Homework 4

AEC 504 - Summer 2007

Fundamentals of Economics

1 General Equilibrium in the Endowment Economy

Consider an economy populated by two identical agents A and B with preferences given by $U = XY$. A is endowed with 2 units of X and 1 unit of Y and B is endowed with 1 unit of X and 5 units of Y.

i. Assume A and B are price takers. Compute their demand and excess demand for X and Y. What is the market demand for X and Y?

ii. Define the equilibrium in the market for X and compute the equilibrium price $P \equiv p_Y/p_X$. What is the market demand for Y at this price? Is it accidental?

iii. What is the equilibrium allocation of goods? What is the utility change brought about by the trade?

iv. Draw the Edgeworth box for the economy and compute the locus of all Pareto-efficient points (aka the contract line). Draw the contract line in the Edgeworth box

v. Draw the initial allocation in the Edgeworth box. What is the set of allocations that improve the welfare of both A and B compared to the initial allocation? Which of these allocations are efficient?

vi. Draw the offer curves and depict the competitive equilibrium in the economy. Is the equilibrium efficient? Why?

vii. Suppose A knows B's preferences and can make a take-it-or-leave-it (TIOLI) offer in terms of X and Y. If B rejects the offer, there is no trade. What will the resulting allocation be? Will it be efficient?

Hint: B will reject offers that make him worse off than the initial allocation, and accept all the other.

viii. Assume that A still knows B's preferences and can make a TIOLI offer, but now she can only offer a price, and B is free to choose the quantities he wants to exchange. What will the resulting allocation be? Will it be efficient?

ix. Suppose the government wants to take away a unit of X. The three proposals are:

a. Take away a unit of X from A.

b. Take away a unit of X from B.

c. Impose a 50% tax on the consumption of X by both A and B.

Compute the equilibrium price and the equilibrium allocation for all three cases. What happens to the welfare of A and B in all three cases? Are the equilibrium allocations efficient?

Hint: Do not confuse the consumption tax and the sales tax. The consumption tax is paid from all X they consume, not from what they trade.

x. Suppose the government decides that A is poorer than B and has to be exempt from the consumption tax on X. Without making any calculations, predict whether the resulting equilibrium will be efficient.

2 Monopoly and the Tragedy of Commons

When Snow White settled with the seven dwarfs, they promised her 10% of their total profits from diamond production. The dwarfs have the only diamond mine in the country, so they can act like a monopoly and face the demand curve $P = 100 - Q$, where Q is the number of diamonds they produce. It costs \$20 to produce a diamond.

i. If Snow White can tell the dwarfs how many diamonds to produce and they will comply, what number of diamonds will she choose? How much money will she get from them?

ii. Suppose now that the dwarfs are greedy and will produce diamonds as long as the production of an additional one gives nonzero profit. How much money will Snow White get from them now? What is the economic intuition behind the result in terms of externalities? What is the intuition in terms of competitive markets?

iii. Suppose now that Snow White can cook tasty food, make dwarfs' home cozy and take other actions to raise their opportunity costs of production (that is, each dwarf will act as if it cost $C > \$20$ to produce a diamond, but this increase in costs will not be reflected in profits). How high should Snow White make the costs in order to get as much money as in (i)?

Hint: What you have to do is to find cost C (per diamond produced), facing which dwarfs will produce in (ii) as many diamonds as they produce in (i) facing cost \$20 per diamond.

iv. What is the loss/gain to diamond buyers as the dwarfs switch from their behavior in (ii) to their behavior in (iii)? What is the loss/gain to the society (the reduction/increase of total welfare of the diamond buyers, the dwarfs and Snow White)? Assume that only monetary benefits count.

v. Suppose diamond buyers know that Snow White can influence dwarfs' behavior and can communicate to her. What would be optimal for them to do?

3 Crime and Punishment

Bret Maverick received some training in finance, which made his utility function equal to the the ratio of expected payoff to the standard deviation of a gamble, that is, his utility from gamble X is now equal to $U = E(X)/\sigma(X)$. But, alas, the training did not change his moral values. When he accepted a position in the industry, he identified at once two fraud opportunities. The first (second) opportunity offered him \$10K (\$38K) payoff and

80% (50%) probability to get away with it. If Bret Maverick gets caught, he suffers a loss of \$25K.

i. Which fraud opportunity is the best for Bret Maverick?

ii. Suppose Bret Maverick's boss Commodore Duvall offers him \$500 fair play bonus: if Bret Maverick does not commit any fraud, he will get the bonus for sure. What will Bret Maverick do?

iii. Suppose now that Commodore offers Maverick nothing, but Bret Maverick's old friend Chief Joseph tempts him to go for a few dollars more and offers him an additional fraud opportunity. The additional fraud can be committed only together with Maverick's choice from (i). It will increase both possible gain and possible loss of Maverick's choice from (i), but will leave its expected gain unchanged. Will Bret Maverick commit the additional fraud? (Do not perform any calculations)

4 Externalities, Property Rights, and Fiscal Illusion

Chief White Halfoat plans to build a wigwam. The value of the wigwam to Chief White Halfoat equals to the level of effort he exerts during production. The cost of level of effort x are $0.25x^2$. The local authorities can vindicate the wigwam from Chief White Halfoat (e.g., to search oil under the wigwam) for a certain price and he cannot reject the offer or bargain. The benefits to the society from taking over Chief White Halfoat's wigwam with probability p are equal to $0.5 \ln p$. Both local authorities and Chief White Halfoat are risk-neutral.

i. Suppose that local authorities can give orders about the effort to Chief White Halfoat and try to maximize the (expected) social welfare, which is equal to $0.5 \ln p + (1 - p) \cdot x - 0.25x^2$ (benefits to the society from taking over the wigwam plus expected value of the wigwam to Chief White Halfoat (given the probability of vindication) minus his costs of building it he incurs anyway, since he does not know if the wigwam will be vindicated or not when he builds it). Find the optimal level of Chief White Halfoat's effort and the optimal probability of vindication.

Hint: Note that the social welfare does not depend on the price Chief White Halfoat will receive for his wigwam, since it is just a transfer from local authorities to Chief White Halfoat, i.e., the decision on the price neither creates nor destroys value.

ii. Suppose now that local authorities can give no orders to Chief White Halfoat and there is a law, which requires that local authorities pay the full value of the wigwam if they choose to vindicate it. (Assume that the value is observable to local authorities). What are the optimal values of Chief White Halfoat's effort and probability of vindication? How do they compare to the socially optimal values? Why? (Think in terms of externalities).

Hint: First, compute the optimal value of Chief White Halfoat's effort given that he gets the value of the wigwam and spends the effort for sure. Then maximize social welfare with respect only to the probability of vindication.

iii. Finally, suppose that the local authorities cannot observe the value of the wigwam and are afraid that Chief White Halfoat will name an extremely high value if he is asked about it. So, they decide to offer him a fixed compensation of 3/4 he cannot reject. Moreover, the local authorities now suffer from fiscal illusion: they have the same benefits from vindication the society has, but think that the only cost of vindication is the (expected) price they pay to Chief White Halfoat. Find the optimal values of Chief White Halfoat's effort and the probability of vindication in this case. Compare them with the results from (i) and (ii). What causes the difference? (Think again in terms of externalities). What is the fixed value of compensation local authorities should offer to get the socially optimal value of effort and probability of vindication even under fiscal illusion?

Hint: Proceed the same way you did in (ii), but now start with maximizing local authorities' welfare, which now consists of the social benefits of vindication and the expected value of the compensation. Find the optimal probability of vindication. Then maximize Chief White Halfoat's welfare, assuming the probability unknown.