NORTH-HOLLAND

Dynamic Asset Pricing and Statistical Properties of Risk

Gloria González-Rivera

Within the framework of the conditional Arbritage Pricing Theory, estimators of conditional risk are not unique. We focus on an estimator of conditional risk based on the conditional volatility of the asset return. Estimates of conditional risk account for: 1) interdependence of conditional and unconditional moments of the asset return; 2) effect of the conditional and unconditional fourth moments of the asset return; and 3) *force* of the conditional volatility process, such that, assets with forceful conditional volatility specifications are empirically riskier, ceteris paribus. This is in contrast to the unconditional theory, where unconditional risk is the unconditional covariance of the asset return with a benchmark portfolio.

Keywords: Arbitrage pricing theory; Conditional volatility

JEL classification: C13, G12

I. Introduction

The classical asset pricing theories, such as the Capital Asset Pricing Model (CAPM) [Sharpe (1964)] and the Arbitrage Pricing Theory (APT) [Ross (1976)], have enlightened our understanding of the concept and the pricing of risk. As they were conceived, they explained the cross-sectional behavior of asset returns, permitting the evaluation and differentiation of assets according to their level of risk. The empirical validation of these theories requires the estimation of risk. In this paper, we distinguish between the economic notion of risk and the statistical estimation of risk. This distinction arises when we consider different estimation methodologies. To understand the difference between economic risk and its statistical content, we need to differentiate between unconditional and conditional estimation of risk.

Department of Economics, University of California at Riverside, Riverside, California.

Address correspondence to: Gloria González-Rivera, University of California at Riverside, Department of Economics, Riverside, CA 92521.

Journal of Economics and Business 1998; 50:461–470

0148-6195/98/$19.00
PII S0148-6195(98)00014-9

First, let us focus on unconditional estimation of risk. Consider the equilibrium pricing relation from a CAPM model:

$$E(R_i) = \beta_i(E(R_m) - r), \tag{1.1}$$

where E is the expectation operator; R_i is the excess return to asset i; R_m is the return to the market portfolio; r is the risk-free rate, and β_i is the *unconditional risk*, defined as:

$$\beta_i = \frac{\text{cov}\,(R_i, R_m)}{\text{var}\,(R_m)}. \tag{1.2}$$

Equations (1.1) and (1.2) constitute the structural model which is derived either from a maximization of a quadratic utility function or from a minimization of the variance of a portfolio of risky and non-risky assets. This theory claims that the unconditional expected return to asset i is a linear function of risk. The economic concept of risk is represented by β_i, which is the ratio of two unconditional moments. The statistical notion of risk corresponds to the estimator of β_i. We need to construct the econometric model which permits the estimation of β_i. From equation (1.2), we observe that running an Ordinary Least Squares (OLS) regression of R_i on R_m provides a consistent estimator of risk, so the econometric model becomes:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \tag{1.3}$$

where

$$\hat{\beta}_{i(ols)} = \frac{\widehat{\text{cov}}\,(R_{it}, R_{mt})}{\widehat{\text{var}}\,(R_{mt})} \qquad p \lim \hat{\beta}_{i(ols)} = \beta_i. \tag{1.4}$$

This is to say that, asymptotically, the estimator of risk, $\hat{\beta}_{i(ols)}$, converges to the economic risk, β_i. A good estimator of the economic risk just requires the knowledge of the *unconditional* mean, variance and covariance of R_{it} with R_{mt}. No other features of the unconditional distribution but the second moments are relevant. In fact, if we wish to exploit the behavior of the variance of R_{it}, from equation (1.3), we obtain:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2, \tag{1.5}$$

in which, once the economic risk is estimated, everything else is known. There is no further information that is not already contained in the estimation of equation (1.3).

In the unconditional CAPM, there is one-to-one correspondence between economic risk and its estimator.

The same reasoning can be applied to the APT model. This theory claims to be more general than CAPM because it recognizes that multiple factors, possibly among them the market portfolio, may affect the behavior of expected returns. In an APT model, economic risk is defined as the exposure of an asset, i, to a set of economic factors, f_j. The excess return can be written as the sum of two components:

$$R_i = \mu_i + \sum_{j=1}^{k} \beta_{ij} f_j + \varepsilon_i. \tag{1.6}$$

The anticipated component is the unconditional expected return, μ_i, and the unanticipated component is the sum of the idiosyncratic noise, ε_i, plus the contribution of k independent factors, f_j. These factors can be understood as innovations in the economy which can drive asset returns in different ways according to the factor loadings, β_{ij}. In equilibrium,

$$\mu_i = \sum_{j=1}^{k} \beta_{ij}\lambda_j, \tag{1.7}$$

where λ_j is the risk premium on the factor f_j. The APT theory is less explicit with respect to the functional form of risk. Economic risk is represented by the factor loadings, β_{ij}, and these are defined as the sensitivity of asset i to factor f_j. The economic risk, β_{ij}, needs to be estimated. To this end, an econometric model based on the economic model (1.7) is:

$$R_{it} = \sum_{j=1}^{k} \beta_{ij}\lambda_{jt} + \nu_{it}. \tag{1.8}$$

Model (1.8) requires some assumptions on λ_{jt}. For the sake of the argument, let us assume that λ_{jt}'s are known and independent, then an estimator of β_{ij} is:

$$\hat{\beta}_{ij(ols)} = \frac{\widehat{\text{cov}}\ (R_{it}, \lambda_{jt})}{\widehat{\text{var}}\ (\lambda_{jt})} \qquad p \lim \hat{\beta}_{ij(ols)} = \beta_{ij}. \tag{1.9}$$

$\hat{\beta}_{ij(ols)}$ is asymptotically a good estimator of the economic risk, β_{ij}. Only *unconditional* moments are relevant to estimate economic risk[1]. Furthermore, according to equation (1.6), we can write:

$$\sigma_i^2 = \sum_{j} \beta_{ij}^2\sigma_{f_j}^2 + \sigma_{\varepsilon}^2, \tag{1.10}$$

where, once the factor loadings are estimated, everything else is known. All the information is contained in the unconditional second moments of R_i and f_j.

During the 1980's, new methodological advances in time series econometrics allowed for an empirical revision of the classical asset pricing theories. In particular, the introduction of the ARCH models by Engle (1982) and GARCH models by Bollerslev (1986) brought a new wave in the estimation and testing of CAPM and APT models[2]. The key element of the ARCH methodology is to focus on *conditional* moments. Investors' decisions are based on time-varying information sets which drive the formation of conditional means and conditional variances of asset returns. The major methodological innovation is to recognize that conditional variances are time-varying as opposed to unconditional variances in equations (1.5) and (1.10). In this sense, we differentiate unconditional estimation from conditional estimation of asset pricing models, unconditional risk from conditional risk.

In a dynamic CAPM, conditional risk is defined as:

$$\beta_{it} = \frac{\text{cov}_t\ (R_{it}, R_{mt})}{\text{var}_t(R_{mt})}. \tag{1.11}$$

[1] If λ_{jt} is estimated, other estimators of β_{ij} are available, such as the instrumental variable estimator.
[2] For a survey on GARCH methodology and its financial applications, see Bollerslev et al. (1992) and Bollerslev et al. (1995).

The estimator of equation (1.11) does not rely on a least squares regression such as equation (1.3). It is necessary to build a time-series model for the time-varying covariance of the individual return with the market return, and for the time-varying variance of the market return. The most popular methodology to carry out this estimation is (G)ARCH. Examples of conditional CAPM estimation are Bollerslev et al. (1988), Ng (1991), and González-Rivera (1996).

In this paper, we focus on conditional estimation of APT models. Examples of dynamic APT models are Engle et al. (1990) and Ng et al. (1992). In the next section, we outline the dynamic APT model. In the unconditional setting, there is a one-to-one correspondence between the economic notion of risk and its estimator. In the conditional setting, we have a variety of estimators which correspond to the same economic definition of risk. We focus on a linear estimator of conditional risk based on the behavior of the conditional volatility of the asset. In contrast to the unconditional setting, where economic risk is estimated as the unconditional covariance of the asset return with a benchmark portfolio, an estimator of conditional risk is a measure which accounts for a richer array of statistical properties of the asset return. We show that *estimates of conditional risk* account for:

1. Interdependence of conditional and unconditional moments of the asset return.
2. Effect of the conditional and unconditional fourth moment of the asset return.
3. *Force* of the conditional volatility process, such that, ceteris paribus, assets with more forceful conditional volatility specifications have larger estimates of conditional risk.

II. Dynamics and Risk

The Economic Model

In this section, we outline the conditional APT model. Let us take the time series version of equation (1.6):

$$R_{it} = \mu_{it} + \beta_i f_t + \varepsilon_{it}, \tag{2.1}$$

where R_{it} is the excess return to asset i; μ_{it} is the conditional expected excess return to asset i; f_t is a dynamic factor with zero mean; β_i is the sensitivity of asset i to the factor, and ε_{it} is the idiosyncratic noise, with zero mean and constant variance. Let us assume that the information set is ψ_{t-1}, containing all the information available up to time $t - 1$. Analogous to the unconditional setting, the anticipated component of the asset return is the conditional mean and the unanticipated component is the sum of the idiosyncratic noise plus the stochastic factor. The theory may account for a set of independent factors. We consider only one factor, but this assumption is not substantially restrictive for the forthcoming arguments. Upon conditioning in the set ψ_{t-1}, a similar set of arguments, as in Ross (1976), leads to the following equilibrium asset pricing formula:

$$\mu_{it} = \beta_i \lambda_t, \tag{2.2}$$

where λ_t is the time-varying risk premium associated with the factor f_t. Furthermore, from equation (2.1), the conditional variance of the excess return to asset i is given by:

$$\mathrm{var}_{t-1}(R_{it}) = \beta_i^2 \, \mathrm{var}_{t-1}(f_t) + \mathrm{var}\,(\varepsilon_{it}). \tag{2.3}$$

The economic concept of risk is represented by the parameter β_i, which, as before, is defined as the sensitivity of the asset, i, to the factor, f. Such sensitivity should be measured from the two first conditional moments of the asset return. From equation (2.2), β_i is the change in the conditional expected return when there is a marginal change in the risk premium of the factor. From equation (2.3), β_i^2 is the change in the conditional variance when there is a marginal change in the conditional variance of the factor. Equations (2.2) and (2.3) constitute the economic model where conditional risk is defined by the parameter, β_i.

The Econometric Model

The estimation of the conditional risk requires the construction of the econometric model. The main difference with the unconditional setting is the relation (2.3). Because the conditional variances need to be estimated, equation (2.3) is stochastic and becomes an integral part of the estimation and testing of the conditional APT model. To abbreviate notation, let us call $\mathrm{var}_{t-1}(R_{it}) = h_{it}$ and $\mathrm{var}_{t-1}(f_t) = h_{ft}$. In order to proceed with the econometric estimation, we need some statistical assumptions. Let us assume that:

$$R_{it}|\psi_{t-1} \overset{d}{\rightarrow} D(\beta_i\lambda_t, h_{it});$$

that is, return R_{it} is conditionally D distributed with conditional mean, $\beta_i\lambda_t$, and conditional variance, h_{it}, where D is some known probability distribution function. The econometric model which corresponds to equations (2.2) and (2.3) becomes:

$$R_{it} = \beta_i\lambda_t + \nu_{it}; \tag{2.4}$$

$$h_{it} = \alpha_i h_{ft} + \sigma_\varepsilon^2, \tag{2.5}$$

where ν_{it} is a conditionally D distributed error with mean zero and conditional variance, h_{it}. The regressors, λ_t and h_{ft}, are considered predetermined variables which have been generated previously. The risk premium, λ_t, is assumed to be a function of the conditional variance of the factor, and h_{ft} is most frequently generated with some univariate GARCH model. The empirical validity of the conditional APT model should be tested with the null hypothesis H_0: $\alpha_i = \beta_i^2$. The econometric model (2.4) and (2.5) can be estimated and tested using different methodologies which can complement each other.

Estimators of the economic risk, β_i, are not unique. Some researchers have used a full information maximum likelihood estimation (FMLE) procedure, where equations (2.4) and (2.5) are jointly estimated with or without imposing the restriction contained in H_0. The researcher needs to choose a functional form for the probability density function, D. The most common assumption is conditional normality. The maximization of the likelihood function yields the FMLE estimator of β_i and α_i. This estimator is highly nonlinear and it does not have a closed functional form. Asymptotically, it is consistent. The restriction contained in H_0 can be tested by standard methods such as a likelihood ratio test or a Lagrange multiplier test. Some examples where this methodology has been used are Engle et al. (1990) and Ng et al. (1992).

There are other methodologies that have been favored by researchers because they are free of distributional assumptions and provide estimators with closed formulations. These are Least Squares (LS) and Instrumental Variables (IV). Examples of these methodologies dealing with financial applications are in Pagan (1984), Pagan and Ullah (1988), and Schwert and Seguin (1990).

Consider equation (2.4). We observe that the error, ν_{it}, is conditionally heteroscedastic, where the heteroscedasticity depends on the conditional variance of the factor. A heteroscedasticity correction is available. Dividing every variable in equation (2.4) by $\sqrt{h_{ft}}$, we can use a Generalized Least Squares (GLS) procedure to obtain a consistent estimator of β_i. But, it is also possible to apply Ordinary Least Squares (OLS) or IV (accounting for the generated regressor problem[3]). The OLS and IV estimators are also consistent but less efficient than GLS.

Furthermore, it is possible to consider equations (2.4) and (2.5) jointly in a simultaneous equations model. The simplest consistent estimator is equation-by-equation OLS or IV. The restriction contained in H_0 amounts to imposing a non-linear restriction across equations. This type of test is considered in Gallant (1987). Assuming that the econometric model is a good approximation to the economic model, we will not be able to reject the restriction contained in H_0.

The above methodologies produce consistent estimators of the parameter of interest, β_i. We focus on the LS or IV estimator of β_i based on the variance equation (2.5) because it helps us to understand further the statistical characteristics of conditional risk.

Estimator of Conditional Risk Based on the Variance Equation

We assume that the null hypothesis, H_0, cannot be rejected using any of the methodologies outlined above, and the fitted econometric model becomes:

$$\hat{R}_{it} = \hat{\beta}_i \lambda_t; \tag{2.6}$$

$$\hat{h}_{it} = \hat{\beta}_i^2 h_{ft} + \hat{\sigma}_\varepsilon^2. \tag{2.7}$$

The OLS estimator (in the instances in which it may be applied) and the IV estimator (if there is a generated regressor problem) of conditional risk, β_i, based on equation (2.7) are:

$$\hat{\beta}^2_{i(ols)} = \frac{\widehat{\text{cov}}\ (h_{it}, h_{ft})}{\widehat{\text{var}}\ (h_{ft})} \qquad \hat{\beta}^2_{i(iv)} = \frac{\widehat{\text{cov}}\ (h_{it}, h_{zt})}{\widehat{\text{cov}}\ (h_{ft}, h_{zt})}, \tag{2.8}$$

where h_{zt} is an instrument for h_{ft}, highly correlated with h_{ft} and uncorrelated with the error term of the regression based on equation (2.7).

We investigate the statistical content of the estimators (2.8). We wish to know which statistical characteristics of the stochastic process $\{R_{it}\}$ are embedded in the estimation of conditional risk. The following proposition summarizes our findings.

Proposition.

Consider the econometric model:

$$R_{it} = \beta_i \lambda_t + \nu_{it}; \tag{2.9}$$

[3] To deal with the generated regressor problem, maximum likelihood estimation is also available [see McAleer and McKenzie (1991); McKenzie and McAleer (1994)].

$$h_{it} = \beta_i^2 h_{ft} + \sigma_\varepsilon^2, \tag{2.10}$$

and information set, ψ_{t-1}. The conditional variances are defined as:

$$h_{it} = E(\nu_{it}^2 | \psi_{t-1}) \qquad h_{ft} = E(\upsilon_{ft}^2 | \psi_{t-1}),$$

such that $E(h_{it}) = E(\nu_{it}^2) = \sigma_\nu^2$ and $E(h_{ft}) = E(\upsilon_{ft}^2) = \sigma_\upsilon^2$, where υ_{ft} is the innovation in the factor model $R_{ft} = \mu_{ft} + \upsilon_{ft}$. Define the conditional kurtosis coefficients as:

$$k_i^c = \frac{E(\nu_{it}^4 | \psi_{t-1})}{h_{it}^2} \qquad k_f^c = \frac{E(\upsilon_{ft}^4 | \psi_{t-1})}{h_{ft}^2}, \tag{2.11}$$

and the unconditional kurtosis coefficients as:

$$k_i^u = \frac{E(\nu_{it}^4)}{\sigma_\nu^4} \qquad k_f^u = \frac{E(\upsilon_{ft}^4)}{\sigma_\upsilon^4}. \tag{2.12}$$

Assume that the distributions of $R_{it}/\sqrt{h_{it}}$ and $R_{ft}/\sqrt{h_{ft}}$ are time-invariant, then $\hat{\beta}_i^2$ can be expressed as:

$$\hat{\beta}_i^2 = c\,\frac{\sigma_\nu^2}{\sigma_\upsilon^2}\left(\frac{k_i^u}{k_i^c} - 1\right)^{1/2}\left(\frac{k_f^u}{k_f^c} - 1\right)^{-1/2}, \tag{2.13}$$

where

$$c = \begin{cases} \rho_1 & \text{if OLS;} \\ \rho_2/\rho_3 & \text{if IV,} \end{cases}$$

with ρ_1 being the correlation coefficient between h_{it} and h_{ft}; ρ_2 the correlation coefficient between h_{it} and h_{zt}; and ρ_3 the correlation coefficient between h_{ft} and h_{zt}.

PROOF

From equations (2.11) and (2.12), we obtain [Engle and González-Rivera (1991)]:

$$k^u = k^c\,\frac{E(h_t^2)}{(E(h_t))^2},$$

and, as $\mathrm{var}(h_t) = E(h_t^2) - (E(h_t))^2$ and $E(h_t) = \sigma^2$, we can write:

$$\mathrm{var}\,(h_t) = \sigma^4\left(\frac{k^u}{k^c} - 1\right). \tag{2.14}$$

Using the definition of correlation coefficient, we can write the covariance as:

$$\mathrm{cov}\,(h_{it}, h_{ft}) = \rho_1 \sqrt{\mathrm{var}\,(h_{it})}\ \sqrt{\mathrm{var}(h_{ft})};$$

$$\mathrm{cov}\,(h_{it}, h_{zt}) = \rho_2 \sqrt{\mathrm{var}\,(h_{it})}\ \sqrt{\mathrm{var}\,(h_{zt})};$$

$$\mathrm{cov}\,(h_{zt}, h_{ft}) = \rho_3 \sqrt{\mathrm{var}\,(h_{zt})}\ \sqrt{\mathrm{var}\,(h_{ft})}.$$

Then,

$$\hat{\beta}_i^2 = c\frac{\sqrt{\text{var}\,(h_{it})}}{\sqrt{\text{var}\,(h_{ft})}}.$$

Substituting equation (2.14) in the last expression, we obtain equation (2.13).

The estimator of conditional risk (2.13) can be decomposed in three components: 1) a function of the coefficients of correlation between the conditional variances of the individual asset, the factor and the instrument for the factor; 2) the relative unconditional variance of the innovation in the asset return with respect to the unconditional variance of the innovation in the factor return; and 3) the relative fourth moment of the innovation in the asset return with respect to the fourth moment of the innovation in the factor return. From equation (2.13), we observe that the magnitude of the estimator, $\hat{\beta}_i^2$, is directly proportional to the magnitude of the ratio, k_i^u/k_i^c. In the next section, we show that the ratio, k_i^u/k_i^c, is a measure which summarizes the amount of conditional volatility.

A comparison between the estimator of unconditional risk (1.9) and the estimator of conditional risk (2.13) reveals that, functionally, they are very different. The main difference lies in the relevance of the fourth moment in assessing conditional risk. This is a direct consequence of the existence of time-varying volatility in the asset return and in the factor return. If the variances are not time-varying, the econometric model becomes (1.8) and the only relevant measure of risk is unconditional risk.

The Meaning of the Ratio $\mathrm{k^u/k^c}$. Because the GARCH methodology has been the most popular in the estimation of conditional volatility, we focus on the meaning of the ratio k^u/k^c in relation to GARCH estimation, but similar arguments can be made in relation to any other methodology which attempts to model a cluster of outliers in the data.

We claim that the ratio k^u/k^c measures the strength of GARCH effects in the data. The key feature of GARCH models lies in their ability to describe random variables with leptokurtic distributions. One of the stylized facts of financial data is that they are characterized by unconditional leptokurtosis (kurtosis larger than three). A successful GARCH specification has to model clusters of outliers such that the standardized random variable $R_t/\sqrt{h_t}$ has a conditional kurtosis smaller than the unconditional kurtosis, $k^u \geq k^c$, hence $k^u/k^c \geq 1$. If the ratio is equal to 1, there is not GARCH in the data. A large value of the ratio means that the GARCH specification is successful in picking up the outliers of the data. The larger the ratio is, the more forceful the GARCH specification is. Let us call the ratio k^u/k^c the *force* of a GARCH specification. Next, we will show that the intuition behind this ratio has a formal justification.

Consider a random variable, y_t, with mean zero and conditional variance h_t, modelled according to a GARCH specification, i.e.,

$$h_t = \omega + \alpha y_{t-1}^2 + \beta h_{t-1}. \tag{2.15}$$

If we add to both sides of equation (2.15) y_t^2, we obtain the ARMA representation of a GARCH process [Bollerslev (1986)]:

$$y_t^2 = \omega + (\alpha + \beta) y_{t-1}^2 - \beta\varepsilon_{t-1} + \varepsilon_t, \tag{2.16}$$

where ε_t is defined as $\varepsilon_t \equiv y_t^2 - h_t$. A criteria which can help to choose a successful specification is to minimize the variance of ε_t with respect to the set of parameters. This variance is:

$$\mathrm{var}\ (\varepsilon_t) = E(\varepsilon_t^2) = E(y_t^4) - E(h_t^2) = \sigma^4\left(k^u - \frac{k^u}{k^c}\right).$$

The var(ε_t) is inversely proportional to the ratio k^u/k^c. Hence, an optimal specification of a GARCH model has k^u/k^c maximized.

The introduction of time-varying volatility defines estimators of conditional risk in two dimensions: in the mean dimension and in the variance dimension. In general terms, risk depends on the exposure of an asset return to a set of factors. This exposure is quantified in terms of statistical moments. Conditional risk accounts for a richer array of statistical properties of the asset return. It requires the assessment of co-movements in the volatility processes of the individual asset and the factor. According to equation (2.13), we should expect larger estimates of conditional risk for those assets which exhibit high conditional volatility with respect to the conditional volatility of the factor. Empirically, assets with larger conditional volatility are riskier, everything else being equal.

III. Conclusions

In this paper, we have reviewed the classical asset pricing theories such as CAPM and APT, distinguishing between conditional and unconditional CAPM and between conditional and unconditional APT. The distinction between conditional and unconditional models depends on the specification of *conditional* or *unconditional* means, variances and covariances of the asset returns. As a consequence, there is a definition of conditional risk and a definition of unconditional risk.

Either if the model is conditional or unconditional, the empirical validation of CAPM and APT requires the estimation of risk. In a CAPM model, the economic concept of risk has a well-defined formulation which permits the construction of a relatively easy estimator, conditional or unconditional. In an APT model, the economic definition of risk is represented by a parameter for which no formulation is provided. Nevertheless, the estimation of an unconditional APT model does not present different problems from those encountered in the estimation of an unconditional CAPM. In the unconditional setting, there is a one-to-one correspondence between the economic concept of risk and its estimator.

However, the econometric formulation of a conditional APT model is more complicated because it requires the estimation of two equations, the conditional mean and the conditional variance. Both equations are tied by a common parameter which represents conditional risk. We have reviewed different methodologies to estimate and test the conditional APT. In contrast to the unconditional setting, estimators of conditional risk are not unique. We have focused on a linear estimator of conditional risk based on the behavior of the conditional volatility of the asset. We have investigated the statistical characteristics of the asset return embedded in this estimator. We have proven that this estimator of conditional risk accounts for the interdependence between conditional and unconditional moments of the asset returns, in particular, the fourth moment is extremely relevant. The estimator of conditional risk is directly proportional to the ratio of unconditional kurtosis to conditional kurtosis. We have shown that this ratio summarizes the amount of conditional volatility in the asset return and that assets with strong conditional volatility are empirically riskier, everything else being equal. In the GARCH framework, we have termed this ratio the *force* of the GARCH specification and we have shown that a successful GARCH model has to maximize the force.

I gratefully acknowledge the Intramural Research Grant of the Academic Senate of the University of California, Riverside. I appreciate the comments of two anonymous referees.

References

Bollerslev, T. 1986. Generalized autoregressive conditional heteroscedasticity. *Journal of Finance* 31:307–327.

Bollerslev, T., Chou, R., and Kroner, K. 1992. ARCH modelling in finance: A review of the theory and empirical evidence. *Journal of Econometrics* 52:5–59.

Bollerslev, T., Engle, R. F., and Nelson, D. 1995. ARCH models. In *Handbook of Econometrics* (R. F. Engle and D. McFadden, eds.), *Vol. IV*. Cambridge University Press.

Bollerslev, T., Engle, R. F., and Wooldridge, J. 1988. A capital asset pricing model with time-varying covariances. *Journal of Political Economy* 96:116–131.

Engle, R. F., and González-Rivera, G. 1991. Semiparametric ARCH models. *Journal of Business and Economic Statistics* 9:345–360.

Engle, R. F., Ng, V., and Rothschild, M. 1990. Asset pricing with a factor ARCH covariance structure: Empirical estimates for Treasury bills. *Journal of Econometrics* 45:213–238.

Engle, R. F. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica* 50:987–1008.

Gallant, A. R. 1987. *Nonlinear Statistical Methods*. New York: John Wiley and Sons, Inc.

González-Rivera, G. 1996. Time-varying risk. The case of the American computer industry. *Journal of Empirical Finance* 2:333–342.

McAleer, M., and McKenzie, C. R. 1991. When are two step estimators efficient? *Econometric Reviews* 10:235–252.

McKenzie, C. R., and McAleer, M. 1994. On the effects of misspecification errors in models with generated regressors. *Oxford Bulletin of Economics and Statistics* 56:441–455.

Ng, L. 1991. Tests of the CAPM with time-varying covariances: A multivariate GARCH approach. *Journal of Finance* 46:1507–1521.

Ng, V., Engle, R. F., and Rothschild, M. 1992. A multi-dynamic factor model for stock returns. *Journal of Econometrics* 52:245–266.

Pagan, A. 1984. Regression with generated regressors. *International Economic Review* 25:221–247.

Pagan, A., and Ullah, A. 1988. The econometric analysis of models with risk terms. *Journal of Applied Econometrics* 3:87–105.

Ross, S. 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13:343–362.

Schwert, G., and Seguin, P. 1990. Heteroskedasticity in stock returns. *Journal of Finance* 45:1129–1155.

Sharpe, W. 1964. A theory of market equilibrium under conditions of risk. *Journal of Finance* 19:425–442.