

# Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns

**Alexander Barinov**

SCHOOL OF BUSINESS

UNIVERSITY OF CALIFORNIA RIVERSIDE

900 University Ave., Riverside, CA 92521 USA

Tel: +1 585 698 7726

E-mail: [abarinov@ucr.edu](mailto:abarinov@ucr.edu)

<http://faculty.ucr.edu/~abarinov/>

**Georgy Chabakauri**

DEPARTMENT OF FINANCE

LONDON SCHOOL OF ECONOMICS

Houghton Street, WC2A 2AE, London, UK

Tel: +44 (0) 20 7107 5374

E-mail: [G.Chabakauri@lse.ac.uk](mailto:G.Chabakauri@lse.ac.uk)

<http://personal.lse.ac.uk/chabakau/>

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## **Abstract**

The paper shows that the value effect and the idiosyncratic volatility (IVol) discount arise because growth firms and high IVol firms beat the CAPM during the periods of increasing aggregate volatility, which makes their risk low. All else equal, growth options' value increases with volatility, and this effect is stronger for high IVol firms, for which growth options take a larger fraction of the firm value and firm volatility responds more to aggregate volatility changes. The factor model with the market factor, the market volatility risk factor and the average IVol factor explains the value effect and the IVol discount.

**JEL Classification:** G12, G13, E44

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# 1 Introduction

Baseline asset pricing models such as the CAPM imply that firm idiosyncratic risks should not affect stock returns. However, growing empirical evidence points towards the importance of idiosyncratic risks for determining asset prices and returns. In particular, Ang, Hodrick, Xing, and Zhang (2006) find that firms with high idiosyncratic volatility (IVol) earn negative abnormal returns, which gives rise to the IVol discount. Furthermore, Ali, Hwang, and Trombley (2003) find that the value effect is about 6% per year larger for high IVol firms. The latter finding poses a challenge to any risk-based explanation of the value effect because any such explanation has to explain why the value effect is related to IVol, which is seemingly not risk. In this paper, we show both theoretically and empirically that the market volatility and average IVol risk factors provide a unifying explanation for the IVol discount, the value effect, and the dependence of the value effect on IVol.<sup>1</sup>

We start with a parsimonious theoretical model that captures the effect of systematic and idiosyncratic volatilities on asset returns and then use it to guide our empirical analysis. More specifically, we consider a finite-horizon economy with one representative investor with power utility over consumption at the terminal date and a continuum of firms. The firms own projects which consist of exogenous streams of cash flows, modelled as Lucas (1978) trees, as well as real growth options written on these trees. The streams of cash flows are subject to systematic and idiosyncratic shocks with time-varying volatilities, which we refer to as systematic and idiosyncratic volatilities, respectively. Moreover, different groups of firms have different levels of IVol, given by a common time-varying component (the same for all firms) multiplied by a constant firm-specific parameter<sup>2</sup>. In line with the empirical evidence (Bartram et al., 2016, and Herskovic et al., 2016), we assume that changes in

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<sup>1</sup>“Aggregate volatility risk” stands for the risk due to both market and idiosyncratic volatility risks.

<sup>2</sup>Empirically, the common time-varying component in IVol can be proxied by average IVol.

the systematic and idiosyncratic volatilities are positively correlated with each other, and negatively correlated with systematic shocks driving the aggregate output.

Our model generates the value effect and the IVol discount in the simulated data. We estimate the standard CAPM in the simulated data and find that stocks with high price-dividend ratios (dubbed growth stocks) have negative alphas whereas stocks with low price-dividend ratios (dubbed value stocks) have positive alphas. Consequently, growth stocks seem overvalued relative to the value stocks. Furthermore, we find that stock price-dividend ratios are increasing functions of IVol. The latter result together with our finding on CAPM alphas implies that stocks with high IVol have negative abnormal returns, giving rise to IVol discount.

Next, we show that controlling for market volatility risk and average IVol risk explains the value effect and IVol discount. First, we show that systematic volatility and average IVol emerge as important state variables affecting asset returns and state price densities. Motivated by the latter finding, in our simulated data, we study the performance of a linear volatility factor model that in addition to the market portfolio also includes shocks to the market volatility and the common component in IVol. We find that the estimates of alphas from the volatility factor model no longer depend on either IVol or price-dividend ratio and are very close to zero. Moreover, volatility betas are increasing functions of IVol and are positive for high IVol stocks and growth stocks, and the corresponding volatility risk premia are negative.

The economic intuition for our results is based on the observation that growth stocks and high IVol stocks are hedges against aggregate volatility<sup>3</sup> increases, holding market return fixed. Growth stocks, for which growth options take a larger fraction of their value, are hedges against aggregate volatility increases because options' value increases in

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<sup>3</sup>“Aggregate volatility” stands for both market and average idiosyncratic volatility.

volatility, all else equal. Higher volatility also increases the likelihood of triggering growth options, which increases stock price-dividend ratios. Moreover, because consumption and volatility (both market volatility and IVol) shocks are negatively correlated in our model, spikes in volatility occur in bad times. Consequently, high IVol stocks, which also happen to be stocks with higher price-dividend ratio, can be used to hedge exposure to aggregate volatility shocks.<sup>4</sup>

In both real and simulated data, we form factor-mimicking portfolios for the market and idiosyncratic volatilities by regressing volatility shocks on asset returns. We find that these portfolios, being hedges against aggregate volatility increases, have negative risk premiums. Negative risk premiums arise because shocks to both market volatility and average IVol are negatively correlated with shocks to aggregate output, and hence, buying volatility hedges compensates investors when aggregate consumption decreases. Moreover, in the volatility factor model with the market factor and the factor-mimicking portfolios for the market and idiosyncratic volatilities, high IVol firms and growth firms have positive betas for the volatility factor-mimicking portfolios, both in simulated and real data, for the reasons described in the previous paragraph. Consequently, positive betas and negative risk premia for the volatility factors cause high IVol firms and growth firms have negative alphas in the standard CAPM, where volatility factors are omitted, and zero alphas in the volatility factor model.

Our main empirical prediction is therefore that the value effect and the IVol discount can be explained by controlling for a market volatility factor and an average IVol factor. Our market volatility risk factor (hereafter - FVIX factor) tracks daily changes in the

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<sup>4</sup>Because market returns are strongly negatively correlated with aggregate volatility, any stock with a positive market beta, including high IVol stocks and growth stocks, will react negatively to increases in aggregate volatility. Our prediction is that high IVol stocks and growth stocks react less negatively to aggregate volatility increases than what their market beta implies. We do not predict, however, that growth stocks and high IVol stocks will go up in value when aggregate volatility increases.

CBOE VIX index, which measures the implied volatility of S&P 100 options.<sup>5</sup> Ang et al. (2016) show that changes in VIX are a good proxy for changes in expected market volatility. Our average IVol factor (hereafter - FIVol) similarly mimics innovations to firm-level IVol averaged within each month across all firms in the economy.

We start our empirical tests by sorting firms on market-to-book and IVol. We find that the volatility factor model with the two volatility factors completely explains the IVol discount and the alpha of the HML factor, just as the volatility factors did in the simulated data. We also find that FVIX is the main driving force behind the IVol discount, whereas FIVol is more helpful in explaining the value effect.

Volatility betas are positive for growth firms, since option value increases in volatility all else equal. In the model, the weight of growth options in total firm value increases in IVol, and, as a result, are volatility betas higher for firms with larger IVol. The latter relation is naturally stronger if the weight of growth options in total firm value is higher. In the empirical tests, we double sort on market-to-book and IVol and find that the IVol discount is zero for value firms and increases in market-to-book. Similarly, the value effect is zero for low IVol firms and monotonically increases with IVol. A similar pattern emerges in FVIX and FIVol betas - the spread in volatility factors betas between low and high IVol firms (value and growth firms) increases in market-to-book (IVol). Controlling for FVIX and FIVol thus eliminates the alpha of the low-minus-high IVol portfolio even among growth firms and reduces by about 70% the alpha of the value-minus-growth strategy for high IVol firms, thus providing a risk-based explanation to the evidence in Ali et al. (2003) that the value effect is stronger for high IVol firms.

Finally, we discuss possible alternative explanations for IVol discount and robustness checks. Several papers suggest mispricing explanations of the IVol discount that are related

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<sup>5</sup>We use the older version of VIX (current ticker VXO) to have a longer sample period.

to short sale constraints. Nagel (2005) uses institutional ownership (IO) as a proxy for supply of shares for shorting and finds that the value effect and the IVol discount are stronger for the firms with low IO. Boehme et al. (2009) find that the IVol discount is higher when short sales are more likely to be very costly. Stambaugh et al. (2015) argue that IVol serves as a limits-to-arbitrage variable and thus is negatively/positively related to future returns for overpriced/underpriced firms, but on average the negative relation dominates, because overpricing is stronger than underpricing due to short sale constraints.

After we control for aggregate volatility risk, the value effect and the IVol discount become insignificant even in the subsample of stocks with the tightest short sale constraints. Aggregate volatility risk also largely explains the strong IVol discount for overpriced firms observed by Stambaugh et al. We conclude that the relation between the value effect/IVol discount and short sale constraints is consistent with our aggregate volatility risk explanation of the value effect/IVol discount and does not necessarily suggest mispricing.

We perform several robustness checks, which confirm that buying value (low IVol) firms and shorting growth (high IVol) firms results in inferior performance during hard times, especially if this strategy is followed in the high IVol (growth) subsample, low IO subsample, or high probability to be on special subsample. We also find that our results are robust to several alternative definitions of volatility risk factors.

The paper proceeds as follows. Section 2 provides a brief literature review. Section 3 develops our model. Section 4 discusses the data sources. Section 5 shows that the value effect and the IVol discount are explained by aggregate volatility risk. Section 6 considers the competing behavioral stories. Section 7 summarizes the robustness checks. Section 8 offers the conclusion.

## 2 Literature Review

The concept of market volatility risk was developed in the ICAPM-type models of Campbell (1993) and Chen (2002). In Campbell (1993), higher market volatility implies lower expected future consumption. Stocks that covary negatively with changes in market volatility command a risk premium because they lose value when the future is also turning bleak. In Chen (2002), investors care not only about future returns, but also about future volatility. Since market volatility is persistent, increases in market volatility imply the need to boost precautionary savings and cut current consumption. The stocks that covary negatively with market volatility changes again command a risk premium, but for a different reason. They lose value exactly when consumption is reduced to build up savings.

A recent paper by Herskovic et al. (2016) argues that the common component in IVol is also a state variable in the ICAPM sense. Herskovic et al. derive the role of the common component in IVol in a model with heterogeneous consumers who possess firm-specific human capital and cannot completely hedge out idiosyncratic shocks to labor income. As average IVol in the economy increases, large adverse shocks to labor income become more likely, and thus consumers value hedges against average IVol increases. Herskovic et al. show empirically that sorting firms on their exposures to changes in the common component of IVol produces the spread in expected returns of 5.4% per year.

Our paper contributes to the literature on pricing of market volatility risk and average IVol risk by predicting which firms (high IVol firms, growth firms) are less exposed to both types of volatility risk and using volatility risk to explain two important anomalies: the value effect and the IVol discount.

A large strand of literature, starting with Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004) assumes that growth options, as levered claims, are riskier

than assets in place, and then argues that market-to-book captures the process of exercising growth options and thus is negatively related to expected returns despite growth options being riskier than assets in place. Our model suggests that this negative relation arises because growth options are inherently less risky as hedges against aggregate volatility risk.

Johnson (2004) shows that disagreement about the value of the underlying asset is negatively related to systematic risk of real options, because the beta of a call option is negatively related to volatility. Babenko et al. (2016) consider an economy where the firm value is additive in idiosyncratic and systematic shocks, and the beta of the firm depends on the weight of the zero-beta “idiosyncratic part” in the firm value. They assume that growth options tend to depend more on idiosyncratic shocks and thus conclude that IVol and market-to-book, as well as their product, will be negatively related to conditional CAPM beta and hence to expected returns. Chen et al. (2020) develop a model with an alternative explanation of procyclical betas of high IVol firms: in their model, distressed high IVol firms engage in risk-shifting by undertaking high-volatility projects, which reduces the risk of their equity (and increases the risk of their debt). Risk-shifting behavior is more common in recessions, when firms are closer to going bankrupt.

The common feature of all three models from the previous paragraph is that the effect of IVol on expected returns works through the beta and the conditional CAPM holds if one measures the beta correctly. Our model is different from the models above because the state price density in our model is endogenous and depends on time-varying systematic and idiosyncratic volatilities. Consequently, these volatilities appear as separate factors that explain the cross-section of returns. Empirically, in Table 4 and in Section 2 of the robustness appendix, we show that while the conditional CAPM cannot fully explain the value effect and IVol discount, our volatility factors can, and in doing so they subsume the



role of time-varying market beta.<sup>6</sup>

McQuade (2018) builds a model with real options, stochastic volatility, and default risks to address the value effect, financial distress and credit spread puzzles. Our paper differs in two ways. First, the main driving force in our model is IVol, whereas all volatility in McQuade (2018) is systematic. Consequently, in addition to the value effect, we also address the IVol discount. Second, while McQuade (2018) assumes the existence of an exogenous risk-neutral measure, our risk neutral measure and state price density (SPD) are endogenous and derived in a general equilibrium. The derivation of the SPD then allows us to follow a standard approach outlined in Cochrane (p. 106, 2005) and identify the priced factors as state variables that determine the SPD.

Campbell et al. (2018) derive an ICAPM with systematic stochastic volatility that explains the value effect and makes progress towards explaining the IVol discount, although IVol alpha is large. In contrast to their model, we incorporate stochastic idiosyncratic volatility and real options, which allows us to simultaneously explain the value effect and IVol discount.

Empirically, both Campbell et al. and McQuade imply that the value effect and IVol discount should be explained by a factor that captures long-run shifts in market volatility. In our empirical tests, we find that, first, an average IVol-based risk factor (FIVol) is necessary to explain the value effect, and second, that the IVol discount is explained by FVIX, the factor capturing short-run market volatility changes.

The empirical study closest to our paper is Ang et al. (2006), which is the first to establish the IVol discount and the pricing of market volatility risk. In their Table 9, they make sorts on IVol conditional on FVIX betas, which reduces the IVol discount by mere 12

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<sup>6</sup>An additional difference between our results and the predictions of the Chen et al. (2020) model is that the latter predicts stronger IVol discount for value, not growth firms, as value firms are more likely to be distressed and engage in risk-shifting.

bp per month, and conclude that market volatility risk cannot explain the IVol discount. In our paper, we perform a more direct test. We use the volatility factor model with the market factor, FVIX, and FIVol and find, in contrast to Ang et al., that the IVol discount is completely explained by aggregate volatility risk. Furthermore, we provide a theory behind the link between the IVol discount and aggregate volatility risk, and the theory generates a novel prediction that the IVol discount is stronger for growth firms.

We also use an improved factor-mimicking procedure to form FVIX. Ang et al. (2006) perform the factor-mimicking regression separately for each month, whereas we perform a single factor-mimicking regression using all available data. In Section 3.1 of the robustness appendix<sup>7</sup>, we find that their FVIX is significantly correlated with our version of FVIX and produces betas of the same sign if used instead of our FVIX. However, the estimates using 22 data points are imprecise, and the imprecise estimation of the constant in Ang et al. makes their FVIX factor premium small and insignificant, which causes the inability of Ang et al.'s FVIX to explain the IVol discount.

Two recent papers, Chen and Petkova (2012) and Herskovic et al. (2016) suggest using different versions of an IVol-based risk factor, and while both show that their IVol factor is priced, Herskovic et al. find that the IVol factor cannot explain the IVol discount, while Chen and Petkova (2012) reach the opposite conclusion. Our empirical results land closer to Herskovic et al. – we find that the role of FIVol in explaining the IVol discount is limited, but also find that FVIX can explain the IVol discount, which is the hypothesis Herskovic et al. do not test. The difference between our results and those of Chen and Petkova stems from the fact that the performance of their IVol factor is not robust to the choice of the base assets in the factor-mimicking regression.<sup>8</sup> Herskovic et al. also do not

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<sup>7</sup>The robustness appendix is available at <http://faculty.ucr.edu/~abarinov/Robustness 2021.pdf>

<sup>8</sup>Section 3.3 of the robustness appendix provides further details on how our FIVol and Chen and Petkova's IVol factor compare.

consider the ability of their IVol factor to explain the value effect, while Chen and Petkova find mixed results. Our contribution is that we find that FIVol reliably explains the value effect and its cross-sectional dependence on IVol.

## 3 The Model

### 3.1 Economic Setup

We consider a continuous-time economy with  $I + 1$  types of firms,  $i = 0, 1, \dots, I$ . There are  $N$  firms of each type, so that the total number of firms is  $N \times (I + 1)$ . Each firm has double index  $(i, n)$ , which characterizes its type and its number within the type. Each firm  $(i, n)$  has assets in place with terminal payoff,  $p_i D_{0,n,T}$ , and growth options with terminal payoff  $q_i (D_{i,n,T} / (K D_{i,n,0}))^\lambda K D_{i,n,0}$ , where  $K D_{i,n,0}$  is an analogue of a strike price and  $\lambda$  captures convexity, as discussed in Remark 1 below. Therefore, firm  $(i, n)$ 's time- $T$  output is given by  $S_{i,n,T} = p_i D_{0,n,T} + q_i (D_{i,n,T} / (K D_{i,n,0}))^\lambda K D_{i,n,0}$ .

Processes  $D_{i,n,t}$  have stochastic systematic and idiosyncratic volatilities, and evolve as

$$dD_{i,n,t} = D_{i,n,t} [\mu_{D,i} dt + h_i \sqrt{v_{1t}} dw_t + g_i \sqrt{v_{2t}} dw_{i,n,t}], \quad (1)$$

where  $n = 1, \dots, N$ ,  $i = 0, \dots, I$ ,  $D_{i,n,0} = 1/N$ , and  $\mu_{D,i}$ ,  $h_i$ , and  $g_i$  are constants,  $w_t$  is a systematic and  $w_{i,n,t}$  is a firm-specific idiosyncratic shock modeled as Brownian motions. Accordingly, we label  $h_i \sqrt{v_{1t}}$  and  $g_i \sqrt{v_{2t}}$  as *systematic volatility* and *idiosyncratic volatility*, respectively, where  $v_{1t}$  and  $v_{2t}$  follow Heston (1993) processes

$$dv_{1t} = \kappa_1 (\bar{v}_1 - v_{1t}) dt + c_1 \sqrt{v_{1t}} dw_t, \quad (2)$$

$$dv_{2t} = \kappa_2 (\bar{v}_2 - v_{2t}) dt + c_2 \sqrt{v_{2t}} d\tilde{w}_t, \quad (3)$$

where  $\kappa_1 > 0$ ,  $\kappa_2 > 0$ ,  $\bar{v}_1 \geq 0$ ,  $\bar{v}_2 \geq 0$ ,  $c_1 < 0$ ,  $c_2 < 0$ , and  $\tilde{w}$  is the Brownian motion driving the idiosyncratic volatility factor in the economy. The coefficients  $c_1$  and  $c_2$  are negative

to capture a stylized fact that volatility and dividend shocks are negatively correlated (see, e.g., Duarte et al., 2012, Herskovic et al., 2016). We also set the correlation between  $dw$  and  $d\tilde{w}$  to  $\rho$ , and assume that  $\rho > 0$  because the processes for the systematic and idiosyncratic volatility are highly correlated in the data (see, e.g., Barinov, 2013, Bartram et al., 2016).

There is one representative investor with CRRA utility over terminal consumption, given by  $u(C_T) = C_T^{1-\gamma}/(1-\gamma)$ . The financial market is complete, and investors can trade shares in firms and invest in a riskless bond with a fixed interest rate  $r_f$ .<sup>9</sup> The state price density (s.p.d.)  $\xi_t$  is endogenous and at terminal date  $T$  is given by the representative investor's marginal utility  $\xi_T = \psi C_T^{-\gamma}$ , where  $\psi$  is a Lagrange multiplier for the investor's static budget constraint. The s.p.d. at time  $t$  is determined from equation  $\xi_t = e^{r_f(T-t)}\mathbb{E}_t[\xi_T]$ , so that  $e^{r_f t}\xi_t$  is a martingale. By  $S_{i,n,t}$  we denote the time- $t$  equilibrium stock price of firm  $(i, n)$ .

**Remark 1 (Modeling Growth Options).** The growth option payoffs can be also modeled as  $(D_{i,n,T} - KD_{i,n,0})^+$ . However, the options with such payoffs are intractable. Therefore, we rewrite option payoff as  $(D_{i,n,T} - KD_{i,n,0})^+ = (D_{i,n,T}/(KD_{i,n,0}) - 1)^+ KD_{i,n,0}$ . Then, similar to Barro (2006), we observe that the convexity of  $(D_{i,n,T}/(KD_{i,n,0}) - 1)^+$  is similar to that of  $(D_{i,n,T}/(KD_{i,n,0}))^\lambda$ . Hence, we obtain the option payoff  $(D_{i,n,T}/(KD_{i,n,0}))^\lambda KD_{i,n,0}$ .

## 3.2 Dynamic Equilibrium

In this subsection we characterize the aggregate consumption  $C_T$ , the state price density  $\xi_T$ , and the stock prices in the economy. By invoking the law of large numbers we demonstrate the  $C_T$  and  $\xi_T$  depend on cumulative idiosyncratic volatility.

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<sup>9</sup>As in the related literature, we note that the interest rate cannot be determined endogenously in a model with consumption over terminal date, and hence we set it exogenously.

### 3.2.1 Aggregate Consumption and State Price Density

Given the state price density, the stock values are given by

$$S_{i,n,t} = \frac{1}{\xi_t} \mathbb{E}_t \left[ \xi_T \left( p_i D_{0,n,T} + q_i \left( \frac{D_{i,n,T}}{K D_{i,n,0}} \right)^\lambda K D_{i,n,0} \right) \right]. \quad (4)$$

To obtain the s.p.d., we first derive the aggregate consumption  $C_T$  from the equilibrium consumption clearing condition  $C_T = \sum_{i=0}^I \sum_{n=1}^N S_{i,n,T}$ , which equates aggregate consumption to aggregate output at the final date. Solving Equation (1) and then substituting  $D_{i,n,T}$  into the expression for consumption  $C_T$ , and setting  $D_{i,n,0} = 1/N$  we obtain:

$$\begin{aligned} C_T &= \left( \sum_{i=0}^I p_i \right) \left( \frac{1}{N} \sum_{n=1}^N \frac{D_{0,n,T}}{D_{0,n,0}} \right) + \sum_{i=0}^I \left( q_i \frac{1}{N} \sum_{n=1}^N \left( \frac{D_{i,n,T}}{K D_{i,n,0}} \right)^\lambda K \right). \\ &= \left( \sum_{i=0}^I p_i \right) \exp \left\{ \mu_{D,0} T - 0.5 h_0^2 \int_0^T v_{1\tau} d\tau - 0.5 g_0^2 \int_0^T v_{2\tau} d\tau + h_0 \int_0^T \sqrt{v_{1\tau}} dw_{1\tau} \right\} \times \\ &\quad \frac{1}{N} \sum_{n=1}^N \exp \left\{ g_0 \int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau} \right\} \\ &+ \sum_{i=0}^I \left( q_i \exp \left\{ \lambda \mu_{D,i} T - 0.5 \lambda h_i^2 \int_0^T v_{1\tau} d\tau - 0.5 \lambda g_i^2 \int_0^T v_{2\tau} d\tau + \lambda h_i \int_0^T \sqrt{v_{1\tau}} dw_{1\tau} \right\} \times \right. \\ &\quad \left. \frac{1}{N} \sum_{n=1}^N \exp \left\{ \lambda g_i \int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau} \right\} \right) K^{1-\lambda}. \end{aligned} \quad (5)$$

We observe that random variables  $\int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau}$  in (5) are i.i.d. normal  $\mathcal{N}(0, \int_0^T v_{2\tau} d\tau)$  in the cross-section of firms, conditional on knowing the realizations of volatilities  $v_{2\tau}$ . This is because volatility  $v_{2\tau}$  is the same for all the firms in the cross-section and is driven by Brownian motion  $\tilde{w}_t$  which is independent of idiosyncratic Brownian motions  $w_{i,n,t}$ . Using this observation, we simplify the expression for the aggregate consumption  $C_T$  by using the law of large numbers. Proposition 1 below reports the results.

**Proposition 1 (Aggregate Consumption).**

Aggregate terminal consumption in the economy is given by:

$$\begin{aligned}
C_T &= \left( \sum_{i=0}^I p_i \right) \exp \left\{ \mu_{D,0} T - 0.5 h_0^2 \int_0^T v_{1\tau} d\tau + h_0 \int_0^T \sqrt{v_{1\tau}} dw_\tau \right\} \\
&+ \sum_{i=0}^I \left( q_i \exp \left\{ \lambda \mu_{D,i} T - 0.5 \lambda h_i^2 \int_0^T v_{1\tau} d\tau - 0.5 \lambda (1 - \lambda) g_i^2 \int_0^T v_{2\tau} d\tau + \lambda h_i \int_0^T \sqrt{v_{1\tau}} dw_\tau \right\} \right) K^{1-\lambda}.
\end{aligned} \tag{6}$$

From Equation (6), we observe that absent any growth options in the economy (i.e.,  $\lambda = 1$  or  $q_i = 0$ ) the aggregate consumption  $C_T$  is not affected by idiosyncratic volatilities  $g_i \sqrt{v_t}$ . However, the idiosyncratic volatility affects the aggregate consumption in the economy with real options. Consumption (6) then determines the state price density in the economy, and allows us to derive stock prices, which are reported in Proposition 2 below.

**Proposition 2 (Characterization of Stock Prices).**

1) *The equilibrium stock price of a type  $i$  firm in the economy is given by*

$$S_{i,n,t} = p_i D_{0,n,t} \hat{F}_t + q_i \left( \frac{D_{i,n,t}}{D_{i,n,0}} \right)^\lambda K^{1-\lambda} D_{i,n,0} F_{i,t}, \tag{7}$$

where

$$\hat{F}_t = \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \exp \left\{ \mu_{D,0} (T - t) - \frac{h_0^2 (\Sigma_{1T} - \Sigma_{1t})}{2} + h_0 (V_T - V_t) \right\} \right], \tag{8}$$

$$F_{i,t} = \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \exp \left\{ \lambda \mu_{D,i} (T - t) - \frac{\lambda h_i^2 (\Sigma_{1T} - \Sigma_{1t})}{2} - \frac{\lambda (1 - \lambda) g_i^2 (\Sigma_{2T} - \Sigma_{2t})}{2} + \lambda h_i (V_T - V_t) \right\} \right], \tag{9}$$

and  $V_t = \int_0^t \sqrt{v_{1\tau}} dw_\tau$ ,  $\Sigma_{1t} = \int_0^t v_{1\tau} d\tau$  and  $\Sigma_{2t} = \int_0^t v_{2\tau} d\tau$ . The state price density  $\xi_t$  is given by  $\xi_t = e^{r_f(T-t)} \mathbb{E}_t [C_T^{-\gamma}]$ .

2) *Consider a large equally weighted portfolio of type- $i$  stocks that consists of stocks that only differ from each other in terms of idiosyncratic shocks  $w_{n,i,t}$ . Then, the time-0 value*

of this portfolio and its time- $T$  payoff are given by:

$$S_{i,0} = D_{i,0} \left[ p_i \widehat{F}_0 + q_i K^{1-\lambda} F_{i,0} \right], \quad (10)$$

$$S_{i,T} = p_i \exp \left\{ \mu_{D,0} T - 0.5 h_0^2 \int_0^T v_{1\tau} d\tau + h_0 \int_0^T \sqrt{v_{1\tau}} dw_\tau \right\} \quad (11)$$

$$+ q_i \exp \left\{ \lambda \mu_{D,i} T - 0.5 \lambda h_i^2 \int_0^T v_{1\tau} d\tau - 0.5 \lambda (1-\lambda) g_i^2 \int_0^T v_{2\tau} d\tau + \lambda h_i \int_0^T \sqrt{v_{1\tau}} dw_\tau \right\} K^{1-\lambda} \quad (12)$$

We use the results of Proposition 2 to discuss the relation between our model and the CAPM. First, we characterize the cross-section of returns using a single-factor model with the state price density (SPD)  $\xi_T$  (see Cochrane (p. 100, 2005)):

$$\mathbb{E}[r_{i,T}] - r_f = -\text{cov}(r_{i,T}, \xi_T) e^{r_f T}, \quad (13)$$

where  $r_{i,T} = S_{i,T}/S_{i,0} - 1$  is the stock return. Because the aggregate consumption  $C_T$  coincides with the value of the market portfolio  $S_{M,T}$  at the final date, we find that the SPD is given by  $\xi_T = S_{M,T}^{-\gamma}$ . From equation (6) for  $C_T$ , we observe that real options introduce an additional term involving idiosyncratic volatility process  $v_{2\tau}$  and given by  $\lambda(1-\lambda)g_i^2 \int_0^T v_{2\tau} d\tau$  into the non-linear SPD  $\xi_T$ . The latter term vanishes in the absence of real options when  $\lambda = 1$ .

We observe that the SPD can be linearized as  $\xi_T \approx S_{M,0}^{-\gamma} (a_0 - \gamma r_{M,T})$ , where  $a_0$  is a constant and  $r_{M,T}$  is the return on the market portfolio, when, for example, time-horizon  $T$  is short. Substituting the latter approximation into equation (13) gives the standard mean-variance CAPM. However, the previous literature (e.g., Bansal and Viswanathan, 1993; Dittmar, 2002) has emphasized the importance of higher order expansions of the SPD for explaining the cross-section of asset returns. Furthermore, in general, real options have long expiry dates, and hence, a linear approximation of the SPD is not feasible in reality and does not adequately capture the effect of real options. With longer horizons

hedging motives also become more important. Equation (6) for the aggregate consumption  $C_T = S_{M,T}$  implies that an expansion of the state price density  $\xi_T = S_{M,T}^{-\gamma}$  for longer horizons depends on total systematic  $\int_0^T v_{1\tau} d\tau$  and idiosyncratic  $\int_0^T v_{2\tau} d\tau$  volatilities, total systematic volatility shock  $\int_0^T \sqrt{v_{1\tau}} dw_\tau$ , and their higher-order products and powers. A theorem in Cochrane (p. 106, 2005) then implies that the latter variables are factors that explain the cross-section of returns.

To mimic our empirical strategy and keep the analysis as parsimonious as possible in our Monte Carlo simulations below we retain only the market portfolio, systematic and idiosyncratic volatilities. We find that these factors are sufficient to explain the cross-section of returns in the model. In the Internet Appendix B, we also consider an extension of the model which additionally incorporates total systematic volatility shock  $\int_0^T \sqrt{v_{1\tau}} dw_\tau$ .

### 3.2.2 Simulations

Stock prices (10) and returns are not available in closed form, and hence, we compute them using Monte Carlo simulations for calibrated baseline parameters. For simplicity, we assume that all stocks differ from each other only in terms of the idiosyncratic shocks and volatility parameter  $g_i$ , which determines the level of idiosyncratic volatility  $g_i \sqrt{v_{2t}}$ . In particular, we set  $\mu_{D,i} = \mu_D$ ,  $h_i = h$ , and  $D_{i,n,0} = D_0$ .<sup>10</sup> As a result, the P/D ratio is a function of IVol (labeled  $g$ ), and hence, the value effect and the IVol discount are both related to the same variable,  $g$ . In the Internet Appendix B we consider a richer setting where firms are also heterogeneous in the systematic volatility parameter  $h$ , and hence,

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<sup>10</sup>We consider a calibration where the horizon is  $T = 1$ , the number of firm types is 30, the risk-free rate is normalized to  $r = 0$ ,  $p_i = 0.9$  and  $q_i = 0.1$  for all firms, where  $p_i$  and  $q_i$  are the weights on non-option and option components of the aggregate output. We also set the volatility mean-reversion  $\kappa = 0.01$ , the average variance  $\bar{v}=0.25$ , and pick volatility-of-volatility parameter  $c = -0.1$ , where the negative sign implies negative correlation between volatility and output shocks so that in bad times volatility goes up and aggregate consumption falls (Bartram et al., 2016, confirm this relation empirically). We further set the expected output growth rate to  $\mu_D = 1.7\%$ , systematic volatility parameter to  $h = 1$ , and idiosyncratic volatility parameter to  $g_i = (i - 1)/10$ .



P/D is no longer driven by the IVol alone. We show that our results remain robust in the latter more general model.

We compute realized stock returns for each stock type  $i$  as  $r_i = S_{i,T}/S_{i,0} - 1$ , as well as for the return on the market portfolio,  $r_M$ . Similar to the mean-variance CAPM, our model has a finite horizon, where project payoffs are realized at date  $T$ . We then repeat our model over time, assuming that each period firms have the same type of projects, except that the realizations of idiosyncratic shocks are different. In particular, at date  $T$  all projects expire and cash flows are realized, and then the firm faces similar investment opportunities next period as at the initial date, except that the realizations of shocks are different. We then simulate the economy 1000 times and obtain 1000 observations of stock returns, market returns, and volatilities.

In line with our empirical strategy in the main part of the paper, first we construct factor mimicking portfolios for the systematic and idiosyncratic variances by regressing the variance changes over period of  $T = 1$  on the stock returns, and then take the fitted values excluding intercept. More specifically, we run the following regression for both volatilities:

$$\Sigma_{kT} = a_0 + b_1 r_1 + b_2 r_2 + \dots + b_I r_I + \varepsilon_v, \quad k = 1, 2 \quad (14)$$

where  $\varepsilon_v$  is an error term,  $\Sigma_{1t} = \int_0^t v_{1\tau} d\tau$  and  $\Sigma_{2t} = \int_0^t v_{2\tau} d\tau$  are measures of the total systematic and idiosyncratic total variance, respectively. We estimate coefficients  $\hat{b}_i$ , and then construct the factor mimicking portfolio as  $r_v = \hat{b}_1 r_1 + \hat{b}_2 r_2 + \dots + \hat{b}_I r_I$ . This way we construct the returns on the factor mimicking portfolios for the systematic and idiosyncratic volatilities, which we denote by  $r_{svol}$  and  $r_{ivol}$ , respectively. Then, we run the two-factor time-series regression

$$r_i - r_f = \alpha + \beta_M (r_M - r_f) + \beta_{svol} r_{svol} + \beta_{ivol} r_{ivol} + \varepsilon, \quad (15)$$

where  $\varepsilon$  is a residual shock. All coefficients in the regressions on simulated data are highly

statistically significant, and hence,  $t$ -statistics are not reported for brevity.

The estimation results for equation (15) are reported on Figure 1. In particular, Figures 1A–1E show  $\alpha$ ,  $\beta_M$ ,  $\beta_{\text{svol}}$  and  $\beta_{\text{ivol}}$  respectively, and Figure 1F shows the date-0 price-output ratio  $S_{i,0}/D_0$  as functions of the idiosyncratic volatility magnitude  $g_i$ . We observe that  $\alpha$  is close to zero, and betas are increasing functions of  $g_i$  when  $g_i$  is sufficiently high.

Figure 1A shows the three-factor  $\alpha$  (solid line) that controls for the market factor and two volatility factors, as well as the CAPM  $\alpha$  (dashed line) as functions of the price-dividend ratio, where the CAPM  $\alpha$  is obtained from the regression of excess stock returns on the excess returns of the market portfolio. Figure 1B shows the same alphas as functions of the IVol magnitude parameter  $g$ . From Figure 1A, we observe that the CAPM  $\alpha$  is a decreasing function of the price-dividend ratio  $S/D$ , and is positive for stocks with low  $S/D$  ratio and negative for stocks with high  $S/D$  ratio. Consequently, the stocks with low  $S/D$  ratio (value stocks) have higher risk-adjusted returns than stocks with high  $S/D$  ratios (growth stocks), and hence, the model generates value effect. Moreover, Figure 1E shows that the price-dividend ratio is an increasing function of IVol, and hence high-IVol stocks have negative abnormal return, consistent with IVol discount. We also observe that the alpha from the volatility factor model is very close to zero, and hence, including the volatility factors eliminates the value effect and IVol discount.

The intuition for the “value effect” and the IVol discount is fully rational. In particular, stocks with high  $S/D$  ratios are those that have higher idiosyncratic volatilities, and hence, more valuable growth options. Such stocks appreciate when aggregate volatility increases and can be used to hedge exposure to volatility shocks, which decreases their risk premiums relative to stocks with low  $S/D$  ratios.

Next, we explain the intuition for volatility betas. First, we note that the factor premiums  $\mathbb{E}[r_{\text{svol}}]$  and  $\mathbb{E}[r_{\text{ivol}}]$  are negative in our simulations. This is because the dividend

shocks  $dD_{i,n,t}$  are negatively correlated with the volatility shocks  $dv_{1t}$  and  $dv_{2t}$  [i.e.,  $c_1 < 0$  and  $c_2 < 0$  in Equations (2)–(3)] because the volatility is typically higher in bad times when the output is low. As a result, the aggregate consumption  $C_T$  is high when the total systematic and idiosyncratic volatilities are low and vice versa. Consequently, the factor mimicking portfolios for volatilities provide a hedge for aggregate consumption, and hence, have negative risk premiums.

From Figures 1D and 1E, we observe that volatility betas have positive signs when the idiosyncratic volatility parameter  $g$  is high. Intuitively, the value of growth options increases in volatility, which makes growth options a hedge against volatility risk (in the sense that they will load positively on the volatility risk factors when the market factor is controlled for). For the same reason, when parameter  $g$  is high, growth options are more valuable and take a larger fraction of the firm value. Hence, firms with higher  $g$  are better hedges against volatility risk due to the fact that growth options, which are a hedge against volatility risk, constitute a larger fraction of high  $g$  firms. As a result, the volatility betas of high  $g$  firms are positive and those firms are hedges against volatility risk ( $\beta_{\text{svol}}\mathbb{E}[r_{\text{svol}}] < 0$  and  $\beta_{\text{ivol}}\mathbb{E}[r_{\text{ivol}}] < 0$ ).

## 4 Data Sources

Our data span the period between January 1986 and December 2017 due to the availability of the VIX index. The VIX index measures the implied volatility of the S&P100 options. Following Ang et al. (2006), we measure IVol as the standard deviation of residuals from the Fama-French (1993) model, which is fitted to daily data. We estimate the model separately for each firm-month, and compute the residuals in the same month. We require at least 15 daily returns to estimate the model and IVol. We obtain the daily and monthly values of the three Fama-French factors and the risk-free rate from Kenneth French web

site at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

We define FVIX, the market volatility risk factor, as a factor-mimicking portfolio that tracks daily changes in the VIX index.<sup>11</sup> Since the autocorrelation of VIX is 0.97 at daily frequency, daily change in VIX is therefore a good proxy for innovation in expected market volatility, and, according to the evidence in Ang et al. (2006), it should be priced. We regress daily changes in VIX on daily excess returns to five portfolios sorted on past sensitivity to VIX changes:

$$\begin{aligned} \Delta VIX_t = & \gamma_0 + \gamma_1 \cdot (VIX1_t - RF_t) + \gamma_2 \cdot (VIX2_t - RF_t) + \\ & + \gamma_3 \cdot (VIX3_t - RF_t) + \gamma_4 \cdot (VIX4_t - RF_t) + \gamma_5 \cdot (VIX5_t - RF_t) + \epsilon_t, \end{aligned} \quad (16)$$

where  $VIX1_t, \dots, VIX5_t$  are the VIX sensitivity quintiles described below, with  $VIX1_t$  being the quintile with the most negative sensitivity. The VIX sensitivity is estimated by regressing daily stock returns in the previous month on the market factor and change in VIX.

The fitted part of the regression above less the constant is our market volatility risk factor (FVIX factor). The returns are then cumulated to the monthly level to get the monthly return to FVIX.<sup>12</sup>

The return sensitivity to VIX changes ( $\gamma_{\Delta VIX}$ ) we use to form the base assets is measured separately for each firm-month by regressing daily stock excess returns in the past month on daily market excess returns and the VIX index change using daily data (at least

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<sup>11</sup>We use the old version of VIX (ticker VXO). Old VIX spans a longer sample and includes the high-volatility episode of October 1987. However, old VIX is a non-tradable estimate of implied market volatility, and thus a factor-mimicking portfolio is needed. The results in the paper are robust to using the tradable new VIX (ticker VIX) and the shorter sample when it is available.

<sup>12</sup>All results in the paper are robust to changing the base assets from the five portfolios sorted on past sensitivity to VIX changes to the ten industry portfolios from Fama and French (1997) or the six size and book-to-market portfolios from Fama and French (1993).

15 non-missing returns are required):

$$Ret_t - RF_t = \alpha + \beta_{MKT} \cdot (MKT_t - RF_t) + \gamma_{\Delta VIX} \cdot \Delta VIX_t + \epsilon_t. \quad (17)$$

Likewise, the IVol risk factor, FIVol, is a similarly defined factor-mimicking portfolio that tracks monthly innovations to average IVol, which is the simple average of the idiosyncratic volatilities (as defined above) of all firms traded during the given month. The innovation is the residual from an ARMA(1,1) model fitted to the average IVol. The base assets used to create FIVol are IVol sensitivity quintiles, where IVol sensitivity is measured in the previous month by regressing monthly stock returns in the most recent 36 months on the market factor and innovations to average IVol.

## 5 Explaining the Puzzles

### 5.1 Is Aggregate Volatility Risk Priced?

In order to be valid volatility factors, FVIX and FIVol have to satisfy two necessary conditions. First, FVIX and FIVol have to be significantly correlated with the variables they mimic - change in the VIX index and innovation to average IVol. Large correlations would imply that FVIX and FIVol are good proxies for the innovations in the state variables, but the correlations can be far from 100% if the innovations are measured with noise or if some information in the state variables is irrelevant for the stock market. Second, FVIX and FIVol have to earn a significantly negative risk premiums, controlling for other sources of risk: FVIX (FIVol) tends to earn positive returns when market (idiosyncratic) volatility increases and consumption drops, thereby providing a valuable hedge.

In Panel A of Table 1, we find that the correlation between FVIX and  $\Delta VIX$  is 0.676, and the correlation between FIVol and  $\overline{IVol}^U$  is 0.424. The relation between FVIX and FIVol and the levels of the respective state variables is expectedly weaker, but still

significantly positive. Second, we find that the correlation between the change in VIX ( $\Delta VIX$ ) and the innovation to average IVol ( $\overline{IVol}^U$ ) is 0.509, validating the assumption of tight positive correlation between idiosyncratic volatility and systematic volatility process we make in our model, and the correlation between FVIX and FIVol is 0.558, meaning that FVIX and FIVol are related, but different factors.

In Panel B, we find that FVIX loses 1.37% per month, with the CAPM and three-factor Fama and French (1993) alphas at -46 bp and -44 bp per month and t-statistics comfortably exceeding 4. The five-factor Fama and French (2015) alpha of FVIX is at -31 bp per month, t-statistic -3.73, indicating a certain overlap between FVIX and the new investment (CMA) and profitability (RMW) factors. Barinov (2020) further discusses the overlap between FVIX and RMW and its causes, and we summarize the discussion in Section 7 and provide more details in robustness appendix.<sup>13</sup>

In Panel C, FIVol has average return of -1.92% per month, the CAPM alpha of -120 bp per month, and the three-factor alpha of -95 bp per month, all statistically significant. The five-factor alpha of FIVol is -74 bp per month, suggesting that, at least on the relative scale, the overlap between FIVol and RMW, if present, is smaller than the overlap between FVIX and RMW. Overall, we conclude from Panels B and C that FVIX and FIVol are priced factors.

We also find that both factors tend to have very negative market betas. This is to be expected, since the market factor is negatively correlated with both changes in VIX (correlation of - 0.675) and innovations to average IVol (correlation of - 0.255). By construction, FVIX and FIVol are positively correlated with the respective innovation, so it is natural that both FVIX and FIVol have large and negative market betas. We also find that FVIX seems unrelated to HML, but positively related to SMB (suggesting that FVIX

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<sup>13</sup>The robustness appendix is available at <http://faculty.ucr.edu/~abarinov/Robustness 2021.pdf>

will be more useful in explaining the IVol discount than the value effect) and FIVol is negatively related to both HML and SMB (suggesting that FIVol will be the main force in explaining the value effect). Lastly, we find that, controlling for the market factor, which is negatively related to both FVIX and FIVol, the relation between FVIX and FIVol is weak, strengthening our view of FVIX and FIVol as empirically very different factors.

## 5.2 Average Idiosyncratic Volatility, Market Volatility, and the State of the Economy

Motivated by empirical evidence in Bartram et al. (2016) and Herskovic et al. (2016), our model assumes that market volatility and average IVol are tightly related to each other, as well as to the state of the economy. In Table 2, we verify in our sample that the average IVol indeed tends to increase in recessions and comove with market volatility (the results are robust to using the median IVol instead). We run pairwise regressions of average IVol on the NBER recession dummy (one during recessions, zero otherwise) and three measures of market volatility. For each business cycle variable we run regressions with it lagged up to four quarters and leaded up to four quarters, and report the slopes in the respective columns of Panel A. For example, in the column labeled “-3” we report  $\gamma_2$  from

$$\log(\overline{IVol}_t) = \gamma_0 + \gamma_1 \cdot t + \gamma_2 \cdot \log(X_{t-3}) + \epsilon_t \quad (18)$$

where  $X_{t-3}$  is one of the business cycle variables lagged by three months.

To account for the fact that IVol has trended up in our sample period, we also add the linear trend into the regressions. Also, to make the slopes on the business cycle variables easier to interpret, we take the log of the average IVol and the log of the market volatility.

The numbers in the first row, where we report the slopes from the regression of average IVol on the NBER recession dummy, represent the percentage increase in IVol during recessions. We find that IVol is on average by 15-20% higher in recessions than in ex-

pansions (the spread between the calmest period in the expansion and the most volatile period in the recession is likely to be much wider). We also notice from looking at the leads and lags that the switch from expansion to recession predicts higher IVol for at least nine months ahead and probably longer, while the increase in IVol can potentially forecast recessions one quarter ahead. This evidence suggests that the increase in average IVol during recessions is not short-lived.

In the next rows of Table 2, we look at the slopes from the regressions of average IVol on the log of the VIX index, on the TARCH(1,1) forecast of market volatility, and on the log of realized market volatility (see Data Appendix for detailed variable definitions).

We find that an increase in market volatility (expected or realized) by 1% triggers the increase in average IVol by 0.15% to 0.35%. The volatility measures have the ratio of the standard deviation to the mean close to 1, hence, a two-standard deviation change in market volatility can trigger the increase in average IVol by 30-70%. We also find that higher market volatility predicts higher IVol for up to a year ahead, and vice versa.<sup>14</sup>

### 5.3 Idiosyncratic Volatility, Market-to-Book, and Volatility Risk

Our model predicts that growth options offer a valuable hedge against aggregate volatility risk, and the hedge, measured as positive betas with respect to the market volatility factor (FVIX) and average IVol factor (FIVol) is stronger if IVol is higher, as evidenced by Figures 1D and 1E. The immediate prediction is that in the single sorts on IVol (the empirical proxy for  $g$  parameter in the model) or market-to-book (the empirical proxy for price-to-dividend,  $S/D$ , in the model) we will observe a positive relation between IVol and both

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<sup>14</sup>In Section 5.1 of the robustness appendix, we also find that FVIX and FIVol can be thought of as ICAPM factors, since they are significantly related to future volatility and future state of the economy, as a valid ICAPM factor should be. Our theoretical model is not ICAPM, and the role of aggregate volatility risk factors in the model arises from non-linearity of firm values, but our empirical volatility factor model can be also interpreted as ICAPM.



volatility betas and market-to-book and both volatility betas.

Figures 1A and 1B plot the alphas as a function of market-to-book and IVol ( $S/D$  and  $g$  in the model) and make a similar prediction about the univariate relation between the CAPM alpha and IVol/market-to-book. However, as the same Figures 1A and 1B suggest, the positive relation between the alpha and IVol/market-to-book should disappear in alphas of the three-factor volatility model that control for the two volatility factors.

Higher IVol in the model makes growth options take a larger fraction of the firm value (Figure 1F), and hence makes the firm a better hedge against aggregate volatility risk. Additionally, existing empirical evidence shows that IVol of higher IVol firms is more responsive to changes in average IVol (see Grullon et al., 2012, and Barinov, 2017). Hence, growth options of high IVol firms will respond most positively (least negatively) to increases in aggregate volatility (i.e., market volatility and IVol).

The cross-sectional prediction from juxtaposition of Figure 1F with Figures 1D and 1E is that the value effect increases with IVol and is likely to be weak/absent for low IVol firms. The prediction about the IVol discount is symmetric and implies that the IVol discount increases with market-to-book and is likely absent for value firms.

In Panel A of Table 3, we look at the value-weighted CAPM alphas in the five-by-five independent portfolio sorts on market-to-book and IVol.<sup>15</sup> Going across rows and confirming Figure 1B, we observe that the magnitude of the IVol discount monotonically increases from 20 bp per month, t-statistic 0.51, in the extreme value quintile to 96.5 bp per month, t-statistic 3.28, in the extreme growth quintile. The difference is statistically significant with t-statistic 2.20. Going down the columns of Panel A, we also observe that the value effect starts with almost exactly zero CAPM alpha in the lowest IVol quintile

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<sup>15</sup>The sorts are performed using NYSE (`exchcd=1`) breakpoints. The results are robust to using conditional sorting and/or CRSP breakpoints, as well as to using raw returns or the Fama-French alphas instead, and/or using equal-weighted returns.

and ends up with the CAPM alpha of 77 bp per month, t-statistic 2.14, in the highest IVol quintile.<sup>16</sup>

Similar to Figures 1A and 1B, which show that in the simulated data controlling for the volatility risk factors resolves the relation between alphas and IVol/market-to-book, alphas of the three-factor volatility model in Panel B suggest that there is no IVol discount and no value effect after we control for volatility risk. The IVol discount turns insignificantly negative in all market-to-book quintiles and no longer depends on market-to-book. The value effect in alphas from the three-factor volatility model fluctuates between -22 bp per month and 5 bp per month, but is never statistically significant and does not depend on IVol.

Panel C of Table 3 shows that the FVIX betas are closely aligned with the CAPM alphas in Panel A. Similar to Figure 1D, which reports the relation between IVol and market volatility beta in simulated data, in Panel C of Table 3 FVIX beta of the low-minus-high IVol portfolio (rightmost column) is always negative and strongly and monotonically increases in absolute magnitude with market-to-book, and the FVIX beta of the value-minus-growth portfolio (bottom row) similarly increases with IVol.

Comparing the FVIX betas of the low-minus-high IVol portfolios and the value-minus-growth portfolios, we find that FVIX is more likely to help in explaining the IVol discount than the value effect. The FVIX beta of the value-minus-growth portfolio is only significantly negative in the highest IVol quintile, the quintile in which the value effect is by far the strongest.

Panel D of Table 3 is the empirical counterpart of Figure 1E, as Panel D reports the FIVol betas and finds that FIVol is the factor that explains the value effect, most

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<sup>16</sup>Section 1.5 in the robustness appendix analyzes several firm characteristics across the double sorts in Table 3 and finds that high IVol growth firms, the negative alphas of which drive both the stronger value effect for high IVol firms and the stronger IVol discount for growth firms, are neither particularly small nor illiquid.

particularly the positive alphas of value stocks (the FIVol betas of these stocks are large and negative), but contributes little to explaining the IVol discount. We also find that the FIVol betas of the value-minus-growth portfolio are equally strong in all IVol quintiles.<sup>17</sup>.

## 5.4 Explaining the Value Effect and the IVol Discount

Table 4 focuses on five arbitrage portfolios. The first portfolio is the HML factor, which is our measure of the value effect. We contrast it with the second portfolio - HMLh, which is the value-minus-growth return spread in the top IVol quintile. The third portfolio - IVol - captures the IVol discount. It goes long (short) in lowest (highest) IVol quintile. The IVolh portfolio does the same for the top market-to-book quintile only in order to capture the stronger IVol discount for growth firms. The IVol55 portfolio is long in the highest IVol growth firms and short in the one-month Treasury bill.

Similar to simulated data in Figures 1A and 1B, in column one the CAPM turns out to be incapable of explaining either the value- or equal-weighted returns to any of the portfolios, except for the HML factor, which still has the CAPM alpha of 31 bp per month with t-statistic above 1.5. The magnitude of the CAPM alphas of other portfolios is about 1% per month<sup>18</sup>.

Other empirical factor models improve compared to the CAPM, but are still unable to completely handle the IVol discount and the relation between IVol and the value effect. The three-factor Fama and French (1993) model makes insignificant the value-weighted alpha of HMLh, though at 37 bp per month, t-statistic 1.57, it is economically large. The other, significant Fama-French alphas are between 53 and 87 bp per month. The Carhart

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<sup>17</sup>In equal-weighted returns in Section 5.4 in the robustness appendix, we do see significantly more negative FIVol betas of the value-minus-growth portfolio in the high IVol quintile

<sup>18</sup>One difference between IVol and IVolh is that IVol includes stocks with non-missing IVol, but missing market-to-book. Therefore, the fact that in Panel A the alphas of IVol are close to the alphas of IVolh does not contradict the increase of the IVol discount with market-to-book.

model reduces the estimates of the IVol effect, but restores the significance of the value-weighted HMLh alpha. The other Carhart alphas in Table 4 are between 42 and 72 bp per month, with all but one t-statistics above 3.

The five-factor Fama and French (2015) model significantly reduces all alphas, which now are 27-42 bp per month, with all but two still significant. As the analysis in the robustness appendix shows (see Section 7 of this paper for a more detailed summary), the increase in the ability of the five-factor model to explain the value effect and IVol discount stems primarily from the overlap between the RMW factor and FVIX. Barinov (2020) further studies this overlap and concludes that FVIX can explain the alpha of RMW, but not the other way around, and the reason for that is that unprofitable firms are distressed, and thus their equity can be thought of as a call option on the assets, and this option-likeness makes equity of unprofitable firms a hedge against volatility increases.

The models in Babenko et al. (2016) and Chen et al. (2020) suggest that the value effect and IVol discount can be explained by conditional CAPM. In the fifth column of Table 4, we estimate conditional CAPM assuming that the market beta is the linear function of the standard conditioning variables: dividend yield, default premium, one-month Treasury bill rate, and term spread<sup>19</sup> and estimates

$$Ret_{it} = \alpha_i + (\beta_{0i} + \beta_{1i} \cdot DIV_{t-1} + \beta_{2i} \cdot DEF_{t-1} + \beta_{3i} \cdot TB_{t-1} + \beta_{4i} \cdot TERM_{t-1}) \cdot MKT_t + \epsilon_{it} \quad (19)$$

Making the beta conditional reduces CAPM alphas by on average 20-30 bp per month and makes some value-weighted alphas marginally significant, but leaves them numerically large.<sup>20</sup>

In the three rightmost columns, we estimate the volatility factor model with the market

<sup>19</sup>The detailed definitions of the four variables can be found in, e.g., Petkova and Zhang (2005).

<sup>20</sup>The robustness appendix at [http://faculty.ucr.edu/~abarinov/Robustness\\_2021.pdf](http://faculty.ucr.edu/~abarinov/Robustness_2021.pdf) also shows that the betas of HML, HMLh, IVol, IVolh are higher in recessions by 0.25-0.4, while the beta of IVol55 is by 0.3 lower in recessions. However, the changing market beta is subsumed by FVIX and FIVol: making the market beta time-varying in the volatility factor model does not change the alphas.

factor, FVIX, and FIVol. Similar to Figures 1A and 1B, in which the two volatility factors explain the relation between the CAPM alphas and parameters  $S/D$  and  $g$ , respectively, column six of Table 4 shows that the volatility factors explain the returns to the IVol, IVolh, and IVol55 portfolios. For example, the IVolh portfolio measures the IVol discount in the extreme growth quintile and possesses the value-weighted CAPM (Fama-French) alphas of 96.5 (67) bp per month, t-statistic 3.28 (2.48). Adding the aggregate volatility risk factors to the CAPM makes the alpha become mere 0.9 bp per month, t-statistic 0.03.

The volatility factors also explain the returns to HML: its CAPM alpha drops from 31 bp per month, t-statistic 1.56, in the CAPM, to -7.4 bp, t-statistic -0.4, in the volatility factor model. The volatility factors also handle HMLh quite well, reducing its value-weighted (equal-weighted) CAPM alpha from 77 (109) bp per month, t-statistic 2.14 (3.64), to -10 (30) bp, t-statistic -0.3 (1.09). Overall, it seems that aggregate volatility risk is able to explain the value effect and its dependence on IVol.

In columns seven and eight, we look at FVIX beta and FIVol beta and find that, as predicted, all portfolios, except for IVol55, load negatively on both factors, while IVol55 loads on them positively, consistent with its negative alpha. The volatility betas in columns seven and eight are consistent with Figures 1D and 1E, which present similar volatility betas in simulated data and express them as functions of parameter  $g$  in the model.

An interesting wrinkle in the real data not predicted by the model is that the IVol discount is explained primarily by FVIX, because FIVol betas of IVol, IVolh, and IVol55, are insignificant, even though they all have the predicted sign. Similarly, FIVol seems to contribute more to explaining the value effect: while FVIX betas of HML and HMLh are significant, they are of the same magnitude as the respective FIVol betas, and FIVol has twice bigger factor risk premium. While the limited ability of an average IVol factor to explain the IVol discount was discovered by Herskovic et al. (2016), both the ability of

FVIX to explain the IVol discount and the ability of FIVol to explain the value effect are new to the literature.

## **6 Behavioral Explanations**

Our model does not necessarily imply that the IVol discount/value effect is unrelated to variables other than market-to-book/IVol. It does imply, however, that after we control for FVIX and FIVol, the cross-sectional relation of the IVol discount and value effect to any variable should disappear. In this section, we verify that FVIX and FIVol can explain the relation of the IVol discount and value effect to short-sale constraints proxies.

### **6.1 Idiosyncratic Volatility Discount, Institutional Ownership, and the Probability to Be on Special**

Several recent empirical papers find evidence consistent with the behavioral explanation of the IVol discount. The behavioral explanation is based on the Miller (1977) argument that under short sale constraints firms with greater divergence of opinion about their value will be more overpriced. Miller (1977) argues that short sale constraints keep pessimistic investors out of the market, and the market price reflects the average valuation of the optimists. The average valuation of the optimists is higher than the fair price and naturally increases with disagreement. Therefore, the overpricing should increase in both short sale constraints and disagreement/volatility, and the negative relation between disagreement/volatility and future returns should be the strongest for the most short sale constrained firms.

Consistent with this idea, Nagel (2005) and Boehme et al. (2009) find that the IVol discount is much stronger for the firms they perceive to be the costliest to short. Nagel (2005) uses low institutional ownership (IO) as a proxy for low supply of shares for shorting

and thus high shorting costs. Boehme et al. (2009) look at high short interest (high demand for shorting).

We follow Nagel (2005) in looking at residual IO, which is orthogonalized to size (see Data Appendix). We do not have access to the short interest data for the full sample period and use instead the estimated probability that the stock is on special.<sup>21</sup> The exact formula for the probability to be on special is in the Data Appendix. It uses the coefficients estimated by D’Avolio (2002) for a short 18-month sample of the stocks with available data on shorting fees. Ali and Trombley (2006) use the same formula to estimate the probability to be on special for a much longer sample and show that it is closely tied to real shorting fees in different sub-periods.

In Panel A of Table 5, we look at the low-minus-high IVol portfolios formed separately in each IO quintile. Consistent with Nagel (2005), the top two rows in Panel A find stronger IVol discount for low IO firms. However, after we control for the FVIX and FIVol factors in Panel A, the IVol discount in all IO quintiles goes away. In the lowest IO quintile in Panel A, it declines from 1.175% per month, t-statistic 3.77, to 0.184% per month, t-statistic 0.46. The difference in the IVol discount between low and high IO firms also declines significantly from 69.1 bp per month, t-statistic 2.41, to 24.9 bp per month, t-statistic 0.76.

To find out why the FVIX beta of the low-minus-high IVol portfolio becomes more negative as IO decreases, we also look at IVol in five-by-five sorts on IVol and IO (results not reported to save space). The sorts suggest that institutions face a trade-off between market volatility risk and IVol, trying to avoid both. Thus, the low IO subsample consists of both stocks with the most negative FVIX betas (and lowest IVol) and the most positive FVIX betas (and the highest IVol).

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<sup>21</sup>The stock is said to be “on special” when the shorting fee exceeds the risk-free rate.

In Panel B of Table 5, we find similar evidence using estimated probability to be on special. The IVol discount, measured as the CAPM alpha, changes from 38 bp per month, t-statistic 1.79, for the stocks that are the cheapest to short, to 121 bp per month, t-statistic 3.17, for the stocks that are the most expensive to short.

After we control for FVIX and FIVol, the IVol discount disappears in all probability to be on special quintiles. The IVol discount in the quintile with the highest probability to be on special declines to -6.1 bp per month.

When we look at the IVol in the single sorts on estimated probability to be on special (results not tabulated to save space), we find that the most expensive to short stocks have twice higher IVol than the cheapest to short stocks. This is to be expected: the losses from lending volatile stocks are potentially greater, and therefore lenders should charge a higher fee. Since stocks with high probability to be on special have higher IVol, sorting these stocks on IVol produces a wider spread in both IVol and aggregate volatility risk.

## **6.2 Value Effect, Institutional Ownership, and the Probability to Be on Special**

Nagel (2005) finds that the value effect is also stronger for the firms with low IO and interprets this result as the evidence that the value effect arises because growth firms are overpriced and some of them are hard to short (for example, when IO and hence the supply of shares for shorting is low).

Analogous to the previous section, we find that institutions prefer to hold firms with intermediate levels of market-to-book and that probability to be on special is much higher for growth firms. Hence, the spreads in market-to-book and, therefore, aggregate volatility risk are mechanically wider if the sorts on market-to-book are performed in the subsample with low IO or high probability to be on special.



In Table 6, we look at the value-minus-growth portfolios formed separately within each IO quintile (Panel A) or estimated probability to be on special quintile (Panel B). In the top row of each part of Table 6, we find that the value effect is indeed stronger for the firms with the lowest IO or the highest probability to be on special.

In the third row of each part of Table 6, we find that controlling for aggregate volatility risk materially reduces the difference in the value effect between stocks that are easy and hard to short. In the IO sorts, the difference in the value effect decreases to 75.7 to 30.7 bp per month, t-statistic 1.01. In the sorts on estimated probability to be on special, the difference decreases from 99.8 to 25.7 bp per month, t-statistic 0.93.

### **6.3 Arbitrage Asymmetry and the IVol Discount**

Stambaugh et al. (2015) argue that the IVol discount arises because of arbitrage asymmetry. If one double-sorts on a comprehensive measure of mispricing and on IVol, Stambaugh et al. argue, the relation between IVol and future returns will be positive for underpriced stocks and negative for overpriced stocks: high IVol underpriced/overpriced stocks have most positive/negative alphas, since IVol is a limits-to-arbitrage proxy. Since shorting is costlier than buying, overpriced high IVol stocks should have larger absolute alphas than underpriced high IVol stocks. Hence, the relation between IVol and future returns will be negative overall.

Stambaugh et al. (2015) propose a comprehensive measure of mispricing, defined as the average rank of the firm from 11 independent sorts on priced firm characteristics, such as accruals, momentum, profitability, etc. Firms that are supposed to have the most positive alphas in each sort receive the highest ranking and vice versa, and the rankings are scaled to be between 0 and 1 and then averaged for all firms that are part of at least five sorts out of 11.

Panel A in Table 7 repeats Stambaugh et al. and finds that in the CAPM alphas, the IVol discount indeed exists only for the most overpriced firms, for which it reaches 1.55% per month, t-statistic 4.08. Consistent with Stambaugh et al. explanation, the driving force is the extremely negative alpha of high IVol overpriced firms, -1.39% per month, t-statistic -5.53.

Panel B presents the alphas from the volatility factor model with the market factor, FVIX, and FIVol, which handles well the IVol discount in the most overpriced subsample, reducing it from 1.55% to 0.345% per month and making it statistically insignificant. The volatility factors also significantly reduce the negative alpha of high IVol overpriced firms to -77.5 bp per month, t-statistic -3.20, and the difference in the IVol discount between overpriced and underpriced firms decreases from 1.54% to 0.97% per month.

On the other hand, the volatility factor model does little to explain the alpha spread created by the sorts on the Stambaugh et al. mispricing measure, which is the reason why we still observe a significantly negative alpha for high IVol overpriced stocks (which is now no longer different from the negative alpha of low IVol overpriced stocks). The volatility factor model also discovers flipped and significant IVol discount (positive alphas of high IVol firms) in the two underpriced quintiles, which would be more consistent with the conventional intuition (see, e.g., Merton, 1987, Boehme et al., 2009) that IVol, if anything, should be positively priced. While the positive pricing of IVol in the two underpriced quintiles can be consistent with both risk-based and mispricing explanations, it is beyond the scope of our paper.

Panel C and D present FVIX and FIVol betas of the double-sorted portfolios and find that, as previously, it is mainly FVIX that explains the IVol discount. The spread in FVIX betas between low and high IVol firms changes from -1.37 to -2.33 between the top underpricing and top overpricing quintiles, with t-statistic for the difference at -4.35.

Likewise, the FVIX beta of the high volatility overpriced firms is at 1.523, t-statistic 4.19, by far the largest FVIX beta in Panel C.

## 7 Robustness Checks

This section provides a brief account of the robustness checks we performed. The results are tabulated and discussed in more detail in the robustness appendix.<sup>22</sup>

**C-GARCH volatility.** In Section 3.2 of the robustness appendix, we follow Adrian and Rosenberg (2008) in separating market volatility forecast into short-run and long-run component using C-GARCH and find that it is only the former that overlaps with FVIX, making FVIX distinct from the long-run volatility factor in Campbell et al. (2018) and a similar long-run volatility factor the model in McQuade (2018) implies. The factor-mimicking portfolio for the short-run market volatility component explains about one-half of the IVol discount, since VIX uses more information than C-GARCH forecast to form its expected volatility part. VIX also contains the risk-aversion part, which is not necessarily related to expected physical volatility, so the explanatory power of the short-run volatility component is the lower bound on the explanatory power that is coming purely from physical volatility forecast. The long-run volatility component, on the other hand, produces economically small loadings, the sign of which is not helpful in explaining either the value premium or the IVol discount.

**Extending the sample period.** Our sample period is constrained by VIX availability. Replacing VIX with short-run volatility component allows extending the sample period back to at least 1963. In the longer sample, covered in Section 3.4 of the robustness appendix, the FIVol factor explains the same fraction of the value effect as in 1986-2017, and the short-run volatility factor explains 70% of the IVol discount. In the same Section

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<sup>22</sup>The robustness appendix is available at <http://faculty.ucr.edu/~abarinov/Robustness 2021.pdf>

3.4, we also experimented with using coefficients from the factor-mimicking regression in 1986-2017 to backfill FVIX in 1963-1985. To do that, we have to use suboptimal base assets, as sorts on prior VIX sensitivity cannot be performed without FVIX. The backfilled FVIX explains roughly one-half of the IVol discount in the longer sample.

**Robustness of the IVol discount.** Bali and Cakici (2008) report that the IVol discount does not exist in the NYSE-only sample. In Section 1.1 of the robustness appendix, we revisit their finding and discover that they used current exchange listing indicator (hexcd on CRSP) instead of historical one (exchcd). That introduces positive look-ahead bias to returns to all IVol quintiles, and the bias is naturally much more pronounced for high IVol firms. When we use the historical listing indicator, we find that the IVol discount is at 67 bp per month for NYSE firms. Huang et al. (2010) argue that the IVol discount disappears controlling for the short-term reversal of Jegadeesh (1990). The short-term reversal lasts only for one month; in Section 1.2 of the robustness appendix, we show that the IVol discount remains visible for at least 9-12 months, which means that the IVol discount is distinct from the short-term reversal.

**Fully Tradable FVIX.** Since Breeden et al. (1989), factor-mimicking regressions are run in the full sample, assuming that investors are more informed than the econometrician. If they are not, the full-sample regression introduce potential look-ahead bias. In Section 3.1 of the robustness appendix, we redo Table 4 using FVIXT and FIVolT, fully-tradable FVIX and FIVol estimated using expanded-window regression (in month  $t$ , FVIXT and FIVolT use coefficients from the factor-mimicking regression estimated with data from months 1 to  $t-1$ ). We find that FVIXT and FIVolT have risk premiums close to those of FVIX and FIVol, and replacing FVIX and FIVol with FVIXT and FIVolT, if anything, makes the alphas of IVol, IVolh, IVol55, HML smaller, while the alpha of HMLh becomes much smaller when FVIXT and FIVolT are used.

**FVIX and RMW.** Barinov (2020) finds that FVIX can explain RMW, but not the other way around, and ascribes that to the fact that unprofitable firms are distressed and their equity is similar to a call option on the assets. The said option benefits from increases in volatility, just as growth firms do in our paper (even though growth firms are on average more profitable and financially healthy than value firms). In Section 4.1 of the robustness appendix, we find that the alphas of IVol, IVolh, IVol55 portfolios survive in the five-factor Fama-French (2015) model, though their magnitude is roughly halved compared to the CAPM or the three-factor Fama-French (1993) model. FVIX and FIVol retain significance if added to the five-factor Fama-French model, but FVIX betas of IVol, IVolh, IVol55 are about half of what is reported in the paper. We find that it is RMW, and not CMA that FVIX overlaps with, and conclude in the spirit of Barinov (2020) that RMW picks up market volatility risk.

**Alternative Versions of the Value Effect.** Section 5.2 in the robustness appendix looks at alternative versions of the value effect, such as the value-minus-growth strategies based on earnings-to-price and cash-flow-to-price ratio, which are more successful in 1986-2017, the “HML devil” strategy of Asness and Franzini, 2013, and the broader “cheap-minus-expensive” strategy of Ilmanen et al., 2019, which combines the value-minus-growth strategy with similar strategies in the bond, futures, and foreign exchange markets. We find that aggregate volatility risk, and in particular negative exposure to FIVol, makes alphas of all those strategies insignificant. Some of those strategies also have significantly negative FVIX betas. Section 5.3 of the robustness appendix uses more measures of growth options, such as the fraction of firm value that comes from future growth (as estimated by Trigeorgis and Lambertides, 2014, and Bali et al., 2020) and confirms our result that growth firms are hedges against aggregate volatility risk using those measures to sort firms into value and growth.

**Stambaugh and Yuan (2017) Mispricing Factors.** Detzel et al. (2019) show that the four-factor model with two mispricing factors suggested in Stambaugh and Yuan (2017) can explain the IVol discount. In Section 4.4 of the robustness appendix, we consider the potential overlap between FVIX, FIVol, and the two mispricing factors, and find virtually no overlap between FIVol and the mispricing factors, while the alpha of the mispricing factors declines by roughly 10 bp per month when FVIX is controlled for, and vice versa. We also confirm in our 1986-2017 sample the conclusion of Detzel et al. that the alphas of IVol, IVolh, IVol portfolios in the Stambaugh-Yuan model are insignificant, but find that the Stambaugh-Yuan model fails to explain the alpha of HMLh. When the Stambaugh-Yuan factors, FVIX and FIVol are used in the same model, none of them loses significance, and their impact on the alphas is largely added on top of each other instead of overlapping, confirming that the aggregate volatility risk explanation of the IVol discount in this paper and the mispricing explanation in Detzel et al. (2019) are two non-mutually-exclusive, independent explanations.

**Aggregate Volatility, Liquidity, and Funding Liquidity** In Section 4.3 of the robustness appendix, we look at the relation between FIVol, FVIX, and a number of liquidity risk factors and funding liquidity measures in the literature. We find that, controlling for the market return, FVIX is largely unrelated to all those factors, and FIVol has small overlap with liquidity risk factors, while the relation between FIVol and funding liquidity measures is mixed. None of those relations help to significantly explain the alpha of either FVIX or FIVol.

## 8 Conclusion

The paper shows both theoretically and empirically that IVol and market-to-book are negatively related to expected returns because high IVol firms and growth firms are hedges

against aggregate volatility risk. The economic mechanism works as follows: the value of growth options is naturally positively related to volatility. For high IVol firms, growth options take a larger fraction of the firm value, which makes those firms better hedges against aggregate volatility risk. Therefore, we also predict that the value effect is stronger for high IVol firms, and the IVol discount is stronger for growth firms, and these regularities are explained by aggregate volatility risk.

In double sorts on IVol and market-to-book, we find that the IVol discount is much stronger for growth firms and absent for value firms. We then introduce two aggregate volatility risk factors: the FVIX factor that tracks innovations to expected market volatility (VIX) and the FIVol factor that tracks innovations to average IVol. We show that FVIX and FIVol are strongly and positively correlated with the innovations they mimic, earn significant premiums controlling for market risk, and can predict future volatility, as Chen (2002) argues a volatility risk factor should do.

We show that high IVol, growth, and especially high IVol growth firms have positive FVIX and FIVol betas. It means that during the periods of increasing aggregate volatility, these firms lose significantly less value than what the CAPM predicts. Augmenting the CAPM with FVIX and FIVol perfectly explains the IVol discount and its dependence on market-to-book, as well as the abysmal returns to the highest IVol growth firms. The volatility factor model with FVIX and FIVol also explains the CAPM alpha of the HML factor and the abnormally large value effect for high IVol firms. Comparing the explanatory power of FVIX and FIVol, we find that FVIX is primarily responsible for explaining the IVol discount and contributes moderately to explaining the value effect, and FIVol is the main driving force behind the value effect, but its contribution to explaining the IVol discount is limited.

We also find that the cross-sectional dependence of the value effect and IVol discount

on institutional ownership (Nagel, 2005) and on short sales constraints (Boehme et al., 2009) can be explained by aggregate volatility risk. Similarly, FVIX and FIVol can largely explain the stronger IVol discount for overpriced firms found in Stambaugh et al. (2015).



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## Appendix A: Data Appendix

**IO (institutional ownership)** – the sum of institutional holdings from Thompson Financial 13F database, divided by the shares outstanding from CRSP. If the stock is above the 20th NYSE/AMEX size percentile, appears on CRSP, but not on Thompson Financial 13F, it is assumed to have zero institutional ownership.

**IVol (idiosyncratic volatility)** – the standard deviation of residuals from the Fama-French model, fitted to the daily data for each month (at least 15 valid observations are required). Average IVol is averaged for all firms within each month.

**MB (market-to-book)** – equity value (share price, *prcc*, times number of shares outstanding, *csho*) divided by book equity (*ceq*) plus deferred taxes (*txdb*), all items from Compustat annual files.

**RInst (residual institutional ownership)** – the residual ( $\epsilon$ ) from the logistic regression of institutional ownership (IO) on  $\log(\text{Size})$  and its square:

$$\log\left(\frac{Inst}{1 - Inst}\right) = \gamma_0 + \gamma_1 \cdot \log(Size) + \gamma_2 \cdot \log^2(Size) + \epsilon. \quad (\text{A1})$$

**Realized (realized market volatility)** - the square root of the average squared daily return to the market portfolio (CRSP value-weighted index) within each given month.

**Size (market cap)** – shares outstanding times price, both from the CRSP monthly returns file.

**Short (probability to be on special)** - defined as in D’Avolio (2002) and Ali and Trombley (2006)

$$Short = \frac{e^y}{1 + e^y}, \quad (\text{A2})$$

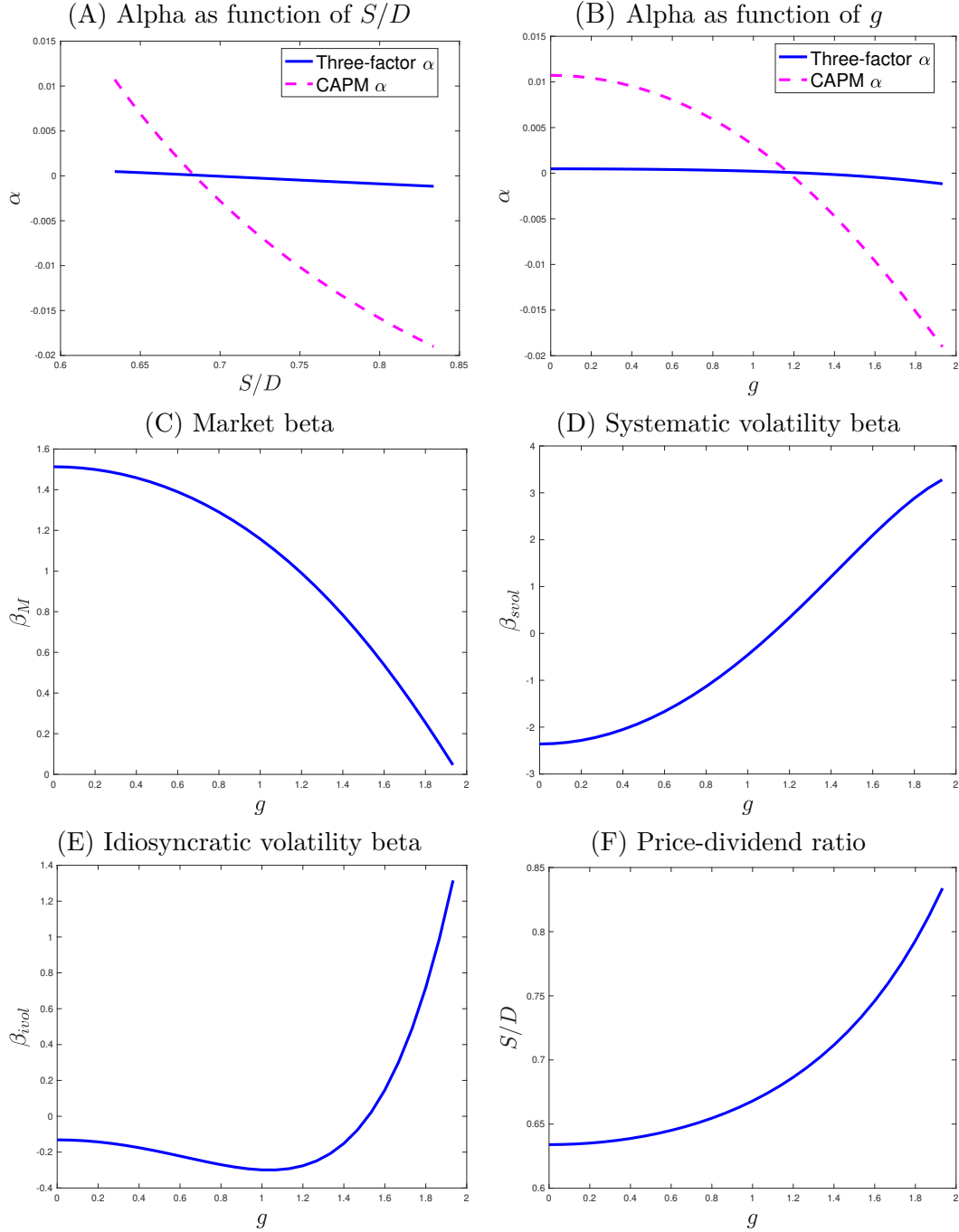
$$y = -0.46 \cdot \log(Size) - 2.8 \cdot IO + 1.59 \cdot Turn - 0.09 \cdot \frac{CF}{TA} + 0.86 \cdot IPO + 0.41 \cdot Glam \quad (\text{A3})$$

*Size* is in million dollars, *Turn* is turnover, defined as the trading volume over shares outstanding (from CRSP). *CF* is cash flow defined as Compustat item OIADP plus Compustat item DP) less non-depreciation accruals, which are change in current assets (Compustat item ACT) less change in current liabilities (Compustat item LCT) plus change in short-term debt (Compustat item DLC) less change in cash (Compustat item CHE). *TA* are total assets (Compustat item AT), *IPO* is the dummy variable equal to 1 if the stock first appeared on CRSP 12 or less months ago, and *Glam* is the dummy variable equal to 1 for three top market-to-book deciles.

**TARCH (expected market volatility)** - from the TARCH(1,1) model (see Glosten, Jagannathan, and Runkle, 1993) fitted to monthly returns to the CRSP value-weighted index:

$$Ret_t^{CRSP} = \gamma_0 + \gamma_1 \cdot Ret_{t-1}^{CRSP} + \epsilon_t, \quad \sigma_t^2 = c_0 + c_1 \sigma_{t-1}^2 + c_2 \epsilon_{t-1}^2 + c_3 \cdot I(\epsilon_{t-1} < 0) \quad (\text{A4})$$

The regression estimated for the full sample. We take the square root out of the volatility forecast to be consistent with our measure of idiosyncratic volatility.



**Figure 1**  
**Effect of idiosyncratic volatility on  $\alpha$ ,  $\beta_M$ ,  $\beta_{svol}$ ,  $\beta_{ivol}$  and  $S/D$  ratio**

This Figure shows alphas and betas of excess returns of portfolios of stocks that ex-ante only differ from each other in terms of idiosyncratic volatility  $g\sqrt{v_t}$ . We show these quantities as functions of the magnitude of idiosyncratic volatility  $g$ . Panel (D) shows the stock-price-output ratios  $S_{i,0}/D_{i,0}$ .

**Table 1. Aggregate and Idiosyncratic Volatility Risk Factors**

Panel A shows the correlations between the new risk factors (FVIX and FIVol), the state variables (VIX and  $\overline{IVOL}$ ), and their innovations ( $\Delta VIX$  and  $\overline{IVOL}^U$ ). FVIX (FIVol) is the aggregate (idiosyncratic) volatility risk factor that tracks innovations to VIX ( $\overline{IVOL}$ ). VIX is the implied volatility of options on S&P 100 index.  $\Delta VIX$  is the monthly change in VIX (FVIX uses daily changes in VIX).  $\overline{IVOL}$  is average IVol averaged across all firms traded during the given month. Idiosyncratic volatility is the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required).  $\overline{IVOL}^U$  is the residual from ARMA(1,1) model fitted to average IVol ( $\overline{IVOL}$ ).

Panels B and C report the average raw return, the CAPM alpha, and the Fama-French alpha, as well as the CAPM and the Fama-French betas for FVIX and FIVol, respectively. Panel B (C) also reports the alpha and the betas from the two-factor model with the market factor and FIVol (FVIX) fitted to the returns to FVIX (FIVol) factor. The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

**Panel A. Correlations**

	VIX	$\Delta VIX$	FVIX	$\overline{IVOL}$	$\overline{IVOL}^U$
$\Delta VIX$	0.291				
t-stat	5.94				
FVIX	0.353	0.676			
t-stat	7.36	17.9			
$\overline{IVOL}$	0.590	0.048	0.115		
t-stat	14.3	0.94	2.25		
$\overline{IVOL}^U$	0.287	0.368	0.224	0.400	
t-stat	5.84	7.73	4.49	8.51	
FIVOL	0.269	0.509	0.558	0.072	0.424
t-stat	5.45	11.5	13.1	1.42	9.15

Panel B. FVIX Factor

	Raw	CAPM	2F	FF	FF4	FF5	FF6
$\alpha$	-1.366	-0.463	-0.480	-0.439	-0.446	-0.305	-0.318
<b>t-stat</b>	<i>-4.77</i>	<i>-4.73</i>	<i>-4.32</i>	<i>-4.00</i>	<i>-3.68</i>	<i>-3.73</i>	<i>-3.50</i>
$\beta_{MKT}$		-1.325	-1.340	-1.358	-1.366	-1.407	-1.427
<b>t-stat</b>		<i>-37.0</i>	<i>-29.9</i>	<i>-35.2</i>	<i>-37.30</i>	<i>-50.70</i>	<i>-51.65</i>
$\beta_{SMB}$				0.170	0.166	0.107	0.095
<b>t-stat</b>				<i>4.94</i>	<i>4.33</i>	<i>4.56</i>	<i>3.49</i>
$\beta_{HML}$				-0.073	-0.078	0.034	0.023
<b>t-stat</b>				<i>-1.41</i>	<i>-2.00</i>	<i>0.59</i>	<i>0.46</i>
$\beta_{FIVol}$			-0.014		-0.007		-0.017
<b>t-stat</b>			<i>-0.47</i>		<i>-0.28</i>		<i>-0.74</i>
$\beta_{CMA}$						-0.142	-0.15
<b>t-stat</b>						<i>-2.31</i>	<i>-2.22</i>
$\beta_{RMW}$						-0.22	-0.23
<b>t-stat</b>						<i>-6.15</i>	<i>-5.89</i>

Panel C. FIVol Factor

	Raw	CAPM	2F	FF	FF4	FF5	FF6
$\alpha$	-1.923	-1.200	-1.361	-0.952	-1.040	-0.743	-0.908
<b>t-stat</b>	<i>-3.70</i>	<i>-2.55</i>	<i>-3.30</i>	<i>-2.62</i>	<i>-3.16</i>	<i>-1.87</i>	<i>-2.59</i>
$\beta_{MKT}$		-1.061	-1.464	-1.094	-1.298	-1.173	-1.785
<b>t-stat</b>		<i>-6.59</i>	<i>-1.49</i>	<i>-10.3</i>	<i>-1.65</i>	<i>-9.87</i>	<i>-2.06</i>
$\beta_{SMB}$				-0.592	-0.569	-0.682	-0.635
<b>t-stat</b>				<i>-3.70</i>	<i>-3.13</i>	<i>-4.71</i>	<i>-3.75</i>
$\beta_{HML}$				-0.800	-0.812	-0.618	-0.618
<b>t-stat</b>				<i>-4.36</i>	<i>-4.87</i>	<i>-3.08</i>	<i>-2.79</i>
$\beta_{FVIX}$			-0.304		-0.151		-0.439
<b>t-stat</b>			<i>-0.47</i>		<i>-0.29</i>		<i>-0.78</i>
$\beta_{CMA}$						-0.268	-0.302
<b>t-stat</b>						<i>-0.99</i>	<i>-0.95</i>
$\beta_{RMW}$						-0.329	-0.416
<b>t-stat</b>						<i>-1.88</i>	<i>-2.09</i>



**Table 2. Idiosyncratic Volatility, Market Volatility,  
and the Business Cycle**

The table presents the slopes from the regressions of the log average IVol ( $\log(\overline{IVOL})$ ) on the business cycle variables. The business cycle variables are the NBER recession dummy, the log of the VIX index, the log market volatility forecast from TARCH(1,1) model, and the log realized market volatility. The numbers in the first row are the number of months by which we lag the business cycle in each column. The slopes indicate the percentage point increase in the average IVol when either the NBER dummy changes from zero to one or any of the other variables increases by 1%. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required). Average IVol is the simple average of the idiosyncratic volatilities of all firms traded during the given month. The NBER recession dummy is one for the months between NBER-announced peak and trough and zero otherwise. VIX index is from CBOE and measures the implied volatility of the one-month options on S&P 100. The TARCH(1,1) model is fitted to monthly returns to the CRSP value-weighted index. The realized market volatility is the square root of the average squared daily return to the market portfolio (CRSP value-weighted index) within each given month. The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

	<b>-12</b>	<b>-9</b>	<b>-6</b>	<b>-3</b>	<b>0</b>	<b>3</b>	<b>6</b>	<b>9</b>	<b>12</b>
<b>NBER</b>	9.706	13.56	16.48	19.07	19.66	14.89	8.608	0.669	-4.196
<b>t-stat</b>	<i>1.61</i>	<i>2.19</i>	<i>2.92</i>	<i>3.25</i>	<i>3.19</i>	<i>2.47</i>	<i>1.40</i>	<i>0.11</i>	<i>-0.69</i>
<b>VIX</b>	0.132	0.180	0.216	0.275	0.351	0.267	0.201	0.178	0.147
<b>t-stat</b>	<i>2.02</i>	<i>2.78</i>	<i>3.33</i>	<i>4.07</i>	<i>5.14</i>	<i>3.73</i>	<i>2.94</i>	<i>2.72</i>	<i>2.21</i>
<b>TARCH</b>	0.117	0.165	0.225	0.304	0.397	0.436	0.367	0.308	0.267
<b>t-stat</b>	<i>1.45</i>	<i>2.15</i>	<i>3.06</i>	<i>4.15</i>	<i>5.40</i>	<i>5.78</i>	<i>4.55</i>	<i>3.77</i>	<i>3.28</i>
<b>Realized</b>	0.096	0.131	0.177	0.216	0.278	0.195	0.150	0.130	0.099
<b>t-stat</b>	<i>2.02</i>	<i>2.82</i>	<i>3.90</i>	<i>4.69</i>	<i>5.90</i>	<i>4.00</i>	<i>3.11</i>	<i>2.70</i>	<i>2.00</i>

**Table 3. Idiosyncratic Volatility and Market-to-Book**

The table presents the value-weighted CAPM alphas (Panel A), alphas from the volatility factor model with the market factor, FVIX and FIVol (Panel B), and FVIX and FIVol betas from the volatility factor model (Panels C and D, respectively) for the 25 IVol - market-to-book portfolios. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required). The portfolios are sorted independently using NYSE (`exchcd=1`) breakpoints. The IVol (market-to-book) portfolios are rebalanced monthly (annually). The FVIX and FIVol betas estimates are from the volatility factor model with the market factor, FVIX and FIVol. FVIX (FIVol) is the aggregate (idiosyncratic) volatility risk factor that tracks innovations to VIX (*IVOL*). The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

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	Panel A. CAPM Alphas						Panel B. Volatility Factor Model Alphas						
	Low	IVol2	IVol3	IVol4	High	L-H	Low	IVol2	IVol3	IVol4	High	L-H	
<b>Value</b>	0.298	0.294	0.427	0.262	0.102	0.196	<b>Value</b>	-0.090	-0.119	0.152	0.133	0.017	-0.107
<b>t-stat</b>	<i>1.25</i>	<i>1.25</i>	<i>1.67</i>	<i>0.98</i>	<i>0.31</i>	<i>0.51</i>	<b>t-stat</b>	<i>-0.43</i>	<i>-0.54</i>	<i>0.58</i>	<i>0.52</i>	<i>0.05</i>	<i>-0.27</i>
<b>MB2</b>	0.311	0.301	0.260	0.438	-0.204	0.514	<b>MB2</b>	0.002	-0.016	-0.091	0.114	-0.031	0.033
<b>t-stat</b>	<i>1.84</i>	<i>1.92</i>	<i>1.21</i>	<i>1.81</i>	<i>-0.94</i>	<i>1.89</i>	<b>t-stat</b>	<i>0.01</i>	<i>-0.11</i>	<i>-0.50</i>	<i>0.59</i>	<i>-0.13</i>	<i>0.11</i>
<b>MB3</b>	0.277	0.318	0.175	-0.092	-0.323	0.600	<b>MB3</b>	-0.085	-0.044	-0.068	-0.315	-0.072	-0.012
<b>t-stat</b>	<i>1.95</i>	<i>1.94</i>	<i>1.06</i>	<i>-0.54</i>	<i>-1.65</i>	<i>2.27</i>	<b>t-stat</b>	<i>-0.56</i>	<i>-0.22</i>	<i>-0.40</i>	<i>-1.82</i>	<i>-0.38</i>	<i>-0.05</i>
<b>MB4</b>	0.441	0.326	-0.031	0.135	-0.473	0.914	<b>MB4</b>	0.088	-0.068	-0.177	0.157	-0.099	0.187
<b>t-stat</b>	<i>3.05</i>	<i>2.48</i>	<i>-0.21</i>	<i>0.88</i>	<i>-2.23</i>	<i>3.25</i>	<b>t-stat</b>	<i>0.68</i>	<i>-0.50</i>	<i>-1.16</i>	<i>1.05</i>	<i>-0.36</i>	<i>0.58</i>
<b>Growth</b>	0.300	0.153	0.064	-0.220	-0.665	0.965	<b>Growth</b>	0.130	0.025	0.183	0.084	0.121	0.009
<b>t-stat</b>	<i>2.31</i>	<i>1.14</i>	<i>0.43</i>	<i>-1.16</i>	<i>-2.89</i>	<i>3.28</i>	<b>t-stat</b>	<i>1.08</i>	<i>0.16</i>	<i>1.08</i>	<i>0.48</i>	<i>0.47</i>	<i>0.03</i>
<b>V-G</b>	-0.002	0.141	0.363	0.482	0.767	0.769	<b>V-G</b>	-0.220	-0.144	-0.032	0.049	-0.104	0.116
<b>t(V-G)</b>	<i>-0.01</i>	<i>0.50</i>	<i>1.18</i>	<i>1.37</i>	<i>2.14</i>	<i>2.20</i>	<b>t(V-G)</b>	<i>-0.86</i>	<i>-0.54</i>	<i>-0.10</i>	<i>0.15</i>	<i>-0.30</i>	<i>0.32</i>

Panel C. FVIX Betas

Panel D. FIVol Betas

	Low	IVol2	IVol3	IVol4	High	L-H		Low	IVol2	IVol3	IVol4	High	L-H
<b>Value</b>	-0.342	-0.262	0.127	0.649	0.749	-1.091	<b>Value</b>	-0.188	-0.240	-0.274	-0.352	-0.354	0.166
<b>t-stat</b>	<i>-1.57</i>	<i>-1.68</i>	<i>0.48</i>	<i>3.87</i>	<i>1.89</i>	<i>-1.93</i>	<b>t-stat</b>	<i>-3.89</i>	<i>-4.78</i>	<i>-5.66</i>	<i>-4.45</i>	<i>-3.56</i>	<i>1.50</i>
<b>MB2</b>	-0.433	-0.324	-0.148	0.090	0.937	-1.370	<b>MB2</b>	-0.089	-0.137	-0.232	-0.300	-0.214	0.125
<b>t-stat</b>	<i>-4.31</i>	<i>-2.69</i>	<i>-0.83</i>	<i>0.33</i>	<i>3.72</i>	<i>-4.21</i>	<b>t-stat</b>	<i>-3.70</i>	<i>-5.07</i>	<i>-9.84</i>	<i>-8.73</i>	<i>-3.22</i>	<i>1.77</i>
<b>MB3</b>	-0.561	-0.488	-0.231	-0.036	0.905	-1.466	<b>MB3</b>	-0.084	-0.112	-0.111	-0.169	-0.138	0.054
<b>t-stat</b>	<i>-4.73</i>	<i>-2.61</i>	<i>-1.38</i>	<i>-0.19</i>	<i>4.90</i>	<i>-5.87</i>	<b>t-stat</b>	<i>-3.08</i>	<i>-3.60</i>	<i>-3.86</i>	<i>-4.66</i>	<i>-5.13</i>	<i>1.30</i>
<b>MB4</b>	-0.722	-0.715	-0.140	0.271	0.989	-1.711	<b>MB4</b>	-0.015	-0.052	-0.067	-0.085	-0.069	0.054
<b>t-stat</b>	<i>-5.04</i>	<i>-3.93</i>	<i>-0.74</i>	<i>2.08</i>	<i>3.06</i>	<i>-4.23</i>	<b>t-stat</b>	<i>-0.50</i>	<i>-2.14</i>	<i>-2.28</i>	<i>-3.89</i>	<i>-1.90</i>	<i>1.11</i>
<b>Growth</b>	-0.517	-0.351	0.192	0.542	1.586	-2.103	<b>Growth</b>	0.057	0.028	0.025	0.044	0.042	0.014
<b>t-stat</b>	<i>-2.76</i>	<i>-2.84</i>	<i>1.40</i>	<i>3.74</i>	<i>4.61</i>	<i>-4.52</i>	<b>t-stat</b>	<i>1.99</i>	<i>1.43</i>	<i>0.95</i>	<i>1.45</i>	<i>0.99</i>	<i>0.23</i>
<b>V-G</b>	0.175	0.089	-0.065	0.107	-0.837	-1.012	<b>V-G</b>	-0.245	-0.268	-0.299	-0.396	-0.397	-0.152
<b>t(V-G)</b>	<i>0.86</i>	<i>0.59</i>	<i>-0.25</i>	<i>0.65</i>	<i>-2.94</i>	<i>-3.67</i>	<b>t(V-G)</b>	<i>-5.55</i>	<i>-5.04</i>	<i>-4.95</i>	<i>-4.04</i>	<i>-3.82</i>	<i>-1.67</i>

**Table 4. Explaining the Idiosyncratic Volatility Effects**

The table reports monthly alphas and betas of the HML factor and the four arbitrage portfolios (IVol, IVolh, HMLh, and IVol55) that measure the IVol discount and the value effect. IVol is the portfolio long in the lowest IVol quintile and short in the highest IVol quintile. IVolh is long in lowest IVol growth portfolio and short in highest IVol growth portfolio. HMLh is long in highest IVol value and short in highest IVol growth portfolio. IVol55 is long in high IVol growth portfolio and short in one-month Treasury bill. The asset-pricing models we fit to their returns are the CAPM, the three-factor Fama-French model (FF3), the Carhart model, the five-factor Fama-French model (FF5), the conditional CAPM (CCAPM), and the volatility factor model with the market factor, FVIX and FIVol. The FVIX and FIVol factors are defined in the heading of Table 3. In the conditional CAPM the conditional betas are assumed to be linear functions of dividend yield, default spread, one-month Treasury bill rate, and term premium. Panel A and B report results for value- and equal-weighted returns, respectively. The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

**Panel A. Value-Weighted Returns**

	$\alpha_{CAPM}$	$\alpha_{FF3}$	$\alpha_{Carhart}$	$\alpha_{FF5}$	$\alpha_{CCAPM}$	$\alpha_{VolF}$	$\beta_{FVIX}$	$\beta_{FIVol}$
<b>HML</b>	0.310				0.244	-0.074	-0.429	-0.152
<b>t-stat</b>	<i>1.56</i>				<i>1.50</i>	<i>-0.40</i>	<i>-1.85</i>	<i>-5.05</i>
<b>HMLh</b>	0.767	0.369	0.641	0.265	0.707	-0.104	-0.837	-0.397
<b>t-stat</b>	<i>2.14</i>	<i>1.57</i>	<i>2.33</i>	<i>1.00</i>	<i>1.82</i>	<i>-0.30</i>	<i>-2.94</i>	<i>-3.82</i>
<b>IVol</b>	0.896	0.660	0.457	0.294	0.580	0.128	-1.915	0.097
<b>t-stat</b>	<i>3.51</i>	<i>4.88</i>	<i>3.25</i>	<i>2.46</i>	<i>2.40</i>	<i>0.51</i>	<i>-4.40</i>	<i>1.62</i>
<b>IVolh</b>	0.965	0.801	0.603	0.356	0.670	0.009	-2.103	0.014
<b>t-stat</b>	<i>3.28</i>	<i>4.16</i>	<i>3.17</i>	<i>1.87</i>	<i>2.48</i>	<i>0.03</i>	<i>-4.52</i>	<i>0.23</i>
<b>IVol55</b>	-0.665	-0.532	-0.416	-0.271	-0.442	0.121	1.586	0.042
<b>t-stat</b>	<i>-2.89</i>	<i>-3.84</i>	<i>-2.94</i>	<i>-2.04</i>	<i>-1.99</i>	<i>0.47</i>	<i>4.61</i>	<i>0.99</i>

**Panel B. Equal-Weighted Returns**

	$\alpha_{CAPM}$	$\alpha_{FF3}$	$\alpha_{Carhart}$	$\alpha_{FF5}$	$\alpha_{CCAPM}$	$\alpha_{VolF}$	$\beta_{FVIX}$	$\beta_{FIVol}$
<b>HMLh</b>	1.092	0.703	0.723	0.417	0.922	0.303	-0.812	-0.339
<b>t-stat</b>	<i>3.64</i>	<i>4.35</i>	<i>5.41</i>	<i>3.37</i>	<i>3.57</i>	<i>1.09</i>	<i>-2.81</i>	<i>-6.82</i>
<b>IVol</b>	0.789	0.633	0.529	0.340	0.565	0.126	-1.409	-0.009
<b>t-stat</b>	<i>4.13</i>	<i>4.74</i>	<i>5.56</i>	<i>4.29</i>	<i>3.16</i>	<i>0.51</i>	<i>-4.05</i>	<i>-0.19</i>
<b>IVolh</b>	1.151	0.865	0.672	0.313	0.864	0.090	-2.082	-0.080
<b>t-stat</b>	<i>4.08</i>	<i>4.94</i>	<i>4.07</i>	<i>2.48</i>	<i>3.29</i>	<i>0.27</i>	<i>-3.70</i>	<i>-1.19</i>
<b>IVol55</b>	-0.888	-0.644	-0.534	-0.369	-0.729	-0.107	1.632	0.021
<b>t-stat</b>	<i>-3.87</i>	<i>-5.75</i>	<i>-4.76</i>	<i>-3.76</i>	<i>-3.11</i>	<i>-0.35</i>	<i>3.72</i>	<i>0.39</i>

**Table 5. Idiosyncratic Volatility Discount: Behavioral Stories**

The table presents the IVol discount (IVol) across the limits-to-arbitrage quintiles. RI is residual IO, defined as the residual from the logistic regression of IO on log size and its square. Sh is the probability to be on special, defined in Data Appendix. IVol is defined as the difference in returns between extreme IVol quintiles. We form all quintiles using NYSE breakpoints. The sorts on IVol are performed separately within each limits to arbitrage quintile. The abnormal returns are from the CAPM and the volatility factor model with the market factor, FVIX, and FIVol. For the volatility factor model, we also report the FVIX and FIVol betas. The FVIX and FIVol factors are defined in the heading of Table 1. The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

Panel A. Residual Institutional Ownership							Panel B. Probability to Be on Special						
	Low	RI 2	RI 3	RI 4	High	L-H		Low	Sh 2	Sh 3	Sh 4	High	H-L
$\alpha_{CAPM}$	1.175	0.976	0.496	0.398	0.484	0.691	$\alpha_{CAPM}$	0.379	0.108	0.415	0.748	1.210	0.831
<b>t-stat</b>	<i>3.77</i>	<i>3.09</i>	<i>1.90</i>	<i>1.87</i>	<i>1.92</i>	<i>2.41</i>	<b>t-stat</b>	<i>1.79</i>	<i>0.40</i>	<i>1.45</i>	<i>2.45</i>	<i>3.17</i>	<i>2.44</i>
$\alpha_{FF3}$	0.993	0.741	0.283	0.289	0.427	0.566	$\alpha_{FF3}$	0.288	-0.132	0.223	0.464	0.880	0.592
<b>t-stat</b>	<i>3.46</i>	<i>3.10</i>	<i>1.42</i>	<i>1.59</i>	<i>1.97</i>	<i>2.06</i>	<b>t-stat</b>	<i>1.44</i>	<i>-0.60</i>	<i>0.99</i>	<i>1.73</i>	<i>2.96</i>	<i>2.00</i>
$\alpha_{VolF}$	0.184	-0.017	-0.297	-0.086	-0.065	0.249	$\alpha_{VolF}$	-0.080	-0.840	-0.513	-0.385	-0.061	0.019
<b>t-stat</b>	<i>0.46</i>	<i>-0.05</i>	<i>-1.14</i>	<i>-0.44</i>	<i>-0.24</i>	<i>0.76</i>	<b>t-stat</b>	<i>-0.38</i>	<i>-2.41</i>	<i>-1.76</i>	<i>-1.02</i>	<i>-0.15</i>	<i>0.05</i>
$\beta_{FVIX}$	-2.258	-2.066	-1.662	-1.309	-1.618	-0.640	$\beta_{FVIX}$	-1.132	-1.762	-1.809	-2.037	-2.280	-1.148
<b>t-stat</b>	<i>-3.83</i>	<i>-3.63</i>	<i>-3.92</i>	<i>-4.22</i>	<i>-4.07</i>	<i>-2.45</i>	<b>t-stat</b>	<i>-5.04</i>	<i>-2.88</i>	<i>-4.43</i>	<i>-3.97</i>	<i>-3.74</i>	<i>-2.18</i>
$\beta_{FIVol}$	0.054	-0.037	-0.014	0.115	0.163	-0.109	$\beta_{FIVol}$	0.046	-0.099	-0.074	-0.155	-0.179	-0.225
<b>t-stat</b>	<i>0.78</i>	<i>-0.47</i>	<i>-0.27</i>	<i>1.65</i>	<i>2.99</i>	<i>-1.87</i>	<b>t-stat</b>	<i>1.03</i>	<i>-1.79</i>	<i>-1.14</i>	<i>-2.73</i>	<i>-2.01</i>	<i>-2.85</i>

**Table 6. Value Effect: Behavioral Stories**

The table presents the value effect across the limits-to-arbitrage quintiles. RI is residual IO, defined as the residual from the logistic regression of IO on log size and its square. Sh is the probability to be on special, defined in Data Appendix. The value effect is defined as the difference in returns between extreme market-to-book quintiles. We form all quintiles using NYSE breakpoints. The sorts on market-to-book are performed separately within each limits to arbitrage quintile. The abnormal returns are from the CAPM and the volatility factor model with the market-factor, FVIX, and FIVol. For the volatility factor model, we also report the FVIX and FIVol betas. The FVIX and FIVol factors are defined in the heading of Table 1. The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

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Panel A. Residual Institutional Ownership							Panel B. Probability to Be on Special						
	Low	RI 2	RI 3	RI 4	High	L-H		Low	Sh 2	Sh 3	Sh 4	High	H-L
$\alpha_{CAPM}$	1.255	0.838	0.526	0.443	0.498	0.757	$\alpha_{CAPM}$	-0.033	0.010	0.223	0.622	0.965	0.998
<b>t-stat</b>	<i>4.01</i>	<i>2.80</i>	<i>2.13</i>	<i>2.34</i>	<i>1.93</i>	<i>2.59</i>	<b>t-stat</b>	<i>-0.13</i>	<i>0.03</i>	<i>0.74</i>	<i>1.63</i>	<i>2.42</i>	<i>3.02</i>
$\alpha_{FF3}$	1.008	0.597	0.308	0.296	0.418	0.590	$\alpha_{FF3}$	-0.267	-0.257	-0.058	0.195	0.513	0.780
<b>t-stat</b>	<i>3.60</i>	<i>2.66</i>	<i>1.63</i>	<i>1.66</i>	<i>1.89</i>	<i>2.16</i>	<b>t-stat</b>	<i>-1.27</i>	<i>-1.10</i>	<i>-0.23</i>	<i>0.71</i>	<i>2.07</i>	<i>2.72</i>
$\alpha_{VolF}$	0.218	-0.143	-0.249	-0.069	-0.089	0.307	$\alpha_{VolF}$	-0.428	-0.520	-0.278	-0.366	-0.171	0.257
<b>t-stat</b>	<i>0.56</i>	<i>-0.46</i>	<i>-0.98</i>	<i>-0.39</i>	<i>-0.32</i>	<i>1.01</i>	<b>t-stat</b>	<i>-1.62</i>	<i>-1.97</i>	<i>-0.92</i>	<i>-1.06</i>	<i>-0.51</i>	<i>0.93</i>
$\beta_{FVIX}$	-2.177	-2.053	-1.611	-1.277	-1.671	-0.505	$\beta_{FVIX}$	-0.322	-0.308	-0.329	-1.071	-1.093	-0.771
<b>t-stat</b>	<i>-4.30</i>	<i>-3.85</i>	<i>-4.00</i>	<i>-5.56</i>	<i>-4.22</i>	<i>-2.13</i>	<b>t-stat</b>	<i>-1.85</i>	<i>-1.40</i>	<i>-1.14</i>	<i>-3.09</i>	<i>-2.52</i>	<i>-2.36</i>
$\beta_{FIVol}$	-0.015	-0.032	-0.018	0.079	0.153	-0.168	$\beta_{FIVol}$	-0.199	-0.330	-0.293	-0.410	-0.523	-0.324
<b>t-stat</b>	<i>-0.25</i>	<i>-0.45</i>	<i>-0.41</i>	<i>1.77</i>	<i>3.49</i>	<i>-2.57</i>	<b>t-stat</b>	<i>-6.98</i>	<i>-5.10</i>	<i>-5.54</i>	<i>-3.97</i>	<i>-6.07</i>	<i>-4.03</i>

**Table 7. Arbitrage Asymmetry and the Idiosyncratic Volatility Discount**

The table presents the value-weighted CAPM alphas (Panel A), alphas from the volatility factor model with the market factor, FVIX and FIVol (Panel B), and FVIX and FIVol betas from the volatility factor model (Panels C and D, respectively) for 25 portfolios sorted on IVol and Stambaugh et al. (2015) mispricing measure. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required). The definition of the Stambaugh et al. mispricing measure is in the Data section. The portfolios are sorted independently using NYSE (`exchcd=1`) breakpoints. The IVol and mispricing portfolios are rebalanced monthly. The FVIX and FIVol betas estimates are from the volatility factor model with the market factor, FVIX and FIVol. FVIX (FIVol) is the aggregate (idiosyncratic) volatility risk factor that tracks innovations to VIX ( $\overline{IVOL}$ ). The t-statistics use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2017.

Panel A. CAPM Alphas							Panel B. Volatility Factor Model Alphas						
	Low	IVol2	IVol3	IVol4	High	L-H		Low	IVol2	IVol3	IVol4	High	L-H
<b>Under</b>	0.247	0.289	0.270	0.558	0.239	0.008	<b>Under</b>	0.012	0.044	0.242	0.693	0.640	-0.628
<b>t-stat</b>	<i>1.99</i>	<i>2.35</i>	<i>1.83</i>	<i>3.80</i>	<i>1.21</i>	<i>0.03</i>	<b>t-stat</b>	<i>0.10</i>	<i>0.31</i>	<i>1.63</i>	<i>4.11</i>	<i>2.76</i>	<i>-2.08</i>
<b>Quint2</b>	0.316	0.356	0.124	0.216	0.117	0.200	<b>Quint2</b>	-0.034	0.024	0.025	0.268	0.622	-0.656
<b>t-stat</b>	<i>2.33</i>	<i>2.35</i>	<i>0.86</i>	<i>1.40</i>	<i>0.52</i>	<i>0.63</i>	<b>t-stat</b>	<i>-0.31</i>	<i>0.15</i>	<i>0.14</i>	<i>2.05</i>	<i>2.43</i>	<i>-2.04</i>
<b>Quint3</b>	0.376	0.322	-0.040	-0.207	-0.113	0.489	<b>Quint3</b>	0.199	0.177	-0.136	-0.234	0.289	-0.091
<b>t-stat</b>	<i>2.26</i>	<i>2.28</i>	<i>-0.25</i>	<i>-1.11</i>	<i>-0.66</i>	<i>1.86</i>	<b>t-stat</b>	<i>1.10</i>	<i>1.08</i>	<i>-0.73</i>	<i>-1.22</i>	<i>1.53</i>	<i>-0.37</i>
<b>Quint4</b>	0.335	0.121	-0.106	-0.162	-0.290	0.625	<b>Quint4</b>	-0.038	-0.139	-0.408	-0.399	0.140	-0.177
<b>t-stat</b>	<i>2.11</i>	<i>0.76</i>	<i>-0.70</i>	<i>-0.93</i>	<i>-1.19</i>	<i>2.42</i>	<b>t-stat</b>	<i>-0.25</i>	<i>-0.78</i>	<i>-2.83</i>	<i>-2.41</i>	<i>0.42</i>	<i>-0.47</i>
<b>Over</b>	0.160	0.078	-0.242	-0.889	-1.389	1.549	<b>Over</b>	-0.429	-0.172	-0.355	-0.889	-0.775	0.345
<b>t-stat</b>	<i>0.66</i>	<i>0.39</i>	<i>-1.38</i>	<i>-4.31</i>	<i>-5.53</i>	<i>4.08</i>	<b>t-stat</b>	<i>-2.19</i>	<i>-0.84</i>	<i>-1.84</i>	<i>-4.28</i>	<i>-3.20</i>	<i>1.20</i>
<b>O-U</b>	0.087	0.211	0.512	1.447	1.628	1.542	<b>O-U</b>	0.441	0.216	0.596	1.581	1.415	0.973
<b>t(O-U)</b>	<i>0.36</i>	<i>0.93</i>	<i>2.10</i>	<i>5.38</i>	<i>5.81</i>	<i>4.28</i>	<b>t(O-U)</b>	<i>1.98</i>	<i>0.90</i>	<i>2.47</i>	<i>5.60</i>	<i>4.76</i>	<i>2.91</i>

Panel C. FVIX Betas

Panel D. FIVol Betas

	Low	IVol2	IVol3	IVol4	High	L-H		Low	IVol2	IVol3	IVol4	High	L-H
<b>Under</b>	-0.580	-0.515	-0.057	0.267	0.793	-1.373	<b>Under</b>	0.027	-0.006	-0.002	0.009	0.028	0.000
<b>t-stat</b>	<i>-3.80</i>	<i>-4.36</i>	<i>-0.31</i>	<i>2.07</i>	<i>3.15</i>	<i>-3.89</i>	<b>t-stat</b>	<i>1.04</i>	<i>-0.24</i>	<i>-0.06</i>	<i>0.26</i>	<i>0.89</i>	<i>-0.01</i>
<b>Quint2</b>	-0.676	-0.447	-0.071	0.285	1.091	-1.767	<b>Quint2</b>	-0.031	-0.103	-0.055	-0.065	0.001	-0.032
<b>t-stat</b>	<i>-4.07</i>	<i>-2.49</i>	<i>-0.38</i>	<i>2.36</i>	<i>4.73</i>	<i>-5.07</i>	<b>t-stat</b>	<i>-1.24</i>	<i>-3.66</i>	<i>-2.30</i>	<i>-1.71</i>	<i>0.01</i>	<i>-0.67</i>
<b>Quint3</b>	-0.387	-0.250	-0.030	0.191	1.043	-1.430	<b>Quint3</b>	0.002	-0.024	-0.068	-0.095	-0.066	0.068
<b>t-stat</b>	<i>-2.75</i>	<i>-1.35</i>	<i>-0.14</i>	<i>1.00</i>	<i>3.83</i>	<i>-4.33</i>	<b>t-stat</b>	<i>0.05</i>	<i>-0.68</i>	<i>-2.69</i>	<i>-2.75</i>	<i>-2.45</i>	<i>1.52</i>
<b>Quint4</b>	-0.520	-0.303	-0.172	-0.017	1.151	-1.670	<b>Quint4</b>	-0.109	-0.099	-0.182	-0.188	-0.084	-0.025
<b>t-stat</b>	<i>-3.15</i>	<i>-1.84</i>	<i>-1.13</i>	<i>-0.10</i>	<i>3.70</i>	<i>-3.87</i>	<b>t-stat</b>	<i>-3.03</i>	<i>-4.38</i>	<i>-5.54</i>	<i>-6.36</i>	<i>-1.55</i>	<i>-0.35</i>
<b>Over</b>	-0.810	-0.189	0.112	0.414	1.523	-2.333	<b>Over</b>	-0.176	-0.134	-0.135	-0.157	-0.074	-0.102
<b>t-stat</b>	<i>-6.11</i>	<i>-1.25</i>	<i>0.47</i>	<i>2.18</i>	<i>4.19</i>	<i>-5.34</i>	<b>t-stat</b>	<i>-4.61</i>	<i>-3.67</i>	<i>-5.46</i>	<i>-3.26</i>	<i>-1.04</i>	<i>-1.38</i>
<b>O-U</b>	0.230	-0.326	-0.169	-0.148	-0.730	-0.960	<b>O-U</b>	0.203	0.128	0.133	0.166	0.102	-0.102
<b>t(O-U)</b>	<i>1.36</i>	<i>-2.03</i>	<i>-0.59</i>	<i>-0.72</i>	<i>-2.78</i>	<i>-4.35</i>	<b>t(O-U)</b>	<i>4.47</i>	<i>3.07</i>	<i>3.63</i>	<i>2.25</i>	<i>1.33</i>	<i>-1.48</i>



# Internet Appendices for “Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns”

## Internet Appendix A: Proofs

**Proof of Proposition 1.** Consider a particular path of Brownian motion  $w_\tau$  that fully determines volatilities  $v_\tau$ . Given the realizations of  $v_\tau$ , random variables  $\int_0^T \sqrt{v_\tau} dw_{i,n,\tau}$  in (5) are i.i.d. normal  $N(0, \int_0^T v_\tau d\tau)$ , and have finite mean and variance. Then, applying strong law of large numbers, we obtain

$$\frac{1}{N} \sum_{n=1}^N \exp\left(\lambda g_i \int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau}\right) \xrightarrow{N \rightarrow +\infty} \tilde{\mathbb{E}}\left[\exp\left(\lambda g_i \int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau}\right)\right], \quad (\text{A1})$$

where  $\tilde{\mathbb{E}}[\cdot]$  is a cross-sectional expectation operator. Using the fact that  $\int_0^T \sqrt{v_\tau} dw_{i,n,\tau}$  is normally distributed in the cross-section and computing the cross-sectional expectation, we obtain

$$\frac{1}{N} \sum_{n=1}^N \exp\left\{\lambda g_i \int_0^T \sqrt{v_{2\tau}} dw_{i,n,\tau}\right\} \xrightarrow{N \rightarrow +\infty} \exp\left\{0.5 \lambda^2 g_i^2 \int_0^T v_{2\tau} d\tau\right\}. \quad (\text{A2})$$

Therefore, passing to the limit  $N \rightarrow +\infty$  in Equation (5) for aggregate consumption, and using the limit (A2) we obtain the aggregate consumption (6). ■

### Proof of Proposition 2.

1) By rewriting equation (4), we obtain:

$$\begin{aligned} \tilde{S}_{i,n,t} &= \frac{1}{\xi_t} \mathbb{E}_t \left[ \xi_T \left( p_i D_{0,n,T} + q_i \left( \frac{D_{i,n,T}}{K D_{i,n,0}} \right)^\lambda K D_{i,n,0} \right) \right] \\ &= p_i D_{0,n,t} \mathbb{E}_t \left[ \frac{\xi_T D_{0,n,T}}{\xi_t D_{0,n,t}} \right] + q_i \left( \frac{D_{i,n,t}}{D_{i,n,0}} \right)^\lambda K^{1-\lambda} D_{i,n,0} \mathbb{E}_t \left[ \frac{\xi_T \left( \frac{D_{i,n,T}}{D_{i,n,t}} \right)^\lambda}{\xi_t \left( \frac{D_{i,n,t}}{D_{i,n,0}} \right)^\lambda} \right], \end{aligned} \quad (\text{A3})$$

which gives us equation (7) for stock prices. Next, using the law of iterated expectations,

we obtain:

$$\begin{aligned}
F_{i,t} &= \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \left( \frac{D_{i,n,T}}{D_{i,n,t}} \right)^\lambda \right] = \mathbb{E}_t \left[ \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \left( \frac{D_{i,n,T}}{D_{i,n,t}} \right)^\lambda \middle| w_\tau \right] \right] \\
&= \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \exp \left\{ \lambda \mu_{D,i}(T-t) - 0.5 \lambda h_i^2 \int_t^T v_{1\tau} d\tau - 0.5 \lambda g_i^2 \int_t^T v_{2\tau} d\tau + \lambda h_i \int_t^T \sqrt{v_{1\tau}} dw_\tau \right\} \right] \\
&\quad \times \mathbb{E}_t \left[ \exp \left\{ \lambda g_i \int_t^T \sqrt{v_{2\tau}} dw_{i,n,\tau} \middle| w_\tau \right\} \right] \\
&= \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \exp \left\{ \lambda \mu_{D,i}(T-t) - \frac{\lambda h_i^2}{2} \int_t^T v_{1\tau} d\tau - \frac{\lambda(1-\lambda)g_i^2}{2} \int_t^T v_{2\tau} d\tau + \lambda h_i \int_t^T \sqrt{v_{1\tau}} dw_\tau \right\} \right], \tag{A4}
\end{aligned}$$

where  $E_t[\cdot|w_\tau]$  is the expectation conditional on all realizations of systematic Brownian motions  $w_\tau$  for all  $\tau \in [0, T]$ . Rewriting (A4) in terms of  $V_t$  and  $\Sigma_t$ , we obtain (9). To derive (A4) we also use the fact that because  $w$  and  $w_{i,n}$  are independent, conditional on knowing  $w$ ,

$$\mathbb{E}_t \left[ \exp \left\{ \lambda g_i \int_t^T \sqrt{v_{2\tau}} dw_{i,n,\tau} \middle| w_\tau \right\} \right] = \exp \left\{ 0.5 \lambda^2 g_i^2 \int_t^T v_{2\tau} d\tau \right\}. \tag{A5}$$

Function  $\widehat{F}_t$  in (8) is obtained along the same lines.

2) Equation (10) is a special case of equation (7), in which we set  $t = 0$  and assume that all  $D_{n,i,0}$  are the same for all firms. Equation (10) is obtained by applying the law of large numbers along the same lines as equation (6) for the aggregate consumption. ■

## Internet Appendix B: Firm heterogeneity across systematic and idiosyncratic volatility

In our benchmark analysis, at date  $t = 0$  the firms are heterogeneous only in terms of their idiosyncratic volatility parameter  $g$ , and hence, all equilibrium processes are functions of parameter  $g$  only. Consequently, the value effect and the idiosyncratic volatility discount in our model are driven by the same variable. We here extend our analysis and make firms heterogeneous both in the idiosyncratic and systematic volatility parameters  $g$  and  $h$ , respectively. Higher  $h$  results in an increase in systematic volatility and hence in an increase in the value of growth options, but leaves idiosyncratic volatility unaffected, thus creating variation in  $P/D$  ratio (the model analogue of market-to-book) unrelated to variation in idiosyncratic volatility.

All the equations describing the equilibrium remain unchanged because they already allow for the heterogeneity in parameter  $h$ . Furthermore, in addition to the market factor

and systematic and idiosyncratic volatility factors we allow for another factor given by  $\int_0^T \sqrt{v_{1\tau}} dw_\tau$ , which we label as the total systematic volatility shock. This factor is motivated by equation (6) for the aggregate consumption  $C_T$ , and also affects the state price density via  $C_T$ .

We consider 100 firms with different combinations of parameters  $g$  and  $h$  and calculate their returns using Monte Carlo simulations. Then, we compute the CAPM alpha and factor alpha and betas. For generality and robustness check, we further extend the set of factors that we use to explain the cross-section of returns. The estimation of alpha and betas follows our baseline analysis, where we first construct the factor mimicking portfolios and then run the following four-factor regression:

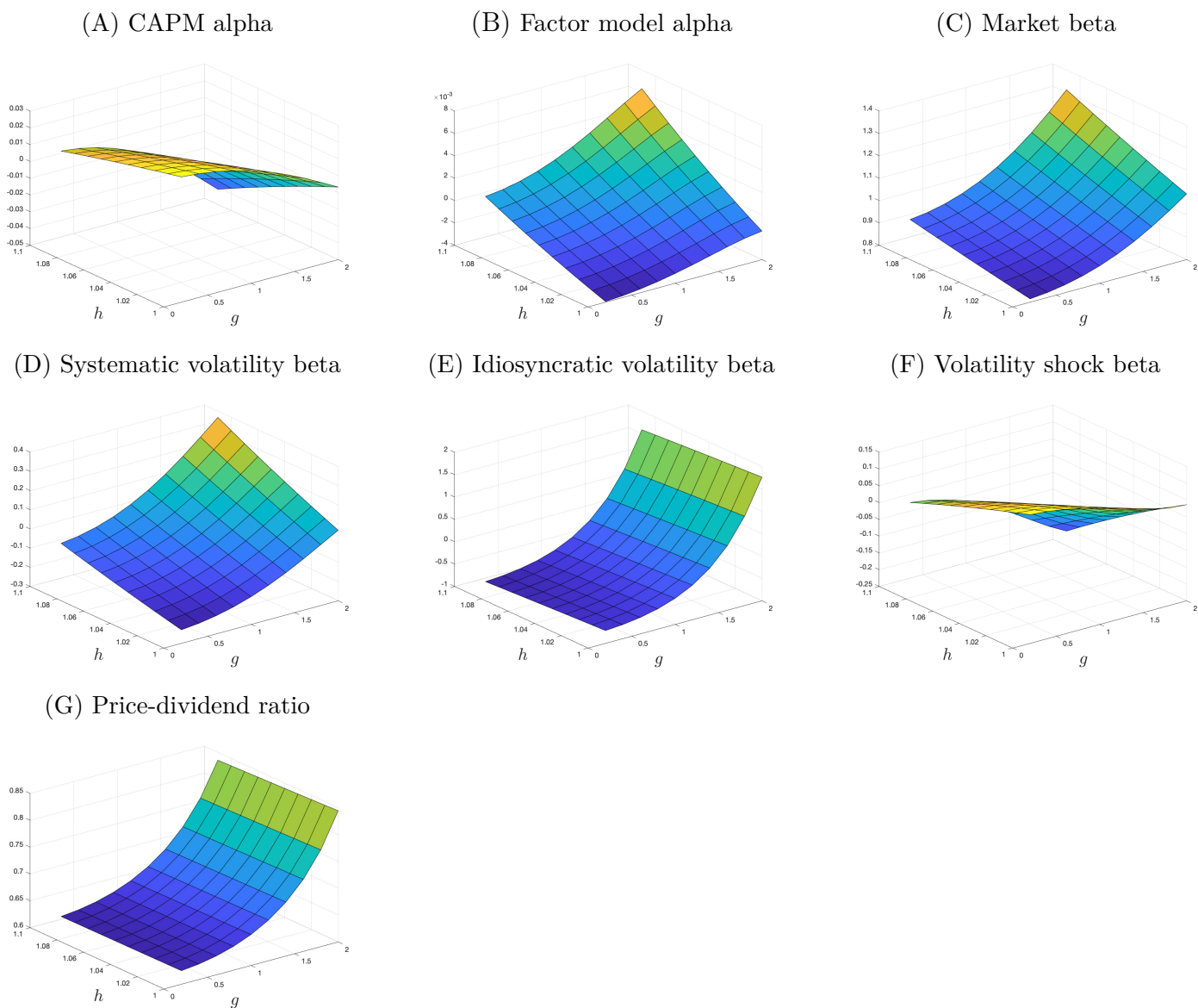
$$r_i - r_f = \alpha + \beta_M(r_M - r_f) + \beta_{\text{svol}}r_{\text{svol}} + \beta_{\text{ivol}}r_{\text{ivol}} + \beta_{\text{vshock}}r_{\text{vshock}} + \varepsilon, \quad (\text{B1})$$

where  $r_{\text{svol}}$ ,  $r_{\text{ivol}}$ ,  $r_{\text{vshock}}$  are the returns of the factor mimicking portfolios.

Figure B1 reports the CAPM and factor model alphas, and factor betas as functions of the idiosyncratic and systematic volatility parameters  $g$  and  $h$ . Larger values of parameters  $g$  and  $h$  signify higher idiosyncratic and systematic volatilities, respectively. The results are consistent with our baseline analysis. First, we observe that firms with the highest idiosyncratic volatility (highest  $g$ ) and highest  $P/D$  (highest  $h$ ) have the most negative CAPM alphas, and therefore the slope of the plane in Figure 2B along  $g$  dimension (the idiosyncratic volatility discount) is the steepest if  $h$  is high, and the slope along  $h$  dimension (the value effect) is the steepest for high values of  $g$ .

Second, Figure 2B shows that after controlling for the volatility factors the alphas are small and much lower than the CAPM alphas (by around 75%). This applies to the model analogues of the value effect and the idiosyncratic volatility discount, as well as the alpha of firms with high values of both  $g$  and  $h$  (the model analogue of growth firms with high idiosyncratic volatility).

Third, idiosyncratic and systematic volatility betas are positive and large for firms with large idiosyncratic ( $g$ ) and systematic ( $h$ ) volatility parameters, mirroring the CAPM alphas and reinforcing our empirical result that growth firms with high idiosyncratic volatility are the best hedges against aggregate volatility risk.



**Figure B1**

**Effect of idiosyncratic volatility on  $\alpha$ , factor betas, and  $S/D$  ratio**

This Figure shows alphas and betas of excess returns of portfolios of stocks that ex-ante only differ from each other in terms of idiosyncratic volatility  $g\sqrt{v_t}$ . We show these quantities as functions of the magnitude of idiosyncratic volatility  $g$ . Panel (D) shows the stock-price-output ratios  $S_{i,0}/D_{i,0}$ .