Solutions to Homework 3

AEC 504 - Summer 2007

Fundamentals of Economics

1 Price Discrimination

Consider a firm with $MC = AC = 2$, which serves two markets with demand functions $Q_1 = 10 - p_1$ and $Q_2 = 15 - 2p_2$.

i. If the firm can price-discriminate (set different prices in different markets), how much it will charge in each market?

$$\max_{P_1} \Pi_1 = (10 - p_1) \cdot (p_1 - 2) \Rightarrow 10 - 2p_1 + 2 = 0 \Rightarrow p_1 = 6 \quad (1)$$

$$\max_{P_2} \Pi_2 = (15 - 2p_2) \cdot (p_2 - 2) \Rightarrow 15 - 4p_2 + 4 = 0 \Rightarrow p_2 = 4.75 \quad (2)$$

ii. Compute the elasticities of demand for both markets at the optimal prices. What is the relationship between the optimal prices and demand elasticities at them? Do you expect this relationship to hold always?

Hint: Think about the optimal markup formula.

$$Q = a - bP \Rightarrow \eta = \frac{bP}{a - bP} \Rightarrow \eta_1 = \frac{1 \cdot 6}{4} = \frac{3}{2}; \quad \eta_2 = \frac{2 \cdot 4.75}{5.5} = \frac{19}{11} > \frac{3}{2} \quad (3)$$

So, we have $\eta_1 < \eta_2$ and $P_1 > P_2$, which is perfectly consistent with the optimal markup formula

$$\frac{P - MC}{P} = \frac{1}{\eta}$$

and will always be true: the monopoly charges more in the market with less elastic demand.

iii. Suppose the firm cannot tell separate the markets and thus views them as one market. What is the demand curve the firm faces in the market?

$$Q = \begin{cases} 25 - 3P, \ P \leq 7.5 \\ 10 - P, \ 7.5 \leq P \leq 10 \end{cases}$$

We just add up the RHS of the demand functions, when the demand is nonzero in both markets, and equate the total demand to demand in market 1, when market 2 shuts down because of too high price.

iv. Compute the optimal price the firm will charge in both markets

Evidently, the demand curve in (iii) is kinked. The firm can choose which part of the demand curve it takes. If it takes the upper part, it solves $\max_P (10 - P) \cdot (P - 2) \Rightarrow P = 6$***, which contradicts to*** $7.5 \leq P \leq 10$***. So, the best the firm can do in the upper part is set*** $P = 7.5$ ***and get*** $\Pi_1 = 2.5 \cdot 5.5 = 13.75$***.***

If it takes the lower part, it solves $\max_P (25 - 3P) \cdot (P - 2) \Rightarrow P = \frac{31}{6} < 7.5$ ***and it gets*** $\Pi_2 = \frac{19}{2} \cdot \frac{19}{6} = \frac{361}{12} = 30.83 > \Pi_1 = 13.75$***. So, the firm takes the lower part, i.e., serves both markets.***

v. Compare the total output (sales) of the firm in (i) and in (iv)

$$Q_{1(i)} = 10 - 6 = 4, \ Q_{2(i)} = 15 - 2 \cdot 4.75 = 5.5 \Rightarrow Q_{(i)} = 9.5;$$
$$Q_{(iv)} = 25 - 3 \cdot \frac{31}{6} = \frac{19}{2} = 9.5 \Rightarrow Q_{(i)} = Q_{(iv)}$$

In fact, it is always true for linear demand: the total sales under price discrimination are equal to the total sales in the common (pooled) market.

vi. Compare social welfare (firm's profit plus the sum of consumer surpluses in both markets) in (i) and (iv). What is better for the society: price discrimination or the absense of such?

Under price discrimination

$$\Pi_1 = 4 \cdot 4 = 16,\ \Pi_2 = \frac{11}{2} \cdot \frac{11}{4} = \frac{121}{8} = 15.125 \Rightarrow \Pi_{(i)} = 31.125$$

$$CS_1 = \frac{1}{2} \cdot 4 \cdot 4 = 8,\ CS_2 = \frac{1}{2} \cdot \frac{11}{4} \cdot \frac{11}{2} = \frac{121}{16} = 7.5625 \Rightarrow CS_{(i)} = 15.5625$$

In fact, under monopoly and linear demand the following is always true: $CS = 0.5 \cdot \Pi$***,*** $SW = 3 \cdot CS = 1.5 \cdot \Pi$***. So,*** $SW_{(i)} \approx 46.7$***.***

In the pooled market

$$\Pi_{(iv)} = 30.83, \quad CS_1 = \frac{1}{2} \cdot (\frac{29}{6})^2 \approx 11.68,\ CS_2 = \frac{1}{2} \cdot \frac{14}{3} \cdot \frac{14}{6} = \frac{196}{36} \approx 5.45 \Rightarrow$$

$$CS_{(iv)} = 17.13,\ SW_{(iv)} = 47.96 > SW_{(i)} = 46.7$$

So, we see that as price discrimination is abolished, the firm and the consumers in the high-elasticity market lose, and the consumers in low-elasticity market win. Under linear demand, the absense of price discrimination is always optimal (when both markets are served).

<u>Remark</u>: Notice that measuring CS in the pooled market as the area under $Q = 25 - 3P$ ***would be misleading because of the kink in the demand curve. Measuring the area under the kinked demand curve from (iii) would give the right CS, but it is hardly easier than the route I followed above and is less insightful.***

vii. Suppose firm's MC=AC jump to 7. How will your answers to (i) and (iv) change?

Under price discrimination the firm solves

$$\max_{P_1}(10 - p_1) \cdot (p_1 - 7) \Rightarrow p_1 = 8.5 \quad (4)$$

$$\max_{P_2}(15 - 2p_2) \cdot (p_2 - 7) \Rightarrow p_2 = 5.75. \quad (5)$$

If price discrimination is prohibited, the firm again chooses between the upper and the lower part of the demand in the pooled market. If it takes the upper part, it solves $\max_{P}(10 - P) \cdot (P - 7) \Rightarrow P = 8.5 \in [7.5,\ 10]$ ***and gets*** $= 1.5^2 = 2.25$***.***

If it takes the lower part, it solves $\max_{P}(25 - 3P) \cdot (P - 7) \Rightarrow P = \frac{23}{3} < 7.5$ ***and gets*** $\Pi = 2 \cdot \frac{2}{3} = \frac{4}{3} < 2.25$***. So, the firm chooses the upper part, i.e. serves only the first market.***

2 Comprehensive Cournot Analysis

Consider two identical firms with *MC=AC=1* facing demand curve $P = 10 - -Q$ and competing a-lá Cournot under duopoly.

i. Solve for the reaction curves of both firms and the market equilibrium (price, quantities, profits)

I use subscript 1 for firm 1 and subscript 2 for firm 2. I also use the fact that $\Pi = PQ - TC = (P - AC)Q$***.***

Firm 1 solves

$$\max_{Q_1}(10 - Q_1 - Q_2 - 1) \cdot Q_1 \Rightarrow FOC:\ 2Q_1 + Q_2 = 9$$

In the same way, for firm 2

$$\max_{Q_2}(10 - Q_1 - Q_2 - 1) \cdot Q_2 \Rightarrow FOC:\ Q_1 + 2Q_2 = 9$$

$2Q_1 + Q_2 = 9$ ***and*** $Q_1 + 2Q_2 = 9$ ***are the reaction curves.***

To find the equilibrium quantities, we have to combine the reaction curves in a system of equations

$$\begin{cases} 2Q_1 + Q_2 = 9 \\ Q_1 + 2Q_2 = 9 \end{cases} \Leftrightarrow \begin{cases} Q_1 = 3 \\ Q_2 = 3 \end{cases} \Rightarrow P = 4, \quad \Pi_1 = \Pi_2 = 9$$

Analyze the impact of the following on the equilibrium price, individual and total production, individual and total profit, and social welfare (you need to show both calculations and economic intuition behind the changes you observe):

ii. MC and AC of the second firm increase from 1 to 2

Now firm 2 solves

$$\max_{Q_2}(10 - Q_1 - Q_2 - 2) \cdot Q_2 \Rightarrow FOC:\ Q_1 + 2Q_2 = 8$$

Its reaction curve shifts downwards. The equilibrium is determined by

$$\begin{cases} 2Q_1 + Q_2 = 9 \\ Q_1 + 2Q_2 = 8 \end{cases} \Leftrightarrow \begin{cases} Q_1 = \dfrac{10}{3} > 3 \\ Q_2 = \dfrac{7}{3} < 3 \end{cases} \Rightarrow Q_1 + Q_2 = \frac{17}{3} < 6$$

$$p = 10 - \frac{17}{3} = \frac{13}{3} > 4, \quad \Pi_1 = \frac{100}{9} > 9, \ \Pi_2 = \frac{49}{9} < 9, \quad \Pi_1 + \Pi_2 = \frac{149}{9} < 18$$

So, we see that firm 1 produces more, because it is relatively more competitive now, and firm 2 produces less because it is relatively less competitive. The same reasoning explains why firm 1 now has higher profits and firm 2 has lower profits. The price rises because on average the production is more costly now. For the same reason total production and total profits drop.

It is obvious (you can draw a graph to see it) that for this demand function $CS = \frac{1}{2}(Q_1 + Q_2)^2$***.***

$$SW = CS + \Pi_1 + \Pi_2 \Rightarrow SW_{(i)} = 18 + 9 + 9 = 36; \quad SW_{(ii)} = \frac{289}{18} + \frac{149}{9} = 32.6$$

Social welfare decreases (both total profit and consumer surplus do) because on average the production is more costly.

iii. The demand function changes to $P = 10 - 2Q$

$$\begin{cases} 2Q_1 + Q_2 = 9/2 \\ Q_1 + 2Q_2 = 9/2 \end{cases} \Leftrightarrow \begin{cases} Q_1 = 3/2 < 3 \\ Q_2 = 3/2 < 3 \end{cases} \Rightarrow Q_1 + Q_2 = 3 < 6$$

$$P = 4 \Rightarrow \Pi_1 = \Pi_2 = 9/2 \Rightarrow \Pi_1 + \Pi_2 = 9 < 18, \quad SW_{(iii)} = 0.5 \cdot 3 \cdot 6 + 9 = 18 < 36$$

The fact that the slope of the demand function has doubled means that consumers are satiated with the product twice as fast as before, that is, the market shrinks in two times. That is why all the variables of interest, except for price, are cut in half. The price stays constant because it reflects the relative strength of buyers and sellers, which does not change as the market size changes.

iv. The firms collude

If the firms collude, they act as one monopoly and divide the profits and production. To get the optimal production level, they solve

$$\max_Q (10 - Q - 1) \cdot Q \Rightarrow Q = \frac{9}{2} < 6, \ Q_1 = Q_2 = \frac{9}{4} < 3, \ \ P = \frac{11}{2} > 4$$

$$\Pi_1 = \Pi_2 = \frac{81}{8} < 9, \;\; \Pi = \frac{81}{4} > 18, \quad CS = \frac{81}{8} < 18, \; SW = \frac{243}{8} < 36$$

Now the market environment is less competitive, so price increases and quantity (individual and total) decreases. The firms get higher profits, because they are able now to coordinate their production effort. However, this profit increase comes at the cost of too high consumer losses, and social welfare decreases, because it always does as competition decreases. (Duopoly means some competition between the firms, monopoly means none).

v. MC and AC of the second firm increase from 1 to 2 and the firms collude

The trick here is to notice that if the firms collude, the low cost firm 1 will produce everything and firm 2 will sit back and get its 50% of total profits just for the fact that is does not interfere. So all the answers remain the same as in (iv).

3 Double Markup

The price of boots ordered online (p) consists of the price of boots charged by Zappos.com (p_1) and the price of shipping charged by UPS (p_2). Assume that there are no other online sales and shipping companies, and Zappos.com and UPS compete a-lá Cournot, but choose prices instead of quantities. The demand for boots ordered online is $20 - p$.

i. Find the equilibrium prices, profits of the firms, and quantity of boots

Zappos.com solves

$$\max_{P_1}(20 - p_1 - p_2) \cdot (p_1 - 7) \Rightarrow FOC : 2p_1 + p_2 = 27$$

UPS solves

$$\max_{P_2}(20 - p_1 - p_2) \cdot (p_2 - 4) \Rightarrow FOC : p_1 + 2p_2 = 24$$

The equilibrium is determined by the following system

$$\begin{cases} 2p_1 + p_2 = 27 \\ p_1 + 2p_2 = 24 \end{cases} \Leftrightarrow \begin{cases} p_1 = 10 \\ p_2 = 7 \end{cases} \Rightarrow Q = 3, \quad \Pi_1 = 9, \; \Pi_2 = 9$$

ii. How will your answer to (i) change if Zappos.com and UPS collude? What is the total loss/gain to the society from the collusion? What is the economic intuition behind the results?

If they collude, they will solve

$$\max_P (20-p)\cdot(p-11) \Rightarrow p = \frac{31}{2} < 17 \Rightarrow Q = \frac{9}{2} > 3 \Rightarrow \Pi_1 = \frac{81}{4} > 18$$

$$CS_{(i)} = \frac{1}{2}\cdot 3^2 = \frac{9}{2},\ \ CS_{(ii)} = \frac{1}{2}\cdot(\frac{9}{2})^2 = \frac{81}{8} > \frac{9}{2};\ \ SW_{(i)} = \frac{45}{2},\ \ SW_{(ii)} = \frac{243}{8} > \frac{45}{2}$$

So, while the collusion is beneficial to the firms, it is also beneficial to the consumers and it looks like it actually increases the amount of competition, because price decreases and quantity increases after the firms collude. The reason is that before the collusion we actually had two monopolies: Zappos.com ripped off UPS and UPS in turn ripped off the consumers. So, the online purchased shoes bore double markup – one from UPS and one from Zappos.com. The collusion eliminated the double markup.

4 Discrete Cournot Duopoly, Collusion and Repeated Interactions

Two hens lay eggs at the cost of 5¢ per egg. Assume they can lay as many eggs as they want and face the demand curve for eggs

$$P = \begin{cases} 35 - Q^2,\ Q < 6 \\ 0,\ Q \geq 6 \end{cases}$$

i. Assuming that hens lay only integer number of eggs, find the equilibrium of the game by eliminating (weakly) dominant strategies

It is easy to notice that Q=0 and $Q \geq 6$ ***are weakly dominated, as they imply*** $\Pi \leq 0$***. For 5 other strategies, we have to draw the table (hen 1 plays rows, hen 2 plays columns):***

Now we see that for each hen laying 4 or 5 eggs is weakly dominated by any of the other strategies. So, we can redraw the table with the three strategies left:

	1	2	3	4	5
1	26; 26	21; 42	14; 42	5; 20	-5; -25
2	42; 21	28; 28	10; 15	-10; -20	-10; -25
3	42; 14	10; 15	-15; -15	-15; -20	-15; -25
4	20; 5	-20; -10	-20; -15	-20; -20	-20; -25
5	-25; -5	-25; -10	-25; -15	-25; -20	-25; -25

	1	2	3
1	26; 26	21; 42	14; 42
2	42; 21	28; 28	10; 15
3	42; 14	10; 15	-15; -15

Here we see that "laying 2 eggs" weakly dominates "laying 3 eggs" and the game is reduced to

	1	2
1	26; 26	21; 42
2	42; 21	28; 28

Now that "laying 3 eggs" is gone, "laying 2 eggs" strongly dominates "laying 1 egg", so it is the equilibrium.

ii. Is the resulting equilibrium Nash equilibrium?

Yes, since it survives the elimination of (weakly) dominated strategies. To check it is Nash equilibrium by definition, go back to the first table and see that if hen 1 chooses row 2, column 2 will be the best choice for hen 2, and vice versa.

iii. In the discrete version of the problem, how many eggs will each hen lay if they collude?

If hens collude, they choose the cell in the tables above, which has the maximum sum of number in it. In fact, there are two such cell (1 egg, 2 eggs) and (2 eggs, 1 egg).

<u>***Remark***</u>***: as you can guess, in the continuous version the collusive outcome will be unique.***

iv. What would prevent them from colluding in a one-shot interaction?

The problem with one-shot collusion is that it is not Nash equilibrium and thus is not sustainable. For example, if the payoffs are as in the tables below and hen 2 promised to lay 2 eggs, hen 1 will want to deviate, lay 2 eggs as well and get 28 instead of 21.

If the hens share the benefits and each gets 31.5 in the collusive equilibrium, hen 2 will have no incentive to give 10.5 to hen 1 in the situation above, because hen 1 cannot punish it once all the eggs are laid and sold.

v. Suppose now that the hens know that in the next period with probability 20% either of them (or both) can be slaughtered irrespective of the number of eggs it lays. Since the interaction between them is now repeated, they have two strategies available – either proceed as in (i), or use tit-for-tat strategy: collude if the other hen colluded in the previous period and proceed as in (i) forever if the other hen did not collude in the next period. Will the hens be able to collude? How high the probability of slaughtering has to be to make them indifferent between colluding and not colluding? Assume that profits in the cartel are shared equally.

This setup is a bit different from the standard one you can find in the textbook. The textbook version says that the deviation in the cartel will be to produce more and get more than half of the monopoly profit while the partner firm sticks to the cartel output. Here in under collusion each hen gets 31.5 and thus the hen, which lays 1 egg in the cartel will not want to lay one more and get 28 instead of 31.5.

However, the other hen will want to deviate, because each period it gives up 10.5 to keep the other hen in the cartel. If she does not deviate, she gets 31.5 forever, which yield expected payoff of $31.5/(1-0.8) = 157.5$***, and if it deviates, it gets 42 in the first period and 28 forever from the next period on, which gives expected payoff of*** $42 + 0.8 \cdot 28/(1-0.8) = 154$***. Thus, collusion is sustainable (the hens will collude).***

To find the threshold slaughtering probability p, after which the hens will not collude, equate the payoff from collusion $31.5/p$ ***and the payoff from the deviation*** $42 + (1-p) \cdot 28/p$***.***

$$42 + (1-p)\cdot\frac{28}{p} = \frac{31.5}{p} \Rightarrow 42p + 28 - 28p = 31.5 \Rightarrow p = \frac{3.5}{14} = 0.25$$

So, if the hens are slaughtered with probability 25% and more after each period, their perspectives are too gloomy to think strategically about the future and collude.

vi. Are there any other Nash equilibria?

Let us go back to the second table (reproduced below with best response strategies shown in bold). We see that not only (2 eggs, 2 eggs) appears in bold, which shows that it is Nash equilibrium, as we noticed in (ii), but also (1 egg, 3 eggs) and (3 eggs, 1 egg) are Nash equilibria. The problem with them is that one player plays a weakly dominated strategy in them. For example, hen 1 chooses row 1 and hen 2 chooses column 3. Deviating to column 2 will leave hen 2 as well off as it was playing column 3, so it does not deviate out of indifference.

	1	2	3
1	26; 26	21; **42**	**14**; **42**
2	**42**; 21	**28**; **28**	10; 15
3	**42**; **14**	10; 15	-15; -15

vii. Recompute the equilibrium in (i) assuming that the hens can lay non-integer number of eggs (that is, the hens can lay 4/3 of an egg or even $\sqrt{2}$ eggs)

Hen 1 solves $\max_{Q_1}(30 - (Q_1+Q_2)^2)\cdot Q_1 \Rightarrow (Q_1+Q_2)^2 + 2Q_1\cdot(Q_1+Q_2) = 30$

Hen 2 solves $\max_{Q_2}(30 - (Q_1+Q_2)^2)\cdot Q_2 \Rightarrow (Q_1+Q_2)^2 + 2Q_2\cdot(Q_1+Q_2) = 30$

$$\begin{cases} (Q_1+Q_2)^2 + 2Q_1\cdot(Q_1+Q_2) = 30 \\ (Q_1+Q_2)^2 + 2Q_2\cdot(Q_1+Q_2) = 30 \end{cases} \Rightarrow \begin{cases} Q_1 = Q_2 \\ 8Q_1 = 30 \end{cases} \Rightarrow Q_1 = Q_2 = \frac{\sqrt{15}}{2} \approx 1.94$$

The continuous case solution is very close to 2 – the outcome in the discrete game.

5 Bundling for Three

A campus bookstore carries three textbooks: in mathematics, in economics, and in psychology. The bookstore buys the textbooks from its supplier at $50 each. All students take mathematics, economics, and psychology. Each student buys at most one textbook for a course. The students in the problem are freshmen and know nothing about Amazon.com and Half.com, neither they are smart enough to buy a used textbook from older students, so the bookstore is a monopoly.

The value assigned by a student to a textbook depends on his/her major, as shown in the table below:

	Maths textbook	Econ textbook	Psy textbook
Maths major	$100	$80	$65
Econ major	$60	$90	$70
Psy major	$60	$80	$95

The number of mathematics, economics, and psychology majors in the university is 100, 200, and 150, respectively.

i. The current bookstore policy is to sell the economics and psychology textbooks as a bundle and offer the mathematics textbook separately. What prices should the bookstore charge? What will be the profit?

   The bookstore should charge $80 for the maths textbook and $165 for the bundle. It will make $9000 on maths textbooks and $22750 on the bundle, total $31750.

   To show that these prices are optimal, notice that the other two alternatives for the maths textbook are $65 and $100 and will bring the profit of $6750 and $5000, respectively. For the bundle, the other two alternatives are $120 and $170, bringing the profit of $9000 and $14000, respectively.

ii. Suppose the bookstore decides to give the students more freedom and begins to sell all textbooks separately. What is the optimal pricing policy and the corresponding profit? Give economic intuition for the change in the profit compared to (i).

The optimal pricing strategy will be to sell the maths textbook for $80, the economics textbook for $90, and the psychology textbook for $80. The respective profits will be $9000, $8000, and $10500, total $27500. It is smaller than $31750 in (i). Bundling the economics and psychology textbooks works because their valuations by the economics and psychology majors, who buy them, are negatively correlated. Unbundling makes the bookstore forego the profit it received in (i) from selling the economics textbooks to psychology majors.

To show that the individual prices of the economics and psychology textbooks are optimal, notice that for the economics textbook the other two alternatives is to sell at $60 or at $70, making $4500 and $7000, respectively. For the psychology textbook, the other two alternatives are selling at $60 and selling at $95, making $4500 and $6750, respectively.

iii. The dean of students visits the bookstore and says that what the students need is not the freedom, but a textbook for each course they take. So, the bookstore decides to sell all three textbooks as one bundle. What is the optimal pricing policy and the corresponding profit? Give economic intuition for the change in the profit compared to (i).

The optimal price for the big bundle is $220 and the profit is $31500. It is smaller than in part (i) because the new pricing policy makes the mathematics majors buy the two textbooks they do not really want (the bookstore would have only $10 profit margin on the economics and psychology textbooks sold to the mathematics majors, if it sold the textbooks separately to all students).

To show that the price is optimal, notice that the two other alternatives are $230 and $250, which give the profit of $28000 and $20000.