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Time-varying risk
The case of the American computer industry

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Abstract

A bivariate GARCH-in-mean model for individual stock returns and the market portfolio is designed to model volatility and to test the conditional Capital Asset Pricing Model versus the conditional Residual Risk Model. We find that a univariate model of volatility for individual stock returns is misspecified. A joint modelling of the market return and the individual stock return shows that a major force driving the conditional variances of individual stocks is the history contained in the market return variance. We find that a conditional residual risk model, where the variance of the individual stock return is used to explain expected returns, is preferred to a conditional CAPM. We propose a partial ordering of securities according to their market risk using first and second order dominance criteria.

JEL classification: C22; G12

Keywords: CAPM; GARCH; Market β risk; Residual risk; Stochastic dominance

1. Introduction

This paper addresses issues of volatility modelling for individual stocks as well as conditional testing of the Capital Asset Pricing Model (CAPM) versus alternative specifications. We design a bivariate GARCH-in-mean system where we pair each individual stock return with the market portfolio return. The CAPM is tested with individual stocks and the approach avoids the need to form portfolios in order to correct for measurement error in the betas. The testing of conditional CAPM is a one-stage procedure in which betas and risk premia are estimated simultaneously. In contrast to the traditional tests of CAPM, there is no assumption of the constancy of the betas and the constancy of the market risk premium. The

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bivariate GARCH-in-mean model is found to be a parsimonious model, which is able to capture enough dynamic structure. Given the availability of the market portfolio, we do not need to specify the set of weights required in order to form the market portfolio as they are already incorporated in the covariance of the individual security with the market return. In previous articles, e.g. Bollerslev et al. (1988), and Chan et al. (1992), the weights have to be calculated or obtained from other sources.

2. The model

Let R_{it} be the stock return to the firm i at time t, R_{Mt} the return to the market portfolio and r_f the return to a riskless asset. The CAPM (Sharpe, 1964), in its conditional formulation, requires

$$E_t(R_{it}) = r_f + \frac{cov_t(R_{it},R_{Mt})}{var_t(R_{Mt})}\big(E_t(R_M) - r_f\big), \tag{1}$$

where $E_t(R_{Mt}) - r_f$ is the market risk premium and $cov_t(R_{it},R_{Mt})/var_t(R_{Mt})$ is the quantity of risk or the beta of the asset i. (1) is stated in terms of conditional moments, reflecting the behavior of the agents who make their investment decisions based on the information available up to time $t-1$ and assumes that investors maximize their utility period by period. In (1) neither the beta nor the risk premium are assumed to be constant over time. Following Merton's intertemporal CAPM (Merton, 1980), there is a positive relation between market risk premium and the variance of the market portfolio. Empirical evidence is provided in Bollerslev et al. (1988), Chou (1988), French et al. (1987) among others. The ratio between risk premium and variance of the market portfolio is given by the aggregate relative risk aversion coefficient δ and is assumed to be constant over time. The testable version of conditional CAPM is given by

$$E_t(R_{it}) = r_f + \delta\, cov_t(R_{it},R_{Mt}). \tag{2}$$

The estimation of (2) requires the specification of the dynamics of $cov_t(R_{it},R_{Mt})$. An econometric model ideally suited to estimate it is the bivariate GARCH-in-mean, which is the multivariate extension of an ARCH-in-mean model as in Engle et al. (1987). We use a bivariate system for the random vector $R'_t = (R_{it},R_{Mt})$ with a variance-covariance matrix H_t. This system allows for a rich structure permitting interaction effects between the market portfolio and the individual security. This is relevant because the consistency of the ARCH-in-mean parameter δ in (2) depends on a well specified volatility process (Pagan and Ullah, 1988). In fact, we will show that a univariate GARCH representation of individual securities is a misspecified model.

We estimate a bivariate GARCH(1,1)-in-mean such as

$$R_t = d + \Delta vech(H_t) + \varepsilon_t$$
$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad \varepsilon_t|\psi_{t-1} \sim N(0,H_t), \tag{3}$$

where *vech* is the column stacking operator of the lower portion of a symmetric matrix,

$$R_t = \begin{pmatrix} R_{it} \\ R_{Mt} \end{pmatrix} \; \varepsilon_t = \begin{pmatrix} \varepsilon_{it} \\ \varepsilon_{Mt} \end{pmatrix} \; \Delta = \begin{pmatrix} 0 & \delta_{12} & 0 \\ 0 & 0 & \delta_{23} \end{pmatrix},$$

d is 2×1 vector of constants, A, B and C are 2×2 parameter matrices, and the elements of H_t are $h_{it} \equiv var_t(R_{it})$, $h_{iMt} \equiv cov_t(R_{it},R_{Mt})$, and $h_{Mt} \equiv var_t(R_{Mt})$, δ_{12} is the ARCH-in-mean parameter in the equation of R_{it} that corresponds to h_{iMt} and δ_{23} is the ARCH-in-mean parameter in the equation of R_{Mt} corresponding to h_{Mt}.

The specification of H_t in (3) guarantees the positive definiteness of H_t, see Engle and Kroner (1995), and allows feed-back between the volatility of the individual firm and the volatility of the market. We do not require symmetry in matrices A and B. If A and B were symmetric and, for instance $a_{12} = 0$, we would be precluding not only firm effects on the variance of the market but also market effects on the variance of the i asset return, that is $a_{21} = 0$. The restrictions $a_{12} = b_{12} = 0$ will be tested.

Since the assumption of conditional normality of ε_t is not appropriate in financial time series, the model (3) is estimated using quasi-maximum likelihood. Given standard conditions, the QMLE estimator is consistent and asymptotically normal distributed with variance-covariance matrix $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}^{-1}$, where $\boldsymbol{A}$ is minus the expectation of the hessian and $\boldsymbol{B}$ is the expectation of the outer product of the score (Bollerslev and Wooldridge, 1992).

3. Results

Our empirical analysis is based on the stock returns of the American computer firms recorded in the CRSP tapes. The computer industry is defined in a wide sense and includes the computing equipment manufacturers and computer software and services. We have selected the sample according to the Standard Industry Classification (SIC) codes of: 3573, 3574, 7372 and 7379. We work with weekly returns constructed from Tuesday to Tuesday and when Tuesday is a holiday, we take the next trading day return. For 89 different companies, the full sample period goes from July 7, 1962 to December 29, 1987 with 1330 observations. Not all companies are active during the same period. In fact, only the largest companies IBM, NCR and Unisys have returns for the whole period 1962–1987. We use the NYSE value-weighted index as a proxy for the market portfolio.

As a preliminary step we analyze the univariate dynamics of every firm in the sample. We have specified different (G)ARCH models going from parsimonious specifications to larger ones. The choice of the best model is based on Wald tests. It is surprising to observe that (G)ARCH effects are not a common feature for all the companies. Out of 89 firms analyzed, 51 do not show any and in 7 a (G)ARCH specification is not appropriate. In general, we find that those companies in the computer software and services segment of the industry do not have any univariate (G)ARCH specification Among the 31 companies with GARCH effects, the most common specification is a GARCH(1,1) model. The persistence of the model, defined as the sum of the ARCH and GARCH coefficients, is high and very similar among firms and ranges from .86 to 1. Generally, univariate GARCH behavior is associated with big firms rather than small ones. The fact that a large proportion of firms do not show any (G)ARCH effect hints that a multivariate approach is more plausible and that the volatility process of individual stock returns may be driven by a set of factors.

The estimation of the bivariate GARCH(1,1)-in-mean model (3) is summarized in Table 2. To evaluate the statistical significance of the estimates of the system (3) we have to consider the problem that arises from the cross-sectional dependence of the t-statistics. Given that all the firms considered belong to the same industry, we should expect from a moderate to a high correlation among the t-statistics.

We use two different tests to analyze the results corresponding to the system (3). If a_{ij}, b_{ij}, δ_{ij} are not statistically significant, for large samples, their corresponding t-statistic can be viewed as draws from a $N(0,1)$. Hence, the hypothesis that we would like to test is H_0:${}^{(t)}\mu = 0, {}^{(t)}\sigma^2 = 1$, where ${}^{(t)}\mu$ is the cross-sectional population mean and ${}^{(t)}\sigma^2$ is the cross-sectional population variance of the t-statistics. The first test is based on the mean of t-statistics (Christie, 1982). Under the null, $Z = \bar{t}/\sigma_{cs}$ is asymptotically distributed as a $N(0,1)$, where $\bar{t}$ is the cross-sectional sample mean of the t-statistics and σ_{cs}^2 is the variance of the cross-sectional mean t-statistic and is equal (under H_0) to

$$\sigma_{cs}^2 = \frac{1}{n} + \frac{n-1}{n}\bar{\rho},$$

where $\bar{\rho}$ is the grand mean of ρ_{ij}, the correlation coefficient of t_i and t_j.

Table 1

	$\bar{\rho}=0$	$\bar{\rho}=0.3$	$\bar{\rho}=0.6$	$\bar{\rho}=0.9$
σ_{cs}	0.10	0.55	0.77	0.97
c	1	8.92	32.68	72.28
ν	89	9.9	2.72	1.2

Table 2
Hypothesis Testing: H_0:$^{(i)}$ $\mu = 0$ $^{(i)}\sigma^2 = 1$ $Z = \bar{t}/\sigma_{cs} \rightarrow N(0,1)$ $Q = ns^2/{}^{(i)}\sigma^2 = \Sigma_i t_i^2 \rightarrow c\chi^2_\nu$ under H_0

		$\bar{\rho} = 0$		$\bar{\rho} = 0.3$		$\bar{\rho} = 0.6$		$\bar{\rho} = 0.9$	
		Z	Q/c	Z	Q/c	Z	Q/c	Z	Q/c
$a_{11}^{(1)}$	0.013	1.44	761.2	0.27	85.33	0.19	23.29	0.15	10.53
$\bar{t}^{(2)}$	0.15	(0.07)$^{(3)}$	(0.0)	(0.39)	(0.0)	(0.42)	(0.0)	(0.44)	(0.0)
a_{12}	−0.011	−3.07	285.3	−0.57	31.98	−0.41	8.73	−0.32	3.94
$\bar{t}$	−0.32	(0.0)	(0.0)	(0.28)	(0.0)	(0.34)	(0.03)	(0.37)	(0.06)
a_{21}	0.077	1.44	650.5	0.27	72.9	0.19	19.90	0.15	8.99
$\bar{t}$	0.15	(0.07)	(0.0)	(0.39)	(0.0)	(0.42)	(0.0)	(0.44)	(0.0)
a_{22}	0.099	11.44	1585.6	2.14	177.7	1.54	48.51	1.22	21.93
$\bar{t}$	1.19	(0.0)	(0.0)	(0.01)	(0.0)	(0.06)	(0.0)	(0.11)	(0.0)
b_{11}	0.601	180.2	109336.1	33.84	12257.4	24.35	3345.6	19.32	1512.6
$\bar{t}$	18.75	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
b_{12}	−0.018	−4.23	279.2	−0.79	31.3	−0.57	8.54	−0.45	3.86
$\bar{t}$	−0.44	(0.0)	(0.0)	(0.21)	(0.0)	(0.28)	(0.03)	(0.32)	(0.07)
b_{21}	0.336	5.19	483.5	0.97	54.2	0.70	14.79	0.55	6.68
$\bar{t}$	0.54	(0.0)	(0.0)	(0.16)	(0.0)	(0.24)	(0.0)	(0.29)	(0.01)
b_{22}	0.678	159.0	57393.9	29.85	6434.2	21.48	1756.2	17.05	794.0
$\bar{t}$	16.54	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
δ_{11}	0.808	4.13	69.09	0.77	7.74	0.55	2.11	0.44	0.95
$\bar{t}$	0.43	(0.0)	(0.94)	(0.22)	(0.64)	(0.29)	(0.49)	(0.32)	(0.41)
δ_{12}	1.278	1.53	52.8	0.28	5.91	0.20	1.61	0.16	0.73
$\bar{t}$	0.16	(0.06)	(0.99)	(0.38)	(0.78)	(0.42)	(0.60)	(0.44)	(0.47)
$\delta_{13}^{(4)}$	−0.39	0.86	49.8	0.16	6.14	0.11	1.69	0.09	0.76
$\bar{t}$	0.09	(0.19)	(0.99)	(0.44)	(0.99)	(0.45)	(0.59)	(0.46)	(0.46)

(1) a_{ij}, b_{ij}, δ_{ij} are cross-sectional averages of time series regressions for the 89 individual stocks.
(2) $\bar{t}$ is the cross sectional mean of the t-statistics.
(3) The numbers in parenthesis are p-values.
(4) The results of this row are based on a trimmed sample (trimmed about 6%) due to heavy outliers.

The second test is based on the variance of the t-statistics. Under the null, $Q = ns^2/{}^{(i)}\sigma^2$ is approximately distributed as $c\chi^2_\nu$, where c and the degrees of freedom ν depend on the degree of correlation of the t-statistics (see appendix). Table 1 shows the values of σ_{cs}, c and ν for different values of $\bar{\rho}$.

When $\bar{\rho} = 0$, we have no correlation in the t-statistics and Q is distributed as a χ^2 with 89 degrees of freedom and $\sigma^2_{cs} = 1/n$ as expected.

From Table 2, in general, both tests, Z and Q, indicate that a_{22}, b_{11} and b_{22} are statistically significant at the conventional levels, and that a_{12} and b_{12} are not significantly different from zero. However, the tests differ on the relevance of a_{11}, a_{21} and b_{21}, with the Z indicating these terms are zero and the test Q indicating that the dispersion of the t-statistics is very different from 1. These results imply that the variance of the market can be modelled as a univariate process with a Factor ARCH structure being appropriate for individual stocks. Also, the volatility of the stock returns is explained by their own past conditional variance, the past market return volatility and the past conditional covariance between the firm and the market. Since the market plays an important role, a univariate representation of the conditional variance of stock returns is a misspecified model. The market is more important in small firms than in big firms and could be related to the cost of information. For small firms, the collection of information is more costly, hence investors use the market in order to forecast the volatility of small firms.

The last three rows of Table 2 show the estimation of the (G)ARCH-in-mean parameters. Although the δ_{12} is of interest, the estimation of the full variance-covariance matrix H_t permits alternative specifications of the mean equation. For instance, empirical evidence on residual risk (Lehmann, 1990) shows that var_t (R_{Mt}), is a better predictor of expected returns than cov_t (R_{it},R_{Mt}), since the variance may be considered as a proxy for omitted risk factors and is represented by the coefficient δ_{11}. Furthermore, the coefficient δ_{13}, which corresponds to the regressor var_t (R_{Mt}), may account for a one-dynamic factor specification, where the factor is the return to the market portfolio.

The estimation results in Table 2 show that there is some support for those theories based on residual risk. The coefficient δ_{11} is significantly different from zero when the cross-correlation of the t-statistics is low. The mean values of δ_{11} is 0.808. If the t-statistics are not correlated, we find a a marginal association between expected returns and the covariance between the individual stock and the market. The mean value of δ_{12} is 1.27. This value is an estimate of the coefficient of relative risk aversion and it agrees with the existing estimates in the literature (Hansen and Singleton, 1983). There is not support for the one-dynamic factor model. The coefficient δ_{13} is not significant at any level of correlation of the t-statistics. In general terms, residual risk models are preferred to conditional CAPM and one-dynamic factor models.

Finally, the bivariate estimation provides a model for the traditional measure of risk, $\beta_{it} = cov_t(R_{it},R_{Mt})/var_t(R_{Mt})$. The econometric specification gives direct estimates of the $h_{iMt} \equiv cov_t(R_{it},R_{Mt})$ and the $h_{Mt} = var_t(R_{Mt})$. We construct the time series β_{it} or quantity of market return risk associated with each stock in the sample. Overall the computer industry is a high risk industry with the mean of β_{it} being well above one; only nine securities have a mean below one.

A previous study by Baillie and Myers (1991) has considered the times series properties of the estimated β. We propose to compare *market* risk among securities from the empirical cumulative distribution function of the β_{it}, using the concepts of first order and second order stochastic dominance criteria. Dominance, in this sense, means less risky. Let $F_x(\beta)$ and $G_y(\beta)$ be the cumulative distribution function of β for assets x and y respectively. We say that x dominates y (first order stochastic dominance) if $F_x(\beta) \geq G_y(\beta)$ for all β. In other words, the cumulative distribution of x lies to the left of the cumulative distribution of y, and they are not allowed to cross each other. If the distributions cut each other, we say that x dominates y (second order stochastic dominance) if $\int_{-\infty}^{\beta}(F_x(\beta) - G_y(\beta))d\beta$

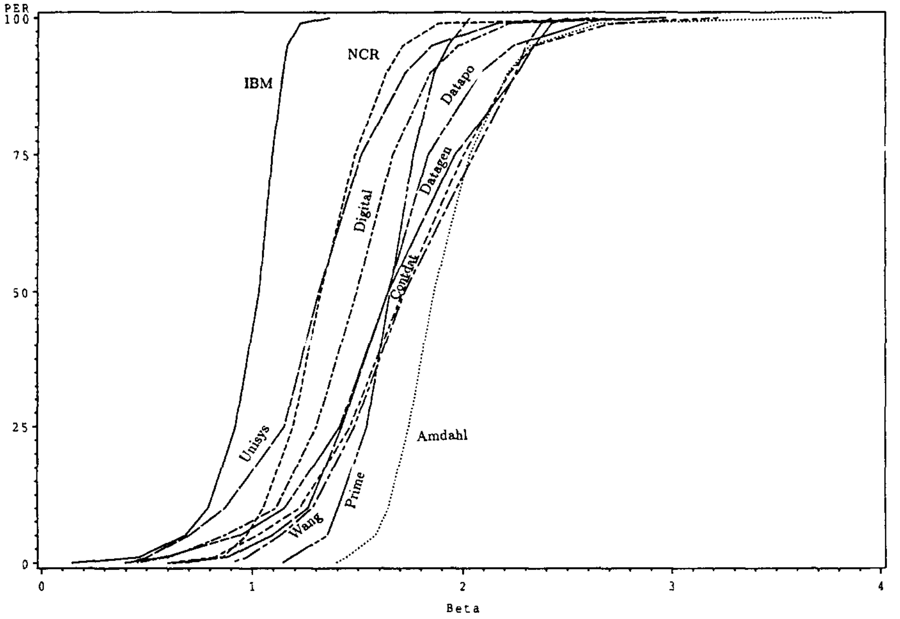

Fig. 1. Cumulative distribution function of beta for the SP500 companies.

≥ 0 for all β. Note the reverse sign of the inequalities compared to the standard definition of first and second order stochastic dominance. In the present context the dominance criteria is established respect to a variable which is non-desirable by the investor.

Fig. 1 presents the empirical cumulative distribution of β_{it} for the SP500 companies. The cumulative distribution function is represented by the 1, 5, 10, 25, 50, 75, 90, 95, 99 and 100 percentiles. For the sample period and in comparison with the SP500 companies, the leader of the industry, IBM, is the least risky, as it is popularly perceived in the financial markets.

4. Conclusions

We test a CAPM versus Residual Risk models for individual stocks using a bivariate GARCH-in-mean specification. Issues of volatility modelling have been addressed and we find that a univariate model of conditional variances is misspecified and that the market variance has information to forecast the individual stock variance. This is of particular importance because as Pagan and Ullah (1988) pointed out misspecification of the variance equation will bias the estimate of the GARCH-in-mean parameter. Individual risk premium is better explained by the variance of the individual security than by the covariance between the market and the security. This finding can be understood as further support for multi-factor theories. The market does not contain enough information to explain the variability of expected returns and the variance of the individual stock is working as a proxy for those omitted factors. Finally we construct the betas for each stock and a partial ordering of the stocks according to the market risk is offered using the cumulative distribution function of the conditional beta.

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Appendix A. Distribution of $Q = ns^2/\sigma^2$ for correlated random variables.

Under the assumption that the $n \times 1$ vector $t \rightarrow N(\mu, \Sigma)$, it can be easily proven (Mathai and Provost, 1992) that $Q = (t-\mu)'(t-\mu)/\sigma^2$ can be written as

$$Q = \sum_{i=1}^{n} \lambda_i U_i^2,$$

where λ_i are the eigenvalues of Σ and U_i are mutually independent standard normal random variables.

The joint null hypothesis that we test is

$$H_0 : \mu = 0 \; \Sigma = \Sigma_{H_0}$$

where

$$\Sigma_{H_0} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{12} & 1 & \rho_{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix}.$$

Under the null

$$Q=\sum_i t_i^2=t't=\sum_{i=1}^{n}\lambda_i U_i^2=\sum_{i=1}^{n}\lambda_i\chi_1^2$$

where λ_i are the eigenvalues of Σ_{Ho}. Approximate eigenvalues of this matrix are

$$\lambda_1\approx\lambda_2\approx\ldots\lambda_{n-2}\approx 1\quad \lambda_{n-1}=1+\sqrt{\sum_i\sum_j\rho_{ij}^2}\quad \lambda_n=1-\sqrt{\sum_i\sum_j\rho_{ij}^2}$$

this approximation satisfies the condition $\Sigma_i\lambda_i=tr(\Sigma_{Ho})=n$.

In order to compute the probability associated with a specific value x, we use the approximation of Patnaik (1949):

$$Pr(Q\le x)\simeq Pr\left(\chi_v^2\le\frac{x}{c}\right),$$

where

$$c=\frac{\Sigma_i\lambda_i^2}{\Sigma_i\lambda_i}=1+(n-1)\bar{\rho}^2\quad v=\frac{(\Sigma_i\lambda_i)^2}{\Sigma_i\lambda_i^2}=\frac{n}{1+(n-1)\bar{\rho}^2}$$

and $\bar{\rho}^2$ is the grand average of ρ_{ij}^2, and χ_v^2 is a Gamma variate with parameters $\alpha=\nu/2$ and $\beta=2$.

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