Solutions to Midterm

AEC 504 - Summer 2007

Fundamentals of Economics

1 Social Welfare and Free Trade (40 points)

Assume that the demand for sporting guns is described by $Q^D = 100 - 2p$ and the supply is described by $Q^S = -20 + p$ (Q^D and Q^S are in millions, p is in \$)

i. *(5 points)* Compute the equilibrium price and quantity. Compute the total value created in the market for sporting guns.

$$\begin{cases} Q^D = 100 - 2p \\ Q^S = -20 + p \end{cases} \Rightarrow \begin{cases} -20 + p = 100 - 2p \\ Q = -20 + p \end{cases} \Rightarrow \begin{cases} p = 40 \\ Q = 20 \end{cases}$$

$$PS = \frac{1}{2} \cdot 20 \cdot 20 = 200;\ CS = \frac{1}{2} \cdot 10 \cdot 20 = 200;\ SW = PS + CS = 300$$

ii. *(5 points)* Suppose that the government views sporting guns as a luxury product and taxes the consumers \$5 for each sporting gun they buy, to redistribute the wealth to the poor. Effectively, it means that the demand for sporting guns is now $Q^D = 100 - 2(p + 5)$. What losses do consumers (of sporting guns) incur as a result of the tax? What losses, if any, do the producers of sporting guns incur? What if the total loss/benefit to the society? What is the optimal tax rate?

Hint: Remember that the poor benefit from the tax, since they get all the revenue the government collects from the buyers of sporting guns.

New equilibrium:

$$\begin{cases} Q^D = 90 - 2p \\ Q^S = -20 + p \end{cases} \Rightarrow \begin{cases} -20 + p = 90 - 2p \\ Q = -20 + p \end{cases} \Rightarrow \begin{cases} p = 110/3 \\ Q = 50/3 \end{cases}$$

$$PS' = \frac{1}{2} \cdot \frac{50}{3} \cdot \frac{50}{3} = \frac{1250}{9} \Rightarrow \Delta PS = \frac{1250}{9} - 200 = -\frac{550}{9} \approx -61.1$$

$$CS' = \frac{1}{2} \cdot \frac{25}{3} \cdot \frac{50}{3} = \frac{625}{9} \Rightarrow\Rightarrow \Delta CS = \frac{625}{9} - 100 = -\frac{275}{9} \approx -30.6$$

$$\Delta SW = \Delta CS + \Delta PS + 5 \cdot \frac{50}{3} = -\frac{25}{3} \approx -8.3$$

The optimal tax rate is zero (you will be able to show it formally in the next section).

iii. *(Hard, 15 points)* Assume (for this part only) that the owners of sporting guns destroy the wildlife and thus generate a negative effects on others. More precisely, the ownership of Q millions of guns creates costs of $3Q^3/64$ to the society. Find the optimal tax rate in this case.

Equilibrium with taxes:

$$\begin{cases} Q^D = 100 - 2t - 2p \\ Q^S = -20 + p \end{cases} \Rightarrow \begin{cases} -20 + p = 100 - 2t - 2p \\ Q = -20 + p \end{cases} \Rightarrow \begin{cases} p = \dfrac{120 - 2t}{3} \\ Q = \dfrac{60 - 2t}{3} \end{cases}$$

$$SW = \frac{1}{2} \cdot (30-t) \cdot \frac{60-2t}{3} + t \cdot \frac{60-2t}{3} = \frac{(30-t)^2}{3} + 2t \cdot \frac{(30-t)}{3} = \frac{(30-t)\cdot(30+t)}{3}$$

$$Tolal\ SW = \frac{(30-t)\cdot(30+t)}{3} - \frac{3}{64} \cdot \frac{8}{27} \cdot (30-t)^3 = \{30 - t \equiv x\} = \frac{x \cdot (60-x)}{3} - \frac{1}{8} \cdot \frac{1}{9} \cdot x^3$$

$$\max_X \frac{x \cdot (60-x)}{3} - \frac{1}{8} \cdot \frac{1}{9} \cdot x^3 \Rightarrow 60 - 2x - \frac{1}{8} \cdot x^2 = 0 \Rightarrow x = 15.32 \Rightarrow t = 14.68$$

We take the largest (positive) root, as the other one would yield the minimum, not maximum to the total social welfare function.

So, the optimal tax rate is \$14.68 per sporting gun, i.e. more than one third of the equilibrium price without taxes. Protection of the environment seems to be a tough thing here.

From the formula for SW above (before taking into account the negative effects the owners of sporting guns produce) one can see that t=0 really maximizes the value created in the market for sporting guns.

iv. *(5 points)* Suppose now that the government is concerned about the dominance of the low-cost foreign producers in the sporting guns market, so it sets the lowest price at which a sporting gun can be sold – \$35. What is the total loss/benefit to the consumers, the producers, and the society as a whole?

Frankly speaking, \$35 was a typo that made your life easier. Since the limiting lowest price is lower than the equilibrium price, it is not really limiting and has no influence on the market, as speed limit of 120 mph would effectively mean "drive as fast as you like". So, the answer is that CS, PS, and SW do not change with the introduction of the lowest price.

If the lowest price was \$45 (which it was meant to be) the things will change, because the consumers will only buy 10 (millions of guns) under this price. The welfare changes are shown in the graph:

$$CS = \frac{1}{2} \cdot 5 \cdot 10 = 25 \Rightarrow \Delta CS = -75;\ PS = \frac{1}{2} \cdot (25 + 15) \cdot 10 = 200 \Rightarrow$$
$$\Rightarrow \Delta PS = 0;\ \Delta SW = -75$$

The fact that the limiting price did not bring any benefit or loss to the producers is, of course, a mere coincidence. If the demand curve was flatter, the quantity would respond more and the limiting price would harm the producers as well as consumers. But it is true that the limiting lowest price harms consumers more than producers.

v. *(10 points)* A famous shooter Bill Tell invents an enhanced version of a sporting gun. Unlike Bill Gates, he is altruistic and commits to supplying the new guns at the fixed price (equal to the cost of production) of \$20. The demand function for the new guns is the same as the demand function for the old ones. However, Bill Tell's invention drastically affect the market for the old guns, the demand for which now becomes $Q^D = 70 - 2p$. What is the value of Bill Tell's invention to the society as a whole?

It is straightforward to find that Bill Tell will sell 60 (millions of new guns) at \$20, and the new equilibrium in the market for old guns will

be $p = 30,\ Q = 10$***.***

The benefit to the society from Bill Tell's invention are equal to the value created in the new market it establishes. Since Bill Tell is completely altruistic, SW is equal to CS there. So, the benefit is

$$CS_{New} = \frac{1}{2} \cdot 30 \cdot 60 = 900$$

However, inventions destroy old markets (the effect of "creative destruction" traced back to Joseph Schumpeter).

$$SW'_{Old} = \frac{1}{2} \cdot 15 \cdot 10 = 75 \Rightarrow \Delta SW'_{Old} = -225$$

We also observe that the reduction of value created in the market for old guns comes at the cost of both producer and consumer surplus. However, it does not mean that the invention harms some consumers, because they now can either buy an old gun at $30 instead of $40, or switch to an older version at even lower price, whichever they find better, so they cannot be worse off. In fact, the graph shows that in the market for old guns all consumers, who were willing to buy and bought at high prices are gone (to the market for new guns, where they enjoy higher surplus because of the low price), since the demand curve intercept in the market for old guns is even lower than the equilibrium price there before the invention. So the small consumer surplus created there goes now to those who bought no guns before (and enjoyed thus zero surplus).

2 Apartment Glut (20 points)

In the aftermath of the dotcom crisis the market of rental apartments on the West Coast suffered a large decrease in the demand

i. *(5 points)* The lendlords were considering offering either rent cuts or free services of landlord's employees (e.g., picking up and dropping off dry cleaning) as means of attracting new clients. Which measure would be more efficient for lendlords of high-quality apartments for the rich and which would be efficient in lending apartments to the poor?

Hint: Think about opportunity costs.

Rent cuts save renter's money, free services save renter's time. Obviously, richer people (who make more money) have higher opportunity cost of time, so they will appreciate time-saving discounts more than poorer people. In fact, the optimal policy for landlords will be to offer time-saving discounts to the clients who earn more than their employees (reallocating time-consuming tasks to those with lower cost of time creates value).

ii. *(5 points)* Assume that the lendlords went with the rent cuts. Compared to the situation before the price cuts (but after the decline in income associated with the crisis), do you think the decrease in rent induced the consumers to spend more on other goods (not apartments)?

The rent cut makes the consumers richer, because they have more money to spend after they paid the rent. It makes them want more of other goods (assuming other goods are on average normal goods). On the other hand, after the price cut other goods become more expensive relative to apartments. It makes the consumers buy less of other goods and rent better apartments. The overall effect on the amount of other goods (and, because the prices of other goods do not change, on the spending on other goods) is ambiguous.

iii. *(5 points)* An article about the aftermath of the dotcom crisis claimed: "In many cities on the coasts, where new construction is more difficult and where an influx of highly educated people over the last two decades has driven up home prices, rents have held up better". Discuss this statement in terms of supply and demand interaction

The more difficult is construction, the higher are the costs of the developers. The higher are the costs of the developers, the lower is the supply

of apartments. High education means high wages, which in turn mean higher income and higher demand for apartments (assuming the apartment is a normal good). So, on the coasts the supply of apartments was growing more slowly, and the demand for them was increasing faster than elsewhere, hence the rents declined less there.

iv. *(5 points)* Assume that the apartment is a necessity good and the own house is a luxury good. Would you rather be a developer of houses or a developer of apartment buildings during the recession after the dotcom boom?

Hint: Necessity goods have income elasticity less than 1, and luxury goods have income elasticity greater than one.

The elasticities mean that the demand for apartments is less sensitive to the changes in income than the demand for houses. In recession, lower sensitivity to changes in income is good (the demand drops less). Therefore, in the recession you would rather be a developer of apartment buildings.

3 Consumer Choice (30 points)

Consider a consumer with utility function $U = (x+1)(y+1)$ and income of 8. The prices of x and y are 1.

i. *(5 points)* Compute the demand functions for x and y and the optimal allocation.

$$\begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ p_x \cdot x + p_y \cdot y = I \end{cases} \Rightarrow \begin{cases} \dfrac{y+1}{x+1} = \dfrac{p_x}{p_y} \\ p_x \cdot x + p_y \cdot y = I \end{cases} \Rightarrow \begin{cases} p_x \cdot x + p_x = p_y \cdot y + p_y \\ p_x \cdot x + p_x \cdot + p_x - p_y = I \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = \dfrac{I + p_y - p_x}{2p_x} \\ y = \dfrac{I + p_x - p_y}{2p_y} \end{cases}$$

To get the optimal allocation, just substitute the values of the prices and the income into the demand functions:

$$x = (8 + 1 - 1)/2 \cdot 1 = 4, \;\; y = (8 + 1 - 1)/2 \cdot 1 = 4$$

ii. *(5 points)* Are the goods normal or inferior? Are they substitutes or complements?

The goods are normal, because their demands increase with income. The demand for x increases as p_y increases, and vice versa, so they are substitutes.

iii. *(5 points)* Compute the own-price, cross-price, and income elasticities of the demand for x at the given prices and income.

$$\eta_I(x) = \frac{1}{2p_x} \cdot \frac{I \cdot 2p_x}{I + p_y - p_x} = \frac{I}{I + p_y - p_x} \Rightarrow \eta_I(x) = 1$$

$$\eta_{p_y}(x) = \frac{1}{2p_x} \cdot \frac{p_y \cdot 2p_x}{I + p_y - p_x} = \frac{p_y}{I + p_y - p_x} \Rightarrow \eta_{p_y}(x) = \frac{1}{8}$$

$$\eta_{p_x}(x) = \frac{-2p_x - 2I - 2p_y + 2p_x}{4p_x^2} \cdot \frac{p_x \cdot 2p_x}{I + p_y - p_x} = \frac{-I - p_y}{I + p_y - p_x} \Rightarrow \eta_{p_x}(x) = -\frac{9}{8}$$

iv. *(5 points)* What is the maximum amount the consumer will pay for the right to consume x at the price of 1?

Hint: Think of what happens if he is precluded from consuming x and how to translate this into an income decrease.

If the consumer cannot buy x, he will spend all his money on y, buy 8 units of it and achieve the utility of 9. If the consumer can buy x, he can achieve this utility level with income of 4, so he will be ready to pay up to 4 out of his current income of 8 to be allowed to buy x.

$$\begin{cases} \min\limits_{x,y} x + y \\ s.t.(x+1)(y+1) = 4 \end{cases} \Rightarrow \begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ (x+1)(y+1) = 4 \end{cases} \Rightarrow \begin{cases} \dfrac{y+1}{x+1} = 1 \\ (x+1)^2 = 4 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 2 \\ y = 2 \end{cases} \Rightarrow I = 4$$

v. *(10 points)* Suppose the price of x jumps to 4. What is the new optimal allocation? What amount of money is needed to compensate the consumer for the price change? What are the income and the substitution effects for x and y?

Again, just substitute the values in the demand functions from (i):

$$x = \frac{8 + 1 - 4}{2 \cdot 4} = \frac{5}{8}, \quad y = \frac{8 + 4 - 1}{2 \cdot 1} = \frac{11}{2}$$

To find the necessary compensation, minimize the expenditure at the new prices needed to reach the old utility level of $(4+1)\cdot(4+1)=25$***:***

$$\begin{cases} \min\limits_{x,y} 4x+y \\ s.t. (x+1)(y+1)=25 \end{cases} \Rightarrow \begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ (x+1)(y+1)=25 \end{cases} \Rightarrow \begin{cases} \dfrac{y+1}{x+1} = 4 \\ 4(x+1)^2 = 25 \end{cases} \Rightarrow$$

$$\begin{cases} x = \dfrac{3}{2} \\ y = 9 \end{cases} \Rightarrow I = 4\cdot\frac{3}{2} + 1\cdot 9 = 15 \Rightarrow \Delta I = 15 - 8 = 7$$

The substitution effect is sliding up the indifference curve from the optimal bundle in (i) to the solution of the expenditure minimization problem:

$$SE_x = \frac{3}{2} - 4 = -\frac{5}{2}, \;\; SE_y = 9 - 4 = 5$$

Because p_x ***went up and*** p_y ***stayed the same,*** $SE_x < 0,\ SE_y > 0$***, exactly as we find.***

The income effect is jumping down from the solution of the expenditure minimization problem to the optimal bundle in (v):

$$IE_x = \frac{5}{8} - \frac{3}{2} = -\frac{7}{8}, \;\; IE_y = \frac{11}{2} - 9 = -\frac{7}{2}$$

Because p_x ***went up and*** p_y ***and*** I ***stayed the same,*** $IE_x < 0,\ IE_y < 0$***, exactly as we find.***