A The Real Options Model

Johnson (2004) and Barinov (2011) develop and successfully test two related models that predict and explain the negative relation between idiosyncratic risk and expected returns. The main idea of these models is that higher idiosyncratic risk makes the systematic risk of real options lower. In the Johnson setup, the natural empirical proxy for idiosyncratic risk is the dispersion of analyst forecasts, which measures analyst disagreement. In the Barinov model, the empirical proxy for idiosyncratic risk is idiosyncratic volatility. In this paper, I show that high turnover variability firms have both higher disagreement and higher idiosyncratic volatility (see Table 1).

In this appendix, I use the Johnson setup, but the model can be easily restated in terms of idiosyncratic volatility. I extend the Johnson model by showing that high idiosyncratic risk means lower aggregate volatility risk (as Barinov, 2011, also does in his setup) and by considering growth options in addition to limited liability.

Consider a firm with unobservable true value of assets C_t that has issued risky debt with the face value K. Assume that the true value of assets follows

$$dC_t = \mu_C C_t dt + \sigma_C C_t dW_C \tag{1}$$

Investors cannot observe the process C_t (the true value of assets) and observe U_t instead, which is C_t contaminated by a stationary noise process η_t. U_t is given by

$$U_t = C_t \cdot \exp(\eta_t) \tag{2}$$

The noise process η_t is an unobservable stationary diffusion process

$$d\eta_t = -\kappa \eta_t dt + \sigma_\eta dW_\eta \tag{3}$$

In addition to U_t, investors observe the stochastic discount factor Λ_t that follows

$$d\Lambda_t = -r\Lambda_t dt + \sigma_\Lambda \Lambda_t dW_\Lambda \tag{4}$$

Johnson (2004) shows that in this economy S_t, the observable price of unlevered claim on C_T (firm's assets)[1], and V_t, the observed value of equity, follow

$$dS_t = (r + \pi_S) S_t dt + \sigma_C S_t d\widetilde{W}_C \tag{5}$$

[1] Note that C_t is the true (unobservable) value of of the underlying asset, and S_t is the observable value of the underlying asset, which moves according to the information about the underlying asset investors are able to filter out of the price and the economy structure.

$$V_t = S_t\Phi(d_1) - \exp(r(T-t))K\Phi(d_2) \tag{6}$$

where $\pi_S = -\rho_{C\Lambda}\sigma_C\sigma_\Lambda$ is the risk premium, $\Phi(\cdot)$ is the normal cdf,

$$d_1 = \frac{log(S/K) + (r(T-t) + \widetilde{\sigma}^2/2)}{\widetilde{\sigma}},\ d_2 = d_1 - \widetilde{\sigma} \tag{7}$$

$$\widetilde{\sigma}^2 = \omega + \sigma_C^2(T-t) \tag{8}$$

$d\widetilde{W}_C$ is the posterior belief of investors about the process governing C_t given the signals they have received, and ω reflects idiosyncratic risk. Johnson (2004) also shows that

$$d\widetilde{W}_C = F(X)(\sigma_C dW_C + \sigma_\eta dW_\eta) + G(X)dW_\Lambda, \tag{9}$$

where $F(X)$ and $G(X)$ are some functions of the model primitives κ, σ_V, σ_C, σ_Λ, $\rho_{C\Lambda}$. (Explicit expressions of $F(X)$ and $G(X)$ are given in Johnson, 2004).

Proposition 1. The risk premium and the CAPM beta of the firm equal

$$\pi_V = \pi_S \cdot \frac{\partial V}{\partial S}\cdot\frac{S}{V};\quad \beta_V = \beta_S\cdot\frac{\partial V}{\partial S}\cdot\frac{S}{V} \tag{10}$$

and its derivatives with respect to idiosyncratic risk and the assets value have the following signs:

$$\frac{\partial \pi_V}{\partial\omega} = \pi_S\cdot\frac{\partial}{\partial\omega}\Big(\frac{\partial V}{\partial S}\cdot\frac{S}{V}\Big) < 0;\quad \frac{\partial\beta_V}{\partial\omega} = \beta_S\cdot\frac{\partial}{\partial\omega}\Big(\frac{\partial V}{\partial S}\cdot\frac{S}{V}\Big) < 0 \tag{11}$$

$$\frac{\partial^2\pi_V}{\partial\omega\partial S} = \pi_S\cdot\frac{\partial^2}{\partial\omega\partial S}\Big(\frac{\partial V}{\partial\quad S}\cdot\frac{S}{V}\Big) > 0 \tag{12}$$

Proof: See the web appendix at http://people.terry.uga.edu/abarinov/Theory.pdf.

The sign of (11) implies that idiosyncratic risk is negatively related to systematic risk and expected returns, which is the main result established in Johnson (2004). The intuition for the sign is that the elasticity of the call option with respect to the underlying asset value, $\Phi(d_1)S_t/V_t$, decreases in the disagreement about the underlying asset, because more disagreement about the underlying asset means that its current value is less informative about the value of the option at the expiration date. Therefore, the current value of the option responds less to the same percentage change in the current value of the underlying asset if there is more disagreement about its true value.

Johnson (2004) also notices that the effect of idiosyncratic risk on the firm's expected returns should be stronger for highly levered firms. The intuition is that disagreement derives its pricing impact from the fact that a levered firm is a call option on the assets, and therefore volatility should matter more for more option-like firms. In algebraic terms it means that as the assets value increases (or as the face value of the debt decreases, or, equivalently, as the firm becomes less distressed), the first derivative (11) becomes less negative, i.e. (12) is positive. Evidently, as the face value of the debt reaches zero, (11) also reaches zero.

If high turnover variability firms have high idiosyncratic risk, one can formulate the following empirical hypothesis:

Hypothesis 1 *High turnover variability firms have low expected returns. The expected returns of high turnover variability firms are especially low if these firms are distressed.*

The main limitation of the Johnson model is that it operates in the CAPM world and therefore cannot produce CAPM alphas. In the rest of this subsection I extend the Johnson (2004) model to show that the expected returns effects Johnson finds (Proposition 1 above) can arise in the CAPM alphas because higher idiosyncratic risk means lower aggregate volatility risk.

The negative correlation between idiosyncratic risk and the exposure of the firm's equity to systematic risk (Proposition 1) is useful during periods of high aggregate volatility. These periods usually coincide with the periods of high idiosyncratic risk (see Barinov, 2011, 2013, and references therein). The next proposition shows that, all else equal, the increased idiosyncratic risk makes the risk premium of option-like firms with high idiosyncratic risk increase less. The smaller increase in the expected risk premium makes the value of these firms drop less during the periods of aggregate volatility.

Proposition 2 The elasticity of the equity risk premium decreases (increases in the absolute magnitude) as idiosyncratic risk increases:

$$\frac{\partial}{\partial \omega}\Big(\frac{\partial \pi_V}{\partial \omega} \cdot \frac{\omega}{\pi_V}\Big) < 0 \tag{13}$$

The second cross-derivative of the elasticity with respect to idiosyncratic risk and the assets value is positive:

$$\frac{\partial^2}{\partial \omega \partial S}\Big(\frac{\partial \pi_V}{\partial \omega} \cdot \frac{\omega}{\pi_V}\Big) > 0 \tag{14}$$

Proof: See the web appendix at http://people.terry.uga.edu/abarinov/Theory.pdf.

As Campbell (1993) and Chen (2002) show, investors require lower risk premium from the stocks that react less negatively to aggregate volatility increases. Hence, Proposition 2 implies that the firm's exposure to aggregate volatility risk decreases with idiosyncratic risk. Because idiosyncratic risk is transformed into lower aggregate volatility risk through the real option created by limited liability, this effect is stronger for distressed firms.

Another effect that ties real options and aggregate volatility risk comes from the fact that, all else equal, higher idiosyncratic risk means higher value of the real option[2]. When both aggregate volatility and idiosyncratic risk increase, this effect makes the value of the real option created by limited liability increase in value (holding other effects fixed). In Proposition 3, I show that, holding constant all other (usually negative) cash flow effects of the aggregate volatility increase, the positive effect of idiosyncratic risk on the real option value is larger for high idiosyncratic risk firms, especially if they are also option-like (distressed).

Proposition 3 The elasticity of the equity value with respect to idiosyncratic risk increases with idiosyncratic risk:

$$\frac{\partial}{\partial\omega}\Big(\frac{\partial V}{\partial\omega}\cdot\frac{\omega}{V}\Big) > 0 \tag{15}$$

The second cross-derivative of the elasticity with respect to idiosyncratic risk and the assets value is negative:

$$\frac{\partial^2}{\partial\omega\partial S}\Big(\frac{\partial V}{\partial\omega}\cdot\frac{\omega}{V}\Big) < 0 \tag{16}$$

Proof: See the web appendix at http://people.terry.uga.edu/abarinov/Theory.pdf.

I define the aggregate volatility factor (the FVIX factor) as the portfolio that tracks changes in expected aggregate volatility. The positive exposure to aggregate volatility factor is then desirable, because it means (relative) gains in response to aggregate volatility increases. If turnover variability proxies for idiosyncratic risk, we can formulate the following empirical hypothesis:

Hypothesis 2 *High turnover variability firms have positive FVIX betas, and the FVIX betas are even more positive if high turnover variability firms are distressed.*

I would like to stress that Propositions 2 and 3 are formulated in terms of partial derivatives. It is beyond doubt that, as almost all risky assets, real options lose value

[2] A recent paper by Grullon, Lyandres, and Zhdanov (2012) presents supporting empirical evidence.

when the market goes down and aggregate volatility increases. During these periods, the risk premium of real options also increases. Moreover, since a real option is a levered claim on the underlying asset, the negative reaction of a real option to an increase in aggregate volatility can be stronger than average. What Propositions 2 and 3 state is that all else equal, firms with abundant real options and high idiosyncratic risk react to aggregate volatility increases less negatively than other firms. That is, firms with abundant real options and high idiosyncratic risk most likely have high market betas and, because changes in aggregate volatility and the market return are strongly negatively correlated, their reaction to aggregate volatility increases is very negative, but it is significantly less negative than the reaction of other firms with the same market beta.

If high turnover variability firms have high idiosyncratic risk, one can formulate the following empirical hypothesis:

Hypothesis 2a *When aggregate volatility increases, high turnover variability firms lose value, but beat the CAPM. This effect is stronger for high turnover variability firms that are distressed*

The results for the option created by limited liability can be easily generalized to growth options, following Barinov (2011). To do that, one needs to consider an all-equity firm that consists of growth options, P_t, and assets in place, B_t and assume that the value of the asset behind growth options is unobservable, just as I assumed before that the value of assets is unobservable. The web appendix analyzes this situation (including the combination of limited liability and growth options in one model) in more detail and comes up with the following empirical hypotheses:

Hypothesis 3 *The expected returns of high turnover variability firms are especially low if these firms have high market-to-book.*

Hypothesis 4 *The FVIX betas of high turnover variability firms are the most positive if the high turnover variability firms also have high market-to-book.*

Hypothesis 4a *When aggregate volatility increases, high turnover variability firms lose value, but beat the CAPM. This effect is stronger for growth firms.*