitment \(p^H\), they would be able to consume their savings in retirement \(x_{2H}\); but with probability \((1 - p^H)\) they expect that the government will re-optimize and their retirement consumption will be \(x\). Finally, incentive-compatibility constraints (3.4) and (3.5) reflect the standard assumption that the government seeks to redistribute from high-skill to low-skill agents, which creates an incentive for each type of high-skill individuals to pretend as low-skill workers, but not vice versa. It follows that the incentive-compatibility constraints for low-skill agents are always slack and can be omitted. In summary, for a broad range of empirically plausible parameter values, we believe that only incentive-compatibility constraints (3.3) – (3.5) will be binding. The numerical examples below also confirm that only (3.3) – (3.5) are binding.

3.1 Homogeneous Beliefs

Before proceeding to our main results, we show that if high-skill agents have the same beliefs regarding the probability of commitment, then the optimal marginal savings tax rates must fall between those under full-commitment and no-commitment:

**Proposition 1** Suppose high-skill individuals have the same probabilistic beliefs, \(p^H = p^L = p\). The optimal marginal tax rates applicable to savings are: \(\tau_{2}^{NC} > \tau_{2} > 0 > \tau_{1} > \tau_{1}^{NC}\).

**Proof.** See Appendix C.1.

In Proposition 1, \(\tau_{2}^{NC}\) and \(\tau_{1}^{NC}\) represent the optimal marginal savings tax rates under no-commitment faced by high-skill and low-skill workers, respectively. Under commitment-without-credibility, if high-skill agents have the same probabilistic beliefs \((p^H = p^L = p)\), the subscript \(j\) denoting belief type is redundant; thus we simply use \(\tau_{2}\) and \(\tau_{1}\) to represent the optimal marginal savings tax rates faced by high-skill and low-skill individuals, respectively. Proposition 1 confirms that without belief heterogeneity, the optimal marginal savings tax rates must fall between those under the polar cases of full-commitment (zero marginal savings taxation) and no-commitment (progressive marginal savings taxation). Indeed, if \(p = 1\) (full-commitment) we obtain \(\tau_{1} = \tau_{2} = 0\), and if \(p = 0\) (no-commitment) we obtain \(\tau_{2} = \tau_{2}^{NC} > 0\) and \(\tau_{1} = \tau_{1}^{NC} < 0\). A full discussion of the intuition underlying the no-commitment results can be found in Farhi, et al. (2012) and Brett and Weymark (2019), but it may be summarized as follows.\(^6\) High-skill agents know that if the government re-optimizes the savings tax in their retirement period, it will redistribute some of their savings toward low-skill workers. This creates an incentive for high-skill agents to mimic as low-skill individuals.

\(^6\)In Farhi, et al. (2012) agents work only in the first period, whereas in Brett and Weymark (2019) individuals work in both periods. In this respect, our analysis is closer to Farhi, et al. (2012).
It follows that in order to deter mimicking, the government brings forward consumption by high-skill workers (through taxing their savings) and delays consumption by low-skill individuals (through subsidizing their savings). The intuition for the non-zero marginal savings tax rates under commitment-without-credibility is qualitatively the same as that under no-commitment, but they are closer to zero because there is some chance of commitment.

It is worth noting that the case of homogeneous beliefs considered here is similar to ‘loose commitment’ in the literature, with the difference being that the loose-commitment government explicitly recognizes that it may re-optimize with some probability. As a result, it is common knowledge among individuals and the government that the probability of commitment is $p \in (0, 1)$ under loose commitment. It is then straightforward to show that the pattern of optimal marginal savings tax rates described in Proposition 1 also holds under loose commitment. Guo and Krause (2014) examine optimal nonlinear income taxation under loose commitment, but in an economy without savings. Debortoli and Nunes (2010) examine capital and labor taxation under loose commitment in a representative-agent model.

3.2 Heterogeneous Beliefs

We now consider the general commitment-without-credibility setting in which $p^H > p^L$, which leads to the following results:

**Proposition 2** Under commitment-without-credibility with heterogeneous beliefs among high-skill agents, the optimal marginal tax rates applicable to savings are: $\tau_{2L} > \tau_{2H} > 0 > \tau_1$.

*Proof.* See Appendix C.2.

High-skill agents face positive marginal savings tax rates, while low-skill individuals face a negative marginal savings tax rate. This is qualitatively identical to that under no-commitment, and the intuition for the progressive pattern of optimal marginal savings taxation is analogous. In addition, those high-skill workers who believe that the probability of commitment is low (the $2L$ type) face the largest marginal savings tax distortion. This is because $2L$ agents think it is relatively more likely that the government will re-optimize the savings tax policy. Accordingly, they must face the highest marginal savings tax rate to deter mimicking by $2H$ individuals.

**Proposition 3** Type $2H$ and low-skill agents always face lower marginal savings tax rate distortions under commitment-without-credibility than under no-commitment, i.e. $\tau_{2H} < \tau_{2NC}$ and $\tau_{1NC} < \tau_1$. However, type $2L$ individuals may face a greater marginal savings tax rate distortion under commitment-without-credibility than under no-commitment, i.e. $\tau_{2L} > \tau_{2NC}$ is theoretically possible.

*Proof.* See Appendix C.3.
Type $2H$ and low-skill individuals always face lower marginal savings tax rate distortions under commitment-without-credibility than under no-commitment. However, it is a theoretical possibility that $2L$ workers face a larger marginal savings tax rate distortion under commitment-without-credibility than under no-commitment. This is our main result, and it may occur even though the only difference between full-commitment and commitment-without-credibility is that individuals in the latter circumstances attach some probability to re-optimization. The intuition is as follows. Agents are distinguished only by their skills under no-commitment, as it is common knowledge that the government cannot commit. Under commitment-without-credibility, however, individuals are distinguished by their skills as well as their beliefs regarding the probability of commitment. The key here is that belief heterogeneity itself calls for marginal savings tax rate distortions. An increase in the degree of belief heterogeneity makes the tax contract intended for $2L$ agents more attractive than that offered to $2H$ individuals. As a result, a higher marginal savings tax rate on $2L$ workers is required to deter mimicking. In particular, Appendix C.3 analytically shows that $2L$ individuals will face a higher marginal tax rate on their savings under commitment-without-credibility than they would under no-commitment if and only if the difference in beliefs among high-skill agents, represented by the ratio of $\frac{p^H}{p^L}$, is sufficiently large.

### 3.3 Numerical Analysis

In this subsection, we provide numerical examples to verify the theoretical possibility stated in Proposition 3 that $\tau_{2L} > \tau_{2NC}$. This possibility is demonstrated with empirically-plausible parameter values, thus illustrating that it cannot be regarded as an unlikely occurrence. These numerical examples also confirm that under commitment-without-credibility, only incentive-compatibility constraints (3.3)–(3.5) are binding and the remaining omitted incentive-compatibility constraints are slack. Our quantitative analysis begins with a benchmark parameterization which postulates equal numbers of high-skill agents believing that the probability of commitment is high or low, i.e. $\pi_H = \pi_L = \frac{1}{2}$, together with the corresponding probabilities given by $p^H = 0.9$ and $p^L = 0.1$. That is, one-half of high-skill individuals think that commitment is highly likely, while the other half think that commitment is highly unlikely. In addition, we specify the quasi-linear utility function (2.1) as:

$$\log(c_{ij}) - l_{ij} + \delta \log(x_{ij}),$$

which follows from Chetty (2006) who concludes that the coefficient of relative risk aversion is one, hence the logarithmic preference formulation. As in Krause (2019), each period is assumed to be 20-years in length and the annual discount rate is assumed to be 2% (Kocherlakota 2010), making $\delta = 0.67$. We also select $n = 0.22$ to reflect an
annual population growth of about 1%. The college wage premium is calibrated to be approximately 60% (Fang 2006; Goldin and Katz 2007), therefore we normalize $a_1 = 1$ and set $a_2 = 1.6$.

The production function (2.6) is postulated to take a constant elasticity-of-substitution (CES) form that exhibits constant returns-to-scale:

$$Q = [\alpha K^\rho + (1 - \alpha) Z^\rho]^{\frac{1}{\rho}}, \quad 0 < \alpha < 1 \text{ and } -\infty < \rho < 1,$$

(3.7)

where $\alpha$ governs the factor income shares, and $\rho$ determines the elasticity of substitution between capital and labor inputs, given by $\sigma = \frac{1}{1-\rho} \in (0, \infty)$. For the baseline configuration, we choose $\rho = 0$ such that the production function (3.7) reduces to the Cobb-Douglas formulation; and $\alpha = \frac{1}{3}$ to match with the capital (and thus labor) share of national income in developed countries.

Table 1 presents the optimal marginal tax rates on savings for our benchmark numerical example. It can be seen that $\tau_{2L} > \tau_{2H} > 0 > \tau_1$, as stated in Proposition 2. We also find that $2H$ agents and low-skill individuals face lower marginal savings tax rate distortions under commitment-without-credibility (CWC) than they would under no-commitment (NC) with $p^H = p^L = 0$ across all high-skill workers; and that $\tau_{2L} > \tau_{2NC}^2$, thus demonstrating the possibility identified in Proposition 3.

<table>
<thead>
<tr>
<th>Table 1: Benchmark Numerical Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Values</td>
</tr>
<tr>
<td>$\pi_H = \frac{1}{2}$, $p^H = 0.9$, $a_1 = 1$, $\delta = 0.67$, $\rho = 0$</td>
</tr>
<tr>
<td>$\pi_L = \frac{1}{2}$, $p^L = 0.1$, $a_2 = 1.6$, $n = 0.22$, $\alpha = \frac{1}{3}$</td>
</tr>
<tr>
<td>Optimal Marginal Tax Rates on Savings</td>
</tr>
<tr>
<td>CWC</td>
</tr>
<tr>
<td>$\tau_{2L} = 0.3$</td>
</tr>
<tr>
<td>$\tau_{2H} = 0.0231$</td>
</tr>
<tr>
<td>$\tau_1 = -0.3$</td>
</tr>
<tr>
<td>NC</td>
</tr>
<tr>
<td>$\tau_{2NC}^2 = 0.2308$</td>
</tr>
<tr>
<td>$\tau_{1NC}^2 = -0.4286$</td>
</tr>
</tbody>
</table>

In terms of sensitivity analyses, we find that the CWC and NC optimal marginal tax rates on savings reported in Table 1 remain numerically unaffected with respect to changes in one of the following four parameters: the population growth rate $n$; the elasticity of substitution between capital and labor in production which is governed by $\rho$; and the capital share of national income $\alpha$; and the discount factor $\delta$. The intuition for this finding is as follows. Stemming from incentive-compatibility constraints of high-skill individuals (3.3) – (3.5), the intertemporal ‘tax wedges’ (2.5) are imposed by the government to impede these workers’ misreporting their types. Since the quasi-linear utility
function (3.6) is postulated to be logarithmic in consumption that is separable from labor hours, together with linear homogeneity of the production technology (3.7), Appendix D first shows that in response to ceteris paribus changes in the parametric space of \( \{ n, \rho, \alpha \} \), the aggregate consistency condition (D.7) and the incentive-compatibility constraints (D.8)–(D.10) will continue to bind when the steady-state levels of total consumption and real wage move at the same direction and by the same rate. Appendix D also shows that upon a change in \( \delta \), the government can adjust individuals’ labor supply and retirement consumption to satisfy all the requisite constraints, given by (D.7)–(D.9) and (D.13), without maneuvering the associated intertemporal tax wedges. As a result, the information rent that high-skill workers may receive from their mimicking behavior will remain the same, which in turn implies that there is no need for the government to alter the original marginal savings tax rate distortions.

Table 2: Sensitivity Analyses on Optimal Marginal Savings Tax Rates

<table>
<thead>
<tr>
<th>Commitment-without-Credibility</th>
<th>No-Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^H = \frac{2}{3} ) and ( p^L = \frac{1}{3} )</td>
<td>( \tau_{2L} = 0.2820; \tau_{2H} = 0.0769; \tau_1 = -0.3333 )</td>
</tr>
<tr>
<td>( p^H = \frac{5}{6} ) and ( p^L = \frac{1}{6} )</td>
<td>( \tau_{2L} = 0.2949; \tau_{2H} = 0.0385; \tau_1 = -0.3095 )</td>
</tr>
<tr>
<td>( \pi_H = \frac{1}{3} ) and ( \pi_L = \frac{2}{3} )</td>
<td>( \tau_{2L} = 0.2723; \tau_{2H} = 0.0231; \tau_1 = -0.3514 )</td>
</tr>
<tr>
<td>( \pi_H = \frac{2}{3} ) and ( \pi_L = \frac{1}{3} )</td>
<td>( \tau_{2L} = 0.3346; \tau_{2H} = 0.0231; \tau_1 = -0.2357 )</td>
</tr>
<tr>
<td>( a_1 = 1 ) and ( a_2 = 1.4 )</td>
<td>( \tau_{2L} = 0.2167; \tau_{2H} = 0.0167; \tau_1 = -0.1750 )</td>
</tr>
<tr>
<td>( a_1 = 1 ) and ( a_2 = 1.8 )</td>
<td>( \tau_{2L} = 0.3714; \tau_{2H} = 0.0286; \tau_1 = -0.4667 )</td>
</tr>
</tbody>
</table>

*Since agents know that the no-commitment government re-optimizes its tax policy, \( \tau_{2NC}^1 \) and \( \tau_{1NC}^1 \) will remain the same for all combinations of \( \{ p^H, p^L \} \) and \( \{ \pi_H, \pi_L \} \).

Next, we consider the cases with variations in the remaining parameters that are related to characteristics of agent types. Table 2 shows that for different combinations of \( \{ p^H, p^L \} \), \( \{ \pi_H, \pi_L \} \) or \( \{ a_1, a_2 \} \) ceteris paribus, the resulting marginal savings tax rates are qualitatively identical to those as in Table 1 under our benchmark parameterization, yielding that \( \tau_{1NC}^1 < \tau_1 < 0 < \tau_{2H} < \tau_{2NC}^1 < \tau_{2L} \). This illustrates that our Propositions 2 and 3 are qualitatively robust with respect to various parametric changes. Moreover, combining Tables 1 and 2 together finds that the highest marginal tax rate on savings \( \tau_{2L} \) is monotonically increasing in (i) the degree of belief heterogeneity among high-skill individuals, represented by \( \frac{b^H}{p^x} \); or (ii) the fraction of high-skill workers who believe that the probability of commitment is high \( \pi_H \); or (iii) the ability/productivity gap between high-skill versus low-skill agents \( a_2 \). Intuitively, \( 2H \) individuals have a stronger
incentive to pretend as $2L$ workers under scenarios (i) or (ii). In order to induce truth-telling, the government will front-load $2L$ agents’ consumption through increasing the marginal tax rate on their savings. In a similar vein, the information rent that high-skill individuals can receive from mimicking low-skill workers becomes higher under scenarios (iii). If follows that the government will impose higher marginal savings rates on high-skill agents (raising both $\tau_{2H}$ and $\tau_{2L}$) and subsidize low-skill individuals more (lowering $\tau_1$). In sum, the preceding numerical results are valuable not only for their confirming the theoretical possibility of $\tau_{2L} > \tau_{NC}^2$, but also for their broad implications about the design and implementation of optimal nonlinear savings taxation within an infinite-horizon OLG model.

3.4 Discussion

Proposition 3 and the subsequent numerical examples demonstrate the possibility that some agents, in our model the $2L$ type, may face a higher marginal savings tax rate under commitment-without-credibility than under no-commitment. This result is somewhat surprising, given that commitment-without-credibility is an intermediate framework between full-commitment and no-commitment; and it does depend upon high-skill workers having different beliefs regarding the probability of commitment. The question then arises as to why these individuals may disagree vis-à-vis the probability of commitment. To this end, based on the empirical evidence that citizens who receive higher education degrees and/or earn higher levels of income are more likely to participate in public affairs, we postulate that high-skill agents may have different opinions regarding the credibility of that promise.

Another issue that arises is with regard to learning. In our model each individual lives for two periods, and observes only one commitment decision by the government. If agents were able to observe many commitment decisions, then it is reasonable to expect that they would learn that the government’s commitment promise is credible, and update their beliefs accordingly. In this case, all workers’ probabilistic beliefs will converge to one, i.e. the full-commitment specification. However, we think that the commitment-without-credibility formulation remains a better description of the observed reality than full-commitment or no-commitment.

Under commitment-without-credibility, the government promises to commit, but agents do not completely believe that promise. This is because re-optimization yields a higher level of social welfare in the short run, so individuals recognize that the government will be tempted to re-optimize. Nevertheless, the government’s motivation in promising to commit is clear, as commitment yields the highest level of social welfare attainable in the long run. The opposite formulation would be one in which the government states that it will re-optimize each period (no-commitment), but workers believe that
there is some chance that the government will not re-optimize. However, the motivations that the government and agents have for their promises/beliefs in this environment are less clear, thus we think that the commitment-without-credibility setting is a better description of the observed reality.

4 Concluding Comments

The cases of full-commitment and no-commitment make strong assumptions regarding the government’s behavior and the beliefs of individuals. Our specification of commitment-without-credibility represents an intermediate as well as a more realistic setting. We find that if agents differ in their beliefs regarding the probability of commitment, some workers may face larger marginal savings tax rate distortions than under no-commitment. This is possible even though all individuals believe that there is some probability of commitment. Therefore, the canonical result that zero marginal savings taxation is optimal under full-commitment (Atkinson and Stiglitz 1976) appears to be very sensitive to the assumption that commitment is completely credible.

5 Appendix A

A.1 Derivation of Equation (2.4)
The relevant first-order conditions from program (2.1) – (2.3) are:

\[ u'(c_{ij}^t) - \lambda^t = 0, \]  
\[ -\lambda^t + \lambda^{t+1}(1 + r^{t+1}) = 0, \]  
\[ \delta u'(x_{ij}^{t+1}) - \lambda^{t+1} = 0, \]

where \( \lambda^t > 0 \) and \( \lambda^{t+1} > 0 \) are the Lagrange multipliers on constraints (2.2) and (2.3), respectively. Algebraic manipulation of (A.1) – (A.3) yields equation (2.4).

A.2 Derivation of Equation (2.10)
In each period, equilibrium can be represented by the national accounting identity:

\[ F(K^t, Z^t) = C^t + I^t, \]  

where \( C^t \) is aggregate consumption and \( I^t \) is aggregate investment in period \( t \). Equation (A.4) can be rewritten as:

\[ Z^t f(k^t) = N_1^t c^t_1 + N_2^t(\pi_{2H} c_{2H}^t + \pi_{2L} c_{2L}^t) + N_1^{t-1} x_1^t + N_2^{t-1}(\pi_{2H} x_{2H}^t + \pi_{2L} x_{2L}^t) + K^{t+1} - K^t, \]  

where \( N_1^t c_1^t + N_2^t (\pi_H c_2^t + \pi_L c_2^t) \) is total consumption by young agents in period \( t \), \( N_1^t-1 x_1^t + N_2^t-1 (\pi_H x_2^t + \pi_L x_2^t) \) is total consumption by old individuals in period \( t \), and investment is equal to the change in the capital stock, \( K^{t+1} - K^t \), assuming for simplicity no capital depreciation. Dividing (A.5) by \( Z^t \) yields:

\[
f(k^t) = \frac{N_1^t c_1^t}{Z^t} + \frac{N_2^t (\pi_H c_2^t + \pi_L c_2^t)}{Z^t} + \frac{N_1^t-1 x_1^t}{Z^t} + \frac{N_2^t-1 (\pi_H x_2^t + \pi_L x_2^t)}{Z^t} + \frac{Z^{t+1}}{Z^t} k^{t+1} - k^t. \tag{A.6}
\]

Under our assumptions that \( N_1^t = N_2^t \) and \( Z^t = N_1^t a_1 l_1^t + N_2^t a_2 (\pi_H L_2^t + \pi_L L_2^t) \), equation (A.6) becomes:

\[
f(k^t) = \frac{c_1^t + \pi_L c_2^t + \pi_H c_2^t + x_1^t + \pi_L x_2^t + \pi_H x_2^t}{a_1 l_1^t + a_2 l_2^t + \pi_H a_2 l_2^t} + \frac{k^{t+1} (1 + n) (a_1 l_1^{t+1} + \pi_L a_2 l_2^{t+1} + \pi_H a_2 l_2^{t+1})}{a_1 l_1^t + a_2 l_2^t + \pi_H a_2 l_2^t} - k^t. \tag{A.7}
\]

Given that \( y_{ij} = w^t a_i l_{ij} \), the steady-state version of (A.7) is:

\[
f(k) = \frac{w \left[ c_1 + \pi_L c_2 + \pi_H c_2 + x_1 + \pi_L x_2 + \pi_H x_2 \right]}{y_1 + \pi_L y_2 + \pi_H y_2} + nk, \tag{A.8}
\]

which is equation (2.10).

### 6 Appendix B

**Optimal Nonlinear Taxation without Commitment**

When it is common knowledge that the government cannot commit, all individuals know that the probability of commitment is zero; and thus individuals differ only by their skills.\(^7\) As savings decisions are made in period \( t \), the stock of savings is fixed come period \( t + 1 \). Hence, when the government re-optimizes in period \( t + 1 \), it will equate low-skill and high-skill consumption (Farhi, et al. 2012; Brett and Weymark 2019). Accordingly, the government’s behavior in steady-state can be described as follows. Choose a steady-state allocation \( c_1, y_1, c_2, y_2, x, \) and \( k \) to maximize:

\[
u(c_1) - \frac{y_1}{wa_1} + \delta u(x) + u(c_2) - \frac{y_2}{wa_2} + \delta u(x), \tag{B.1}
\]

\(^7\)In addition to Farhi, et al. (2012) and Brett and Weymark (2019), there have been a number of recent studies that examine dynamic nonlinear taxation without commitment. See, for example, Apps and Rees (2006), Berliant and Ledyard (2014), Krause (2009, 2017), Guo and Krause (2011, 2013, 2015a, 2015b), Aronsson and Sjogren (2016), and Morita (2016). However, the focus of those papers is not on savings taxation, but rather on how the government may use skill-type information revealed in earlier periods to implement first-best taxation in latter periods.
subject to:

\[ f(k) \geq w \left[ c_1 + c_2 + \frac{2x}{1+n} \right] + nk, \quad (B.2) \]

\[ u(c_2) - \frac{u_2}{wa_2} + \delta u(x) \geq u(c_1) - \frac{u_1}{wa_2} + \delta u(x), \quad (B.3) \]

where \( w = f(k) - f'(k)k \) by profit maximization (equation 2.8), and \( x \) denotes the equalized level of consumption that both low-skill and high-skill individuals obtain in their retirement period. Equation (B.1) is a utilitarian social welfare function, equation (B.2) is the steady-state equilibrium condition, and equation (B.3) is the high-skill type’s incentive-compatibility constraint. As all individuals receive the same level of consumption (and hence utility) in their retirement period, the last term on both sides of equation (B.3) is redundant. But to assist interpretation of the incentive-compatibility constraint, we do not delete this term.

We first show that \( r = n \). The first-order condition on \( k \) can be written as:

\[ \lambda[f'(k) - n] + \frac{\partial w}{\partial k} \left[ \frac{y_1}{w^2a_1} - \frac{\theta_2 y_1}{w^2a_2} + \frac{(1 + \theta_2)y_2}{w^2a_2} - \frac{\lambda w}{y_1 + y_2} \left( c_1 + c_2 + \frac{2x}{1+n} \right) \right] = 0, \quad (B.4) \]

where \( \lambda > 0 \) is the Lagrange multiplier on constraint (B.2), and \( \theta_2 > 0 \) is the Lagrange multiplier on constraint (B.3). The first-order conditions on \( y_1 \) and \( y_2 \) are:

\[ \frac{-1}{wa_1} + \frac{\theta_2}{wa_2} + \frac{\lambda w}{(y_1 + y_2)^2} = 0, \quad (B.5) \]

\[ -\frac{(1 + \theta_2)}{wa_2} + \frac{\lambda w}{(y_1 + y_2)^2} \left( c_1 + c_2 + \frac{2x}{1+n} \right) = 0, \quad (B.6) \]

which together imply that:

\[ \frac{y_1}{w^2a_1} - \frac{\theta_2 y_1}{w^2a_2} + \frac{(1 + \theta_2)y_2}{w^2a_2} = \frac{\lambda w}{y_1 + y_2} \left( c_1 + c_2 + \frac{2x}{1+n} \right). \quad (B.7) \]

Therefore, equation (B.4) reduces to \( f'(k) - n = 0 \). Profit maximization implies that \( f'(k) = r \) (equation 2.9), which then yields \( r = n \) that corresponds to the modified golden rule under zero capital depreciation rate. For future reference, note also that (B.5) and (B.6) imply that \( \theta_2 = \frac{a_2/a_1 - 1}{2} \).

The first-order conditions on \( c_1 \), \( c_2 \), and \( x \) are, respectively:

\[ (1 - \theta_2)u'(c_1) - \frac{\lambda w}{y_1 + y_2} = 0, \quad (B.8) \]

\[ (1 + \theta_2)u'(c_2) - \frac{\lambda w}{y_1 + y_2} = 0, \quad (B.9) \]
2\delta u'(x) - \frac{2\lambda w}{(1 + n)(y_1 + y_2)} = 0. \quad (B.10)

Algebraic manipulation of equations (B.8) and (B.10) yields:

\begin{equation}
1 - \frac{u'(c_1)}{\delta(1 + n)u'(x)} = \frac{-\theta_2}{1 - \theta_2}, \quad (B.11)
\end{equation}

where $1 - \theta_2 > 0$ by equation (B.8). Using equation (2.5) and $r = n$, (B.11) implies that $\tau_{NC1} < 0$. Likewise, algebraic manipulation of (B.9) and (B.10) yields:

\begin{equation}
1 - \frac{u'(c_2)}{\delta(1 + n)u'(x)} = \frac{\theta_2}{1 + \theta_2}. \quad (B.12)
\end{equation}

Using equation (2.5) and $r = n$, (B.12) implies that $\tau_{NC2} > 0$.

7 Appendix C

C.1 Proof of Proposition 1

If $p^H = p^L = p \in (0, 1)$, the commitment-without-credibility optimal tax problem becomes the following: choose a steady-state allocation $c_1, y_1, x_1, c_2, y_2, x_2,$ and $k$ to maximize:

\begin{equation}
\begin{aligned}
& u(c_1) - \frac{y_1}{wa_1} + \delta u(x_1) + u(c_2) - \frac{y_2}{wa_2} + \delta u(x_2), \\
& \text{subject to:} \\
& f(k) \geq \frac{w [c_1 + c_2 + \frac{x_1 + x_2}{1 + n}]}{y_1 + y_2} + nk, \\
& u(c_2) - \frac{y_2}{wa_2} + \delta [pu(x_2) + (1 - p)u(x)] \geq u(c_1) - \frac{y_1}{wa_2} + \delta [pu(x_1) + (1 - p)u(x)],
\end{aligned} \quad (C.1)
\end{equation}

where equation (C.1) is the utilitarian social welfare function, equation (C.2) is the steady-state equilibrium condition, and equation (C.3) is the high-skill type’s incentive-compatibility constraint which reflects the common belief that the probability of commitment is $p$.

The first-order conditions with respect to $y_1$ and $y_2$ can be written as:

\begin{equation}
\begin{aligned}
& -\frac{1}{wa_1} + \frac{\theta_2}{wa_2} + \frac{\lambda w}{(y_1 + y_2)^2} \left( c_1 + c_2 + \frac{x_1 + x_2}{1 + n} \right) = 0, \\
& -(1 + \theta_2) + \frac{\lambda w}{(y_1 + y_2)^2} \left( c_1 + c_2 + \frac{x_1 + x_2}{1 + n} \right) = 0,
\end{aligned} \quad (C.4)
\end{equation}

where $\lambda > 0$ and $\theta_2 > 0$ are the Lagrange multipliers on constraints (C.2) and (C.3), respectively. Solving equations (C.4) and (C.5) for $\theta_2$ yields $\theta_2 = \frac{a_2/a_1 - 1}{2}$, which is the same as that under no-commitment. Moreover, as under no-commitment, it can be
shown that \( r = n \).

The first-order conditions on \( c_1, x_1, c_2, \) and \( x_2 \) are, respectively:

\[
(1 - \theta_2)u'(c_1) - \frac{\lambda w}{y_1 + y_2} = 0, \tag{C.6}
\]

\[
\delta (1 - \theta_2 p)u'(x_1) - \frac{\lambda w}{(1 + n)(y_1 + y_2)} = 0, \tag{C.7}
\]

\[
(1 + \theta_2)u'(c_2) - \frac{\lambda w}{y_1 + y_2} = 0, \tag{C.8}
\]

\[
\delta (1 + \theta_2 p)u'(x_2) - \frac{\lambda w}{(1 + n)(y_1 + y_2)} = 0. \tag{C.9}
\]

Algebraic manipulation of equations (C.6) and (C.7) leads to:

\[
1 - \frac{u'(c_1)}{\delta (1 + n)u'(x_1)} = \frac{-\theta_2(1 - p)}{1 - \theta_2}, \tag{C.10}
\]

where \( 1 - \theta_2 > 0 \) by equation (C.6). Comparing (B.11) and (C.10) establishes that \( \tau_1^{NC} < \tau_1 < 0 \). Likewise, algebraic manipulation of (C.8) and (C.9) yields:

\[
1 - \frac{u'(c_2)}{\delta (1 + n)u'(x_2)} = \frac{\theta_2(1 - p)}{1 + \theta_2}. \tag{C.11}
\]

Comparing (B.12) and (C.11) establishes that \( \tau_2^{NC} > \tau_2 > 0. \]

### C.2 Proof of Proposition 2

We first show that \( r = n \) holds under commitment-without-credibility. The first-order condition with respect to \( k \) in program (3.1) – (3.5) can be written as:

\[
\lambda [f'(k) - n] + \frac{\partial w}{\partial k} \left[ y_1 \left( \frac{1}{w^2 a_1} - \frac{\theta_{2H} + \theta_{2L}}{w^2 a_2} \right) + \frac{y_{2H}(\pi_{H} + \gamma_{2H} + \theta_{2H})}{w^2 a_2} + \frac{y_{2L}(\pi_{L} - \gamma_{2H} + \theta_{2L})}{w^2 a_2} \right] - \frac{\partial w}{\partial k} \left\{ \lambda \left[ c_1 + \pi_{L} c_{2L} + \pi_{H} c_{2H} + \frac{x_{1} + \pi_{L} x_{2L} + \pi_{H} x_{2H}}{1+n} \right] \right\} = 0, \tag{C.12}
\]

where \( \lambda > 0, \gamma_{2H} > 0, \theta_{2H} > 0, \) and \( \theta_{2L} > 0 \) are the Lagrange multipliers on constraints (3.2) – (3.5), respectively. The first-order conditions on \( y_1, y_{2H}, \) and \( y_{2L} \) are:

\[
-\frac{1}{w a_1} + \frac{\theta_{2H} + \theta_{2L}}{w a_2} + \lambda \frac{\pi_{L} c_{2L} + \pi_{H} c_{2H} + \frac{x_{1} + \pi_{L} x_{2L} + \pi_{H} x_{2H}}{1+n}}{(y_1 + \pi_{L} y_{2L} + \pi_{H} y_{2H})^2} = 0, \tag{C.13}
\]

\[
-\frac{(\pi_{H} + \gamma_{2H} + \theta_{2H})}{w a_2} + \frac{\pi_{H} \lambda \left[ c_1 + \pi_{L} c_{2L} + \pi_{H} c_{2H} + \frac{x_{1} + \pi_{L} x_{2L} + \pi_{H} x_{2H}}{1+n} \right]}{(y_1 + \pi_{L} y_{2L} + \pi_{H} y_{2H})^2} = 0. \tag{C.14}
\]
\[- \frac{(\pi_L - \gamma_{2H} + \theta_{2L})}{w_{02}} + \pi_L \lambda w \left[ c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1 + n} \right] = 0. \quad \text{(C.15)}\]

Using (C.13) – (C.15), equation (C.12) reduces to \( f'(k) - n = 0 \), which by profit maximization (equation 2.9) establishes that \( r = n \).

The first-order conditions on \( c_1 \) and \( x_1 \) are, respectively:

\[
(1 - \theta_{2H} - \theta_{2L}) u'(c_1) - \frac{\lambda w}{y_1 + \pi_L y_{2L} + \pi_H y_{2H}} = 0, \quad \text{(C.16)}
\]

\[
\delta(1 - \theta_{2H} p^H - \theta_{2L} p^L) u'(x_1) - \frac{\lambda w}{(1 + n)(y_1 + \pi_L y_{2L} + \pi_H y_{2H})} = 0. \quad \text{(C.17)}
\]

Algebraic manipulation of equations (C.16) and (C.17) yields:

\[
1 - \frac{u'(c_1)}{\delta(1 + n) u'(x_1)} = \frac{\left[ \theta_{2H} (1 - p^H) + \theta_{2L} (1 - p^L) \right]}{1 - \theta_{2H} - \theta_{2L}}. \quad \text{(C.18)}
\]

where \( 1 - \theta_{2H} - \theta_{2L} > 0 \) by equation (C.16). Using equation (2.5) and \( r = n \), equation (C.18) implies that \( \tau_1 < 0 \).

The first-order conditions on \( c_{2H} \) and \( x_{2H} \) are, respectively:

\[
(\pi_H + \gamma_{2H} + \theta_{2H}) u'(c_{2H}) - \frac{\pi_H \lambda w}{y_1 + \pi_L y_{2L} + \pi_H y_{2H}} = 0, \quad \text{(C.19)}
\]

\[
\delta(\pi_H + \gamma_{2H} p^H + \theta_{2H} p^H) u'(x_{2H}) - \frac{\pi_H \lambda w}{(1 + n)(y_1 + \pi_L y_{2L} + \pi_H y_{2H})} = 0. \quad \text{(C.20)}
\]

Algebraic manipulation of equations (C.19) and (C.20) yields:

\[
1 - \frac{u'(c_{2H})}{\delta(1 + n) u'(x_{2H})} = \frac{(1 - p^H)(\gamma_{2H} + \theta_{2H})}{\pi_H + \gamma_{2H} + \theta_{2H}}. \quad \text{(C.21)}
\]

Using equation (2.5) and \( r = n \), equation (C.21) implies that \( \tau_{2H} > 0 \).

The first-order conditions on \( c_{2L} \) and \( x_{2L} \) are, respectively:

\[
(\pi_L - \gamma_{2H} + \theta_{2L}) u'(c_{2L}) - \frac{\pi_L \lambda w}{y_1 + \pi_L y_{2L} + \pi_H y_{2H}} = 0, \quad \text{(C.22)}
\]

\[
\delta(\pi_L - \gamma_{2H} p^H + \theta_{2L} p^L) u'(x_{2L}) - \frac{\pi_L \lambda w}{(1 + n)(y_1 + \pi_L y_{2L} + \pi_H y_{2H})} = 0. \quad \text{(C.23)}
\]

Algebraic manipulation of equations (C.22) and (C.23) yields:

\[
1 - \frac{u'(c_{2L})}{\delta(1 + n) u'(x_{2L})} = \frac{\theta_{2L}(1 - p^L) - \gamma_{2H}(1 - p^H)}{\pi_L - \gamma_{2H} + \theta_{2L}}, \quad \text{(C.24)}
\]

where \( \pi_L - \gamma_{2H} + \theta_{2L} > 0 \) by equation (C.22). Recall that \( p^H > p^L \). Also, using equations
(C.14) and (C.15) we obtain:

\[
\frac{\theta_{2L} - \gamma_{2H}}{\pi_L} = \frac{\theta_{2H} + \gamma_{2H}}{\pi_H},
\]

which establishes that \( \theta_{2L} > \gamma_{2H} \). Using equation (2.5) and \( r = n \), equation (C.24) now implies that \( \tau_{2L} > 0 \).

Finally, using (C.21) and (C.24) we obtain:

\[
\tau_{2L} - \tau_{2H} = \frac{\theta_{2L}(1 - p^L) - \gamma_{2H}(1 - p^H)}{\pi_L - \gamma_{2H} + \theta_{2L}} - \frac{(1 - p^H)(\gamma_{2H} + \theta_{2H})}{\pi_H + \gamma_{2H} + \theta_{2H}},
\]

which can be manipulated to yield:

\[
\tau_{2L} - \tau_{2H} = \frac{(p^H - p^L)[(\pi_L + \theta_{2L})(\gamma_{2H} + \theta_{2H}) + \pi_H \gamma_{2H}]}{(\pi_L - \gamma_{2H} + \theta_{2L})(\pi_H + \gamma_{2H} + \theta_{2H})}.
\]

which establishes that \( \tau_{2L} > \tau_{2H} \).

**C.3 Proof of Proposition 3**

Using (B.11) and (C.18) we obtain:

\[
\tau_1 - \tau_{NC}^1 = \frac{[\theta_{2H}(1 - p^H) + \theta_{2L}(1 - p^L)]}{1 - \theta_{2H} - \theta_{2L}} + \frac{\theta_{2}}{1 - \theta_{2}},
\]

which can be manipulated to yield:

\[
\tau_1 - \tau_{NC}^1 = \frac{(1 - \theta_{2})[\theta_{2H}p^H + \theta_{2L}p^L] + \theta_{2} - \theta_{2H} - \theta_{2L}}{(1 - \theta_{2H} - \theta_{2L})(1 - \theta_{2})}.
\]

Using (C.13) and (C.14) we obtain:

\[
\frac{1}{wa_1} - \frac{1}{wa_2} - \frac{\theta_{2H} + \theta_{2L}}{wa_2} = \frac{\theta_{2H} + \gamma_{2H}}{\pi_H wa_2},
\]

which leads to:

\[
\frac{1}{2} \left( \frac{a_2}{a_1} - 1 \right) - \frac{\theta_{2H} + \theta_{2L}}{2} = \frac{\theta_{2H} + \gamma_{2H}}{2\pi_H}.
\]

Recall that \( \theta_{2} = \frac{a_2/a_1 - 1}{2} \). Therefore, equation (C.31) becomes:

\[
\theta_{2} - \theta_{2H} - \theta_{2L} = \frac{\theta_{2H} + \gamma_{2H}}{2\pi_H} - \frac{\theta_{2H} + \theta_{2L}}{2}.
\]

Algebraic manipulation reveals that the right-hand side of equation (C.32) equals zero if and only if:

\[
\theta_{2H}(1 - \pi_H) = \pi_H \theta_{2L} - \gamma_{2H}.
\]
From (C.25) we obtain:

\[ \pi_H \theta_{2L} = (\theta_{2H} + \gamma_{2H}) \pi_L + \gamma_{2H} \pi_H . \quad (C.34) \]

Thus, after algebraic manipulation:

\[ \theta_{2H}(1 - \pi_H) = \pi_H \theta_{2L} - \gamma_{2H} \quad \text{iff} \quad \pi_H + \pi_L = 1, \quad (C.35) \]

which is true. Therefore, \( \theta_2 - \theta_{2H} - \theta_{2L} = 0 \) and equation (C.29) reduces to:

\[ \tau_1 - \tau_1^{NC} = \frac{(1 - \theta_2) \left[ \theta_{2H} p^H + \theta_{2L} p^L \right]}{(1 - \theta_{2H} - \theta_{2L}) (1 - \theta_2)}, \quad (C.36) \]

which establishes that \( \tau_1 > \tau_1^{NC} \).

Using (B.12) and (C.21) we obtain:

\[ \tau_2^{NC} - \tau_{2H} = \frac{\theta_2}{1 + \theta_2} - \frac{(1 - p^H)(\gamma_{2H} + \theta_{2H})}{\pi_H + \gamma_{2H} + \theta_{2H}}, \quad (C.37) \]

which can be manipulated to yield:

\[ \tau_2^{NC} - \tau_{2H} = \frac{p^H (1 + \theta_2) (\gamma_{2H} + \theta_{2H}) + \theta_2 \pi_H - \gamma_{2H} - \theta_{2H}}{(1 + \theta_2) (\pi_H + \gamma_{2H} + \theta_{2H})}. \quad (C.38) \]

From (C.31) we obtain:

\[ \theta_2 \pi_H - \frac{\pi_H (\theta_{2H} + \theta_{2L})}{2} = \frac{\theta_2 H + \gamma_{2H}}{2}, \quad (C.39) \]

which can be rewritten as:

\[ \theta_2 \pi_H - \gamma_{2H} - \theta_{2H} = \frac{\pi_H (\theta_{2H} + \theta_{2L})}{2} - \frac{\theta_2 H + \gamma_{2H}}{2}. \quad (C.40) \]

Algebraic manipulation, along with use of (C.34), reveals that the right-hand side of equation (C.40) equals zero if and only if:

\[ (\pi_H + \pi_L)(\theta_{2H} + \gamma_{2H}) = \theta_{2H} + \gamma_{2H}, \quad (C.41) \]

which is true given that \( \pi_H + \pi_L = 1 \). Therefore, equation (C.38) reduces to:

\[ \tau_2^{NC} - \tau_{2H} = \frac{p^H (1 + \theta_2) (\gamma_{2H} + \theta_{2H})}{(1 + \theta_2) (\pi_H + \gamma_{2H} + \theta_{2H})}, \quad (C.42) \]

which establishes that \( \tau_2^{NC} > \tau_{2H} \).
Using (B.12) and (C.24) we obtain:

\[ \tau_{2L} - \tau_2^{NC} = \frac{\theta_{2L}(1 - p^L) - \gamma_{2H}(1 - p^H)}{\pi_L - \gamma_{2H} + \theta_{2L}} - \frac{\theta_2}{1 + \theta_2}, \]  

(C.43)

which can be manipulated to yield:

\[ \tau_{2L} - \tau_2^{NC} = \frac{\theta_{2L}(1 - p^L) - \gamma_{2H}(1 - p^H) - \theta_2(\pi_L - \gamma_{2H} + \theta_{2L})}{(\pi_L - \gamma_{2H} + \theta_{2L})(1 + \theta_2)} + \frac{2L}{1 + \theta_2}; \]  

(C.44)

Algebraic manipulation of (C.44) reveals that \( \tau_{2L} > \tau_2^{NC} \) if and only if \( \frac{\theta_{2L}}{\gamma_{2H}} > \frac{2L}{1 + \theta_2} \). Section 3.3 provide numerical examples with \( \tau_{2L} > \tau_2^{NC} \). ■

8 Appendix D

Optimal Marginal Savings Tax Rates under Changes in \( \{n, \rho, \alpha, \delta\} \)

In the presence of taxation, the present-value formulation of type \( ij \)'s budget constraint over its lifetime is:

\[ c_{ij}^t + \frac{x_{ij}^{t+1}}{1 + r^{t+1}} \leq w^t a_i l_i^t - \left( T_{ij}^t + \frac{T_{ij}^{t+1}}{1 + r^{t+1}} \right), \]  

(D.1)

where \( T_{ij}^t \) and \( T_{ij}^{t+1} \) are agent \( ij \)'s tax expenditures at periods \( t \) and \( t + 1 \), respectively. Taking aggregation of the steady-state version of (D.1), together with \( r = n \) (see the proof of Proposition 2) and the government’s balanced budget, leads to:

\[ c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1 + n} = y_1 + \pi_L y_{2L} + \pi_H y_{2H}, \]  

(D.2)

which states that aggregate consumption is equal to total pre-tax income across all individuals. Using the steady-state versions of (2.8) and (A.7) yields:

\[ f(k) - nk = \frac{c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1 + n}}{a_1 l_1 + \pi_L a_{2L} + \pi_H a_{2H}}, \]  

(D.3)

where \( a_1 l_1 + \pi_L a_{2L} + \pi_H a_{2H} \) represents total effective labor. On the other hand, it is straightforward from the per effective-labor form of the production technology (3.7) to derive that the steady-state rental rate and real wage are:

\[ r = \alpha k^{\rho-1} (\alpha k^\rho + 1 - \alpha)^{\frac{1-\rho}{\rho}} = n, \]  

(D.4)

\[ w = (\alpha k^\rho + 1 - \alpha)^{\frac{1}{\rho}} - nk, \]  

(D.5)

which indicate that \( k \) and thus \( w \) are functions of \( n, \rho \) and \( \alpha \).
Next, per the utility function (3.6) and the definition of \( y_{ij}^t = w^t a_i l_{ij}^t \), the commitment-without-credibility optimal tax problem becomes the government’s choosing a steady-state allocation \( \langle c_1, l_1, x_1 \rangle, \langle c_{2L}, l_{2L}, x_{2L} \rangle, \langle c_{2H}, l_{2H}, x_{2H} \rangle \) and \( k \) to maximize:

\[
\log(c_1) - l_1 + \delta \log(x_1) + \sum_{j=L,H} \pi_j \left[ \log(c_{2j}) - l_{2j} + \delta \log(x_{2j}) \right], \tag{D.6}
\]

subject to:

\[
w = \frac{c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1+n}}{a_1 l_1 + \pi_L a_2 l_{2L} + \pi_H a_2 l_{2H}}, \tag{D.7}
\]

\[
\log \left[ \frac{c_{2H}}{c_{2L}} \left( \frac{x_{2H}}{x_{2L}} \right)^{\delta p_H} \right] = l_{2H} - l_{2L}, \tag{D.8}
\]

\[
\log \left[ \frac{c_{2H}}{c_1} \left( \frac{x_{2H}}{x_1} \right)^{\delta p_H} \right] = \frac{a_2 l_{2H} - a_1 l_1}{a_2}, \tag{D.9}
\]

\[
\log \left[ \frac{c_{2L}}{c_1} \left( \frac{x_{2L}}{x_1} \right)^{\delta p_L} \right] = \frac{a_2 l_{2L} - a_1 l_1}{a_2}, \tag{D.10}
\]

where (D.7) restates (D.3) that is the aggregate consistency condition, and equations (D.8) – (D.10) are incentive-compatibility constraints of high-skill agents.

In response to a change in either \( n \) or \( \rho \) or \( \alpha \), the real wage \( w \) given by (D.5) will rise/fall by a certain percentage denoted as \( g_w = \frac{\Delta w}{w} \). In this case, the government may consider the following variation exercise: keeping labor hours \( \langle l_1, l_{2L}, l_{2H} \rangle \) unchanged and adjusting individuals’ consumption \( \langle c_1, c_{2L}, c_{2H}, x_1, x_{2L}, x_{2H} \rangle \) such that the resulting total consumption \( = c_1 + \pi_L c_{2L} + \pi_H c_{2H} + \frac{x_1 + \pi_L x_{2L} + \pi_H x_{2H}}{1+n} \) moves at the same direction and by the same rate as the real wage.\(^8\) It is straightforward to see that this alternative allocation will satisfy constraints (D.7) – (D.10). Therefore, the resulting steady-state intertemporal ‘tax wedges’:

\[
\tau_{ij} := 1 - \frac{x_{ij}}{\delta(1+n)c_{ij}}, \tag{D.11}
\]

remains unaffected because movements in \( \langle c_1, \frac{x_1}{1+n} \rangle, \langle c_{2L}, \frac{x_{2L}}{1+n} \rangle \) and \( \langle c_{2H}, \frac{x_{2H}}{1+n} \rangle \) are cancelled out. It follows that the optimal marginal savings tax rates will be numerically identical to those reported in Table 1 under the benchmark parameterization.

\(^8\)Upon a change in \( \rho \) or \( \alpha \), this can be attained by setting each element of agents’ consumption \( \langle c_1, c_{2L}, c_{2H}, x_1, x_{2L}, x_{2H} \rangle \) to vary at the same direction and by the same rate as the real wage. When the population growth rate \( n \) is changed to \( n' \), this can be achieved by (i) letting individuals’ young-age consumption \( \langle c_1, c_{2L}, c_{2H} \rangle \) comove with the real wage at the same rate \( g_w \), and (ii) altering individuals’ old-age consumption \( \langle x_1, x_{2L}, x_{2H} \rangle \) such that \( x_{ij}' = x_{ij}(1+g_w) \), where \( x_{ij}' \) represents type \( ij \)'s retirement consumption at the alternative allocation.
When there is a change in the discount factor $\delta$, the rental rate and real wage as in (D.4) – (D.5) remain the same. Given our postulated quasi-linear preference formulation (3.6), taking the difference between (D.8) and (D.9) leads to:

$$\log \left[ \frac{c_2L}{c_1} \left( \frac{x_{2L}}{x_1} \right)^{\delta p^H} \right] = \frac{a_2l_{2L} - a_1l_1}{a_2},$$  \hspace{1cm} \text{(D.12)}

which can then be combined with equation (D.10) to obtain:

$$x_{2L} = x_1.$$ \hspace{1cm} \text{(D.13)}

In this environment, the government may consider the following variation exercise: (i) keeping $\langle c_1, c_{2L}, c_{2H} \rangle$ unchanged, (ii) letting $\langle x_1, x_{2L}, x_{2H} \rangle$ move at the same direction and by the same rate as $\delta$, and (iii) choosing $\langle l_1, l_{2L}, l_{2H} \rangle$ to satisfy constraints (D.7) – (D.9). For all possible values of $\delta \in (0, 1)$, we have verified that the simultaneous equation system (D.7) – (D.9) always exhibits a unique solution in agents’ labor supply $\langle l_1, l_{2L}, l_{2H} \rangle$. As a result, the above-mentioned alternative allocation, characterized by (i) – (iii), is feasible and does not influence the steady-state intertemporal tax wedges (D.11) since movements in $x_{ij}$ and $\delta$ are cancelled out. It follows that the original marginal savings tax rate distortions will not be altered.
References


