Theory Appendix to

"Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns"

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Abstract

This document contains the formal derivation of the aggregate volatility story in "Idiosyncratic Volatility, Growth Options,and the Cross-Section of Returns" and shows why aggregate volatility risk should explain the value effect and the idiosyncratic volatility discount. The aggregate volatility story suggests high idiosyncratic volatility firms, growth firms, and especially high idiosyncratic volatility growth firms beat the CAPM in the periods of increasing aggregate volatility. Section 1 of this document presents the setup of the model and derives the main predictions. Section 2 collects the proof of the propositions in Section 1. Section 3 presents simulations to back up the proofs and to evaluate the magnitude of the effects in the model.

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1 The Model

1.1 Cross-Sectional Effects

Consider a firm that consists of growth options, P_t, and assets in place, B_t. The growth options are represented by a European call option, which gives the right to receive at time T S_T for price K. Both S_t, the price of the asset underlying the growth options, and B_t follow geometric Brownian motions:

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_S + \sigma_I S_t dW_I \tag{1}$$

$$dB_t = \mu_B B_t dt + \sigma_B B_t dW_B \tag{2}$$

The stochastic discount factor process is given by

$$d\Lambda_t = -r\Lambda_t dt + \sigma_\Lambda \Lambda_t dW_\Lambda \tag{3}$$

dW_I is the purely idiosyncratic component of S_t and is assumed to be uncorrelated with the pricing kernel and, for simplicity, with W_S and W_B, though relaxing the second assumption will not change the results. I also assume for simplicity that there is no purely idiosyncratic component in B_t (relaxing this assumption also does not change anything).

dW_I represents firm-specific shocks to growth options value. While the part of dW_S that is orthogonal to the pricing kernel is also firm-specific, I need dW_I to be able to increase the variance of the firm-specific shocks without increasing the covariance of S_t with the pricing kernel.

I do not assume anything about the correlation between W_S and W_B. The underlying asset of growth options and assets in place in my model are driven by two different processes, but these processes can be highly correlated.

The no-arbitrage condition and the definition of the pricing kernel imply that

$$dB_t = (r + \pi_B)B_t dt + \sigma_B B_t dW_B \tag{4}$$

$$dS_t = (r + \pi_S)S_t dt + \sigma_S S_t dW_S + \sigma_I S_t dW_I \tag{5}$$

where $\pi_B = -\rho_{B\Lambda}\sigma_B\sigma_\Lambda$ and $\pi_S = -\rho_{S\Lambda}\sigma_S\sigma_\Lambda$ are the risk premiums. The idiosyncratic risk is not priced for the unlevered claim on the asset behind growth options and it will not be priced for assets in place if I assume that they also carry some purely idiosyncratic risk.

However, for growth options the idiosyncratic risk is priced:

Proposition 1. The value of the firm is given by

$$dV_t/V_t = \mu_V dt + \Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S + \sigma_I dW_I) + \sigma_B \frac{B_t}{V_t} dW_B \tag{6}$$

$$\text{where} \quad \mu_V = r + \pi_B - (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{P_t}{V_t} \tag{7}$$

$$d_1 = \frac{log(S/K) + (r + \sigma_S^2/2 + \sigma_I^2/2)(T-t)}{\sqrt{(\sigma_S^2 + \sigma_I^2)\cdot(T-t)}} \tag{8}$$

If assets in place are riskier than growth options, $\pi_B - \pi_S \Phi(d_1) S_t/P_t > 0$, then the expected rate of return to the firm (the drift in the firm value, μ_V) decreases in idiosyncratic risk, σ_I, and increases in the value of assets in place, B.

Proof: See Section 2.

The intuition of the proof is that the idiosyncratic risk discount consists of two parts and relies on the existence of the value effect. First, an increase in idiosyncratic risk reduces the expected return by reducing elasticity of the growth options value with respect to the underlying asset value ($\Phi(d_1)S_t/P_t$). Second, an increase in idiosyncratic risk increases the relative value of growth options (P_t/V_t) and makes the firm more growth-like, which decreases expected returns if the value effect exists[1]

By definition, the beta of the option is determined by, first, how responsive the underlying asset is to a percentage change in the risk factor and, second, how responsive the price of the option is to a percentage change in the price of the underlying asset. Hence, the beta of the option is equal to the product of the elasticity and the beta of the underlying asset. The elasticity decreases as volatility increases because if volatility is high, a change in the underlying asset price is less informative about its value at the expiration date. When idiosyncratic volatility goes up, the elasticity declines and the beta of the underlying asset stays constant, hence their product - the beta of growth options - decreases.

[1]The condition that assets in place are riskier than growth options is sufficient (but not necessary) for the existence of the idiosyncratic volatility discount in my model. Zhang (2005) argues that assets in place are riskier in recessions because of costly divesture. Campbell and Vuolteenaho (2004) that shows that value firms have higher cash flow betas and growth firms have low cash flow betas, and the cash flow risk earns a much higher risk premium. Barinov (2010) and this appendix (Proposition 4) show that growth firms can be less risky than value firms because growth firms beat the CAPM when aggregate volatility increases.

The idiosyncratic risk in my model is idiosyncratic at the level of the underlying assets, but its presence changes the systematic risk of growth options. If one pools the underlying assets, the risk will be diversified away, and this is the reason it is not priced for the unlevered claim on any of them. However, if one pools the underlying assets and then creates an option on them, the decrease in the idiosyncratic volatility will lead to the systematic risk of the option being greater than the systematic risk of the portfolio of separate options on each of the underlying assets.

Corollary 1. Define $IVar$ as the variance of the part of the return generating process (6), which is orthogonal to the pricing kernel. Then the idiosyncratic variance $IVar$ is

$$\begin{aligned} IVar = \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1-\rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1-\rho_{B\Lambda}^2) + \\ + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \end{aligned} \tag{9}$$

I show that for all reasonable parameter values σ_I

$$\frac{\partial IVar}{\partial \sigma_I} > 0, \tag{10}$$

which implies that my empirical measure of idiosyncratic volatility - the standard deviation of Fama-French model residuals - is a noisy but valid proxy for σ_I.

Proof: See Section 3.3.

Corollary 1 shows that the idiosyncratic volatility depends positively on the idiosyncratic risk parameter. It is also impacted by some other factors, which means that it is a valid, although noisy, proxy for the idiosyncratic risk parameter. I do not claim that idiosyncratic volatility is the best proxy for idiosyncratic risk. All I need to tie my model to the data is that it is positively correlated with idiosyncratic risk, and Corollary 1 shows that it should be true.

Leaning on Corollary 1, in the rest of the section I use the terms "idiosyncratic volatility" and "idiosyncratic risk" interchangeably.

Corollary 2. The expected return differential between assets in place and growth options, $\pi_B - \pi_S \Phi(d_1) S_t/P_t$, is increasing in idiosyncratic risk.

Proof: Follows from the well-known fact that the option price elasticity with respect to the price of the underlying asset, $\Phi(d_1) S_t/P_t$, is decreasing in volatility.

Corollary 2 suggests a simple reason why in the rational world the value effect is higher for high volatility firms, as Ali et al. (2003) show. High volatility reduces the expected returns to growth options by reducing their elasticity with respect to the value of the underlying asset (and therefore reducing their beta) and leaves assets in place unaffected.

Corollary 2 implies that the observed value effect can wholly be an idiosyncratic volatility phenomenon. The return differential between growth options and assets in place can take different signs at different levels of idiosyncratic volatility. If the value effect is actually negative at zero idiosyncratic volatility, and positive at the majority of its empirically plausible values, the value effect will be on average positive even though growth options are inherently (absent idiosyncratic volatility) riskier than assets in place. In this case, the observed part of the value effect will be created only by the interaction between idiosyncratic volatility and growth options captured by my model.

Proposition 2. The effect of idiosyncratic volatility on returns, $\left|\frac{\partial \mu_V}{\partial \sigma_I}\right|$, is decreasing in the value of assets in place, B.

Proof: See Section 2.

The main idea behind Proposition 2 is that without growth options or with very large B_t idiosyncratic volatility will not have any impact on returns. As growth options take a greater fraction of the firm, the impact of idiosyncratic volatility on returns becomes stronger, since it works through growth options. Also, more idiosyncratic volatility makes growth options less risky, while the risk of assets in place stays constant. It means a wider expected return spread between growth options and assets in place. The positive cross-derivative captures both effects.

Proposition 2 implies that the observed value effect can wholly be an idiosyncratic volatility phenomenon. The return differential between growth options and assets in place can take different signs at different levels of idiosyncratic volatility. If the value effect is actually negative at zero idiosyncratic volatility, and positive at the majority of its empirically plausible values, the value effect will be on average positive even though growth options are inherently (absent idiosyncratic volatility) riskier than assets in place. In this case, the observed part of the value effect will be created only by the interaction between idiosyncratic volatility and growth options captured by my model.

The sign of the excess return derivative in Proposition 2 implies that in the cross-sectional regression the product of market-to-book and volatility is negatively related to future returns. In portfolio sorts Proposition 2 predicts large and significant idiosyncratic volatility discount for growth firms and no idiosyncratic volatility discount for value firms. Proposition 2 also predicts stronger value effect for high volatility firms.

Hypothesis 1. The cross-sectional regression implied by my model is

$$Ret \approx a - b \cdot M/B + c \cdot (M/B)_0 \cdot IVol - c \cdot M/B \cdot IVol + \delta Z,\ a,\ c > 0 \quad (11)$$

where $(M/B)_0$ is the market-to-book ratio for the firm with no growth options and Z are other priced characteristics.

It implies that

$$\frac{\partial Ret}{\partial M/B} \approx -b - c \cdot IVol < 0 \quad (12)$$

$$\frac{\partial Ret}{\partial IVol} \approx -c \cdot (M/B - (M/B)_0) < 0 \quad (13)$$

I predict that in cross-sectional regressions the coefficient of idiosyncratic volatility, $c \cdot (M/B)_0$, is positive. The coefficient of the volatility product with market-to-book, c, is negative. The ratio of the coefficients equals to $(M/B)_0$, the market-to-book of the firm with no growth options. For the firm with no growth options, as (13) shows, the two terms cancel out and idiosyncratic volatility has no impact on returns. While the lowest possible market-to-book is 1 in my model, in Hypothesis 1 I replace 1 with an unknown $(M/B)_0$. $(M/B)_0$ is likely to be lower than 1, because book values lag market values and losses in the market value may be unrecognized in the book value for some time.

Equation (11) divides the observed value effect into two parts. The first one is denoted by b and represents the part of the value effect, which is unrelated to idiosyncratic volatility and comes from the difference in expected returns to assets in place and growth options absent idiosyncratic volatility. The second one is denoted $c \cdot IVol$ and represents the part of the value effect, which is driven by the interaction between growth options and idiosyncratic volatility. My model makes no prediction about the magnitude of the first part and even its sign.

The theoretical results in this section rely on the fact that growth options are call options on the projects behind them. In theory, any option-like dimension of the firm can be used to generate similar results, i.e. the idiosyncratic volatility discount that increases as the firm becomes more option-like. One well-known option-like dimension of the firm is leverage, which can replace growth options in the discussion above.

Empirically, market-to-book and leverage are strongly inversely related. One reason is the mechanical correlation created by the market value being in the numerator of market-to-book and in the denominator of leverage. There are also several corporate finance theories predicting that growth firms should choose lower leverage (e.g., the free cash flow problem). Hence, in empirical tests the possible link between the idiosyncratic volatility discount and leverage should work against finding any relation between the idiosyncratic volatility discount and market-to-book.

1.2 The Idiosyncratic Volatility Hedging Channel

In the previous subsection I developed predictions about the impact of idiosyncratic volatility on the cross-section of returns. I derived from my model the three idiosyncratic volatility effects: the idiosyncratic volatility discount, the stronger idiosyncratic volatility discount for growth firms, and the higher value effect for high volatility firms. In this subsection, I sketch the ICAPM-type explanation of why the link between idiosyncratic volatility and expected returns cannot be captured by one-period models.

Campbell (1993) develops a model of aggregate volatility risk, where aggregate volatility increase means higher future risk premium. In Campbell (1993) the assets that react less negatively to aggregate volatility increases, offer an important hedge against adverse business-cycle shocks. These stocks earn a lower risk premium, because they provide consumption when future investment opportunities become worse.

Chen (2002) develops a model offering another reason why the assets that react less negatively to aggregate volatility increases can be valuable. In his model, investor care not only about future investment opportunities, but also about future volatility. An increase in expected aggregate volatility means the need to reduce current consumption in order to build up precautionary savings. The stocks that do not go down as aggregate volatility goes up provide consumption when it is most needed and therefore earn a lower risk premium.

My model goes further by predicting what types of firms will have the lowest, probably negative, aggregate volatility risk. I show that the presence of idiosyncratic volatility and its close time-series correlation with aggregate volatility[2] creates the economy-wide

[2]See Campbell, Lettau, Malkiel, and Xu, 2001, and Goyal and Santa-Clara, 2003

idiosyncratic volatility hedging channel that consists of two parts. One part comes from the impact of idiosyncratic volatility on expected returns, and the other comes from the impact of idiosyncratic volatility on the value of growth options. This subsection shows that the idiosyncratic volatility hedging channel makes the prices of high volatility, growth, and high volatility growth firms covary least negatively with aggregate volatility, which means lower exposure to aggregate volatility risk.

In unreported findings I show that the idiosyncratic volatility of low and high volatility firms respond to aggregate volatility movements by changing by the same percentage rather than by the same amount. Therefore, the key variable in the time-series dimension is the elasticity of risk premium with respect to volatility, instead of the derivative, which was the focus of the cross-sectional analysis in the previous subsection.

Proposition 3 The elasticity of the risk premium in my model decreases (increases in the absolute magnitude) as idiosyncratic volatility increases:

$$\frac{\partial}{\partial \sigma_I}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) < 0 \tag{14}$$

The elasticity of the risk premium in my model increases (decreases in the absolute magnitude) as the value of assets in place increases:

$$\frac{\partial}{\partial B}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) > 0 \tag{15}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is positive:

$$\frac{\partial^2}{\partial \sigma_I \partial B}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) > 0 \tag{16}$$

Proof: See Section 2.

Proposition 3 summarizes the first part of the idiosyncratic volatility hedging channel. As aggregate volatility increases, the future risk premium and idiosyncratic volatility also increase. The previous subsection shows that, all else equal, high idiosyncratic volatility means lower risk and lower expected returns. By Proposition 3, when the economy slides into recession and idiosyncratic volatility of all firms increases, the future risk premium of high volatility firms goes up less than the future risk premium of low volatility firms. The impact on current stock prices is exactly opposite, because higher expected return means lower current price, all else equal. So, Proposition 3 implies that when both aggregate volatility and idiosyncratic volatility increase in recessions, high idiosyncratic volatility firms beat the CAPM, and low idiosyncratic volatility firms perform worse than the CAPM prediction. The identical reasoning can be repeated for growth firms and high volatility growth firms.

A 50% increase and even a 100% increase in idiosyncratic volatility is not uncommon in recessions (see e.g., Figure 4 in Campbell, Lettau, Malkiel, and Xu, 2001). The simulations in Section 3 show that the impact of such idiosyncratic volatility changes on the risk premium is substantial. In the simulations, the risk premium elasticity with respect to idiosyncratic volatility varies from zero for low volatility value firms to -0.5 for high volatility firms. It means that, net of any other effects of the recession on the risk premium, in recessions the idiosyncratic volatility hedging channel can reduce the expected returns to high volatility growth firms by a quarter or even a half.

Proposition 4 The elasticity of the firm value with respect to idiosyncratic volatility increases with idiosyncratic volatility:

$$\frac{\partial}{\partial\sigma_I}\left(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}\right) > 0 \tag{17}$$

The elasticity of the firm value decreases in the value of assets in place:

$$\frac{\partial}{\partial B}\left(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}\right) < 0 \tag{18}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is negative:

$$\frac{\partial^2}{\partial\sigma_I\partial B}\left(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}\right) < 0 \tag{19}$$

Proof: See Section 2.

Proposition 4 summarizes the second part of the idiosyncratic volatility hedging channel. The value of growth options, like the value of any option, tends to increase with volatility. As the economy enters the recession and volatility increases, growth options will beat the assets with similar market beta. This hedging channel is naturally stronger for growth firms, because their return is more affected by the changes in the growth options value. This is a new explanation of why growth firms are less risky than value firms.

Based on simulations (see Section 3.6), I conclude that this hedging channel is also stronger for high volatility firms than for low volatility firms and that it is the strongest for high volatility growth firms. The simulations also show that the firm value elasticity with respect to idiosyncratic volatility is substantial. It varies from 0 for low volatility value firms to -0.3 and higher for high volatility growth firms. Therefore, net of any other (negative) cash flow effects of the recession, the increase in idiosyncratic volatility during the recession can increase the value of high volatility growth firms by 15-20%.

The bottom line of Propositions 3 and 4 is that, controlling for the market risk, high volatility, growth, and high volatility growth firms load most positively on changes in aggregate volatility. Hence, these three types of firms hedge against aggregate volatility risk. The reason is the idiosyncratic volatility channel, which predicts that the value of volatile growth options goes up the most as aggregate volatility and idiosyncratic volatility increase, and the expected risk premium of volatile growth options increases the least during volatile times.

Hypothesis 2. High idiosyncratic volatility firms, growth firms, and especially high idiosyncratic volatility firms hedge against aggregate volatility risk. Their betas with respect to the aggregate volatility risk factor are negative and lower than those of low volatility, value, and low volatility value firms.

The difference in the loadings on the aggregate volatility risk factor between high and low volatility firms should totally explain the idiosyncratic volatility effect and the stronger idiosyncratic volatility effect for growth firms. The aggregate volatility factor should also significantly contribute to explaining the value effect and why it is stronger for high volatility firms. I can also use Proposition 3 to test the hedging ability of high volatility, growth, and high volatility growth firms against adverse business-cycle shocks in a more conventional fashion. In the CAPM, lower risk premium means lower betas. Proposition 3 can be rephrased in terms of betas to show that in the conditional CAPM the betas of high volatility, growth, and high volatility growth firms are lower in recessions than in booms (details are available from the author).

Theoretically, the ICAPM is a more fruitful framework to explain the three idiosyncratic volatility effects than the conditional CAPM. The conditional CAPM assumes investors have no hedging demands and only care about the market risk. The idiosyncratic volatility hedging channel in the conditional CAPM is limited to the negative correlation between the market beta and the market risk premium, which produces negative unconditional CAPM alphas for high volatility, growth, and high volatility growth firms.

Beyond that, in the ICAPM the hedging channel also means that these three types of firms provide additional consumption when it is most needed to increase savings. The reasons to increase savings after volatility increases are worse future investment opportunities and lower future consumption (Campbell, 1993) and higher future volatility and the precautionary motive (Chen, 2002). Also, the ICAPM captures the hedge coming from the fact that the value of growth options increases with volatility.

As in the previous subsection, the results in this subsection can be reformulated using any option-like dimension of the firm. The implication is that no matter which option-like dimension of the firm (market-to-book, leverage, etc.) is creating the idiosyncratic volatility discount, it should be explained by lower sensitivity of high volatility firms to negative business-cycle news and their lower risk in recessions.

2 Proofs

This section collects the proofs of the propositions in Section 1. Some prepositions refer to the simulations described in Section 3

Proposition 1. The value of the firm is given by

$$dV_t/V_t = (r+\pi_B-(\pi_B-\pi_S\Phi(d_1)\frac{S_t}{P_t})\cdot\frac{P_t}{V_t})dt+\Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S+\sigma_I dW_I)+\sigma_B\frac{B_t}{V_t}dW_B \quad (20)$$

where

$$d_1 = \frac{log(S/K)+(r+\sigma_C^2/2+\sigma_I^2/2)(T-t)}{\sqrt{(\sigma_C^2+\sigma_I^2)\cdot(T-t)}} \quad (21)$$

The expected rate of return to the firm decreases in idiosyncratic risk, σ_I, and increases in the value of assets in place, B.

Proof: Black and Scholes (1973) formula in my case yields

$$P_t = S_t\Phi(d_1) - \exp(r(T-t))K\Phi(d_2) \quad (22)$$

where $\Phi(\cdot)$ is the normal cdf, d_1 is as defined in (21), and $d_2 = d_1 - \widetilde{\sigma}$.

Applying the Ito's lemma and the no-arbitrage condition to the value of the firm, $V_t = P_t + B_t$, I find that the value of the firm follows

$$dV_t/V_t = (r+\pi_S\cdot\Phi(d_1)\frac{S_t}{V_t}+\pi_B\frac{B_t}{V_t})dt+\Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S+\sigma_I dW_I)+\sigma_B\frac{B_t}{V_t}dW_B \quad (23)$$

Then I rearrange the expression for the drift

$$\mu_V = r + \pi_S \Phi(d_1)\frac{S_t}{V_t} + \pi_B \frac{B_t}{V_t} = r + \pi_B - [\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}] \cdot \frac{P_t}{V_t} \tag{24}$$

Determining the sign of the drift's derivatives with respect to idiosyncratic risk and assets in place is now simple and intuitive. The term in the square brackets is positive if assets in place earn higher returns than growth options, which is a sufficient condition to derive the value effect. The changes in assets in place, B_t, influence only the denominator of the last term in (24). As B_t increases, V_t increases as well, and the whole last term decreases if $(\pi_B - \pi_S \Phi(d_1) S_t/P_t) > 0$, meaning that an increase in B_t causes an increase in expected returns. Algebraically,

$$\frac{\partial \mu_V}{\partial B} = \frac{P_t}{V_t^2} \cdot (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) > 0, \tag{25}$$

An increase in idiosyncratic risk, σ_I, increases the price of growth options, P_t, and their fraction in the value of the firm, P_t/V_t. An increase in idiosyncratic risk also leads to a decrease in the option elasticity with respect to the price of the underlying asset, $\Phi(d_1)S_t/P_t$, (see Galai and Masulis, 1976, for a proof). Therefore, both parts of the last term in (24) increase as idiosyncratic risk increases, and expected returns decrease. Algebraically,

$$\frac{\partial \mu_V}{\partial \omega} = \pi_S \frac{\partial(\Phi(d_1)S_t/P_t)}{\partial \omega} \cdot \frac{P_t}{V_t} - (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{B_t}{V_t^2} \cdot \frac{\partial P_t}{\partial \omega} < 0 \ , \tag{26}$$

where the first term captures the effect of idiosyncratic risk on the option elasticity, and the second term captures the increase in the relative weight of growth options.

QED

Corollary 1. Define $IVar$ as the variance of the part of the return generating process (6), which is orthogonal to the pricing kernel. Then the idiosyncratic variance $IVar$ is

$$\begin{aligned} IVar = {} & \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1 - \rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1 - \rho_{B\Lambda}^2) + \\ & + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \end{aligned} \tag{27}$$

I show that for all reasonable parameter values σ_I

$$\frac{\partial IVar}{\partial \sigma_I} > 0, \tag{28}$$

which implies that my empirical measure of idiosyncratic volatility - the standard deviation of Fama-French model residuals - is a noisy but valid proxy for σ_I.

Proof: The orthogonal to dW_Λ part of any diffusion is $dW_\bullet - \rho_{\bullet\Lambda} \cdot dW_\Lambda$. Therefore, (20) can be rewritten as

$$\begin{aligned} dV_t/V_t = (r + \pi_B - (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{P_t}{V_t})dt + \\ +[\Phi(d_1)\frac{S_t}{V_t} \cdot (\sigma_S(dW_S - \rho_{S\Lambda}dW_\Lambda) + \sigma_I dW_I) + \sigma_B \frac{B_t}{V_t} \cdot \\ \cdot(dW_B - \rho_{B\Lambda}dW_\Lambda)] + [\sigma_S \Phi(d_1)\frac{S_t}{V_t} \cdot \rho_{S\Lambda} + \sigma_B \frac{B_t}{V_t} \cdot \rho_{B\Lambda}]dW_\Lambda \end{aligned} \tag{29}$$

where the first square bracket contains the part orthogonal to dW_Λ and the second square bracket contains the part driven by dW_Λ. The standard deviation of the first square bracket is the model measure of idiosyncratic volatility, and its most natural empirical estimate is the standard deviation of an asset-pricing model's residuals (in the empirical part I choose the Fama-French model).

Applying Fubini's theorem and collecting terms yields, as claimed in Corollary 1, that the idiosyncratic variance is

$$\begin{aligned} IVar = \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1 - \rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1 - \rho_{B\Lambda}^2) + \\ + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \end{aligned} \tag{30}$$

The analytical expression for the derivative of $IVar$ wrt σ_I is complicated, and its sign cannot be determined without simulations. The simulations in Section 3.3 show that at all empirically plausible parameter values the idiosyncratic volatility increases with the idiosyncratic risk parameter σ_I. The idiosyncratic volatility is also impacted by other parameters, so it is a noisy, but valid proxy for σ_I.

QED.

Proposition 2. The effect of idiosyncratic risk on returns, $\left|\frac{\partial \mu_V}{\partial \sigma_I}\right|$, is increasing in the value of assets in place B.

Proof:

$$\frac{\partial^2 \mu_V}{\partial \sigma_I \partial B} = -\pi_S \frac{\partial(\Phi(d_1)S_t/P_t)}{\partial \sigma_I} \cdot \frac{P_t}{V_t^2} + (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{B - P}{V^3} > 0 \tag{31}$$

The first term is always positive, and the second term is positive if $B > P$ and negative otherwise. However, for small B the first term becomes relatively large. Simulations in Section 3 show that the derivative is positive except for the parameter value that imply total volatility of 70% per annum or more and and market-to-book higher than 5. The simulations also show that for these extreme parameter values the expected return is about the same as for the parameter values yielding the positive derivative.

QED

Proposition 3 The elasticity of the risk premium in my model decreases (increases in the absolute magnitude) as idiosyncratic volatility increases:

$$\frac{\partial}{\partial \sigma_I}(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) < 0 \tag{32}$$

The elasticity of the risk premium in my model increases (decreases in the absolute magnitude) as the value of assets in place increases:

$$\frac{\partial}{\partial B}(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) > 0 \tag{33}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is positive:

$$\frac{\partial^2}{\partial \sigma_I \partial B}(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) > 0 \tag{34}$$

Proof: It turns out that the derivative in (33) is the easiest to sign:

$$\frac{\partial}{\partial B}(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) = \frac{1}{\lambda_V^2} \cdot (\frac{\partial^2 \lambda_V}{\partial \sigma_I \partial B} \cdot \sigma_I \cdot \lambda_V - \frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\partial \lambda_V}{\partial B} \cdot \sigma_I) > 0 \tag{35}$$

The derivative in the first term of (35) is positive at reasonable parameter values (see Proposition 2) and the derivatives in the second term of (35) are positive and negative, respectively (see Proposition 1). So, at reasonable parameter values (35) is a sum of two positive terms.

$$\begin{aligned}\frac{\partial}{\partial \sigma_I}(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) &= \frac{1}{\lambda_V^2} \cdot ((\frac{\partial^2 \lambda_V}{\partial \sigma_I^2} \cdot \sigma_I + \frac{\partial \lambda_V}{\partial \sigma_I}) \cdot \lambda_V - (\frac{\partial \lambda_V}{\partial \sigma_I})^2 \cdot \sigma_I) = \\ &= \frac{1}{\lambda_V} \cdot (\frac{\partial^2 \lambda_V}{\partial \sigma_I^2} \cdot \sigma_I + \frac{\partial \lambda_V}{\partial \sigma_I}(1 - \frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}))\end{aligned} \tag{36}$$

The first term in (36) has an ambiguous sign and the second term is always negative. Simulations in Section 3 show that the first term is positive but small for empirically plausible parameters, and the overall sign of (32) is negative.

Taking the cross-derivative (34) is tedious and, as in the previous case, there is no obvious way to sign it without simulations. The simulations in Section 3 show that at reasonable parameter values it is positive.

QED

Proposition 4 The elasticity of the firm value with respect to idiosyncratic volatility increases with idiosyncratic volatility:

$$\frac{\partial}{\partial \sigma_I}\left(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}\right) > 0 \tag{37}$$

The elasticity of the firm value decreases in the value of assets in place:

$$\frac{\partial}{\partial B}\left(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}\right) < 0 \tag{38}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is negative:

$$\frac{\partial^2}{\partial \sigma_I \partial B}\left(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}\right) < 0 \tag{39}$$

Proof: It turns out that the derivative in (38) is the easiest to sign. The value of growth options increases in idiosyncratic volatility, and the effect of idiosyncratic volatility is weaker if assets in place take a larger share of the firm value. Algebraically, the elasticity is the firm value derivative with respect to idiosyncratic volatility scaled by the firm value. The derivative is always positive and does not depend on the value of assets in place[3]. The firm value increases in the value of assets in place, which makes the whole ratio (i.e., the elasticity) decrease in assets in place.

The derivatives in (37) and (39) are complicated. The simulations in Section 3 show that their values are always positive except for the extreme growth firms (in the model, the market-to-book higher than 5 and annual total volatility higher than 50% per annum). However, the elasticity for those firms is still much larger than the elasticity of most other firms.

[3]The fact that the call option value increases in volatility is widely known in finance. The respective derivative is called vega and equals to $P \cdot \exp(-r \cdot (T-t))\phi(d_1)\sqrt{T-t}$.

3 Simalations

3.1 Parameter Values

In the simulations, I fix two sets of parameter values. The first set is the moments of the three processes: the pricing kernel, Λ_t, the value of the assets in place, B_t, and the value of the asset behind the growth options, S_t. The values of the parameters are chosen so that the value effect roughly matches its empirical magnitude (about 6% per year). In the current setup, to keep things simple, I assume that the difference in expected returns between B_t and S_t is large enough to produce the positive value effect. It turns out that because the growth options are a highly levered claim on S_t, I have to assume quite large difference in the expected returns to S_t and B_t. The way to avoid it is to formally model the idiosyncratic volatility hedging channel, which I leave for future research. In my world, the idiosyncratic volatility hedging channel is responsible for explaining why the value effect can ever be positive, but the model is potentially open for incorporating other explanations.

I fix the volatility of the pricing kernel, σ_Λ, at 50% per year, the volatility of the asset behind the growth options, σ_S, at 10% per year, and the correlation between the asset behind the growth options and the pricing kernel, $\rho_{S\Lambda}$, at -0.8, which yields the risk premium $\pi_S = -\rho_{S\Lambda}\sigma_\Lambda\sigma_S = 4\%$. I also fix the volatility of the assets in place, σ_B, at 40% per year, and their correlation with the pricing kernel, $\rho_{B\Lambda}$, at -0.7, which yields the risk premium $\pi_B = -\rho_{B\Lambda}\sigma_\Lambda\sigma_B = 14\%$. All simulations produce similar results for other combinations of the parameters values that yield the risk premiums of 4% and 14%. In the simulations of the idiosyncratic variance from Corollary 1, equation (8) or (36), I assume that the correlation between S_t and B_t is 0.5, but setting it to another value does not affect the results.

The second set of parameters describes the growth options. I assume that the current value of the asset behind them is 100 and the strike price is 90. My model is scale-invariant, so these values only mean that the asset is slightly in the money. The expiration period is set at 4 years. In what follows, I will discuss how the change in the maturity and the moneyness of the growth options affect my results. The overall conclusion is that my results are robust to reasonable variations in the maturity and the moneyness.

The two other parameters that vary freely in my tests are σ_I, the volatility of the purely idiosyncratic part in S_t, and B, the value of the assets in place. Varying these two parameters gives me a rich cross-section in terms of idiosyncratic volatility, $\sqrt{IVar}$, and market-to-book, V/B. As σ_I varies from 0% to 70% per annum, and B varies from 0 to 150, the idiosyncratic volatility varies between 20% and 80% per annum, and the market-to-book varies from 1.5 to above 6[4].

3.2 The Magnitude of the Three Idiosyncratic Volatility Effects

The top figure in Figure 1 shows the variation in the expected return as a function of the idiosyncratic volatility parameter, σ_I, and the value of the assets in place, B. First, I notice that idiosyncratic volatility is always negatively related to returns. Consistent with what my model predicts, the idiosyncratic volatility discount varies from 7% per year for growth firms ($B = 10$, $V/B \in [4,\ 7]$) to 2% per year for value firms ($B = 150$, $V/B \in [1.2,\ 1.4]$). The value effect varies with idiosyncratic volatility from 0.2% per year for low volatility firms ($\sigma_I = 5\%$ per year, $\sqrt{IVar} \in [20\%,\ 25\%]$) to 5.5% per year for high volatility firms ($\sigma_I = 70\%$ per year, $\sqrt{IVar} \in [50\%,\ 100\%]$).

Overall, my model produces numerically large effects of idiosyncratic volatility on expected returns. These effects are smaller than their empirical counterparts, because the simulations do not account for the aggregate volatility risk. I also fix the baseline parameters quite conservatively. For example, for some firms in the data the risk premium spread between the assets in place and the asset behind the growth options can be larger, which would magnify the idiosyncratic volatility effects.

In the bottom two graphs I look at the effect of varying the parameters of the growth options on the three idiosyncratic volatility effects. In the bottom left graph I reproduce the top graph with a higher strike price, $K = 100$, which makes the growth options exactly at the money. As expected, the idiosyncratic volatility effects become stronger, because at-the-money options are the most sensitive to volatility. The idiosyncratic volatility discount now varies from 9% per year for growth firms to 2% per year for value firms. I also see the negative value effect of -2% per year for low volatility firms. The value effect becomes positive as idiosyncratic volatility goes up and reaches 6% per year for high volatility firms.

Naturally, if I push the growth options deeper in the money, the effect is the reverse

[4] The lowest possible value of market-to-book in my model is 1. The market value, or the firm value V_t, differs from the book value, or the value of the assets in place B_t, by the always positive value of the growth options, P_t.

of what is in the bottom left graph in Figure 1, i.e., the value effect for low volatility firms increases, and the three idiosyncratic volatility effects decline. However, even if the value of the asset behind the growth options exceeds the strike price by a factor of 1.5, the idiosyncratic volatility effects are at least 3% per year.

In the bottom right graph, I reproduce the top graph with a shorter maturity of the growth options equal to 2 years. It makes the idiosyncratic volatility effects stronger. The reason is the slight convexity of expected return in idiosyncratic volatility that can be seen in the top graph in Figure 1. If the total life-time volatility of the option is smaller, the effect of the changes in it is stronger. With the maturity of the growth options equal to 2 years the idiosyncratic volatility discount varies from 10% per year for growth firms to 2.5% per year for value firms. The value effect changes from -3.5% per year for low volatility firms to 5.5% per year for high volatility firms. If I increase the maturity to 8 years, the idiosyncratic volatility discount varies from 4% to 1.5%, and the value effect varies from 6.5% to 4.5%.

The slight convexity of the graphs in idiosyncratic volatility does not contradict the empirical finding that the idiosyncratic volatility discount is driven by the firms in the highest volatility quintile. Because idiosyncratic volatility in the data is extremely positively skewed, the highest volatility quintile spans a huge spread in the idiosyncratic volatility, about half of the values in the graph.

3.3 Simulations for Corollary 1

In Corollary 1 I claim that the idiosyncratic variance, $IVar$, monotonically increases with the idiosyncratic volatility parameter, σ_I. The idiosyncratic variance is defined as the variance of the part of the firm value that is orthogonal to the pricing kernel. The idiosyncratic volatility parameter measures the volatility of the purely idiosyncratic part of the process for the asset behind the growth options.

The top graph in Figure 2 shows that the idiosyncratic variance indeed increases with σ_I. The value of the assets in place is fixed at 50, which implies the market-to-book between 1.6 and 2.2. The increase is quite strong and becomes stronger, as σ_I increases and begins to take a larger fraction of the idiosyncratic variance.

In unreported results, I tried the values of the assets in place between 10 and 150, which spans the market-to-book values between 1.15 and 7, and the relation between $IVar$ and σ_I never turned negative.

3.4 Simulations for Proposition 2

In Proposition 2 I claim that the idiosyncratic volatility discount increases with market-to-book and the value effect increases with idiosyncratic volatility. Algebraically, it means that the second cross-derivative of the expected return with respect to idiosyncratic volatility and the value of the assets in place is positive. The more assets in place the firm has, the weaker is the negative relation between the expected return and the idiosyncratic volatility, because the idiosyncratic volatility effects work through the growth options.

In Figure 1, I show that this assertion is true for all reasonable parameter values and the highest volatility growth firms have the lowest expected returns. In the bottom of Figure 2, I look at the cross-derivative graph and, expectedly, find that the derivative is positive almost everywhere. The exception is the bottom right corner, where the derivative dips to zero. The corner is populated by the extremely high volatility firms (total volatility of more than 70% per year) with extremely high market-to-book (more than 6). For these values, which are, at least, quite uncommon empirically, the derivative can become negative and the relations claimed in Proposition 2 can reverse. However, the rest of the bottom graph in Figure 2 and the graphs in Figure 1 show that the claimed relations embrace almost all empirically plausible parameter values.

3.5 Simulations for Proposition 3

Proposition 3 asserts that the elasticity of the risk premium with respect to idiosyncratic volatility decreases in idiosyncratic volatility and market-to-book. I use this fact to state that the increase in the expected risk premium in recessions, when idiosyncratic volatility is high, is the smallest for high volatility, growth, and high volatility growth firms. Proposition 3 implies that these firms have lower betas in recession and their value decreases the least when the economy slides into recession. In the paper, I use this fact to predict that these firms hedge against aggregate volatility risk.

In the simulations, I need to determine the sign of two derivatives of the elasticity - one with respect to idiosyncratic volatility, and one with respect to idiosyncratic volatility and the value of the assets in place. I start with looking at the graph of the elasticity in the top part of Figure 3.

The graph shows that indeed the elasticity generally declines (increases in the absolute magnitude) in market-to-book and idiosyncratic volatility. The elasticity is the lowest for high volatility growth firms. The value of the elasticity is substantial and can reach -0.5. Given that the 50% increase in idiosyncratic volatility is not uncommon in recessions, the expected risk premium of high volatility growth firms can easily be cut by a quarter in bad times compared to what it could have been in the absence of idiosyncratic volatility.

I also see on the graph that the elasticity can increase (decrease in the absolute magnitude) in idiosyncratic volatility as both idiosyncratic volatility and market-to-book are high enough. In the bottom left graph, which shows the derivative of the elasticity with respect to idiosyncratic volatility, I see that the derivative is negative in the bottom right corner. The corner is populated by the firms with total volatility exceeding 50% per year and market-to-book exceeding 5, which is quite rare empirically. Even for these firms, as the top graph in Figure 3 shows, the elasticity remains very large, much larger than the elasticity for the firms with more usual values of volatility and market-to-book (the center of the graph).

In the bottom right graph I plot the cross-derivative of the elasticity with respect to idiosyncratic volatility and market-to-book. Proposition 3 says that the derivative should be positive, which is the sufficient condition for the elasticity being the highest for high volatility growth firms. I see in the graph that the derivative turns negative for high volatility growth firms. The region of the wrong sign is broader than in the bottom left graph. The derivative can in fact be negative for total volatility of 45% per year and market-to-book of 3.5, which is empirically plausible. However, the graph of the elasticity itself shows that high volatility growth firms do have large negative elasticity, which is much higher than the elasticity of most firms.

3.6 Simulations for Proposition 4

In Proposition 4 I look at the elasticity of the firm value with respect to idiosyncratic volatility, which is always positive, because higher idiosyncratic volatility increases the value of growth options. I claim that the elasticity is the highest for high volatility, growth, and especially high volatility growth firms. Algebraically, it means that the derivative of the elasticity with respect to idiosyncratic volatility is positive, and the cross-derivative with respect to idiosyncratic volatility and the value of the assets in place should be negative[5]. Economically, it means that the value of high volatility, growth, and high volatility growth firms increases, as the idiosyncratic volatility increases and the economy slides into recession. In the paper, I use this fact as another way to explain why these firms hedge against aggregate volatility risk.

The top graph in Figure 4 plots the elasticity of the firm value with respect to idiosyncratic volatility. The elasticity is substantial and increases with idiosyncratic volatility and the value of assets in place. The elasticity values of 0.3 and higher are not unusual and start at the parameter values implying total volatility of 40% and market-to-book of 2.5. The elasticity of 0.3 implies that the volatility increase in recessions can increase the firm value by 15%, just because growth options are more valuable in a volatile environment.

In the bottom left graph I plot the derivative of the elasticity with respect to idiosyncratic volatility and find that it is always positive. It the bottom right part I plot the derivative of the elasticity with respect to idiosyncratic volatility and the value of the assets in place. The derivative does become negative in the bottom right corner, populated by the firms with extremely high volatility and extremely high market-to-book. The area of the wrong sign is populated by the firms with total volatility higher than 50% per year and market-to-book exceeding 4, which is quite unusual empirically. However, the top graph shows that even the wrong sign of the second derivative does not really compromise the conclusion of Proposition 4 that the firm value of high volatility growth firms responds most positively to volatility increases.

[5]The fact that the derivative with respect to the value of the assets in place is negative was proven in Section 2

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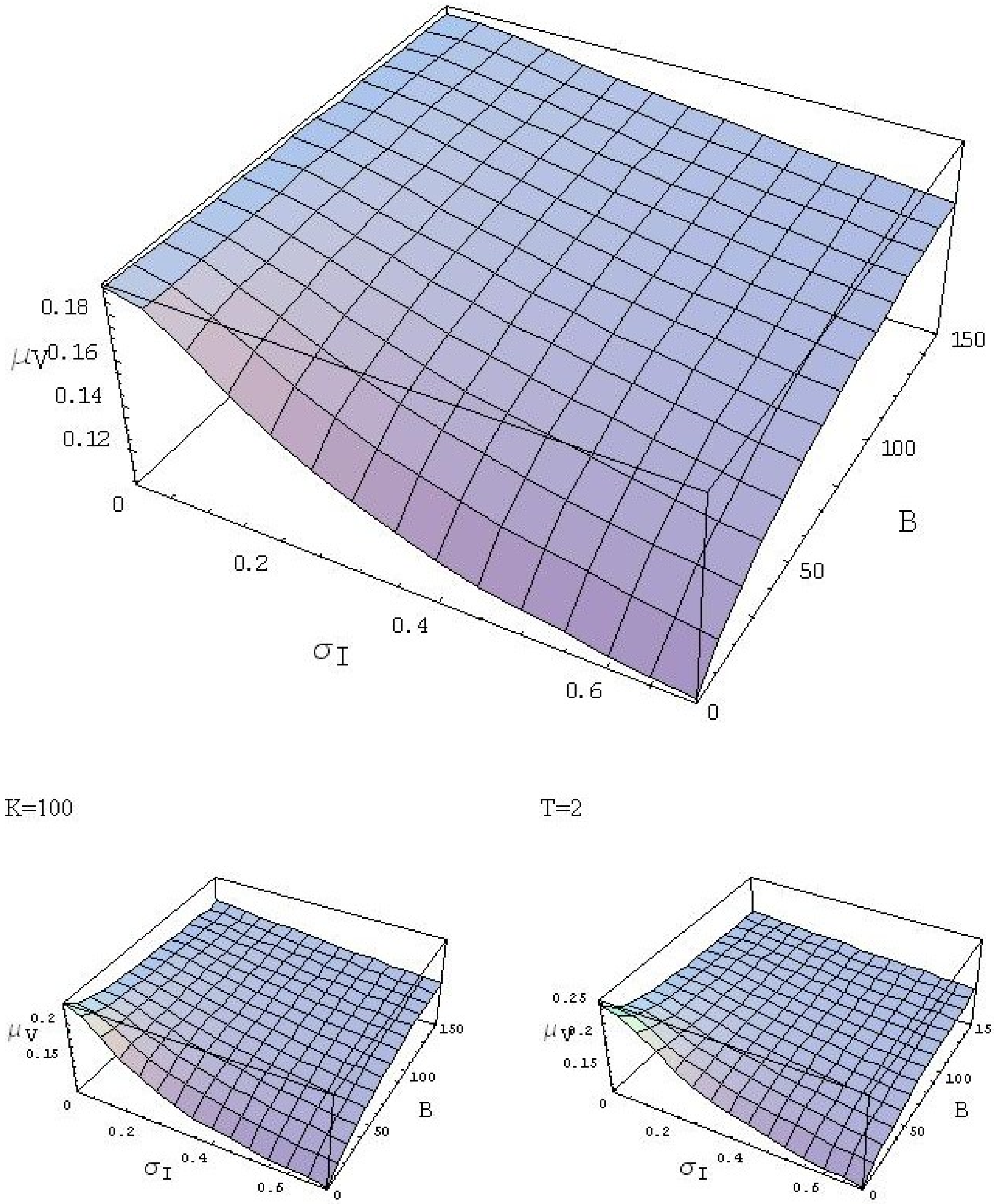

Figure 1. Expected Return as a Function of Idiosyncratic Volatility and the Value of Assets in Place.
The figures show the expected return, μ_V, for the firm in my model on the vertical axis. Idiosyncratic volatility, σ_I, is plotted on the left axis and the value of assets in place, B, are on the right axis. The top figure shows the expected return for the baseline values of the parameters $S = 100$, $K = 90$, $T - t = 4$, $r = 5\%$, $\sigma_S = 10\%$, $\sigma_B = 40\%$, $\sigma_\Lambda = 50\%$, $\rho_{S\Lambda} = -0.8$, $\rho_{B\Lambda} = -0.7$, $\rho_{SB} = 0.5$. The two bottom figures show the effect of setting $K = 100$ (left) or $T - t = 2$ (right).

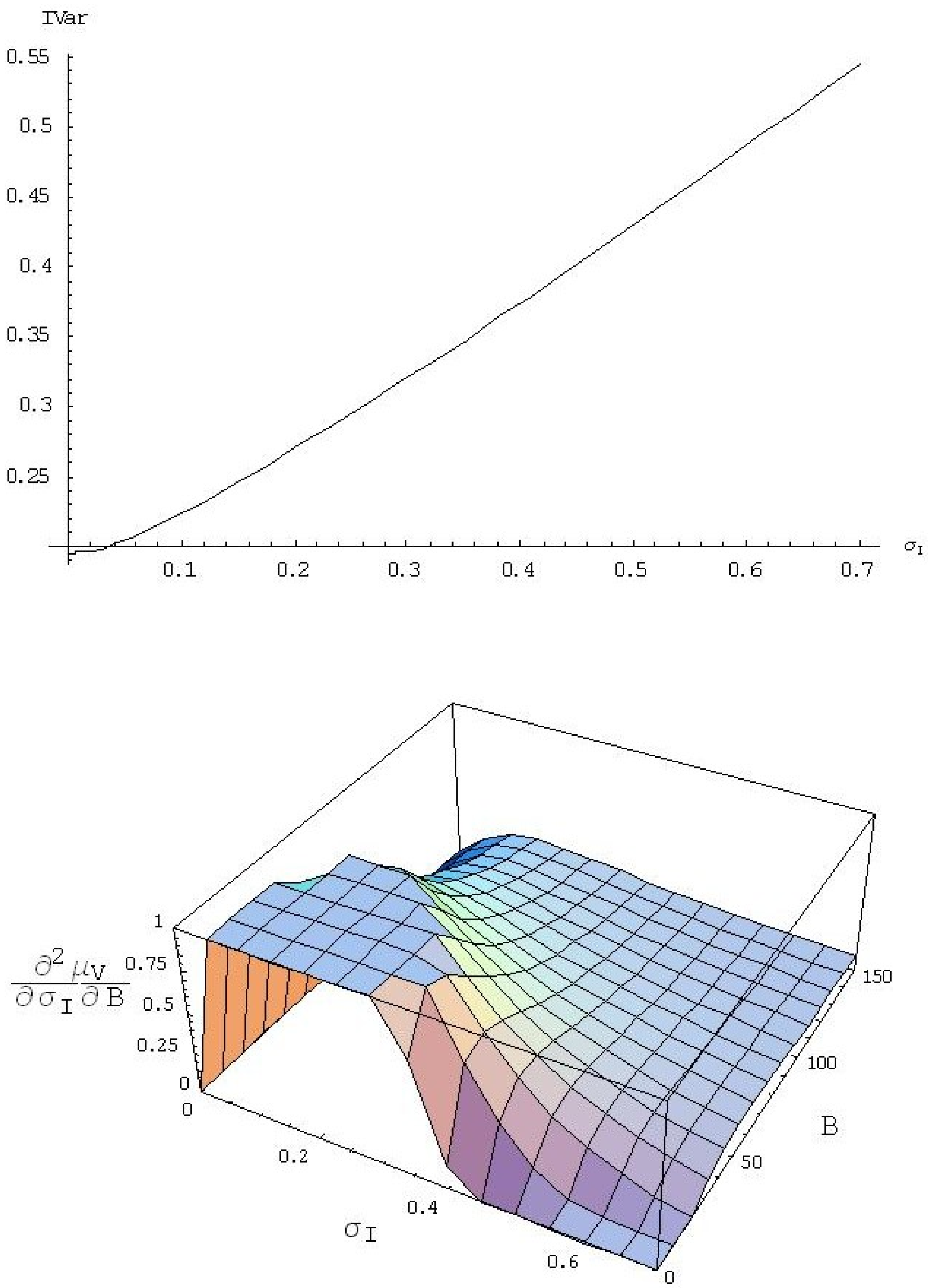

Figure 2. Idiosyncratic Variance, (27), and the Derivative of the Expected Return with respect to Idiosyncratic Volatility and the Value of Assets in Place, (31).

The top figure plots the idiosyncratic variance, $IVar$, of the firm's return as a function of σ_I. The idiosyncratic variance is defined as the variance of the part of the firm value process that is orthogonal to the pricing kernel. σ_I measures the volatility of the idiosyncratic part in the process for the asset behind the growth options. The bottom figure plots the derivative (31) as a function of σ_I and the value of the assets in place B. Other parameters are at the baseline values $S = 100$, $K = 90$, $T - t = 4$, $r = 5\%$, $\sigma_S = 10\%$, $\sigma_B = 40\%$, $\sigma_\Lambda = 50\%$, $\rho_{S\Lambda} = -0.8$, $\rho_{B\Lambda} = -0.7$, $\rho_{SB} = 0.5$. In the top figure B is fixed at 50.

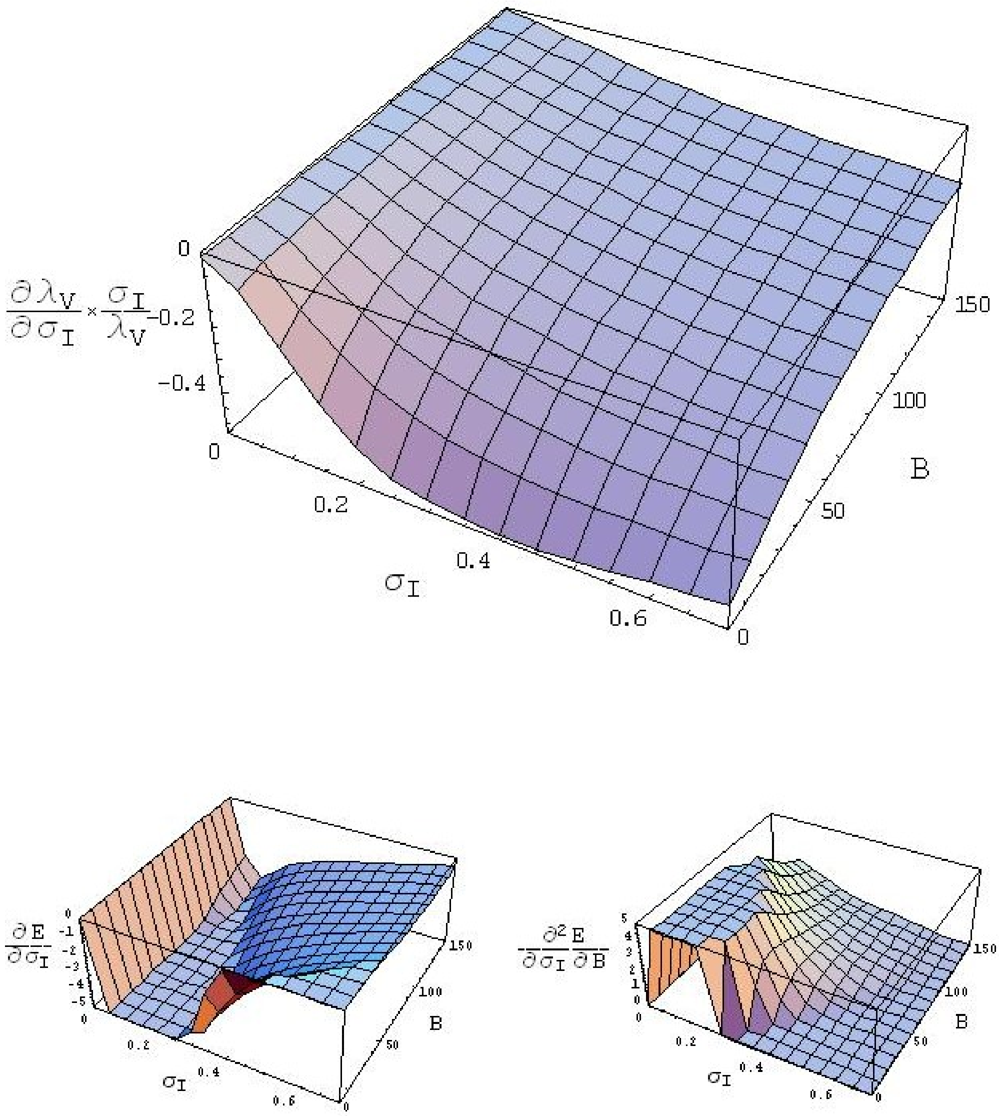

Figure 3. Risk Premium Elasticity with respect to Idiosyncratic Volatility
The top figure plots the risk premium elasticity as a function of idiosyncratic volatility, σ_I, and the value of assets in place, B. The bottom left figure plots the derivative of the elasticity with respect to idiosyncratic volatility, (32). The bottom right figure plots the second cross-derivative of the elasticity with respect to idiosyncratic volatility and the value of assets in place, (34). Other parameters are at the baseline values $S = 100$, $K = 90$, $T - t = 4$, $r = 5\%$, $\sigma_S = 10\%$, $\sigma_B = 40\%$, $\sigma_\Lambda = 50\%$, $\rho_{S\Lambda} = -0.8$, $\rho_{B\Lambda} = -0.7$, $\rho_{SB} = 0.5$.

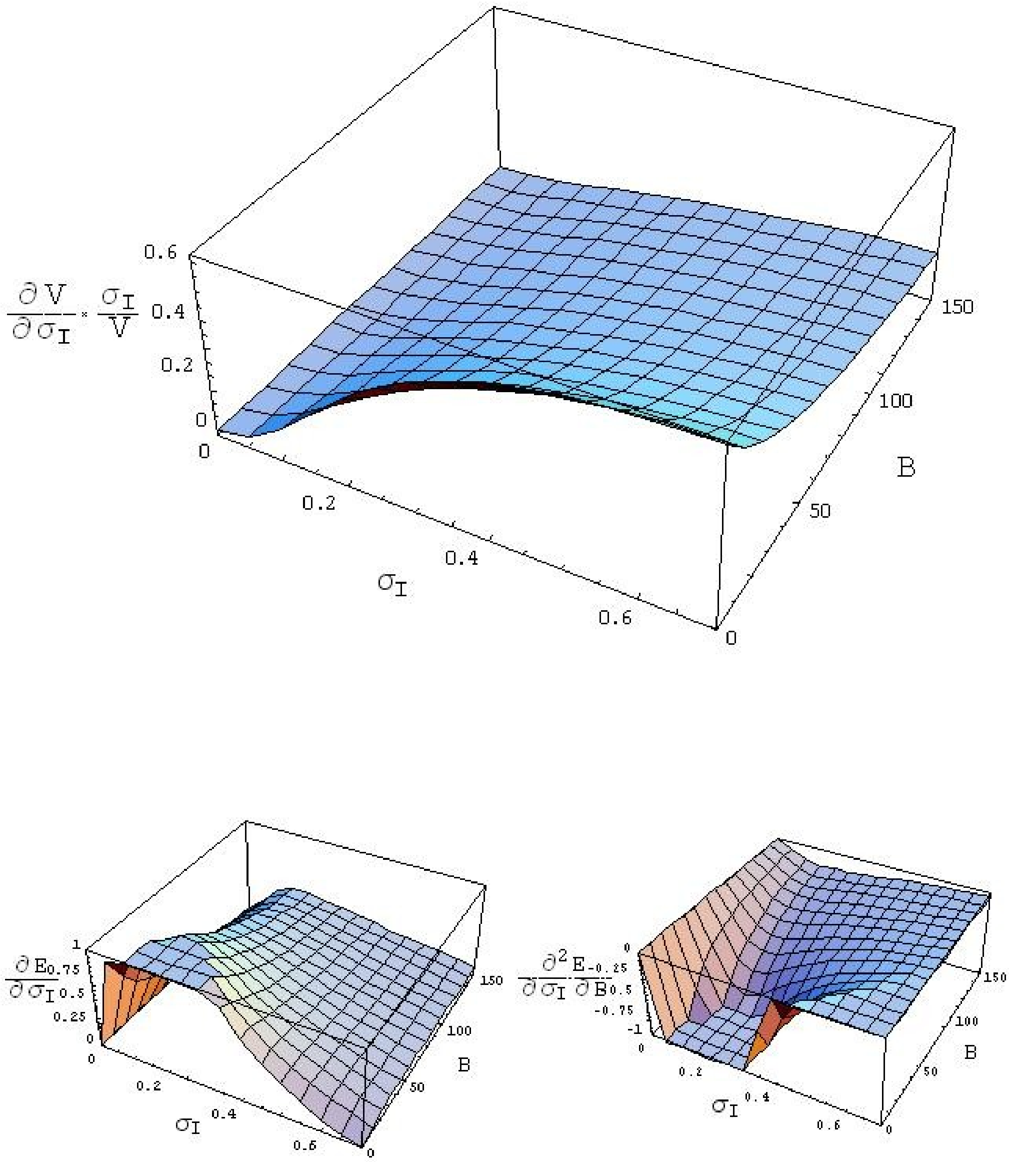

Figure 4. Firm Value Elasticity with respect to Idiosyncratic Volatility
The top figure plots the firm value elasticity as a function of idiosyncratic volatility, σ_I, and the value of assets in place, B. The bottom left figure plots the derivative of the elasticity with respect to idiosyncratic volatility, (37). The bottom right figure plots the second cross-derivative of the elasticity with respect to idiosyncratic volatility and the value of assets in place, (39). Other parameters are at the baseline values $S = 100$, $K = 90$, $T - t = 4$, $r = 5\%$, $\sigma_S = 10\%$, $\sigma_B = 40\%$, $\sigma_\Lambda = 50\%$, $\rho_{S\Lambda} = -0.8$, $\rho_{B\Lambda} = -0.7$, $\rho_{SB} = 0.5$.