Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns

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The Three Puzzles

- Low future returns to high idiosyncratic volatility firms (IVol discount, Ang et. al, 2006)
- Stronger value effect for high idiosyncratic volatility firms (Ali et. al, 2003)
- Stronger IVol discount for growth firms (this paper)

Aggregate Volatility Risk

- Market volatility increase means worse future investment opportunities (Campbell, 1993)
- Market volatility increase means the need to increase precautionary savings (Chen, 2002)
- Firms with most positive return sensitivity to aggregate volatility changes have lower expected returns (Ang et al, 2006)
- Herskovic et al. find that average idiosyncratic volatility is also priced

Contribution

- General equilibrium model with Lucas trees and options on the trees generates the three puzzles
- The model implies that firms with high idiosyncratic volatility and growth firms are hedges against aggregate volatility risk
- Empirically, the market volatility factor explains the IVol discount and the average IVol factor explains the value premium

Model Setup

- Continuous-time economy a-la Lucas with $I+1$ types of firms, N firms of each type
- One representative investor with CRRA utility over terminal consumption, given by

$$u(C_T) = C_T^{1-\gamma}/(1-\gamma)$$

- Each firm (i,n) has assets in place with terminal payoff, $p_i D_{0,n,T}$, and growth options with terminal payoff

$$q_i(D_{i,n,T}/(KD_{i,n,0}))^{\lambda}KD_{i,n,0}$$

- $KD_{i,n,0}$ is an analogue of a strike price and λ captures convexity

Aggregate Volatility

- Dividend processes $D_{i,n,t}$ have stochastic systematic and idiosyncratic volatilities

$$dD_{i,n,t} = D_{i,n,t}[\mu_{D,i}dt + h_i\sqrt{v_{1t}}dw_t + g_i\sqrt{v_{2t}}dw_{i,n,t}]$$

- w is a systematic Brownian motion that affects all firms in the economy, whereas $w_{i,n,t}$ is a firm-specific idiosyncratic shock
- $h_i\sqrt{v_{1t}}$ and $g_i\sqrt{v_{2t}}$ as *systematic* and *idiosyncratic volatility*, respectively, and follow Heston processes

$$dv_{1t} = \kappa_1(\bar{v}_1 - v_{1t})dt + c_1\sqrt{v_{1t}}dw_t$$

$$dv_{2t} = \kappa_2(\bar{v}_2 - v_{2t})dt + c_2\sqrt{v_{2t}}d\tilde{w}_t$$

Main Assumptions

- We assume that both systematic and idiosyncratic volatility are negatively correlated with dividend shocks
- We also assume that shocks to systematic and idiosyncratic volatility are positively correlated
- This is consistent with empirical evidence in Barinov (2013), Bartram et al. (2016)
- Note that idiosyncratic shocks are uncorrelated across firms, but their volatilities are correlated (through systematic volatility)

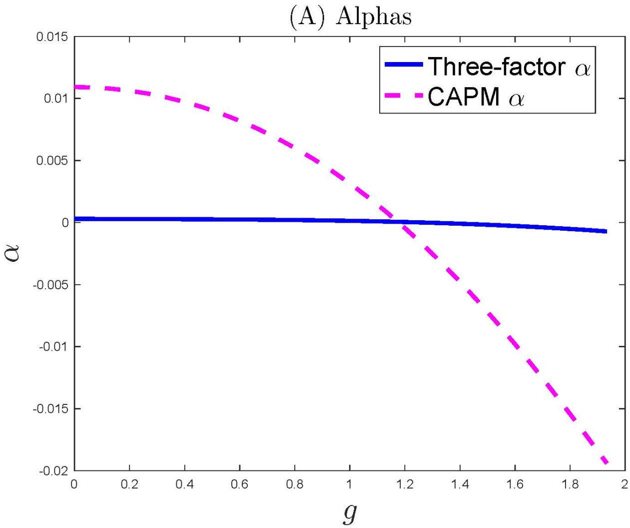
Main Result

$$C_T = \Big(\sum_{i=0}^{I} p_i\Big) \exp\Big\{\mu_{D,0} T - 0.5 h_0^2 \int_0^T v_{1\tau} d\tau + h_0 \int_0^T \sqrt{v_{1\tau}} dw_\tau\Big\}$$

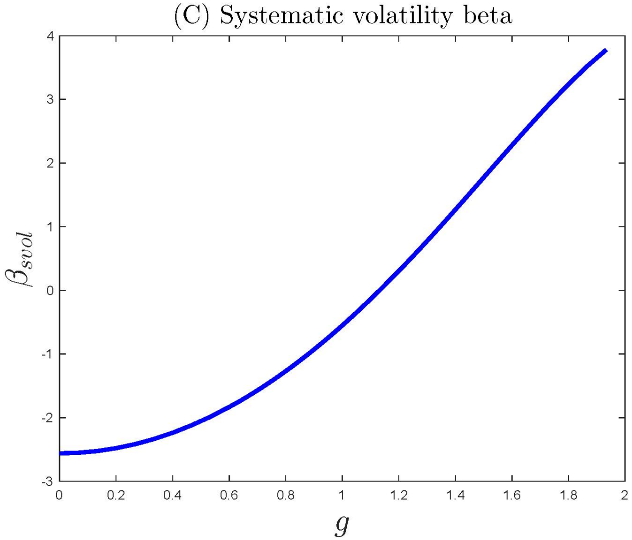
$$+ \sum_{i=0}^{I} \Big(q_i \exp\Big\{\lambda \mu_{D,i} T - 0.5 \lambda h_i^2 \int_0^T v_{1\tau} d\tau - 0.5 \lambda (1-\lambda) g_i^2 \int_0^T v_{2\tau} d\tau + \lambda h_i \int_0^T \sqrt{v_{1\tau}} dw_\tau\Big\}\Big) K^{1-\lambda}$$

- The formula above is for aggregate terminal consumption
- If we look at the exponent in the second sum
  - The first piece yields the standard CAPM risk premium
  - The next two pieces show the need for the systematic and idiosyncratic volatility factors
  - The fourth piece looks like conditional market beta, but is empirically subsumed by the aggregate volatility factors

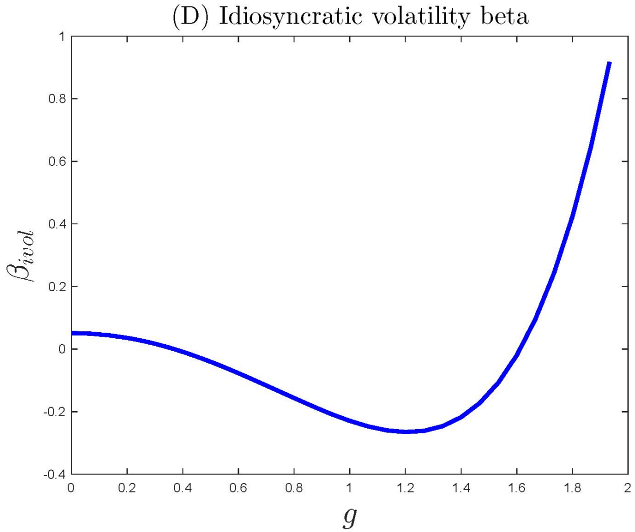
Simulations: Alphas



Simulations: Market Volatility Betas



Simulations: IVol Factor Betas



ICAPM Interpretation

- Our model is a one-period model and has no hedging demand
- Volatility factors arises because of non-linearities and stochastic volatility
- However, the empirical aggregate volatility risk factors can also be viewed in the ICAPM sense, since both market volatility and average IVol are related to the state of the economy

Average IVol and Business Cycle

	-12	-6	-3	0	3	6	12
NBER	9.706	16.48	19.07	19.66	14.89	8.608	-4.196
t-stat	*1.61*	*2.92*	*3.25*	*3.19*	*2.47*	*1.40*	*-0.69*
VIX	0.132	0.216	0.275	0.351	0.267	0.201	0.147
t-stat	*2.02*	*3.33*	*4.07*	*5.14*	*3.73*	*2.94*	*2.21*
TARCH	0.117	0.225	0.304	0.397	0.436	0.367	0.267
t-stat	*1.45*	*3.06*	*4.15*	*5.40*	*5.78*	*4.55*	*3.28*
Realized	0.096	0.177	0.216	0.278	0.195	0.150	0.099
t-stat	*2.02*	*3.90*	*4.69*	*5.90*	*4.00*	*3.11*	*2.00*

Summary of Results

- If the economy slows down and the market becomes more volatile, average IVol in the next 9-12 months increases by 15-20%
- Conversely, a spike in average IVol can predict recession and higher market volatility up to 12 months ahead
- A two-standard deviation change in market volatility can trigger an increase in average IVol by 30-70%
- We also show that FIVol and FVIX can predict the state of the economy and market volatility/average IVol in the following 3-6 months

Defining the State Variables

- Our proxy for shocks to expected market volatility is change in VIX
- We use old VIX (implied volatility of S&P100 one-month near-the-money options) to increase the sample period
- Our proxy for shocks to IVol is residuals for ARMA(1,1) model fitted to average IVol in the economy
- IVol is standard deviation of residuals from FF3 model fitted to daily returns in each firm-month
- Then IVol is averaged across all firms in the market each month

Defining the Risk Factors

- FVIX mimics daily changes in VIX
- We regress daily changes in VIX on excess returns to five quintile portfolios sorted on prior month sensitivity to VIX changes
- The fitted part of the regression less the constant is the FVIX factor
- The correlation between FVIX and the change in VIX is 0.676
- FIVol mimics ARMA(1,1) shocks to average IVol in the same way
- The correlation between FIVol and shocks to average IVol is 0.424

FVIX Factor: Alphas and Betas

	Raw	CAPM	ICAPM	FF	FF4	FF5	FF6
α	-1.366	-0.463	-0.480	-0.439	-0.446	-0.305	-0.318
t-stat	*-4.77*	*-4.73*	*-4.32*	*-4.00*	*-3.68*	*-3.73*	*-3.50*
β_{MKT}		-1.325	-1.340	-1.358	-1.366	-1.407	-1.427
t-stat		*-37.0*	*-29.9*	*-35.2*	*-37.30*	*-50.70*	*-51.65*
β_{SMB}				0.170	0.166	0.107	0.095
t-stat				*4.94*	*4.33*	*4.56*	*3.49*
β_{HML}				-0.073	-0.078	0.034	0.023
t-stat				*-1.41*	*-2.00*	*0.59*	*0.46*
β_{FIVol}			-0.014		-0.007		-0.017
t-stat			*-0.47*		*-0.28*		*-0.74*
β_{CMA}						-0.142	*-0.15*
t-stat						*-2.31*	*-2.22*
β_{RMW}						*-0.22*	*-0.23*
t-stat						*-6.15*	*-5.89*

FIVol Factor: Alphas and Betas

	Raw	CAPM	ICAPM	FF	FF4	FF5	FF6
α	-1.923	-1.200	-1.361	-0.952	-1.040	-0.743	-0.908
t-stat	*-3.70*	*-2.55*	*-3.30*	*-2.62*	*-3.16*	*-1.87*	*-2.59*
β_{MKT}		-1.061	-1.464	-1.094	-1.298	-1.173	-1.785
t-stat		*-6.59*	*-1.49*	*-10.3*	*-1.65*	*-9.87*	*-2.06*
β_{SMB}				-0.592	-0.569	-0.682	-0.635
t-stat				*-3.70*	*-3.13*	*-4.71*	*-3.75*
β_{HML}				-0.800	-0.812	-0.618	-0.618
t-stat				*-4.36*	*-4.87*	*-3.08*	*-2.79*
β_{FVIX}			-0.304		-0.151		-0.439
t-stat			*-0.47*		*-0.29*		*-0.78*
β_{CMA}						-0.268	-0.302
t-stat						*-0.99*	*-0.95*
β_{RMW}						-0.329	-0.416
t-stat						*-1.88*	*-2.09*

FVIX and FIVol: Alphas and Betas

- CAPM/FF3 alpha of FVIX is -46 bp/-44 bp per month
- CAPM/FF3 alpha of FIVol is -120 bp/-95 bp per month
- Controlling for market beta, there is little overlap between FVIX and FIVol
- FVIX does not overlap with HML, but does overlap with RMW (see Barinov, 2020 on the latter)
- The reverse is true for FIVol
- FF5 alphas of FVIX and FIVol are significant at -31 bp and -74 bp per month, respectively

Double Sorts: Hypotheses

- In the model without growth options, the CAPM holds
- Growth options create a hedge against aggregate volatility risk
- If a firm has higher IVol, growth options take a larger fraction of the firm's value
- Hence, high IVol growth firms are the best hedges against aggregate volatility risk

Portfolio Sorts: CAPM alphas

	Low	IVol2	IVol3	IVol4	High	L-H
Value	0.298	0.294	0.427	0.262	0.102	0.196
t-stat	*1.25*	*1.25*	*1.67*	*0.98*	*0.31*	*0.51*
MB2	0.311	0.301	0.260	0.438	-0.204	0.514
t-stat	*1.84*	*1.92*	*1.21*	*1.81*	*-0.94*	*1.89*
MB3	0.277	0.318	0.175	-0.092	-0.323	0.600
t-stat	*1.95*	*1.94*	*1.06*	*-0.54*	*-1.65*	*2.27*
MB4	0.441	0.326	-0.031	0.135	-0.473	0.914
t-stat	*3.05*	*2.48*	*-0.21*	*0.88*	*-2.23*	*3.25*
Growth	0.300	0.153	0.064	-0.220	-0.665	0.965
t-stat	*2.31*	*1.14*	*0.43*	*-1.16*	*-2.89*	*3.28*
V-G	-0.002	0.141	0.363	0.482	0.767	0.769
t(V-G)	*-0.01*	*0.50*	*1.18*	*1.37*	*2.14*	*2.20*

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Portfolio Sorts: Alphas Controlling for Volatility Risk

	Low	IVol2	IVol3	IVol4	High	L-H
Value	-0.090	-0.119	0.152	0.133	0.017	-0.107
t-stat	*-0.43*	*-0.54*	*0.58*	*0.52*	*0.05*	*-0.27*
MB2	0.002	-0.016	-0.091	0.114	-0.031	0.033
t-stat	*0.01*	*-0.11*	*-0.50*	*0.59*	*-0.13*	*0.11*
MB3	-0.085	-0.044	-0.068	-0.315	-0.072	-0.012
t-stat	*-0.56*	*-0.22*	*-0.40*	*-1.82*	*-0.38*	*-0.05*
MB4	0.088	-0.068	-0.177	0.157	-0.099	0.187
t-stat	*0.68*	*-0.50*	*-1.16*	*1.05*	*-0.36*	*0.58*
Growth	0.130	0.025	0.183	0.084	0.121	0.009
t-stat	*1.08*	*0.16*	*1.08*	*0.48*	*0.47*	*0.03*
V-G	-0.220	-0.144	-0.032	0.049	-0.104	0.116
t(V-G)	*-0.86*	*-0.54*	*-0.10*	*0.15*	*-0.30*	*0.32*

Portfolio Sorts: FVIX Betas

	Low	IVol2	IVol3	IVol4	High	L-H
Value	-0.342	-0.262	0.127	0.649	0.749	-1.091
t-stat	*-1.57*	*-1.68*	*0.48*	*3.87*	*1.89*	*-1.93*
MB2	-0.433	-0.324	-0.148	0.090	0.937	-1.370
t-stat	*-4.31*	*-2.69*	*-0.83*	*0.33*	*3.72*	*-4.21*
MB3	-0.561	-0.488	-0.231	-0.036	0.905	-1.466
t-stat	*-4.73*	*-2.61*	*-1.38*	*-0.19*	*4.90*	*-5.87*
MB4	-0.722	-0.715	-0.140	0.271	0.989	-1.711
t-stat	*-5.04*	*-3.93*	*-0.74*	*2.08*	*3.06*	*-4.23*
Growth	-0.517	-0.351	0.192	0.542	1.586	-2.103
t-stat	*-2.76*	*-2.84*	*1.40*	*3.74*	*4.61*	*-4.52*
V-G	0.175	0.089	-0.065	0.107	-0.837	-1.012
t(V-G)	*0.86*	*0.59*	*-0.25*	*0.65*	*-2.94*	*-3.67*

Portfolio Sorts: FIVol Betas

	Low	IVol2	IVol3	IVol4	High	L-H
Value	-0.188	-0.240	-0.274	-0.352	-0.354	0.166
t-stat	*-3.89*	*-4.78*	*-5.66*	*-4.45*	*-3.56*	*1.50*
MB2	-0.089	-0.137	-0.232	-0.300	-0.214	0.125
t-stat	*-3.70*	*-5.07*	*-9.84*	*-8.73*	*-3.22*	*1.77*
MB3	-0.084	-0.112	-0.111	-0.169	-0.138	0.054
t-stat	*-3.08*	*-3.60*	*-3.86*	*-4.66*	*-5.13*	*1.30*
MB4	-0.015	-0.052	-0.067	-0.085	-0.069	0.054
t-stat	*-0.50*	*-2.14*	*-2.28*	*-3.89*	*-1.90*	*1.11*
Growth	0.057	0.028	0.025	0.044	0.042	0.014
t-stat	*1.99*	*1.43*	*0.95*	*1.45*	*0.99*	*0.23*
V-G	-0.245	-0.268	-0.299	-0.396	-0.397	-0.152
t(V-G)	*-5.55*	*-5.04*	*-4.95*	*-4.04*	*-3.82*	*-1.67*

Portfolio Sorts: Alphas Summary

- High IVol growth portfolio has the most negative CAPM alpha of -67 bp per month
- This alpha is largely responsible for stronger value effect for high IVol firms (77 bp per month) than for low IVol (-0.2 bp per month)
- The same alpha is largely responsible for stronger IVol discount for growth firms (97 bp per month vs. 20 bp per month for value firms)
- All those alphas are reduced to roughly 10 bp per month after we control for FVIX and FIVol

Portfolio Sorts: Betas Summary

- FVIX betas of low-minus-high IVol portfolios are all negative and become more negative in the growth subsample
- FVIX beta of the value-minus-growth portfolio is zero outside of the high IVol quintile
- FIVol betas of the value-minus-growth portfolio are all negative and become more negative in the high IVol subsample
- FIVol betas of low-minus-high IVol portfolios are all zero

Arbitrage Portfolios: Three Idiosyncratic Volatility Effects

- IVol - low IVol quintile minus high IVol quintile
- IVolh - same only for growth firms
- HMLh - value minus growth for high IVol only
- IVol55 - excess return to the highest IVol and highest M/B portfolio

Explaining the IVol Effects

	α_{CAPM}	α_{FF3}	$\alpha_{Carhart}$	α_{FF5}	α_{CCAPM}	α_{VolF}	β_{FVIX}	β_{FIVol}
HML	0.310				0.244	-0.074	-0.429	-0.152
t-stat	*1.56*				*1.50*	*-0.40*	*-1.85*	*-5.05*
IVol	0.789	0.633	0.529	0.340	0.565	0.126	-1.409	-0.009
t-stat	*4.13*	*4.74*	*5.56*	*4.29*	*3.16*	*0.51*	*-4.05*	*-0.19*
IVolh	1.151	0.865	0.672	0.313	0.864	0.090	-2.082	-0.080
t-stat	*4.08*	*4.94*	*4.07*	*2.48*	*3.29*	*0.27*	*-3.70*	*-1.19*
HMLh	1.092	0.703	0.723	0.417	0.922	0.303	-0.812	-0.339
t-stat	*3.64*	*4.35*	*5.41*	*3.37*	*3.57*	*1.09*	*-2.81*	*-6.82*
IVol55	-0.888	-0.644	-0.534	-0.369	-0.729	-0.107	1.632	0.021
t-stat	*-3.87*	*-5.75*	*-4.76*	*-3.76*	*-3.11*	*-0.35*	*3.72*	*0.39*

Explaining the IVol Effects: Summary

- CAPM/FF3/Carhart/CCAPM model cannot explain the IVol effects
- FF5 cuts them roughly in half because of RMW
- Barinov (2020) shows that FVIX can explain RMW, but not the other way around
- The volatility factor model with FVIX and FIVol makes the IVol effects insignificant and reduces the alphas from roughly 60-80 bp per month to 10 bp per month

Conclusion

- Growth options provide a hedge against increases in market volatility and average IVol
- Growth options of high IVol firms are particularly valuable, thus high IVol firms provide this hedge too
- We predict and higher IVol discount for growth firms, which can be explained controlling for aggregate volatility risk
- Similarly, aggregate volatility risk explains higher value effect for high IVol firms
- Aggregate volatility risk also explains the value effect and the IVol discount across the board