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A Game Theoretical Approach for Hospital Stockpile in Preparation for Pandemics

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Abstract

This paper explores the problem of medical stockpiling at hospitals in preparing for a flu pandemic. Taking into account the uncertain demand that may occur under various possible pandemic scenarios, we consider the problem of determining the stockpile quantity of one critical medical item. We take a game theoretical approach in order to capture any mutual aid agreement that a group of hospitals may have, and its impact on the stockpile decisions made. The existence of a Nash equilibrium will be examined and discussion of how to evaluate it will be presented.

Keywords
Healthcare, stockpile, flu pandemic, game theory.

1. Introduction

A pandemic influenza outbreak presents a threat to healthcare facilities and a challenge to public health officials across the country. Such an event is expected to cause a large surge of patients over a short period of time and therefore potentially overwhelm the healthcare systems. As a consequence, public health and healthcare industries have recently begun putting in place their flu pandemic preparedness planning. These planning efforts include the establishment of flu pandemic response procedure, protocols, and cooperation between government agencies and healthcare providers. Some government sources, such as the World Health Organization (WHO) and the the Department of Health and Human Services (HHS), provide general checklists and recommendations on how to achieve preparedness goals [1]. However, they lack detailed guidelines on the amount of supplies that healthcare providers should maintain in order to reach the desired level of surge capacity should a pandemic flu break out.

This research addresses the issue of hospital stockpile levels in preparing for a flu pandemic. Great uncertainties lie in when and how the flu would spread due to the lack of scientific understanding of flu strain characteristics (such as the attack rate and mortality rate) in the pre-pandemic stage. Therefore, this paper takes an analytical approach in determining the appropriate hospital stockpile levels of certain critical medical items (excluding vaccines) under various flu pandemic scenarios. Specifically, it focuses on group decision making for a community of hospitals. Hospitals commonly have mutual aid relationships (such as a memorandum of understanding, MOU) with one another for emergency situations, and these partnerships may certainly be beneficial in a pandemic scenario. As a consequence, a game theoretical approach is applied to the stockpile problem in order to capture the effect of stockpile sharing between hospitals with mutual aid agreements on stockpile quantities. Specifically, this hospital stockpiling game is a non-cooperative strategic game with perfect information [2].

The organization of this paper is as follows: Section 2 describes the game model of stockpiling for a flu pandemic. Section 3 discusses the existence of a Nash equilibrium. Section 4 provides a numerical study of the game, and discussion of this work is presented in Section 5.

2. Model Description

Consider a group of $n+1$ hospitals as players in a stockpiling game, where each stockpiles in preparation for a pandemic. That is, the strategy or choice of each player is the decision on its own stockpile level. Assume the hospitals have mutual aid agreements in place with which each may provide medical supplies to other participating hospital (upon request) in a medical emergency. For any hospital i, even though it plans on responding to a pandemic, assume there is some probability, ρ_i, that the hospital ends up not responding. We assume that hospitals decide on whether to respond or not independently. In addition, a hospital is assumed to have the capability of providing aid (a fraction of its supplies) to another hospital only when it is not responding to the pandemic. When evaluating hospital i's best response function given other hospitals, $j \neq i$, decisions on stockpile, we will assume that hospital i plans its stockpile assuming that it will respond to the pandemic.

2.1 Data and Decision Variable

Note that only a unique critical medical item is considered in this model. The following are the data:

c_1^i	:	hospital i's ordering cost per unit item from supplier
c_2	:	borrowing cost per unit item from another hospital; for simplicity, assumed to be same for all hospitals
h_i	:	hospital i's holding cost per unit item per unit time
D_i	:	hostpial i's overall demand during a pandemic (in excess of regular demand under normal condition); a random variable with given discrete probability distribution: $Pr(D_i = d_i^l) = q_i^l, \quad l = 1, \ldots, L$, where L is the number of demand scenarios.
ρ_i	:	probability that hospital i responds to the pandemic
α_i	:	fraction of stockpile that hospital i is ready to share if it is not responding to the pandemic
p_i	:	hospital i's penalty cost per unit of unsatisfied demand during the pandemic
T	:	time until the pandemic starts (random variable with assumed mean value)

The decision variable of each hospital is the stockpile level, s_i, it maintains for the critical medical item considered. Note that this model can easily be generalized to a game with continuous demand distribution.

2.2 Best Response Objective Function

In this section, we are considering the best response problem of hospital i given the stockpile decision s_j of other hospitals $j \neq i$. In this problem, hospital i assumes that it will respond to the pandemic. Since each one of the other hospitals $j \neq i$ may or may not respond to a pandemic, (with probability ρ_j and $1-\rho_j$ respectively) independently of other hospitals, there are 2^n possible response outcomes. In a given response outcome, hospitals $i, m_1, \ldots, m_k$ respond (where $\{m_1, \ldots, m_k\}$ is a subset of $\{1, \ldots, n+1\} \setminus \{i\}$, possibly the empty set) and request respectively the random amount $(D_i - s_i)^+, (D_{m_1} - s_{m_1})^+, \ldots, (D_{m_k} - s_{m_k})^+$, to be able to meet their pandemic demand. Hospitals $m_{k+1}, \ldots, m_n$ do not respond and can collectively supply up to $\sum_{j=k+1}^n \alpha_{m_j} s_{m_j}$. Assuming that, according to the MOU, the overall number of supplies available for sharing is distributed in proportion of hospital sizes (denoted as ms; such as the number of beds of a hospital). Let $w^i_{(m_1,\ldots,m_k)}$ be the fraction of the total supply that is available to hospital i when the other responding hospitals are $(m_1, \ldots, m_k)$. In such case, $w^i_{(m_1,\ldots,m_k)} = \frac{ms(i)}{ms(i)+ms(m_1)+\ldots+ms(m_k)}$. Moreover, if all of the amount requested is not available, the penalty cost of p_i per unit of unsatisfied demand (diffence between amount requested by the hospital and the amount available to the hospital) will be imposed.

Let Y_i be the discrete random variable representing the amount available to hospital i. We assume that Y_i and D_i are independent. Since the probability distribution of Y_i is a function of all the stockpiles, it can be expressed as:

$$Pr\Big(Y_i = w^i_{(m_1,\ldots,m_k)} \sum_{j=k+1}^{n} \alpha_{m_j} s_{m_j}\Big) = \prod_{j=1}^{k} \rho_{m_j} \prod_{j=k+1}^{n} (1-\rho_{m_j}), \tag{1}$$

for each of the 2^n outcomes mentioned above. Therefore, the probability distribution of Y_i can be denoted by

$$Pr(Y_i = y_i^t) = r_i^t, \quad t = 1, \ldots, 2^n, \tag{2}$$

Note that $y_i^t \geq 0$ depends on $s_j,\ j \neq i$; we omit to write this dependency explicitly for ease of reading. Therefore, in a pandemic, hospital i borrows $\min\{Y_i, D_i - s_i\}^+$ (at a cost of c_2 per unit) and falls short by $(D_i - s_i - Y_i)^+$ (which incurs a penalty of p_i per unit).

In hospital i's best response problem, the objective is determining the stockpile level that minimizes the expected total cost, assuming given the other hospitals' stockpile levels. In this model, the total cost includes the purchasing cost from a supplier, $c_1^i s_i$ to build the stockpile, the holding cost until the onset of the pandemic, $h_i\ s_i\ T$, the cost of borrowing from other hospitals, $c_2\ \min\{Y_i, D_i - s_i\}^+$, and the penalty cost, $p_i(D_i - s_i - Y_i)^+$ should a shortage occur. Without loss of generality, we assume that the possible stockpile level s_i is bounded from above by a large constant M_i (for example, the storage capacity). Therefore, hospital i's best response to the other hospitals' stockpiles is as follows:

$$\begin{aligned}\pi(s_j, j \neq i) &= \min_{0\leq\ s_i \leq M_i} E[c_1^i s_i + h_i s_i T + c_2 \min\{Y_i, D_i - s_i\}^+ + p_i(D_i - s_i - Y_i)^+] \\ &= \min_{0\leq\ s_i \leq M_i} c_1^i s_i + h_i s_i E[T] + c_2 E[\min\{Y_i, D_i - s_i\}^+] + p_i E[(D_i - s_i - Y_i)^+] \end{aligned} \tag{3}$$

3. Model Analysis

The structure of a strategic game is primarily characterized by each player's best response problem's objective (cost or payoff) function and strategy space. This section identifies the properties of each hospital's objective function in the pandemic stockpile game. By using these properties, the (non)existence of a Nash equilibrium of the game can be verified.

3.1 Best Response Problem Objective Function Properties

The best response problem objective function (3), can be further evaluated by the known distributions of D_i and Y_i (for given $s_j, j \neq i$). First, for fixed $s_j, j \neq i$ let

$$J(s_i) = c_1^i s_i + h_i s_i E[T] + c_2 E[\min\{Y_i, D_i - s_i\}^+] + p_i E[(D_i - s_i - Y_i)^+] \tag{4}$$

The definitions of D_i and Y_i provide the following

$$\min\{Y_i, D_i - s_i\}^+ = \begin{cases} y_i^t & \text{if } y_i^t \leq d_i^l - s_i \\ d_i^l - s_i & \text{if } y_i^t \geq d_i^l - s_i \geq 0 \\ 0 & \text{if } d_i^l - s_i \leq 0 \end{cases} \qquad \text{with probability } r_i^t q_i^l \tag{5}$$

Therefore,

$$E[\min\{Y_i, D_i - s_i\}^+] = \sum_{l,\ t:\ s_i \leq d_i^l - y_i^t} y_i^t r_i^t q_i^l + \sum_{l,\ t: 0\leq\ d_i^l - y_i^t \leq s_i \leq d_i^l} (d_i^l - s_i) r_i^t q_i^l\ , \tag{6}$$

which is a continuous piecewise linear non-increasing function of s_i. The slope, while non-positive, is not necessarily monotonic. Similarly,

$$(D_i - s_i - Y_i)^+ = \begin{cases} d_i^l - s_i - y_i^t & \text{if } d_i^l - s_i - y_i^t \geq 0 \\ 0 & \text{if } d_i^l - s_i - y_i^t \leq 0 \end{cases} \qquad \text{with probability } r_i^t q_i^l\ , \tag{7}$$

which is also a continuous piecewise linear non-increasing function of s_i. Its (non-positive) slope is non-increasing. Therefore, equation (4) can be rewritten as:

$$\begin{aligned} J_i(s_i) &= c_2 \sum_{l,\ t:\ s_i \leq d_i^l - y_i^t} y_i^t r_i^t q_i^l\ +\ c_2 \sum_{l,\ t:\ 0\leq\ d_i^l - y_i^t \leq s_i \leq d_i^l} (d_i^l - s_i) r_i^t q_i^l\ +\ p_i \sum_{l,\ t:\ s_i \leq d_i^l - y_i^t} (d_i^l - s_i - y_i^t) r_i^t q_i^l \\ &\quad + c_1^i s_i\ +\ h_i s_i E[T] \\ &= \sum_{l,\ t:\ s_i \leq d_i^l - y_i^t} \big(c_2 y_i^t + p_i(d_i^l - s_i - y_i^t)\big) r_i^t q_i^l\ +\ c_2 \sum_{l,\ t:\ 0\leq\ d_i^l - y_i^t \leq s_i \leq d_i^l} (d_i^l - s_i) r_i^t q_i^l \\ &\quad +\ c_1^i s_i\ +\ h_i s_i E[T] \end{aligned} \tag{8}$$

Therefore, the resulting objective function is a continuous piecewise linear function of s_i. The slope at $s_i \geq 0,\ \ s_i \notin$

$\{d_i^l - y_i^t,\ \ l = 1,\dots,L,\ t = 1,\dots,2^n\} \cup \{d_i^l,\ \ l = 1,\dots,L\}$ (which are points of discontinuity of the slope) is:

$$\alpha_i(s_i) = c_1^i + h_i E[T] - c_2 \sum_{l,\ t:\ 0 \le\ d_i^l - y_i^t < s_i < d_i^l} r_i^t q_i^l - p_i \sum_{l,\ t:\ s_i < d_i^l - y_i^t} r_i^t q_i^l \tag{9}$$

In addition, since it is reasonable to assume that the penalty per cost per unit of unsatisfied demand, p_i, is greater than the cost of borrowing one unit of supply from another hospital, i.e., $p_i > c_2$, $J_i(\cdot)$ is a convex function. It can be shown by examining its slope being, by definition, constant on every linear piece and increasing at every slope discontinuity point.

3.2 Solution of the Best Response Problem

Since $J_i(\cdot)$ is shown to be piecewise linear and convex, and since it has a positive slope $c_1^I + h_i E[T]$ for large s_i, its minimum value on $[0, M_i]$ depends on the sign of its slope at 0. If the slope at zero is non-negative, then 0 is a minimum. Otherwise, the minimum is reached when the slope changes sign. Thus the best response, $s_i^* = \text{Arg } \min_{0 \le\ s_i \le M_i} J_i(s_i)$, for hospital i can be defined as:

$$s_i^* = \begin{cases} 0 & \text{if } c_1^i + h_i E[T] - c_2 \sum_{l,t:\ d_i^l - y_i^t < 0} r_i^t q_i^l - p_i \sum_{l,t:\ d_i^l - y_i^t > 0} r_i^t q_i^l \ge 0 \\ \bar{s}_i & \text{else} \end{cases} \tag{10}$$

where $\bar{s}_i$ satisfies $\alpha_i(\bar{s}_i^{\,-}) < 0$ and $\alpha_i(\bar{s}_i^{\,+}) \ge 0$.

3.3 Existence of a Nash Equilibrium

A Nash equilibrium (NE) of a game is the set of all players' decision such that none of them has any incentive to deviate from his decision. In other words, each player plays his best response strategy to other players' strategies. Since each players acts in his own best interest, it is a common assumption in the economics and operations research literature that the outcome of the game will be a Nash equilibrium.

In this hospital stockpiling game: (1) the strategy set of hospital i's best response problem is the non-empty compact convex set $[0,\ M_i]$; (2) the cost function of hospital i's best response problem is continuous and convex, as shown in section 3.1. Therefore, based on Debreu's Theorem [3], there exists at least one pure strategy NE in the game.

4. Example

In order to illustrate the mathematical properties and concepts of such a game, a simple, two-hospital stockpiling game is set up and shown in this section.

4.1 Illustration of Objective Function

First illustration is the objective function. In this two-hospital game, hospital 1's and 2's probabilities of responding to pandemic, ρ_1 and ρ_2, are both set to 0.7. Consequently, $r_1^1 = \rho_2 = 0.7$, and $r_1^2 = 1 - \rho_2 = 0.3$. In addition, since we only consider two hospitals, $w_{\{\emptyset\}}^1 = \frac{0.5}{0.5} = 1$, and $w_{\{hospital\ 2\}}^1 = \frac{0.5}{0.5+0.5} = 0.5$ by assuming each hospital has equal size of market share. The available amount of sharing to hospital 1, denoted by y_1^1, when hospital 2 is not responding to the pandemic is $\alpha_2 \cdot s_2$; while y_1^2 is 0 in the case when hospital 2 is also responding. We set $\alpha_1 = 0.8$ and $\alpha_2 = 0.9$.

As expressed analytically in the previous section, each hospital's best response objective function, i.e. the overall cost function, is piecewise linear and convex. Figure 1 shows these properties. The slope of the objective function increases as s_1 increases, and it turns from negative to positive at $s_1 = 570,\ 450,\ 400$ where s_2 = 350, 500, and 870 respectively. It means that, in this game set-up, the best response stockpile level for hospital 1, s_1^*, is 570 units when hospital 2 stockpiles 350, and so forth. These best response points in turn give hospital 1 the lowest expected cost considering all demand scenarios. We observe that as hospital 2 chooses a higher stockpile level, s_2, hospital 1's best response, s_1^*, decreases, because more supply is expected to be available for hospital 1 to borrow.

4.2 Illustration of Nash Equilibrium

As mentioned earlier, a NE exists when each of the players in the game selects his best response strategy. In a two-player game such as this example, a NE lies where the two best response functions intersect. In Figure 2, the solid line is the best response, s_1^*, as a function of s_2, while the dotted line is the best response, s_2^*, as a function of s_1. It clearly

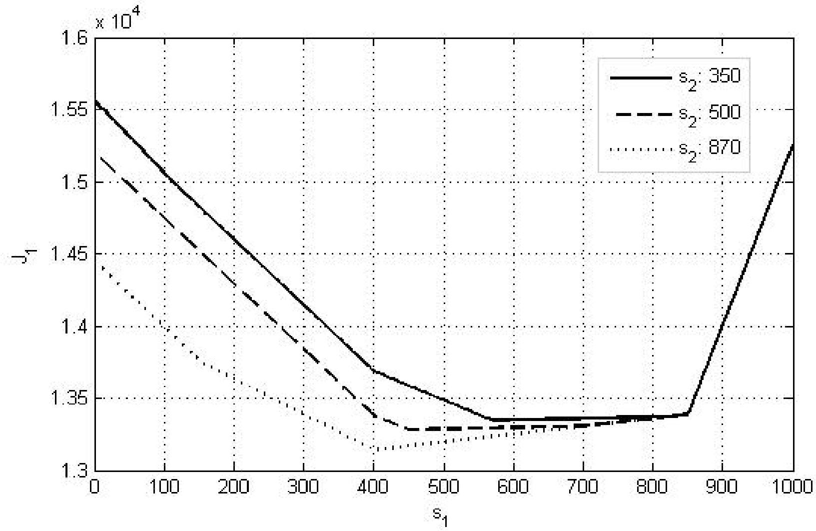

Figure 1: Data used in the example: $c_1^1 = 5$; $c_2 = 10$; $p_1 = 20$; $h_1 = 1$; $E[T] = 10$; $L = 3$; $q_1 =$ [0.15 0.7 0.15]; $d_1 =$ [400 850 1100].

shows that these two lines intersect at at least one point.

A closer look at the plot reveals that the two best response functions intersect at one point. As shown in Figure 3, the NE in this two-hospital stockpiling game is at (s_1^*, s_2^*)=(580, 300).

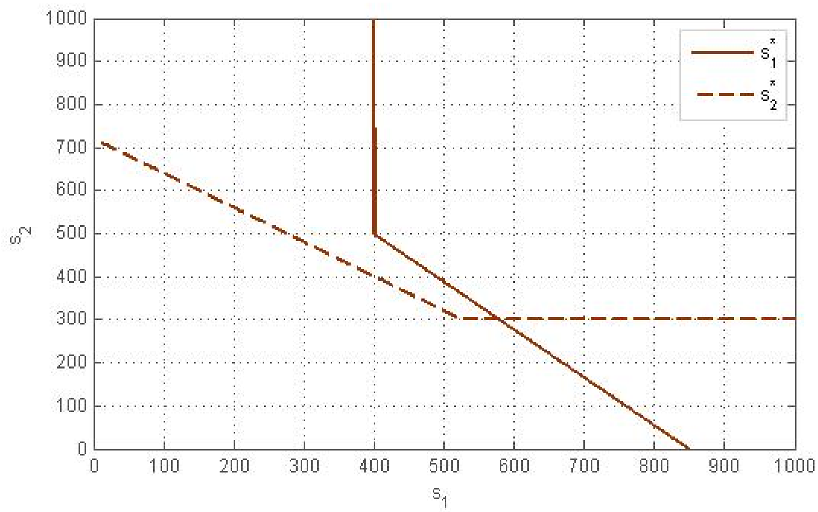

Figure 2: Best response functions. Data used in the example: $c_1^1 = 5$; $c_1^2 = 6$; $c_2 = 10$; $p_1 = p_2 = 20$; $h_1 = h_2 = 1$; $E[T] = 10$; $r_1 = [0.7\ 0.3]$; $r_2 = [0.7\ 0.3]$; $L = 3$; $q_1 = q_2 = [0.15\ 0.7\ 0.15]$; $d_1 = [400\ 850\ 1100]$; $d_2 = [300\ 720\ 980]$.

5. Conclusion

In this research, a game theoretical approach is used to study a group of hospitals' stockpiling of critical medical supplies in preparation for a flu pandemic. A hospital's decision on how much to stockpile one item depends not only on the uncertainties associated with the flu pandemic that may break out, but also on its mutual aid relationship with other partner hospitals. The analytical results of this game model show the existence of a Nash equilibrium, and

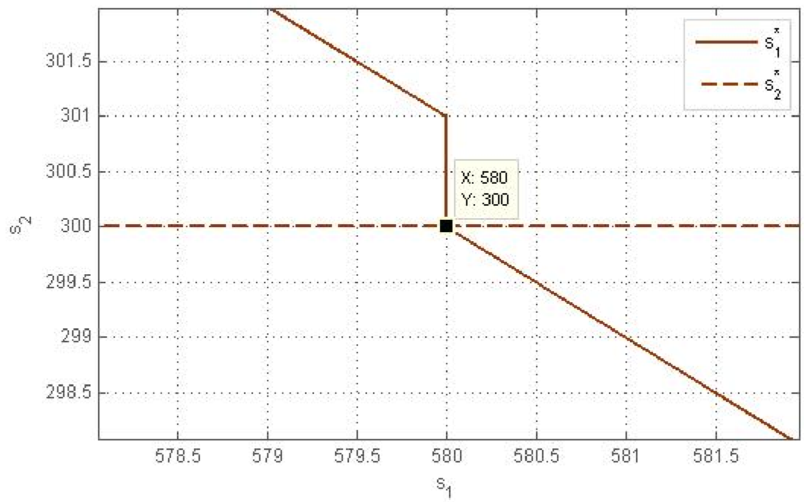

Figure 3: The intersection points of the best response functions.

the two-hospital numerical example verifies it. Further analytical investigation on the uniqueness of the NE is indeed necessary and presently ongoing.

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