Vaccine Market Coordination Using Subsidy*

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Abstract

Prevention of infectious diseases is an important concern for managing public health. Although vaccines are the most effective means for preventing infectious diseases, the existence of a negative network externality often makes it difficult for vaccine coverage to reach a level that is socially optimal. In this research, we consider how a subsidy program can induce a socially optimal vaccine coverage. We consider an oligopoly market with identical vaccine producers and derive a subsidy that leads to a socially efficient level of coverage. We also derive a tax-subsidy combination that is revenue neutral, but achieves the same effect. Overall, our results provide useful insights for governments and policy makers with respect to an important issue related to public health.

Keywords: Vaccine coverage, network externality, negative network effect, vaccine effectiveness, vaccine pricing, vaccine subsidy.

1 Introduction

The market for vaccines to prevent the spread of infectious diseases is a large one, both in the US as well as globally, with a double digit growth rate. Global vaccine sales by major manufacturers have grown from US$3.6 billion in 1999 to US$9.9 billion in 2004, representing a compound annual growth rate of about 29% (Belsey et al. 2006). The total global sales are projected to grow to US$30–42 billion by 2015. Even though vaccines are widely recognized for their efficiency and cost-effectiveness and often touted as the "ultimate weapon against infection and drug resistance" (World Health Organization 2000), vaccine uptakes in populations have typically been low (Blue 2008), and often well below the socially optimal level.[1] Such undesirably low vaccine coverage in a population can be attributed to the negative network externality on the demand side, as well as operational issues on the supply side (such as yield uncertainty and a lack of sufficient incentives for vaccine manufacturers). As demonstrated by many shortages in the past few years, influenza vaccine is a prime example of the supply-side issues. Several papers in the literature argue that the operational issues have been one of the main factors for the influenza vaccine shortage in the market seen in the past (Chick et al. 2008, Deo and Corbett 2009).

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[1]We use the term "socially optimal level" to mean a level of coverage that achieves the maximum total social welfare.

In addition to the supply-side issues, one could argue that the negative network effect facing the consumer plays an important role in the relatively low vaccine coverage . This is because the chance of infection reduces as the network of vaccinated individuals grows. Therefore, for an individual who is not yet vaccinated, the willingness to pay for the vaccine reduces accordingly. This effect is exactly the opposite of the positive network effect identified in (Katz and Shapiro 1985), where the willingness to pay increases with the size of the network. Viewed differently, a voluntary vaccination policy presents a dilemma to an individual: if a sufficient proportion of the population is vaccinated and already immune, then the chance of becoming infected becomes low, and even the slightest risk or cost associated with vaccination outweighs the expected risk or cost from infection. In this situation, an individual may choose to free-ride off the herd immunity instead, and, consequently, voluntary vaccination is unlikely to reach a socially optimal level in the population. Several research questions arise in this context:

- For an infectious disease, what should be the socially optimal level of coverage for its vaccine, and how can that be achieved by a social planner?
- Since many vaccines are not fully effective, what impact does the effectiveness of vaccines play in the above decisions?
- Do governments or global non-profit organizations have a role to play in this context and, if so, how should they intervene?

Prior research has recognized a few of these issues related to the consumption-side externality and sub-optimal market coverage (e.g., Bauch and Earn 2004, Brito et al. 1991). However, prior work suffers from several limitations. First and foremost, existing literature on this topic is somewhat compartmentalized. Based on well-studied epidemic models, Bauch and Earn (2004) provide a game-theoretic framework of vaccine coverage. However, their model fails to identify the appropriate social welfare function. Brito et al. (1991), on the other hand, lack the epidemiological details, but are more comprehensive in their economic analysis of the situation. Second, in prior work, vaccine production is considered as exogenous and is not part of the total social welfare. However, we consider production costs and find that they play a very significant role in the vaccine market and its coverage.

Geoffard and Philipson (1997) study the issue of disease eradication using vaccines by considering a compartmental differential equation model of disease dynamics in the population. They show that the market competition, by itself, cannot eradicate an infectious disease from the population. They consider a number of intervention strategies by the government, such as subsidies and (partial) mandatory vaccination programs and show that price subsidies in a perfect market can eradicate the disease, while a monopoly manufacturer has an incentive to keep the disease alive. Our model is different from that of Geoffard and Philipson (1997) in several ways. First, Geoffard and Philipson focus primarily on the eradication of a disease and do not consider the best social outcome, as we do in this paper. Second, they consider a slightly different epidemic model in which consumer price elasticity changes over time. We, on the other hand, assume that the price elasticity is static over

the time horizon; this allows us to focus on other important issues—such as mechanism design and effects of imperfect vaccines on the proposed subsidies—that they do not consider.

Our work is also related to the work on vaccine supply chain. For example, Chick et al. (2008) study a supply chain with a monopolistic manufacturer selling influenza vaccines to a government when vaccine yields are uncertain. They design a cost sharing agreement that provides incentives to all the players in the supply chain in order to achieve an optimal balance between for-profit manufacturer incentives and public health incentives of the government. While production yield is random for their supply chain, Chick et al. do not consider the consumer behavior or consumers' incentives and assume that vaccine prices are exogenous to their model. We relax this assumption by explicitly modeling the consumers' incentives, by considering vaccine price as their decision variable, and by evaluating its combined effect with network externalities in determining the vaccine coverage in the population. Furthermore, unlike Chick et al., we analyze an oligopoly market.

In another related article, Deo and Corbett (2009) use Cournot competition to investigate the role of production yield in explaining the limited number of players in the influenza vaccine market. Similar to the context of this paper, Deo and Corbett study the effect of vaccine price on the vaccination coverage in the population. Also, in a recent article, Arifoglu et al. (2011) study a monopoly supply chain with consumer externalities, and show that the limited availability of vaccines lead to demand inflation. Similarly, in this paper, we study the effect of vaccine price and negative network externality on the vaccine coverage as well as the level of inefficiency in a market-based system. However, our research questions are quite different from these papers—we consider mechanism design issues targeted towards better market efficiency and determine a subsidy program that coordinates the vaccine market, whereas the above papers evaluate the effects of production uncertainty and/or consumption externalities on the manufacturers' profits, and demand for vaccines in the market.

In another work, Ovchinnikov and Raz (2010) study a number of mechanisms to (partially) coordinate the supply chain by aligning either price or quantity decisions in a market for public goods. They show the effectiveness of subsidies compared to buyback and rebate mechanisms by considering a monopoly manufacturer facing random demand. In this paper, we consider an oligopoly market with deterministic demand and derive a coordinating mechanism to coordinate the market. Furthermore, we focus on a specific class of products—vaccines—rather than generic public goods. This allows us to derive a demand function that is not only based on economic parameters such as the vaccine price but also on other epidemiological factors such as the population's perception of the vaccine effectiveness and infection dynamics.

Drawing upon the prior literature, in this paper, we combine basic economic modeling concepts with epidemiological findings to generate insights about public policy issues related to vaccine pricing, coverage, and subsidies, aimed at maximizing the total social welfare. Our paper makes the following contribution to the existing literature:

- We consider the costs and benefits of the general consumers as well as the vaccine producers to derive the total social surplus, and show that this never reaches an optimal level with a

free market.

- Unlike the previous literature on this topic, we consider situations where vaccine programs are not completely effective by considering the infection dynamics of the vaccinated and non-vaccinated fractions separately.
- We introduce governmental intervention in the form of consumer subsidies (that can be easily implemented as a co-pay) which can lead to socially-optimal levels of production and coverage.

We show that a simple fixed subsidy scheme can indeed provide appropriate incentive for individuals to guarantee a vaccine coverage that is socially optimal in the sense that it maximizes the total social welfare. When vaccines are relatively cheap to produce, it is optimal to reach the critical vaccination fraction in the population, and the subsidy is increasing with the vaccine production cost. For moderate values of the per-unit cost of production, however, we see that the subsidy is a decreasing function of the production cost. The result in this region is interesting and somewhat counter-intuitive—the subsidy for the vaccine of a disease decreases as the disease becomes more infectious. This is somewhat counter-intuitive because one might expect a more active role of the government (i.e., a larger subsidy), when a disease is more infectious. However, in this situation, the negative network externality diminishes as the disease becomes more infectious,[2] and individuals have a higher incentive to get the vaccine because they rely more on vaccination to avoid the infection rather than benefiting from the free-riding effect. Thus, more individuals would be willing to vaccinate on their own, and a lower subsidy can coordinate the market. We also show that, for prohibitively high production costs, vaccination becomes a wasteful spending, so no subsidy should be provided. Finally, we consider the issue of revenue neutrality and find a tax-subsidy combination that achieves the social optimum in a revenue neutral manner. Our results have important implications in terms of the role of governments and global health organizations in vaccine programs and provide directions for appropriate incentive schemes that work towards a greater good.

The remainder of the paper is organized as follows. Section 2 develops the modeling framework and identifies the socially optimal outcome. The oligopoly market is examined in Section 3. These results are then used in Section 4 to identify how a social planner can intervene using a subsidy to drive the market towards social optimality. Section 5 concludes the paper and offers directions for future research.

2 Basic Model

In this section, we introduce the notations used in this paper and develop necessary constructs of our model. We assume the following sequence of events, along with the complete information structure:

[2]If a disease is more infectious, the probability of infection is higher and does not go down quickly enough with higher vaccine coverage. In other words, in this situation, a consumer needs the vaccine more to avoid infection and is willing to pay a higher price.

- The vaccine effectiveness and the probability of infection are common knowledge for all parties.
- Manufacturers know the distribution of consumers' loss from infection, and decide on a production quantity based on that information.
- Production happens and vaccines become available to the consumer. Consumers expect a certain level of market coverage and make individual vaccination decisions based on that.
- The rational expectations equilibrium is reached, in which the market clears at the equilibrium price and consumers expectation about the market coverage is fulfilled.

Before we can develop a complete characterization of the consumer behavior and analyze the basic market structure, we need to estimate the probability of infection. We start this section with a discussion of how the probability of infection depends on vaccine coverage.

2.1 Infection Probability

The probability of infection depends on the specifics of the epidemic model and infection dynamics. We use a deterministic compartmental model of homogeneous and randomly mixing individuals that start out as *susceptible* (S) to infection, may become infected and *infectious* (I), and eventually be *removed* (R) upon recovery from infection. Such epidemic modeling is known as a standard SIR compartmental model and is a widely accepted model of the natural history of many infectious diseases such as influenza, tuberculosis, and smallpox (Bauch and Earn 2004, Bauch et al. 2003, Hethcote 2000, Hill and Longini 2003).

Because many of the vaccines available today are not always perfect, we would like to examine the situation where a vaccinated consumer, albeit less likely, can still get and transmit the infection (Longini et al. 1996). In this paper, we use an approach similar to the one adopted by Longini et al. (1978), and define a parameter ϕ, $0 < \phi \leq 1$, to reflect the combined vaccine effect on transmission, including susceptibility and infectiousness effects (Chick et al. 2008). When the vaccine is completely effective, of course, $\phi = 1$. For imperfect vaccines, even though $\phi < 1$, it is still typically a high number. For example, $\phi = 0.9$ is a reasonable estimate for seasonal influenza vaccines (Weycker et al. 2005).

Infection dynamics are different for the vaccinated and unvaccinated fractions of the population (Anderson and Hanson 2005, Hughes et al. 2002). Hence, we use different infection probability functions for each subgroup. Let $p(f)$, and $P(f)$ be the infection probability for the vaccinated and unvaccinated fractions, respectively, when the vaccine coverage for the population is f. Let $r(f)$ be the infection probability for the entire population; it can also be viewed as the overall fraction of infected individuals in the population and must satisfy:

$$r(f) = fp(f) + (1 - f)P(f).$$

Therefore, we can eliminate $P(f)$ completely from the rest of the discussion because:

$$P(f) = \frac{r(f) - fp(f)}{1 - f}.$$

In the epidemiology literature, $r(\cdot)$ is known to have the following form (see Appendix A for more details):

$$r(f) = \zeta \left(\max \left\{ 1 - \phi f - \frac{1}{R_0}, 0 \right\} \right).$$

For the rest of the discussion, we employ the above form of $r(\cdot)$, without considering any specific form for $p(\cdot)$, except a few reasonable assumptions that we make later. We also set $\zeta = 1$, without any loss of generality to obtain:

$$r(f) = \begin{cases} 0, & \text{if } f > \frac{R_0 - 1}{\phi R_0} \\ 1 - \phi f - \frac{1}{R_0}, & \text{otherwise.} \end{cases} \tag{1}$$

In Equation (1), R_0 stands for the basic reproduction number and is a measure of the infectiousness of a disease, since it represents the number of infections expected from an infectious individual (Anderson and May 1991, Murray 1993). The larger this number, the higher is the rate at which the disease is likely to spread. It must be noted that, if $R_0 \leq 1$, the infection probability is always zero, and the disease is not considered a serious epidemic threat. Therefore, for the remainder of the paper, we only consider $R_0 > 1$. For $1 < R_0 < \infty$, the minimum vaccination fraction at which the overall infection probability, $r(f)$, drops to zero is $F = \frac{R_0 - 1}{\phi R_0}$. In epidemiology, F is called the *critical vaccination fraction*—it represents the minimum level of vaccine coverage necessary for providing *herd immunity*, a situation that arises when the vaccination level is sufficiently high so that it eliminates the disease from the population completely (Anderson and May 1985).

We now turn our attention to the estimation of $p(f)$. In Appendix B, we show how $p(f)$ should behave based on the infection dynamics. As mentioned above, our model does not assume any specific form for $p(f)$, but makes the following three reasonable assumptions:

- $P(f) = \frac{r(f) - fp(f)}{1 - f}$ is non-increasing in f,
- $(r(f) - p(f))$ is a convex non-increasing function of f, and
- $f(r(f) - p(f))$ is a concave function of f in $[0, F]$.

The first assumption is intuitive; it simply means that the probability of infection for a non-vaccinated individual decreases with vaccine coverage. The second assumption implies that, as f increases, the infection probability gap between the vaccinated and the general population decreases till the critical vaccination fraction is reached, after which this gap vanishes. Furthermore, this assumption also means that the rate of decrease of this gap is a non-increasing function of f. The third assumption is commonly made in economics to ensure that the revenue is a concave function. To see this more clearly, note that $(r(f) - p(f))$ also indicates the price of the vaccine

in Equation (3). In order to ensure that these assumptions are valid in a real-world setting, we have done extensive numerical testing with the actual $p(f)$ that can be obtained from the infection dynamics and have found that these three assumptions always hold; please see Appendix B for further details.

Although we work with a general $p(f)$, in order to plot and illustrate our results, throughout the paper, we use the following approximation for $p(f)$:

$$p(f) = \eta(1-\phi)r(f), \tag{2}$$

where η is a constant that can be adjusted to obtain a good fit for the approximation. The approximation in (2) abides by the above properties and is quite close to the actual function; please refer to Appendix B for details.

2.2 Characterization of the Consumer Behavior

Vaccine consumers are considered to be heterogeneous because the amount of benefit from immunization would vary from user to user. In order to capture this, consumers are indexed by a parameter u that indicates their relative loss from an infection; we assume that u is uniformly distributed over the interval $[0, 1]$. The absolute cost to consumer u from an infection can then be expressed as Lu, where L is a constant. Let the price of the vaccine be W. Then, the total expected cost of getting vaccinated is simply $W + Lup(f)$, where f is the vaccine coverage, i.e., the fraction of the population that has been immunized. On the other hand, a consumer who has not been immunized does not have to pay the price W, but has a higher probability of infection, $P(f)$. Thus, the total expected cost of not getting vaccinated is $LuP(f)$. As a result, for the marginal consumer u, who is indifferent between getting vaccinated and not vaccinated, we must have: $W + Lup(f) = LuP(f)$, or $W = Lu(P(f) - p(f))$.

It is interesting to observe that the willingness to pay for any consumer u depends on $P(f) - p(f) = \frac{r(f)-p(f)}{1-f}$, which, in turn depends on f, the market coverage. In other words, $P(f) - p(f)$ determines the strength of the network effect, which has two parameters: R_0 and ϕ. When R_0 is finite or when $\phi < 1$, the term $\frac{r(f)-p(f)}{1-f}$ can be shown to be less than one, meaning that an unvaccinated individual benefits from other individuals obtaining the vaccine. In other words, unlike other typical products or services, in the vaccine market, a consumer does enjoy some of the benefits of a vaccine, without having to actually purchase it. Therefore, the decision to purchase depends not only on the consumer's own preferences, but also on decisions made by other consumers. As R_0 becomes infinitely large and ϕ approaches one, however, the term $\frac{r(f)-p(f)}{1-f}$ approaches 1 for all values of coverage level, f, implying that, in this limiting case, the actions of other individuals become irrelevant (i.e., there is no externality effect), and the only way to enjoy the benefits of the vaccine is to acquire it. Thus, the smaller is R_0 or ϕ, the stronger is the externality effect of the vaccine.

As shown in Figure 1, any consumer to the right of the marginal consumer, u, gets immunized,

whereas anyone to the left does not. Therefore, $u = 1 - f$. Furthermore, as mentioned above, $P(f) - p(f) = \frac{r(f)-p(f)}{1-f}$. Substituting these and letting $w = \frac{W}{L}$, we get:

$$w = r(f) - p(f). \tag{3}$$

In other words, w in Equation (3) represents the normalized price associated with a vaccine coverage of f. For the rest of the paper, we will use this normalized price. It is interesting to note that the equilibrium price of the vaccine, w, is the marginal reduction in probability of infection due to the vaccine.

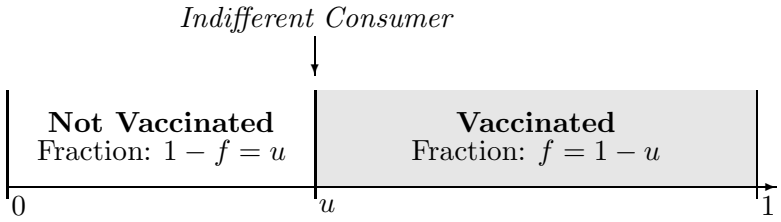

Figure 1: Consumers Get Immunized Based on Their Relative Benefit

Before we proceed, a word is in order about the assumption of uniform distribution for the relative loss, u. This assumption is equivalent to assuming a linear demand curve for a market without externalities. While this assumption makes it easier for the development and exposition of our results, the process described in this paper can be extended to a general cumulative distribution function $G(\cdot)$, with an inverse $G^{-1}(\cdot)$. In that case, Equation (3) simply changes to: $w = \frac{G^{-1}(1-f)}{1-f}(r(f) - p(f))$. Furthermore, for most of the common distributions, the uniqueness of $\tilde{f}$, $\hat{f}$, and $\hat{f}(s)$, as needed in Propositions 1, 2, and 4, respectively, can be assured from the log-concavity of the revenue (Bagnoli and Bergstrom 2005). Therefore, all our results hold with suitable modifications.

2.3 The First-Best Problem

In order to define the first-best problem, we need to determine the total social welfare for a market coverage of f and a price $w = r(f) - p(f)$, and maximize it. For this, we consider the normalized surplus accruing to the different parties.

Producer Surplus

Let the unit cost of producing a vaccine be C, and let the unit normalized cost of production be $c = \frac{C}{L}$. Therefore:

$$\text{Producer surplus} = (w - c)f.$$

Vaccinated Surplus

First we consider the surplus for all the individuals who decide to get vaccinated. When these individuals do get vaccinated, their probability of infection reduces and the expected infection cost is lower, but they all pay wf for purchasing the vaccine. If, on the other hand, these individuals do not get the vaccine, the total vaccinated fraction would be zero and their probability of getting infected would be $r(0)$; subsequently, if they do get infected, they incur a normalized infection cost $u \in (1-f, 1)$ (see Figure 1). The expected surplus gained from vaccination for this fraction of the population is then the difference between the costs with and without vaccination. Therefore, the vaccinated fraction f gets a surplus of:

$$\text{Vaccinated surplus} = \int_{1-f}^{1} x(r(0) - p(f))dx - wf = [r(0) - p(f)]\left(\frac{1-(1-f)^2}{2}\right) - wf.$$

Non-vaccinated Surplus

Similarly, each individual in the non-vaccinated fraction, with a normalized infection cost $u \in (0, 1-f]$ (see Figure 1), has a probability $P(f)$ of becoming infected when the other fraction is vaccinated, and a probability $r(0)$ of becoming infected when the other fraction is not vaccinated. Calculating the difference in the total expected costs between these two situations yields the total surplus gained for this fraction of the population from the presence of the vaccine. Therefore, the non-vaccinated fraction $(1-f)$ gets a surplus of:

$$\text{Non-vaccinated surplus} = \int_{0}^{1-f} x(r(0) - P(f))dx = [r(0) - P(f)]\frac{(1-f)^2}{2}.$$

Societal Surplus

In addition to the cost incurred by the manufacturer for producing the vaccine and the costs of disease borne by the infected individuals, we also consider another cost that may accrue on the society as a whole. When an individual gets the disease, there are costs borne by the society that are not borne by the individual. For example, the loss of work time may accrue on the society as a whole, whereas the individual can simply get a sick leave. Similarly, the burden on the public health system caused by the individual may not be fully borne by the individual. These are instances of negative externalities that must be part of any debate on public health. Therefore, to obtain the total social welfare, we assume that every individual who gets infected poses a loss of Λ (a normalized cost of $\lambda = \frac{\Lambda}{L}$) on the society; of course, the possibility that $\lambda = 0$ is not excluded, in case these losses are not important within a context. Since, in an expected sense, a fraction $r(f)$ may get infected, we can write the societal surplus as:

$$\text{Societal surplus} = -\lambda r(f).$$

Combining all these together, we get the normalized total social welfare as:

$$\text{Total Social Welfare} = \frac{r(0)}{2} - cf - \lambda r(f) - \frac{(1-f)r(f) + fp(f)}{2}. \tag{4}$$

Maximizing the total social welfare in (4) is equivalent to solving the following decision problem:

$$\min_{0 \le f \le \bar{F}} \text{SC} = \frac{(2\lambda+1)r(f) - f(r(f) - p(f))}{2} + cf, \tag{5}$$

where $\bar{F} = \min\{F, 1\}$.

The upper bound ($f \le \bar{F}$) in (5) comes from the fact that, if $f > F = \frac{R_0 - 1}{\phi R_0}$, $r(f) = 0$ and SC becomes an increasing function of f. Therefore, $0 \le f \le F$, and we have:

PROPOSITION 1 *Let $f = \tilde{f}$ be the solution of:*

$$c - \frac{\phi(2\lambda+1)}{2} - \frac{1}{2}\frac{d}{df}\left(f(r(f) - p(f)\right) = 0.$$

Then, the socially optimal level of vaccine coverage can be written as:

$$f^* = \begin{cases} 0, & \text{if } c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0} - \frac{p(0)}{2}, \\ \tilde{f}, & \text{if } \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right] \le c \le \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0} - \frac{p(0)}{2}, \\ \min\left\{\frac{R_0-1}{\phi R_0}, 1\right\}, & \text{otherwise.} \end{cases}$$

When the vaccine is perfect, $\phi = 1$ and $p(f) = 0$. For that special case, we have:

COROLLARY 1 *When the vaccine is perfect, the socially optimal level of vaccine coverage is given by:*

$$f^* = \begin{cases} 0, & \text{if } c > 1 + \lambda - \frac{1}{2R_0}, \\ 1 + \lambda - c - \frac{1}{2R_0}, & \text{if } \lambda + \frac{1}{2R_0} \le c \le 1 + \lambda - \frac{1}{2R_0}, \\ \frac{R_0-1}{R_0}, & \text{otherwise.} \end{cases}$$

Figure 2 plots the result in Proposition 1 for three different values of ϕ.[3] In these plots, $R_0 = 2$ and $\lambda = 0.1$. This figure shows that the result is in line with what one may intuitively expect in this situation. When the cost of production is high, i.e., $c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0} - \frac{p(0)}{2}$, it is better not to produce the vaccine at all. Since this threshold increases with the vaccine effectiveness, ϕ, vaccine production should stop at a lower c when ϕ decreases. On the other hand, when this cost is reasonably low, i.e., $c < \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right]$, it is the best to have a coverage equal to the critical vaccination fraction, $\bar{F}$, which is a decreasing function of vaccine effectiveness, ϕ. Between these two extreme cases, the socially optimal level is non-zero, but it is below the critical vaccination fraction.

As expected, vaccine effectiveness, ϕ, has a direct impact on the socially optimal vaccine coverage. When the production cost is low, a less effective vaccine (i.e., one with a lower ϕ) causes

[3] As mentioned earlier, for this as well as all the other plots in the paper, we make use of the approximation: $p(f) = \eta(1-\phi)r(f)$, for $\phi < 1$.

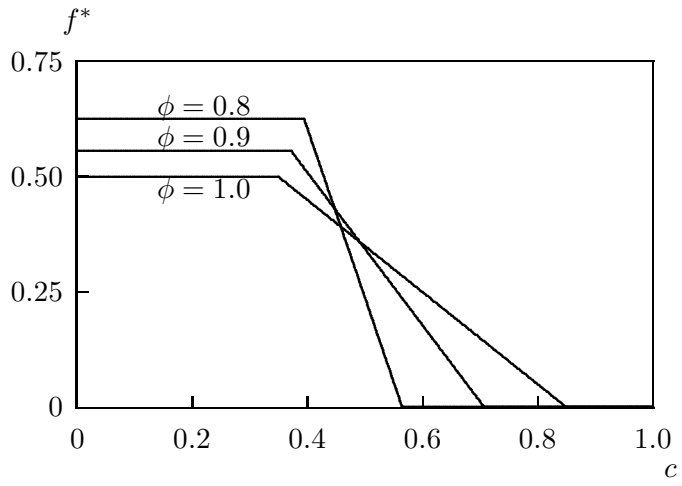

Figure 2: Socially Optimal Vaccine Coverage as a Function of Production Cost

the socially optimal coverage to increase—it takes more vaccination to reach the desired level of immunity in the population, since the critical vaccination fraction is inversely proportional to ϕ. As the per-unit production cost increases, however, a less effective vaccine leads to switching to zero production—indeed, administering a poorly matched and expensive vaccine cannot be justified from the society's welfare perspective, and the less effective the vaccine is, the lower is the cost threshold.

In order to see the impact of vaccine effectiveness more clearly on market coverage and total social welfare, we plot the optimal market coverage, f^*, and the optimal social cost, $\mathrm{SC}(f^*)$, as a function of ϕ, for $R_0 = 2$, $c = 0.45$, and $\lambda = 0.1$, in Figure 3. When the vaccine effectiveness,

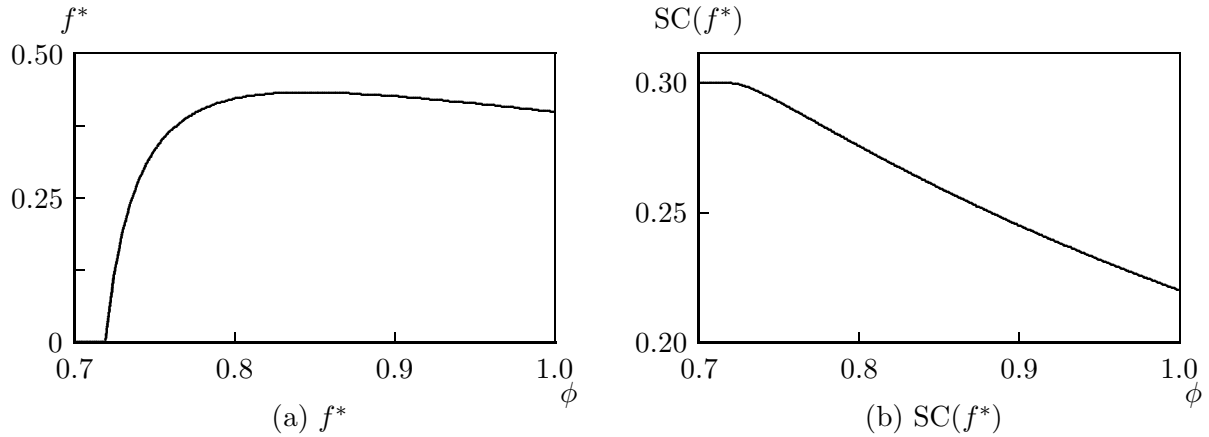

Figure 3: Optimal Market Coverage and Social Cost as a Function of Vaccine Effectiveness

ϕ, is below a certain threshold (about 0.719298 for the choice of parameter values in Figure 3), it is better not to administer the vaccine at all. The social cost in this region reflects the total cost to the society if no one is vaccinated. Beyond this threshold, the optimal vaccine coverage

increases, but the rate of increase slows down as ϕ increases. Once the optimal vaccine coverage reaches the critical vaccination fraction, F, it stays at that level. However, since $F = \frac{R_0-1}{\phi R_0}$ is a decreasing function of ϕ, the optimal coverage decreases in this region. Overall, the relationship between f^* and ϕ is not monotonic. This is because vaccine effectiveness has two impacts (Xu 1999). On one hand, a vaccine with a higher effectiveness is more desirable, because it combats the infection better; this is the positive effect observed at low levels of coverage—f^* increases with ϕ in this region. On the other hand, as the effectiveness increases, at higher coverage levels, the herd immunity can be reached much sooner, and the vaccine becomes less desirable in this range. The negative network effect dominates this region, and f^* decreases with ϕ. As expected, in both these regions, the optimal social cost goes down as the vaccine effectiveness increases—for a given cost of production, as vaccine effectiveness increases, everybody (irrespective of whether they get the vaccine or not) is better off because the vaccine is more effective in combating the disease.

3 Oligopoly Market

We now consider a Cournot competition among n identical vaccine producers, each with a normalized unit production cost of c, and apply the concept of rational expectations Cournot equilibrium (Katz and Shapiro 1985).[4] Let f_i be the market share of producer i. Clearly, $\sum_{i=1}^{n} f_i = f$. The ith producer faces the following decision problem:

$$\max_{f_i} \pi_i = (w - c)f_i = (r(f) - p(f) - c)f_i. \tag{6}$$

Solving (6), we get:

Proposition 2 *In an oligopoly vaccine market with n identical producers engaged in a Cournot competition, the total market coverage is given by:*

$$f = \begin{cases} 0, & \text{if } c > \frac{R_0-1}{R_0} - p(0), \\ \hat{f}, & \text{otherwise,} \end{cases}$$

where $f = \hat{f}$ is the solution of:

$$-c + r(f) - p(f) + \frac{f}{n}\left[r'(f) - p'(f)\right] = 0.$$

Corollary 2 *In an oligopoly vaccine market with n identical producers engaged in a Cournot*

[4]The vaccine market is a perfect real-world setting to apply Cournot competition. In this market, the production decisions are made long before the actual production starts and the vaccine becomes available to the consumer. As a result, the manufacturer cannot react to a change in demand quickly, as is evident from common stories about vaccine shortages and huge back orders (Chick et al. 2008, Deo and Corbett 2009). Therefore, a Cournot-like quantity competition is more suitable for this market than a Bertrand-like price competition. Furthermore, as shown by Kreps and Scheinkman (1983), when manufacturers plan for a certain quantity and capacity in the first stage, a Bertrand-like price competition would still lead to a Cournot equilibrium.

competition, if the vaccine is perfect, the total market coverage is given by:

$$f = \begin{cases} 0, & \text{if } c > \frac{R_0-1}{R_0}, \\ \frac{n}{n+1}\left(\frac{R_0-1}{R_0} - c\right), & \text{otherwise.} \end{cases}$$

The result in Proposition 2 is plotted in Figure 4 for $\phi = 0.9$, $R = 2$, $\lambda = 0.1$, and $n = 2$, along with the socially optimal coverage (f^*) as a benchmark. In this figure, we also plot the limiting market coverage $f_\infty = \lim_{n\to\infty} f$, a situation that would arise if there were many vaccine producers in the market. It is clear from the figure that a free market under-produces the vaccine. The plot

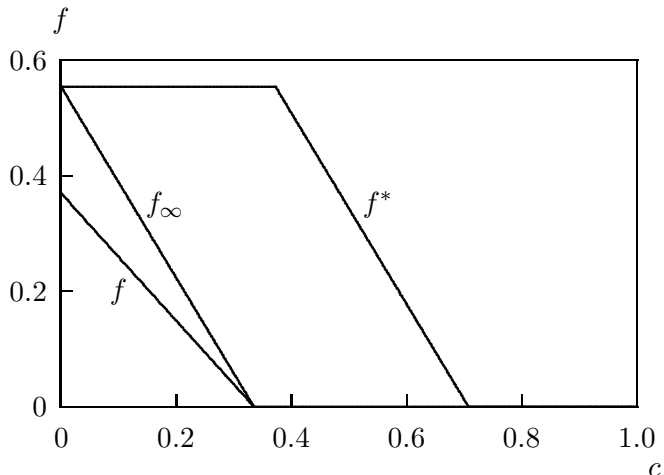

Figure 4: Free Market Vaccine Coverage as a Function of Production Cost

of f rotates to the right (towards f_∞) as n increases, but it can never reach the social optimum.

PROPOSITION 3 *Even in a very competitive market, the vaccine coverage cannot reach the socially optimal level, i.e., $f_\infty \leq f^*$. In fact, except in the trivial cases, where either $c = 0$ or $f_\infty = f^* = 0$, f_∞ is strictly less than f^*.*

Proposition 3 highlights the need for some kind of government intervention, and we will examine the role of subsidy, in the next section, as an instrument to coordinate this market.

4 Role of Subsidy

Having established in the previous section that a free market for vaccines may not be socially efficient, we turn our attention to the role of subsidy, either provided by the government or some other organization. A subsidy given to consumers at the time of vaccination simply changes the effective price consumers pay and changes their effective demand. Let S be the per-unit subsidy, and let $s = \frac{S}{L}$. This changes the demand equation in (3) to:

$$w(s) = s + r(f) - p(f). \tag{7}$$

Of course, this implies that $w \geq s$, implying that a consumer cannot make a net monetary profit from the vaccine.

4.1 Vaccine Producers' Response

Let us now consider the optimization problem facing producer i, among n identical ones engaged in a Cournot competition:

$$\max_{f_i} \pi_i = (w(s) - c)\, f_i = (s - c + r(f) - p(f))\, f_i. \tag{8}$$

Solving (8), we get:

PROPOSITION 4 *When a per-unit normalized subsidy of s is offered, the total market coverage is given by:*

$$f(s) = \begin{cases} 0, & \text{if } c > s + \frac{R_0 - 1}{R_0} - p(0), \\ \hat{f}(s), & \text{if } s - \frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right] \leq c \leq s + \frac{R_0 - 1}{R_0} - p(0), \\ \min\left\{\frac{R_0 - 1}{\phi R_0}, 1\right\}, & \text{otherwise,} \end{cases}$$

where $f = \hat{f}(s)$ is the solution of:

$$s - c + r(f) - p(f) + \frac{f}{n}\left[r'(f) - p'(f)\right] = 0. \tag{9}$$

As before, we consider the special case of perfect vaccines:

COROLLARY 3 *In an oligopoly vaccine market with n identical producers engaged in a Cournot competition, when a per-unit normalized subsidy of s is offered, if the vaccine is perfect, the total market coverage is given by:*

$$f(s) = \begin{cases} 0, & \text{if } c > s + \frac{R_0 - 1}{R_0}, \\ \frac{n}{n+1}\left(\frac{R_0 - 1}{R_0} + s - c\right), & \text{if } s - \frac{1}{n}\frac{R_0 - 1}{R_0} \leq c \leq s + \frac{R_0 - 1}{R_0}, \\ \frac{R_0 - 1}{R_0}, & \text{otherwise.} \end{cases}$$

4.2 Optimal Subsidy

We now explore whether the government can obtain the first-best outcome by providing a subsidy. Intuitively, this amounts to finding an s that solves $f(s) = f^*$.

THEOREM 1 *Let s be the normalized per-unit subsidy given by:*

$$s = \begin{cases} 0, & \text{if } c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0 - 1}{2R_0} - \frac{p(0)}{2}, \\ \frac{\phi(2\lambda+1) + (n-2)c - (n-1)[r(f^*) - p(f^*)]}{n}, & \text{if } \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right] \leq c \leq \frac{\phi(2\lambda+1)}{2} + \frac{R_0 - 1}{2R_0} - \frac{p(0)}{2}, \\ c + \frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right], & \text{otherwise.} \end{cases}$$

This subsidy coordinates the market, i.e., induces the producers to cover the market to the socially optimal level.

The following result is for the special case of perfect vaccines:

COROLLARY 4 *Let s be the normalized per-unit subsidy given by:*

$$s = \begin{cases} 0, & \text{if } c > 1 + \lambda - \frac{1}{2R_0}, \\ \frac{\left(1+(n+1)\lambda - c + \frac{n-1}{2R_0}\right)}{n}, & \text{if } \lambda + \frac{1}{2R_0} \leq c \leq 1 + \lambda - \frac{1}{2R_0}, \\ c + \frac{1}{n}\frac{R_0-1}{R_0}, & \text{otherwise.} \end{cases}$$

If the vaccine is perfect, this subsidy coordinates the market.

There are several implications of Theorem 1. First, it tells us that, given a context, an appropriate subsidy can be derived and implemented to coordinate the market to the socially optimal level. This is useful. Given the importance of vaccine programs and the associated network effects, governments, social planners, and charitable organizations have a tool to drive these programs towards the greater good. Second, there exists a subsidy for every value of $n \geq 1$. In other words, irrespective of the number of producers in the market, the social planner can find a suitable subsidy that would work in a given situation. Finally, it is clear that the subsidy is a decreasing function of n. This is intuitive. As n increases, the market becomes more competitive and the coverage grows. Therefore, the incentive necessary to drive it towards the socially efficient solution also decreases.

It is also interesting to observe that our result reduces to the standard result in welfare economics for markets with no network externality. For such a market, $\phi = 1$ and $R_0 \to \infty$. Furthermore, in traditional welfare economics, usually the additional societal cost of λ is ignored. Indeed, we find that when $\lambda = 0$, $\phi = 1$, and $R_0 \to \infty$, s approaches $\frac{1-c}{n}$ and becomes zero as $n \to \infty$. Clearly, the policy maker has no role to play in that case, because a Cournot market without externalities is efficient when $n \to \infty$.

Figure 5 illustrates the result in Theorem 1 for three different values of ϕ. In these plots, $R_0 = 2$ and $\lambda = 0.1$. As can be observed from the figure, the subsidy is first increasing in c, and then it is decreasing. When c is small, i.e., when $c < \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right]$, it is socially optimal to reach a vaccination level equal to the critical vaccination fraction, F. In this region, $p(f) = 0$ and, from Equation (7), $w = s$. As a result, the price and, hence, the optimal subsidy increases with c. The subsidy, in this region—let us call it Region I—essentially has two components. The first component, c, simply covers the marginal cost of production. The second component, $\frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right]$, is the oligopoly premium arising from the inefficiency in a Cournot competition, which vanishes as n becomes large. In the other extreme, when c is large, i.e., when $c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0} - \frac{p(0)}{2}$, we are in Region III where the cost of production outweighs any potential benefit of the vaccine, and it is socially optimal to not produce the vaccine at all. The subsidy in this region is zero. Between these two regions lies Region II, where the subsidy is a decreasing function of c, which ensures that the vaccine producers do not overproduce for higher values of c. Special attention should be paid to

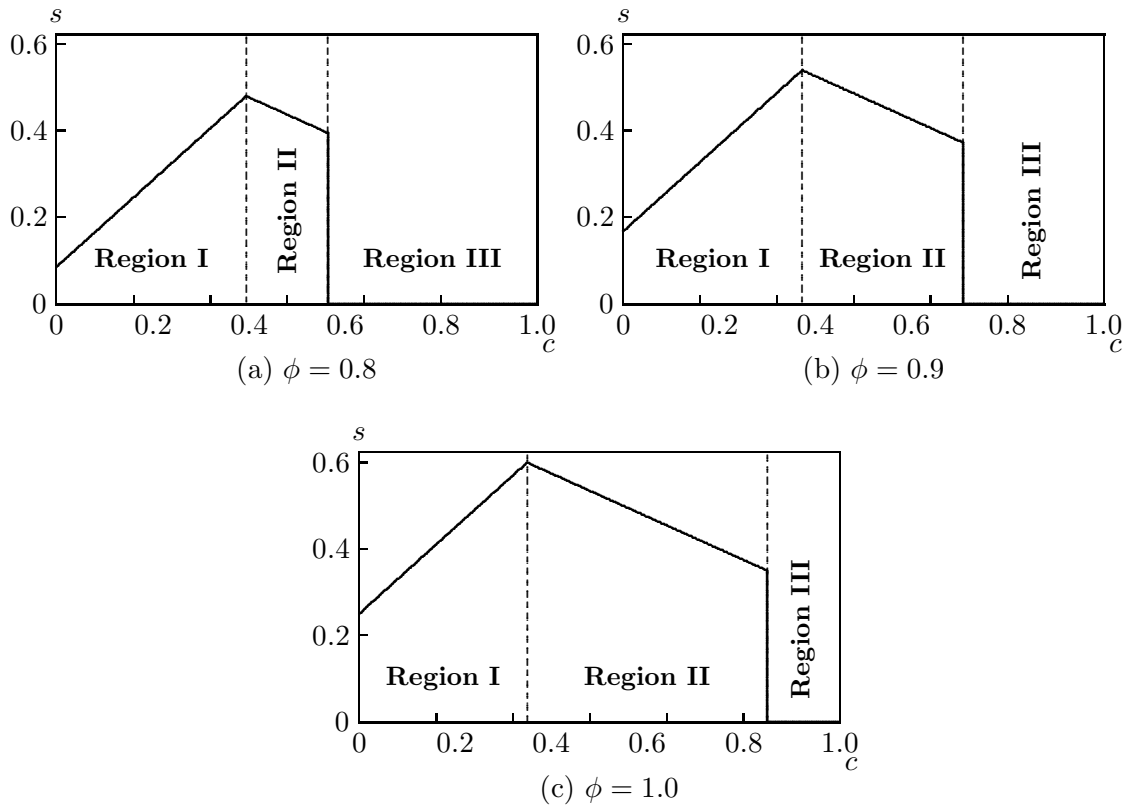

Figure 5: Optimal Subsidy as a Function of Production Cost

the discontinuity at the boundary between Regions II and III. In Region II, the optimal subsidy, s, decreases with the per-unit cost of production, c. As s decreases, so does $f(s)$, the market coverage induced by s. At the boundary, the subsidy reduces to such an extent that the manufacturers reduce their production to zero, i.e. $f(s) = 0$. Reducing s any further at this point does not change the socially optimal outcome anymore. In particular, $s = 0$ also leads to the social optimal, which is what Region III represents.

In order to see the impact of vaccine effectiveness on the optimal subsidy, in Figure 6, we plot the result in Theorem 1 for three different values of n, when $c = 0.45$. It is interesting to observe that the optimal subsidy increases with ϕ for low values of n, but decreases for high values of n. In order to understand why this is so, we need to understand the two factors behind the inefficiency within the vaccine market. First, as mentioned earlier, in all Cournot competition models, there is an inherent inefficiency in the market. This inefficiency disappears only when the market becomes fully competitive with a large number of manufacturers. The second source of inefficiency in the market is, of course, the negative network externality. The subsidy scheme presented in Theorem 1 addresses both these factors simultaneously. Now, as ϕ increases, the first factor—underproduction by manufacturers— become worse. This is because a higher effectiveness exacerbates the free-riding problem and reduces the demand for the vaccine. On the other hand,

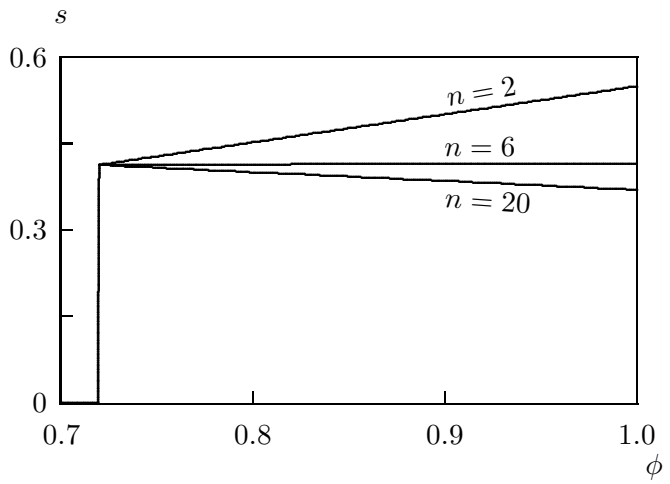

Figure 6: Optimal Subsidy as a Function of Vaccine Effectiveness

the impact of the negative network externality lessens as ϕ increases. Recall that, for very high values of R_0, the negative network effect disappears completely if $\phi = 1$. The optimal subsidy is determined by the interplay between these two factors. When n is small, the first effect dominates and, consequently, the optimal subsidy increases with n. For large values of n, on the other hand, the market becomes quite competitive, and the second effect dominates. Here the trend reverses, and the optimal subsidy, after the initial increase, decreases with ϕ. For moderate values of n—for example, for $n = 6$ with our choice of parameter values—the two effects counterbalance each other, and the optimal subsidy remains flat. Another observation from Figure 6 is that, for very low values of ϕ, the subsidy is zero. This is intuitive. At very low values of ϕ, it is socially optimal to not produce any vaccine at all, which is also what a profit-maximizing manufacturer adopts. In the absence of any misalignment, it is better not to provide any subsidy. This case is shown as Region III in Figure 5.

4.3 Subsidy, Taxation, and Revenue Neutrality

The use of a subsidy or a tax to account for an economic externality is nothing new in welfare economics (e.g., Jones 2005). Subsidies as a public policy instrument have been discussed in a variety of situations, ranging from education and R&D to agriculture and international trade. Many of these ideas have been successfully implemented in several countries all over the world.

To the best of our knowledge, we are the first to develop an implementable subsidy structure in the context of vaccine production, by taking into account the costs of vaccine production as well as the unique characteristics of the spread of a disease, as discussed in existing epidemiology literature. However, as is the case with any subsidy, a natural implementation question that may arise has to do with how a social planner should finance the subsidy. It is well-known that additional tariffs or taxes may have a distortionary effect elsewhere in the economy (Jones 2005, Mishan 1971, Salanie 2003). Naturally, a social planner looking for policy advice in this context would be interested in

finding answers to: (i) whether a combination of a subsidy and a tax exists that can achieve the same effect, and (ii) whether a revenue-neutral solution is possible. We answer these questions in the section.

The answer to the first question is easy—irrespective of the actual values of the subsidy and the tax, a tax-subsidy combination can make the market efficient, as long as the demand equation—Equation (7)—remains the same. In order to see this more clearly, consider the situation where a social planner imposes a tax of τL, $\tau \geq 0$, on people who are not vaccinated and provides a subsidy of σL, $\sigma \geq 0$, to the vaccinated. Then, a consumer u who is indifferent between the two choices must satisfy:

$$w + p(f)u - \sigma = P(f)u + \tau.$$

As before, we substitute $u = 1 - f$ and use the definition of $r(f)$, to obtain:

$$w = (\sigma + \tau) + r(f) - p(f).$$

In other words, if $\sigma + \tau = s$, then the demand equation reduces to the one given by Equation (7). Therefore, for any given s, there is an infinite number of subsidy-tax combinations $(\sigma, s - \sigma)$ which can coordinate the market.

The answer to the second question then hinges on whether there exists a subsidy-tax combination such that the revenue from the tax exactly finances the subsidies given out to the vaccinated consumers. The following proposition provides an answer to that question:

PROPOSITION 5 *Let s be as given in Theorem 1. Then $\sigma = (1 - f^*)s$ and $\tau = f^*s$ coordinates the market in a revenue neutral manner.*

The importance of Proposition 5 lies in the fact that it provides the social planner a way of exactly financing the desired subsidy without having to impose any distortionary taxes in this context, as well as other contexts within an economy. Furthermore, it works for all choices of parameters including production cost, vaccine effectiveness, and the level of competition. This makes it easier to implement this scheme as a public policy instrument, as long as taxes can be collected from the non-vaccinated population. One way to collect this tax would be to impose an up front tax of τ on everyone in the population—in the US, this can be done very easily, for example, by reducing the personal exemption by τ; subsequently, when a person gets vaccinated, he or she can be returned an amount $s = \tau + \sigma$ at the time of the vaccination.

4.4 Sensitivity Analysis

While we have shown that the subsidy derived earlier can coordinate the vaccine market and can do so in a revenue-neutral manner, we need to address one final issue before our analysis can be implemented in a real-world situation: how robust is our analysis and what impact do inaccurate estimates of parameters have on the social welfare? In order to address this concern, we perform extensive sensitivity analysis with respect to the two main parameters in the model: R_0 and c.

In order to investigate the impact of an inaccurate parameter, say R_0, we consider the situation where the actual R_0 is different from the estimated one, and derive the social cost (SC) with a subsidy calculated from the erroneous R_0. This social cost can then be compared with the optimized social cost, when the actual R_0 is known with certainty. Figure 7(a) provides one such plot (marked with $\times$) of this social cost with an estimated $R_0 = 2$ for $\lambda = 0.1$, $c = 0.45$, and $\phi = 0.9$. This figure also shows the social cost (marked as $\bullet$) if the subsidy is implemented with the actual R_0, as well as a plot of the social cost under free market (with no subsidy) for $n = 2$. As can be

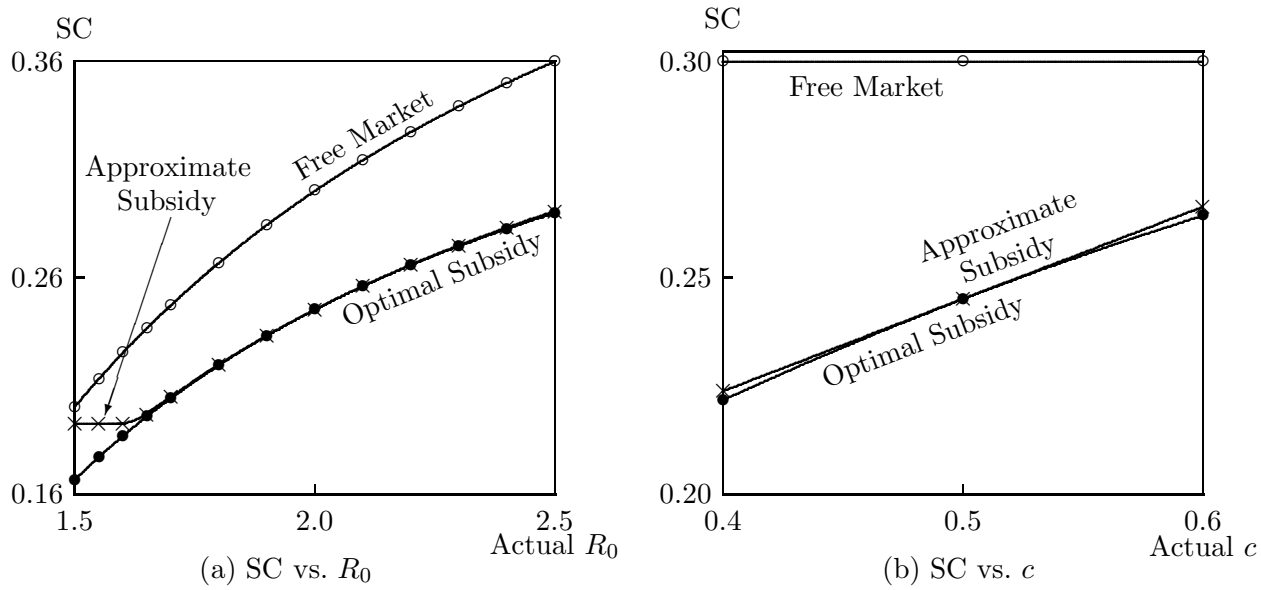

Figure 7: Sensitivity Analysis With R_0 and c

seen clearly from this figure, the implemented subsidy with an approximate value of $R_0 = 2$ leads to a total social cost that is quite close to the optimal level (calculated with the exact value of R_0), even when the approximate value of R_0 is quite different from the actual value. As the estimated value approaches the exact value of R_0, the two curves get even closer to each other. Furthermore, this figure clearly shows that the social cost with a subsidy calculated from the approximate R_0 is a lot less when compared to the social cost in the free market situation, i.e., with no subsidy. Similar trends were observed for other combinations of parameter values as well. This analysis clearly tells us that our results are robust with respect to the estimation of R_0, and it is better to provide a subsidy even with an inaccurate estimate of R_0 than not providing any subsidy at all.

Similar sensitivity analysis was also done with respect to the vaccine production cost parameter, c, and the results are quite encouraging there as well. Figure 7(b) provides a plot of the social cost with an estimated $c = 0.45$ for $\lambda = 0.1$, $R_0 = 2$, and $\phi = 0.9$. This figure also shows the social cost if the subsidy is implemented with the actual c, as well as a plot of the social cost under free market (with no subsidy) for $n = 2$. From this figure, once again, we see that our results are robust with respect to the estimation of c, and it is better to provide a subsidy even with an inaccurate

estimate of c than not providing any subsidy at all.

Finally, we examine the situation where the social planner decides to implement a subsidy different from the one obtained from our model. To illustrate this, we consider the following parameters: $c = 0.45$, $\lambda = 0.1$, $R_0 = 2$, and $\phi = 0.9$. In this case, the optimal subsidy can be found to be $s = 0.50125$, and the optimal market coverage is given by $f^* = 0.427$. However, we allow the actual subsidy to vary from 0.3 to 0.8, and plot the social cost in Figure 8. As expected,

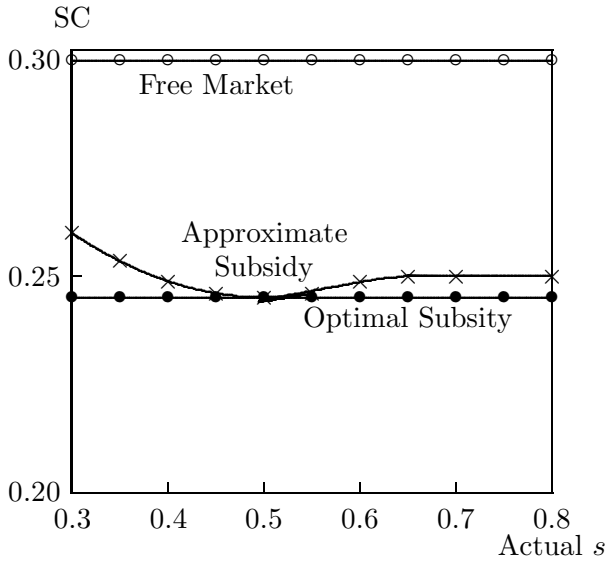

Figure 8: Sensitivity Analysis With s

the social cost increases as s deviates from the optimal subsidy, although it appears to be better than the free market situation, where no subsidy is offered. An interesting observation from this figure is that the social cost becomes flat as soon as s reaches the value of 0.6175. Actually, at this point, the subsidy induces a market coverage equal to the critical vaccination fraction. When the subsidy is increased beyond this point, it does not provide any additional incentive for people to get vaccinated, as the price $w(s)$ increases to ensure $w(s) - s \geq 0$. Since the price is simply a transfer from the consumer to the manufacturer, the social cost becomes flat beyond $s = 0.6175$. The point to note here is that a full subsidy (equivalent to a free immunization) may not be socially optimal. A full subsidy always induces a market coverage equal to the critical vaccination fraction. If, on the other hand, the socially optimal market coverage is actually below the critical vaccination fraction (because of the high production cost), as is the case in Figure 8, the social cost will be higher than the optimal level.

5 Conclusions

The vaccine market suffers from a negative network externality effect—as more individuals get vaccinated, the chance of infection, and hence the willingness to pay for a vaccine decreases for an

unvaccinated individual. This externality leads to socially suboptimal outcome from a free market setting. Government intervention via subsidy programs can provide consumers some incentive to get vaccinated by lowering their net cost of the vaccine and thus increase the overall level of vaccine coverage. In this paper, we use existing notions from welfare economics and analyze how subsidies should be designed in an oligopolistic vaccine market to maximize the societal benefits, by balancing vaccine producers profits, consumers welfare, and infection cost on society.

We show that a simple fixed subsidy scheme can indeed provide appropriate incentive for individuals to guarantee a socially optimal vaccine coverage that maximizes the total social welfare. If vaccines are relatively cheap to produce, it is optimal to reach the critical vaccination fraction in the population, and the subsidy is increasing with the vaccine production cost. For a moderate cost of vaccine production, however, we see that the subsidy is a decreasing function of the production cost. In this region, an interesting and somewhat counter-intuitive result is that the subsidy decreases with the basic reproduction number, R_0. A higher basic reproduction number indicates a rapid spread of the disease in the population, so one might expect a more active role of the government in terms of a larger subsidy. However, with higher R_0, the negative network externality diminishes, and individuals have a higher incentive to get the vaccine because they rely more on vaccination to avoid the infection rather than benefiting from the free-riding effect. Thus, the population on its own would be willing to vaccinate more, and a lower subsidy is sufficient to coordinate the market. We also show that, for prohibitively high production costs, vaccination becomes a wasteful economic spending, so no subsidy should be provided. Finally, we consider the issue of revenue neutrality and find a tax-subsidy combination that achieves the social optimum in a revenue neutral manner.

This work focuses on a specific type of intervention, namely government subsidies, for coordinating the vaccine market and perhaps increasing the overall vaccine coverage. Our result that an optimal subsidy can coordinate the market relies upon an implicit assumption that consumers are well-informed and rational. In many underdeveloped and poor countries where literacy levels are typically quite low, or where religious and cultural beliefs obfuscate scientific truth, this assumption may not hold. Indeed, the vaccine uptake in such countries (for example, in many African nations) is often well below the critical vaccination fraction, even when the disease is deadly and the vaccine is offered for free by some charitable organizations (Centers for Disease Control and Prevention 2009). Therefore, besides subsidy programs, there are other types of interventions that public health agencies and governments may also consider. For instance, it is possible to make vaccination mandatory (i.e., fine individuals that refuse to be vaccinated). Clearly this would increase vaccine uptake with the obvious drawback of taking away the population's freedom to choose to vaccinate and the lack of popularity and political difficulties that go along with it (and hence make it less easy to implement). Another, less extreme intervention would be to improve public information by education about vaccines and increase quicker and easier access to the vaccine. Marketing campaigns and the creation of neighborhood clinics could, for example, achieve these goals and could change individuals willingness to vaccinate or their valuation of the vaccine. The effect of such

interventions is beyond the scope of this paper, but should be considered in future research.

As is the case with any quantitative model, our model is an abstraction of the real world. In this model, we assume that the parameters determining the probability of infection—R_0 and ϕ—do not change during the epidemic season and the vaccine uptake occurs primarily right at the start of the season. This results in a static price elasticity of demand. This is a reasonable assumption for epidemics that have a short seasonal cycle, such as seasonal influenza. However, if the epidemic period is long, this assumption may be violated and, in such cases, our results should be applied with proper caution. Extending our model to a more dynamic environment would be an excellent future research topic.

There are several other directions in which our work could be extended. For example, in this paper we consider oligopolistic vaccine producers with identical production cost. It would be interesting to study the effect of asymmetric production cost on the optimal subsidy structure. Second, in this study, we assume a constant unit cost of production. In reality, there are usually large fixed costs and economies of scale in vaccine production. Furthermore, the production of vaccines often faces a great deal of uncertainty. It would be interesting to study how these factors affect the market coordination and the subsidy structure. Perhaps, a two-tier subsidy structure that incorporates production subsidy along with price subsidy would be warranted. Finally, in this work, we do not model the impact of the treatment of infectious diseases (such as HIV and malaria) and incentives that may be provided for these treatments. When subsidies are given for a non-vaccine medical items (such as drugs and surgical procedures), they would impact the spread of disease, the infection probability, the distribution of individual loss from an infection, and the overall fraction of individuals with immunity from the disease. These changes must be reflected in the model for it to provide comprehensive picture of the real world. We are examining all these issues in an ongoing project to develop a more complete understanding of the public policy challenges in this area.

References

[1] Anderson, R. and M. Hanson (2005). Potential Public Health Impact of Imperfect HIV Type 1 Vaccines. *Journal of Infectious Diseases* **191**(Supp. 1), S85–S96.

[2] Anderson, R. and R.M. May (1985). Vaccination and Herd Immunity to Infectious Diseases. *Nature* **318**, 323–329.

[3] Anderson, R. and R.M. May (1991). *Infectious Diseases of Humans: Dynamics and Control*. Oxford University Press, Oxford.

[4] Arifoglu, K., S. Deo, and S. Iravani (2011). Consumption Externality and Yield Uncertainty in the Influenza Vaccine Supply Chain: Interventions in Demand and Supply Sides. *Management Science*. Forthcoming.

[5] Bagnoli, M. and T. Bergstrom (2005). Log-Concave Probability and Its Applications. *Economic Theory* **26**(2), 445–469.

[6] Bauch, C.T. and D.J.D. Earn (2004). Vaccination and the Theory of Games. *Proceedings of the National Academy of Sciences* **101**(36) 13391–13394.

[7] Bauch, C.T., A.P. Galvani, and D.J .D. Earn (2003). Group Interest Versus Self-interest in Smallpox Vaccination Policy. *Proceedings of the National Academy of Sciences* **100**(18) 10564–10567.

[8] Belsey, M.J., B. de Lima, A.K. Pavlou, and J.W. Savopoulos (2006). Influenza Vaccines. *Nature Reviews Drug Discovery* **5** 183–184, March.

[9] Blue, L. (2008). Why Don't Adults Get Vaccinated? *Time*, January 24.

[10] Brito D.L., E. Sheshinski, M.D. Intriligator (1991). Externalities and Compulsory Vaccination. *Journal of Public Economics* **45**, 69–90.

[11] Centers for Disease Control and Prevention (2009). Epidemiology of the Unimmunized Child. Accessed December 20, 2011 at: http://www.who.int/immunization/sage/CDC_UNVACC_REPORT_FINAL_v2.pdf.

[12] Chick, S.E., H. Mamani, and D. Simchi-Levi (2008). Supply Chain Coordination and Influenza Vaccination. *Operations Research* **56**(6), 1493–1506.

[13] Deo, S. and C.J. Corbett (2009). Cournot Competition Under Yield Uncertainty: the Case of the US Influenza Vaccine Market. *Manufacturing and Service Operations Management* **11**(4), 563–576.

[14] Geoffard, P. and T. Philipson (1997). Disease Eradication: Private Versus Public Vaccination. *American Economic Review* **87**(1), 222–230.

[15] Hethcote, H. W. (2000). The Mathematics of Infectious Diseases. *SIAM Review* **42**(4), 599–653.

[16] Hill, A.N. and I.M. Longini (2003). The Critical Fraction for Heterogeneous Epidemic Models. *Mathematical Biosciences* **181**, 85–106.

[17] Jones, C. (2005). *Applied Welfare Economics*. Oxford, New York, NY.

[18] Katz, M.L., C. Shapiro. 1985. Network Externalities, Competition and Compatibility. *American Economic Review* **75**(3) 424–440.

[19] Kreps, D.M., J.A. Scheinkman. 1983. Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes. *The Bell Journal of Economics* **14**(2) 326–337.

[20] Longini I.M., S. Datta, M.E. Halloran (1996). Measuring Vaccine Efficiency for Both Susceptibility to Infection and Reduction in Infectiousness for Prophylactic HIV-1 Vaccines. *Journal on AIDS and Human Retrovirology* **13**, 440–447.

[21] Longini I.M., E. Ackerman, L.R. Elveback (1978). An Optimization Model for Influenza A Epidemics. *Mathematical Biosciences* **38**, 141–157.

[22] Mishan, E.J. (1971). The Postwar Literature on Externalities: An Interpretative Essay. *Jounal of Economic Literature* bf 9(1) 1–28.

[23] Murray, J.D. (1993). *Mathematical Biology* (2nd Ed.). Springer, Berlin.

[24] Ovchinnikov A.S. and G. Raz (2010). A Newsvendor Model with Pricing for Public Interest Goods. *Working Paper*. Darden School of Business, University of Virginia, Charlottesville, VA.

[25] Salanie, B. (2003). *The Economics of Taxation*. MIT Press, Boston, MA.

[26] Weycker, D., J. Edelsberg, M.E. Halloran, I.M. Longini, A. Nizam, V. Ciuryla, and G. Oster (2005). Population-Wide Benefits of Routine Vaccination of Children Against Influenza. *Vaccine* **23**, 1284–1293.

[27] World Health Organization (2000). Overcoming Antimicrobial Resistance. Accessed February 03, 2010 at http://www.who.int/infectious-disease-report/2000/index.html.

[28] Xu, X. (1999). Technological Improvements in Vaccine Efficacy and Individual Incentive to Vaccinate. *Economic Letters* **65**(3) 359–364.

Appendices

A Estimating the Overall Infection Probability, $r(f)$

The analysis presented in the paper, relies on an underlying epidemic model to obtain the infection probability $r(f)$. This appendix presents two variants of the textbook SIR epidemic model, one with vital dynamics and one without. We derive a closed form solution for infection probability of an SIR model with vital dynamics which gives rise to formula used in the paper. Unfortunately, for an SIR model without vital dynamics, there is no closed-form solution. However, we show that, for diseases of interest and for realistic values

of infection model parameters, the same formula can be used as an effective approximation for the true probability function. Using the notation in the paper, a vaccine's effectiveness is denoted by a parameter ϕ, where $0 < \phi \leq 1$.

A.1 An SIR Model with Vital Dynamics (Endemic Model)

In this model, births and deaths are explicitly taken into account. The birth and death rates are equal so that the total size of the population would remain constant. Notice that such models can be used for diseases which can stay in the population for a long time period (e.g., smallpox), but may not be appropriate for diseases that can invade a population for a relatively short period of time (e.g., influenza).

A standard formulation for the SIR epidemic with vital dynamics (endemic model) uses a three-compartment model in which individuals are either susceptible to the disease (S), infectious (I), or recovered (R). The fractions of individuals in these three states at time t are represented as $S(t)$, $I(t)$, and $R(t)$, respectively. When expressed as a set of differential equations, epidemiologists tend to parameterize the endemic model by:

$$\begin{aligned} \frac{dS}{dt} &= \mu(1-\phi f) - \beta SI - \mu S, \\ \frac{dI}{dt} &= \beta SI - \frac{I}{\delta} - \mu I, \text{ and} \\ \frac{dR}{dt} &= \frac{I}{\delta} + \mu\phi f - \mu R, \end{aligned}$$

where μ is the birth and death rate, β is the mean transmission rate, and $\delta > 0$ is the mean duration of the infection.

Let $\theta = \mu\delta$ be the infectious period as a fraction of mean lifetime, $R_0 = \frac{\beta\delta}{\mu\delta+1}$ be the basic reproduction number, and $F = \frac{R_0-1}{R_0\phi}$ be the critical vaccination fraction.

Let $\hat{S}$ and $\hat{I}$ be the number of susceptible and infected individuals (vaccinated and unvaccinated) in the population in the equilibrium. One can show that if $f \geq F$, then the system converges to the disease-free state $(\hat{S}, \hat{I}) = (1-\phi f, 0)$, whereas if $f < F$, then the system converges to the following stable endemic state $(\hat{S}, \hat{I}) = \left(1-\phi f, \frac{\phi\theta}{\theta+1}(F-f)\right)$ (Bauch and Earn 2004).

As a result, the infection probability of a susceptible individual can be expressed as the proportion of susceptible individuals becoming infected versus dying in any unit time. Therefore, the total fraction of infected individuals in the population is:

$$r(f) = (1-\phi f)\frac{R_0(\theta+1)\hat{I}}{R_0(\theta+1)\hat{I}+\theta} = 1 - \phi f - \frac{1}{R_0},$$

which is the same as the probability function presented in the paper with $\zeta = 1$.

A.2 An SIR Model without Vital Dynamics (Epidemic Model)

This is an epidemic model which does not include newborns or regular deaths in the population due to the relative small window of the infection period. This model has been shown to be consistent with history of many diseases including influenza. As a result, letting $\mu = 0$ in the above-mentioned system of differential equations leads to:

$$\begin{aligned} \frac{dS}{dt} &= -\beta SI, \\ \frac{dI}{dt} &= \beta SI - \frac{I}{\delta}, \text{ and} \\ \frac{dR}{dt} &= \frac{I}{\delta}. \end{aligned}$$

Timely vaccination followed by the onset of (instantaneous) infections from exogenous sources results in initial conditions $R(0) = f\phi$, $S(0) = (1-f\phi)(1-\chi)$, $I(0) = (1-f\phi)\chi$, where χ is the initial infected fraction

of the population. As expected, this number is typically very small ($0 < \chi \ll 1$).

It can be shown that the attack rate in this case does not have an explicit form solution like the previous section. However, it can be characterized as follows (Chick et al. 2008):

$$r(f) = (1 - \phi f)\left[1 - (1 - \chi)e^{-R_0 r(f)}\right].$$

Moreover, it can be shown that the critical vaccination fraction for this case is the same as the one for the endemic model, i.e., $F = \frac{R_0 - 1}{R_0 \phi}$, where $R_0 = \beta\delta$ is the basic reproduction number. Chick et al. (2008) also argue that, for a small enough value of χ, the overall infected fraction of the population, $r(f)$, can be well estimated by a piecewise linear function. Using the first-order Taylor series approximation, we get:

$$r(f) = \max\left\{1 - \frac{1}{R_0} - f\phi, 0\right\},$$

which is identical to the endemic case. While this equation has an attractive epidemiological interpretation and models the critical vaccination fraction fairly accurately, it estimates the actual $r(0)$ poorly due to the Taylor series approximation. We can account for this by adjusting the function at point $f = 0$ which leads to $r(f) = \max\{\frac{R_0 r(0)}{R_0 - 1}(1 - \frac{1}{R_0} - f\phi), 0\}$, where $r(0)$ is the attack rate in the absence of any vaccination, i.e.,

$$r(0) = 1 - (1 - \chi)e^{-R_0 r(0)}.$$

Thus, $r(f)$ can be approximated by the formula in this paper with $\zeta = \frac{R_0 r(0)}{R_0 - 1}$.

B Estimating the Infection Probability of the Vaccinated Population, $p(f)$

In this appendix, we extend the SIR models developed in Appendix A to account for the disease dynamic within the vaccinated and unvaccinated populations. Such compartmental models are mainly discussed in the context of HIV and HPV vaccines in the epidemiology literature (Anderson and Hanson 2005, Hughes et al. 2002). In this section, we use a variant of the mentioned models to fit other diseases of interest such as influenza.

B.1 An SIR model with imperfect vaccines

As before, we consider two variants of the SIR epidemic model, one with vital dynamics and one without. Unlike the total fraction of the infected population, we are not able to find a closed-form analytical solution for the infection probability of vaccinated population. However, we can show that a simple piecewise linear function can provide a fairly effective approximation for $p(f)$ for both scenarios.

In order to differentiate between the vaccinated and non-vaccinated populations, we define the following five states for the individuals at any time t. Let $S_n(t)$ be the fraction of non-vaccinated susceptible individuals, $I_n(t)$ the fraction of non-vaccinated infected individuals, $S_v(t)$ the fraction of vaccinated susceptible individuals, $I_v(t)$ the fraction of vaccinated infected individuals, and $R(t)$ the fraction of recovered population. With imperfect vaccination and vital dynamics, assuming that vaccination occurs at birth, the system of differential equations characterizing the epidemic model can be written as:

$$\begin{aligned}
\frac{dS_n}{dt} &= (1 - f)\mu - \mu S_n - \beta S_n I_n - \beta S_n I_v, \\
\frac{dS_v}{dt} &= f\mu - \mu S_v - \beta(1 - \phi) S_v I_n - \beta(1 - \phi) S_v I_v, \\
\frac{dI_n}{dt} &= \beta S_n I_n + \beta S_n I_v - \frac{I_n}{\delta} - \mu I_n, \\
\frac{dI_v}{dt} &= \beta(1 - \phi) S_v I_n + \beta(1 - \phi) S_v I_v - \frac{I_v}{\delta} - \mu I_v, \text{ and}
\end{aligned}$$

$$\frac{dR}{dt} = \frac{I_n}{\delta} + \frac{I_v}{\delta} - \mu R,$$

with the initial conditions $R(0) = 0$, $S_v(0) = 0$, $S_n(0) = 1 - \chi$, $I_v(0) = 0$, $I_n(0) = \chi$, where χ is as defined earlier.

Setting $\mu = 0$ in the set of differential equations above would lead to the infection dynamics for an infection without vital dynamics:

$$\begin{aligned}
\frac{dS_n}{dt} &= -\beta S_n I_n - \beta S_n I_v, \\
\frac{dS_v}{dt} &= -\beta(1-\phi) S_v I_n - \beta(1-\phi) S_v I_v, \\
\frac{dI_n}{dt} &= \beta S_n I_n + \beta S_n I_v - \frac{I_n}{\delta}, \\
\frac{dI_v}{dt} &= \beta(1-\phi) S_v I_n + \beta(1-\phi) S_v I_v - \frac{I_v}{\delta}, \text{ and} \\
\frac{dR}{dt} &= \frac{I_n}{\delta} + \frac{I_v}{\delta}.
\end{aligned}$$

Timely vaccination followed by the onset of (instantaneous) infections from exogenous sources results in initial conditions $R(0) = 0$, $S_v(0) = f$, $S_n(0) = 1 - f - \chi$, $I_v(0) = 0$, $I_n(0) = \chi$, where χ is as defined earlier.

B.2 Assumptions validation

While we are not able to find a closed-form solution for the infection probability of the vaccinated population, $p(f) = \left(\int_0^\infty I_v(t)dt\right)/f$, in either one of the epidemic models, our numerical experiments illustrate that assumptions made in the paper on functions $p(f)$ and $T(f) = r(f) - p(f)$ are realistic; namely, we assumed in the paper that: (i) $P(f)$ is a non-increasing function of f, (ii) $T(f)$ is a convex non-increasing function of f, and (iii) $fT(f)$ is a concave function of f.

In order to validate these three assumptions, we ran extensive numerical experiments for different sets of parameter values. In each run, we solve numerically the system of ordinary differential equations (ODE) above and evaluate the probability of infection for vaccinated individuals $p(f)$. Figure B1 represents functions $P(f)$, $T(f)$, and $fT(f)$ as a function of f based on the numerical solution of the ODEs for the case without vital dynamics, with $\phi = 0.9$. Similar figures were obtained for other values of ϕ, as well as for the case with vital dynamics; these results are not included here for the sake of brevity. This figure clearly illustrates that the assumptions made in the paper that $P(f)$ is non-increasing and $T(f)$ is convex non-increasing in f are validated by our numerical results. Similarly, the assumption that $fT(f)$ is a concave function of f appears valid for $f \in [0, F]$.

B.3 Further Approximation of $p(f)$

Throughout the paper, all our illustrations make use of an approximation of $p(f)$:

$$p(f) = \eta(1-\phi) r(f).$$

Because $r(f)$ is known in closed form and is a piecewise linear function of f, it follows that this approximation of $p(f)$ is also a piecewise linear function of f. We now demonstrate that there exists a value of η such that this is a reasonably close approximation for $p(f)$ both with and without vital dynamics. In Figure B2, the actual values of $p(f)$—obtained from numerical solutions of the ODEs for the case without vital dynamics with $\phi = 0.9$—are plotted as points (represented by $\times$). The approximation of $p(f)$ is drawn as a solid line. In this particular case, the best fit is obtained for $\eta = 3.3$. This is figure clearly shows that our approximation is quite close to the reality. Similar plots were obtained for other values of ϕ and for the case with vital dynamics as well, but are not included here for the sake of brevity.

With the above approximation, all of our results can be simplified further. For example, the socially

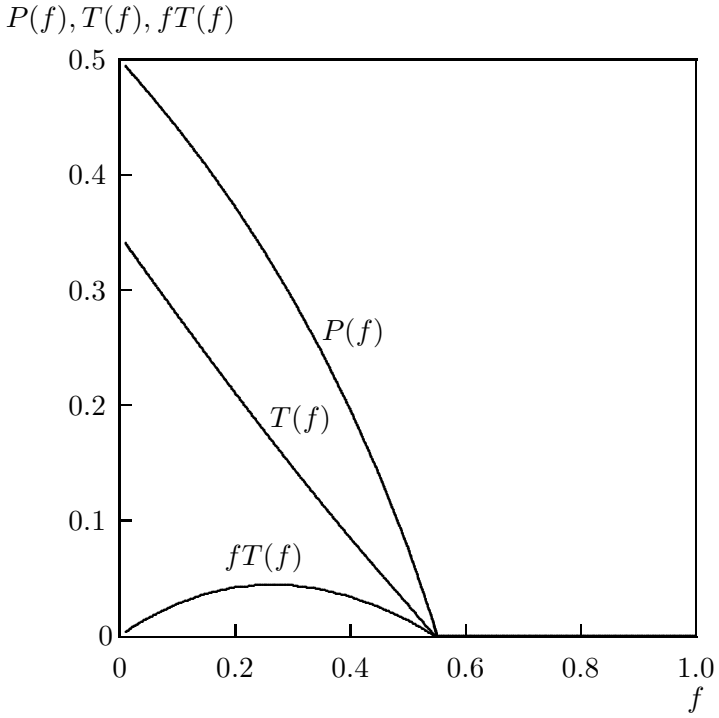

Figure B1: $T(f)$ and $fT(f)$ As Obtained Numerically from the ODEs

optimal level of vaccine coverage, as given in Proposition 1, can be further simplified to:

$$f^* = \begin{cases} 0, & \text{if } c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0}(1-\eta(1-\phi)), \\ \frac{R_0-1}{2\phi R_0} + \frac{\phi(2\lambda+1)-2c}{2\phi(1-\eta(1-\phi))}, & \text{if } \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}\phi}{2}(1-\eta(1-\phi)) \le c \le \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0}(1-\eta(1-\phi)), \\ \min\left\{\frac{R_0-1}{\phi R_0}, 1\right\}, & \text{otherwise.} \end{cases}$$

Similarly, in the case of Cournot competition with n identical producers with no subsidy, the total market coverage (from Proposition 2) is given by:

$$f = \begin{cases} 0, & \text{if } c > \frac{R_0-1}{R_0}(1-\eta(1-\phi)), \\ \frac{n}{n+1}\left(\frac{R_0-1}{\phi R_0} - \frac{c}{\phi(1-\eta(1-\phi))}\right), & \text{otherwise.} \end{cases}$$

When a per-vaccine normalized subsidy of s is offered in the above market, the total market coverage in Proposition 4 becomes:

$$f(s) = \begin{cases} 0, & \text{if } c > s + \frac{R_0-1}{R_0}(1-\eta(1-\phi)), \\ \frac{n}{n+1}\left(\frac{R_0-1}{\phi R_0} - \frac{c-s}{\phi(1-\eta(1-\phi))}\right), & \text{if } s - \frac{\bar{F}\phi}{n}(1-\eta(1-\phi)) \le c \le s + \frac{R_0-1}{R_0}(1-\eta(1-\phi)), \\ \min\left\{\frac{R_0-1}{\phi R_0}, 1\right\}, & \text{otherwise} \end{cases}$$

Finally, the optimal subsidy, presented in Theorem 1, that coordinates the market can now be written as:

$$s = \begin{cases} 0, & \text{if } c > \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0}(1-\eta(1-\phi)), \\ \frac{\left(\frac{\phi(n+1)(2\lambda+1)}{2} - \frac{(n-1)(R_0-1)(1-\eta(1-\phi))}{2R_0} - c\right)}{n}, & \text{if } \frac{\phi(2\lambda+1)}{2} - \frac{\bar{F}\phi}{2}(1-\eta(1-\phi)) \le c \le \frac{\phi(2\lambda+1)}{2} + \frac{R_0-1}{2R_0}(1-\eta(1-\phi)), \\ c + \frac{\bar{F}\phi}{n}(1-\eta(1-\phi)), & \text{otherwise.} \end{cases}$$

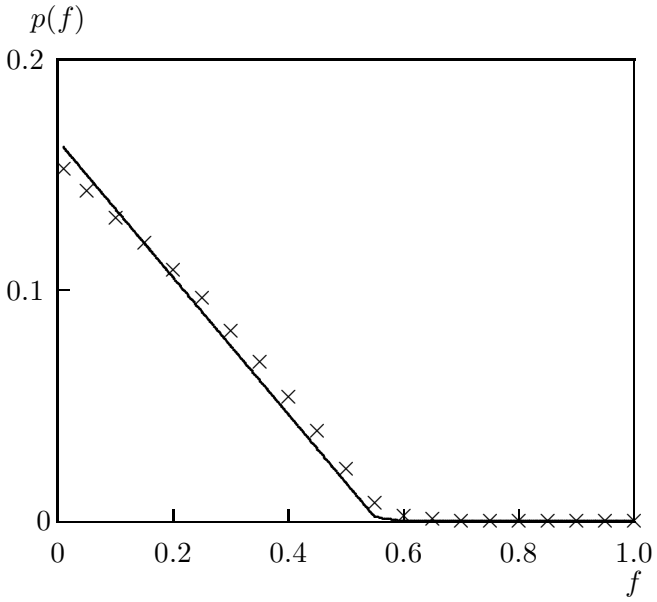

Figure B2: Actual and Approximated Values of $p(f)$

C Proofs

Proof of Proposition 1

Since $0 \leq f \leq F$, $r(f) = 1 - \phi f - \frac{1}{R_0}$. Substituting this into SC and differentiating SC twice with respect to f, we get:

$$\frac{d^2(\text{SC})}{df^2} = -\frac{1}{2}\frac{d^2}{df^2}(f(r(f) - p(f))) \geq 0.$$

The last inequality follows from the fact that $f(r(f) - p(f))$ is a concave function of f. Therefore, SC is a convex function of f over $0 \leq f \leq F$, and we can find the solution to the first-best problem using the first-order optimality condition:

$$c - \frac{\phi(2\lambda + 1)}{2} - \frac{1}{2}\frac{d}{df}\left(f(r(f) - p(f)\right) = 0.$$

Let $f = \tilde{f}$ be the solution of the above first-order condition. Then the socially optimal level of vaccine coverage should be $f^* = \max\{\tilde{f}, 0\}$. Finally, to get the appropriate regions of c, let $y(f) = \frac{1}{2}\frac{d}{df}\left[f\left(r(f) - p(f)\right)\right]$. It is easy to see that, if $c > \frac{\phi(2\lambda+1)}{2} + y(0)$, $\tilde{f} < 0$ and, hence, $f^* = 0$. Also, if $c < \frac{\phi(2\lambda+1)}{2} + y(\bar{F})$, $f^* = \bar{F}$. Between these two extremes, $f^* = \tilde{f}$. In order to complete the proof, we note that $y(0) = \frac{1}{2}\left[1 - \frac{1}{R_0} - p(0)\right]$ and $y(\bar{F}) = -\frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right]$. ■

Proof of Corollary 1

Straightforward from Proposition 1. ■

Proof of Proposition 2

Notice that supplier i's profit function is a decreasing function of f_i when the cumulative vaccination fraction in the population is more than the critical vaccination level (i.e., $f \geq F$) as $\pi_i = -cf_i$. Therefore, we direct our attention to the region where $f \leq F$. Taking the second derivatives of the supplier i's profit with respect

to f_i we get:

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial f_i^2} = \frac{\partial^2}{\partial f_i^2}\left[(r(f) - p(f) - c)f_i\right] &= 2(r'(f) - p'(f)) + (r''(f) - p''(f))f_i \\ &= 2(r'(f) - p'(f)) - p''(f)f_i\end{aligned}$$

The first term is less than or equal to zero because $r(f) - p(f)$ is a non-increasing function of f. Since $r''(f) = 0$, if $p''(f) \geq 0$, then clearly $\frac{\partial^2 \pi_i}{\partial f_i^2} \leq 0$. If, on the other hand, $p''(f) < 0$, we have:

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial f_i^2} &= 2[r'(f) - p'(f)] + f_i(r''(f) - p''(f)) \\ &\leq 2[r'(f) - p'(f)] + f(r''(f) - p''(f)) \\ &= \frac{\partial^2}{\partial f^2}(f(r(f)) - p(f)) \leq 0,\end{aligned}$$

since $f(r(f)) - p(f))$ is a concave function of f. Therefore, π_i is a concave function of f_i as long as $0 \leq f \leq F$. This means that, in equilibrium, the decision of producer i must satisfy the first-order optimality condition:

$$\frac{\partial \pi_i}{\partial f_i} = -c + r(f) - p(f) + f_i\left(r'(f) - p'(f)\right) = 0,$$

which results in:

$$f_i = \frac{c - r(f) + p(f)}{r'(f) - p'(f)}. \tag{C1}$$

Since (C1) is true for any i, it is clear that only a symmetric outcome is possible in this case. Furthermore, concavity of π_i w.r.t. f_i ensures that this outcome is also unique. Viewed differently, the equilibrium solution is symmetric (all f_i's are equal) and unique, and no vendor has any incentive to deviate from this equilibrium. Therefore, the equilibrium solution must satisfy $f = nf_i$, and we have the statement of this proposition. ■

Proof of Corollary 2

Straightforward from Proposition 2. ■

Proof of Proposition 3

In order to prove this proposition, we need the following two results:

- $\phi \geq \frac{R_0 - 1}{R_0} - p(0)$, and
- $f_\infty(r'(f_\infty) - p'(f_\infty)) \geq c - \frac{R_0 - 1}{R_0} + p(0)$.

In order to prove the first result, we make use of the definition of $r(f)$:

$$r'(f) = \frac{d}{df}\left[fp(f) + (1 - f)P(f)\right] = fp'(f) + p(f) - P(f) + (1 - f)P'(f).$$

Now, for $0 \leq f \leq F$, $r'(f) = -\phi$. Therefore, for $f = 0$, we get: $\phi = -p(0) + P(0) - P'(0)$. However, by definition, $P(0) = r(0) = \frac{R_0 - 1}{R_0}$, and $P'(0) \leq 0$, leading to the stated result.

For the second result, we use Proposition 2 to characterize f_∞ as: $r(f_\infty) - p(f_\infty) = c$. Using the Mean Value Theorem for the function $(r(f) - p(f))$, we know that there exists an $x \in [0, f_\infty]$ such that:

$$\frac{[r(f_\infty) - p(f_\infty)] - [r(0) - p(0)]}{f_\infty - 0} = r'(x) - p'(x) \leq r'(f_\infty) - p'(f_\infty),$$

where the inequality is obtained from the convexity of $(r(f) - p(f))$. The proof is complete by replacing $r(f_\infty) - p(f_\infty) = c$.

Now, to prove Proposition 3, i.e., to prove that $f_\infty \leq f^*$, we need to show that $\frac{d\text{SC}}{df} \leq 0$ at $f = f_\infty$. The desired result then follows due to the convexity of the social welfare function. Using the derivative of SC from Proposition 1, we have:

$$\begin{aligned}
\left.\frac{d\text{SC}}{df}\right|_{f=f_\infty} &= c - \frac{\phi(2\lambda+1)}{2} - \frac{1}{2}\frac{d}{df}\left[f(r(f) - p(f))\right]_{f=f_\infty} \\
&= c - \frac{\phi(2\lambda+1)}{2} - \frac{1}{2}\left[f_\infty(r'(f_\infty) - p'(f_\infty)) + r(f_\infty) - p(f_\infty)\right] \\
&= c - \frac{\phi(2\lambda+1)}{2} - \frac{1}{2}\left[f_\infty(r'(f_\infty) - p'(f_\infty)) + c\right] \\
&= \frac{1}{2}\left[c - \phi(2\lambda+1) - f_\infty(r'(f_\infty) - p'(f_\infty))\right] \\
&\leq \frac{1}{2}\left[c - \phi(2\lambda+1) - \left(c - 1 + \frac{1}{R_0} + p(0)\right)\right] \\
&= \frac{1}{2}\left[-\phi(2\lambda+1) + 1 - \frac{1}{R_0} - p(0)\right] \\
&\leq -\phi\lambda \leq 0.
\end{aligned}$$

Note that the two inequalities used above were established at the beginning. ■

Proof of Proposition 4

Once again, we note that the supplier i's profit function is a decreasing function of f_i when the cumulative vaccination fraction in the population is more than the critical vaccination level (i.e., $f \geq F$). Therefore, $0 \leq f \leq F$ is the appropriate range, and within this range:

$$\frac{\partial^2 \pi_i}{\partial f_i^2} = 2[r'(f) - p'(f)] + f_i(r''(f) - p''(f)).$$

The first term is less than or equal to zero because $r(f) - p(f)$ is a non-increasing function of f. Since $r''(f) = 0$, if $p''(f) \geq 0$, then clearly $\frac{\partial^2 \pi_i}{\partial f_i^2} \leq 0$. If, on the other hand, $p''(f) < 0$, we have:

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial f_i^2} &= 2[r'(f) - p'(f)] + f_i(r''(f) - p''(f)) \\
&\leq 2[r'(f) - p'(f)] + f(r''(f) - p''(f)) \\
&= \frac{\partial^2}{\partial f^2}(f(r(f)) - p(f)) \leq 0,
\end{aligned}$$

since $f(r(f)) - p(f))$ is a concave function of f. Therefore, π_i is a concave function of f_i as long as $0 \leq f \leq F$. The optimal solution for producer i can then be found from the first-order optimality condition:

$$\frac{\partial \pi_i}{\partial f_i} = s - c + r(f) - p(f) + f_i\left(r'(f) - p'(f)\right) = 0,$$

which results in:

$$f_i = \frac{c - s - r(f) + p(f)}{r'(f) - p'(f)}. \tag{C2}$$

(C2) is true for any i. Therefore, similar to the proof of Proposition 2, the equilibrium solution is symmetric (all f_i's are equal) and unique, and no vendor has any incentive to deviate from this equilibrium. Therefore, the equilibrium solution must satisfy $f = nf_i$, which leads to the desired result.

In order to identify the regions of solution. let $z(f) = r(f) - p(f) + \frac{f}{n}\left[r'(f) - p'(f)\right]$. It is easy to see that,

if $c > s + z(0)$, $\hat{f} < 0$ and, hence, $f(s) = 0$. Also, if $c < s + z(\bar{F})$, $f(s) = \bar{F}$. Between these two extremes, $f(s) = \hat{f}$, which satisfies Equation (9). The result follows from the observation that $z(0) = 1 - \frac{1}{R_0} - p(0)$ and $z(\bar{F}) = -\frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right]$. ■

Proof of Corollary 3

Straightforward from Proposition 4. ■

Proof of Theorem 1

We examine, the three cases one at a time. First, if $c > \phi\lambda + \frac{\phi+1}{2} - \frac{1}{2R_0} - \frac{p(0)}{2}$, and $s = 0$, we claim that the producers would not produce at all, i.e., $f(s) = f^* = 0$. This would be true if $A - B > 0$, where $A = \phi\lambda + \frac{\phi+1}{2} - \frac{1}{2R_0} - \frac{p(0)}{2}$ and $B = \frac{R_0-1}{R_0} - p(0)$. We note that:

$$A - B = \phi\lambda + \frac{1}{2}\left[\phi + \frac{1}{R_0} - 1 + p(0)\right].$$

Now, if $F \leq 1$, $\phi \geq 1 - \frac{1}{R_0}$ and, hence, $\phi + \frac{1}{R_0} - 1 \geq 0$. On the other hand, if $F > 1$, $p(0) > p(1) = r(1) = 1 - \phi - \frac{1}{R_0}$. In either case, $A - B > 0$.

Next, if $c < \phi\lambda + \frac{1}{2R_0} + \frac{\phi-1}{2} - \frac{R_0-1}{2\phi R_0}p'(\frac{R_0-1}{\phi R_0})$, and $s = c + \frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right]$, from Proposition 4, we can see that $f(s) = f^* = \min\{F, 1\}$. So, the first-best is achieved here as well.

Finally, we consider the case $\phi\left(\lambda + \frac{1}{2}\right) - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right] \leq c \leq \phi\lambda + \frac{\phi+1}{2} - \frac{1}{2R_0} - \frac{p(0)}{2}$. In this case, $s = \frac{\phi(2\lambda+1)+(n-2)c-(n-1)[r(f^*)-p(f^*)]}{n}$. We substitute this s into Equation (9) to obtain:

$$\frac{n-1}{n}\left[r(f) - p(f) - r(f^*) + p(f^*)\right] - \frac{2}{n}\left[c - \frac{\phi(2\lambda+1)}{2} - \frac{1}{2}\frac{d}{df}\left(f(r(f) - p(f))\right)\right] = 0.$$

Clearly, $f = f^*$ is the unique root of the above equation—the first term becomes zero at $f = f^*$, and, from Proposition 1, we can see that the second term vanishes as well at $f = \hat{f} = f^*$.

In order to complete the proof, we need to check the producers' incentive compatibility from Proposition 4, i.e., whether s satisfies:

$$s - \frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right] \leq c \leq s + \frac{R_0 - 1}{R_0} - p(0).$$

We first note that:

$$\begin{aligned} c - \left[s - \frac{\bar{F}}{n}\left[\phi + p'(\bar{F})\right]\right] \\ = \frac{2}{n}\left[c - \left[\phi\left(\lambda + \frac{1}{2}\right) - \frac{\bar{F}}{2}\left[\phi + p'(\bar{F})\right]\right]\right] + \frac{n-1}{n}\left[r(f^*) - p(f^*)\right]. \end{aligned}$$

The first term is non-negative from the lower bound on c and the second term is non-negative because, by definition, $r(f) \geq p(f)$. Next, we find that:

$$\begin{aligned} c - \left[s + \frac{R_0 - 1}{R_0} - p(0)\right] \\ = \frac{2}{n}\left[c - \left(\phi\lambda + \frac{\phi+1}{2} - \frac{1}{2R_0} - \frac{p(0)}{2}\right)\right] + \frac{n-1}{n}\left[(r(f^*) - p(f^*)) - (r(0) - p(0))\right]. \end{aligned}$$

The first term is zero or negative from the upper bound on c and the second term is not positive because $r(f) - p(f)$ is decreasing function of f. ■

Proof of Corollary 4

Straightforward from Theorem 1. ■

Proof of Proposition 5

That (σ, τ) coordinates the market is easy to see because $\sigma + \tau = s$. This combination is also revenue neutral since the total tax collection by the government, net of the total subsidy doled out, is:

$$(1 - f^*)\tau - f^*\sigma = f^*(1 - f^*)s - f^*(1 - f^*)s = 0.$$

This completes the proof. ■