Solutions to Homework 2

AEC 504 - Summer 2007

Fundamentals of Economics

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1 Marionette Theater

Pinocchio and Harlequin established their own Marionette Theater and began competing with Fire Eater. At first, the admission fee was $5 in both theaters. After that, Pinocchio and Harlequin decided to undercut Fire Eater and reduced their admission fee to $4. Before the price cut, Fire Eater was selling 10000 tickets a year. He knows that the cross-price *arc elasticity* of demand for his tickets with respect to the admission fee in the competing theater is 1, and own-price *arc elasticity* of the demand is -2.

i. How many tickets a year will Fire Eater sell after the price cut?

I use subscript PH for Pinocchio and Harlequin and FE for Fire Eater, superscript 0 for the status quo price and quantity and 1 for the price and quantity after the price cut by Pinocchio and Harlequin.

$$E_{P_{PH}}(D_{FE}) = \frac{Q^1_{FE} - Q^0_{FE}}{(Q^1_{FE} + Q^0_{FE})/2} \cdot \frac{(P^1_{PH} + P^0_{PH})/2}{P^1_{PH} - P^0_{PH}} = \frac{Q^1_{FE} - 10,000}{Q^1_{FE} + 10,000} \cdot \frac{9}{-1} = 1 \Rightarrow \quad (1)$$

$$\Rightarrow -9Q^1_{FE} + 90,000 = Q^1_{FE} + 10,000 \Rightarrow Q^1_{FE} = 8,000 \quad (2)$$

<u>Remark</u>: The simplistic approach "OK, they cut their price by 20%, the elasticity is 1, so his sales drop by 20%, i.e. to 8000" works here only by chance (because the elasticity is 1) - to see it, change the elasticity to 2 in the calculations above or look at part b.

ii. By how much will he have to cut his price to sell the same number of tickets as before?

Now I introduce superscript 2 for the price and quantity after Fire Eater cuts his price.

$$\eta_{P_{FE}}(D_{FE}) = \frac{Q^2_{FE} - Q^1_{FE}}{(Q^2_{FE} + Q^1_{FE})/2} \cdot \frac{(P^2_{FE} + P^1_{FE})/2}{P^2_{FE} - P^1_{FE}} = \frac{P^2_{FE} + \$5}{P^2_{FE} - \$5} \cdot \frac{2}{18} = -2 \Rightarrow \quad (3)$$

$$\Rightarrow -18P^2_{FE} + \$90 = P^2_{FE} + \$5 \Rightarrow P^2_{FE} = \$85/19 \quad (4)$$

Remark: The simplistic approach here can take two routes "He needs to go up 25%, from 8000 back to 10000, the elasticity is -2, so he has to cut the price by 12.5%, i.e to 4.375" or "They cut the price by 20%, his demand is twice as sensitive to own price as it is to their price, so he has to cut the price by 10%, i.e. to 4.5". Those are the minimum and the maximum bound on the right answer, which is 4.47.

iii. Assuming that Fire Eater's theater is not full and the marginal cost of admitting an additional customer is 0, was Fire Eater right in his decision to sell 10000 tickets a year in the first place? If not, should he cut or increase the ticket price?

If the marginal cost of admitting an additional customer is 0, Fire Eater should maximize the revenue and have the own price elasiticity of -1. His elasticity is -2, suggesting that selling 10000 tickets per year is suboptimal. He can gain from cutting the price and increasing the sales.

iv. You can verify for yourself that as the cross-price elasticity increases, Fire Eater's sales will drop more and more, and he will have to undertake more and more serious price cuts to sell 10000 tickets a year after Pinocchio and Harlequin cut their price. What does the problem tell you about the importance of customer loyalty?

The cross-price elasticity shows the degree of customer loyalty – large cross-price elasticity means that customers very likely to switch to the competitor's product even if the competitor makes a small price cut, i.e. they are not brand-loyal. The problem shows you why Fire Eater would care about customers' loyalty: as you can verify, if the cross-price elasticity was 2, the answer to (i) would be $Q^1_{FE} = 70K/11 < 8K$ ***and the answer to (ii) would be*** $P^2_{FE} = \$4 < \$85/19$. ***So, the increase in the cross-price elasticity means that Fire Eater's sales will be more***

vulnerable to Pinocchio and Harlequin's price cut and he will have to cut price more in response to Pinocchio and Harlequin's price cut to maintain the same level of sales.

2 Cost Minimization

Consider a firm with production function $Y = K^{1/3}L^{1/3}$, which faces the cost of capital $r = 8$ and the wage of $w = 1$. Assume that the firm plans to produce $Y = 8$.

i. Find the optimal production plan (the cost-minimizing allocation of K and L).

$$\begin{cases} \min_{K,L}(8K + L) \\ s.t.\ K^{1/3}L^{1/3} = 8 \end{cases} \Rightarrow \begin{cases} \dfrac{\partial U/\partial K}{\partial U/\partial L} = \dfrac{MPK}{MPL} = \dfrac{L}{K} = \dfrac{8}{1} \\ K^{1/3}L^{1/3} = 8 \end{cases} \Rightarrow \begin{cases} L = 8K \\ K^{1/3}L^{1/3} = 8 \end{cases}$$

where MPK and MPL) are the marginal products of capital and labor. The optimal production plan is found by solving:

$$\begin{cases} L = 8K \\ K^{1/3}L^{1/3} = 8 \end{cases} \Rightarrow \begin{cases} L = 8K \\ 2K^{2/3} = 8 \end{cases} \Rightarrow \begin{cases} K = 8 \\ L = 64 \end{cases}$$

<u>Remark</u>: You can verify that you would get the same solution if you solved $\max_{K,L} K^{1/3}L^{1/3}$ $s.t.\ 8K + L = 128 = 8 \cdot 8 + 1 \cdot 64$***, that is, you can learn to solve a quantity maximization problem or a cost minimization problem. This nice property is called "duality".***

ii. What is the breakeven price of the output, that is, the price of Y at which the firm makes zero profit?

$$\Pi = p \cdot K^{1/3}L^{1/3} - 8K - L = 8p - 8 \cdot 8 - 1 \cdot 64 = 0 \Rightarrow p = 16 \tag{5}$$

iii. What is the average (marginal) product of labor (L) (APL & MPL) when the input mix is the one in (i)? And the average (marginal) product of capital (K) (APK & MPK)? Clearly and concisely, please explain how you would interpret these four numbers.

$$APL \equiv \frac{Y}{L} = \frac{K^{1/3}}{L^{2/3}} = \frac{8^{1/3}}{64^{2/3}} = \frac{2}{16} = \frac{1}{8}; \quad MPL \equiv \frac{\partial Y}{\partial L} = \frac{1}{3} \cdot \frac{K^{1/3}}{L^{2/3}} = \frac{1}{24} \tag{6}$$

$$APK \equiv \frac{Y}{K} = \frac{L^{1/3}}{K^{2/3}} = \frac{64^{1/3}}{8^{2/3}} = \frac{4}{4} = 1; \quad MPK \equiv \frac{\partial Y}{\partial K} = \frac{1}{3} \cdot \frac{L^{1/3}}{K^{2/3}} = \frac{1}{3} \tag{7}$$

The average product of labor (capital) is the average output generated by one unit of labor (capital). The marginal product of capital (labor) is the change in total output associated with a one (infinitesimal) unit change in capital (labor), holding labor (capital) fixed. So, for example, $MPK = 1/3$ ***tells you that if you increase*** K ***by*** ϵ***,*** Y ***will increase by*** $\epsilon/3$***.***

iv. What happens to the value of marginal product of labor (MPL) at the optimal production plan as w increases, assuming that r stays constant? What happens with the average product (APL)? What happens to MPK and APK? What is the economic intuition behind the changes?

As w ***increases, the firm chooses to employ less labor and more capital.***

$$\begin{cases} \min\limits_{K,L}(8K + wL) \\ s.t.\ K^{1/3}L^{1/3} = 8 \end{cases} \Rightarrow \begin{cases} wL = 8K \\ K^{1/3}L^{1/3} = 8 \end{cases} \Rightarrow \begin{cases} K = 8w^{1/2} \\ L = 64/w^{1/2} \end{cases}$$

You can see from formulae in (iii) that if L decreases and K increases, the numerators in MPL and APL increase and the denominators decrease, which means that both ratios (MPL and APL) increase. The opposite is true for MPK and APK.

The intuition is that a unit of labor gets more productive if labor becomes scarce (the law of diminishing returns) and capital becomes abundant (complementarity between K and L, which can be inferred from $\partial^2 U/\partial K \partial L > 0$***). Conversely, a unit of capital gets less productive if capital becomes abundant and labor becomes scarce.***

v. What are the cost shares of labor and capital $\frac{rK}{C}$ and $\frac{wL}{C}$? Do they depend on the relative price of K and/or L?

$$K = 8,\ L = 64 \Rightarrow C = 8 \cdot 8 + 1 \cdot 64 = 128 \Rightarrow \frac{rK}{C} = \frac{wL}{C} = \frac{1}{2} \tag{8}$$

The cost shares do not depend on r ***and*** w***, since you can see from the FOC that for arbitrary values of*** r ***and*** w

$$\frac{\partial U/\partial K}{\partial U/\partial L} = \frac{MPK}{MPL} = \frac{L}{K} = \frac{r}{w} \Rightarrow wL = rK \Rightarrow \frac{rK}{C} = \frac{wL}{C} = \frac{1}{2} \tag{9}$$

Alternatively, you could just say that it is the property of the Cobb-Douglas function we discussed so much when we talked about consumer choice.

vi. Find the factor demands and the cost function for this firm

First, plug the FOC into the production function and rearrange it to express K and L in terms of the output and factor prices, which will give you the factor demands:

$$\begin{cases} Y = K^{1/3}L^{1/3} \\ wL = rK \end{cases} \Rightarrow \begin{cases} Y = (w/r)^{1/3}L^{2/3} \\ Y = (r/w)^{1/3}K^{2/3} \end{cases} \Rightarrow \begin{cases} K = (\frac{w}{r})^{1/2}Y^{3/2} \\ L = (\frac{r}{w})^{1/2}Y^{3/2} \end{cases}$$

Now, plug the factor demands into the definition of cost. The result is the cost function

$$C \equiv rK + wL \Rightarrow C = 2Y^{3/2}r^{1/2}w^{1/2} \tag{10}$$

NB: In the consumer problem, we would call this function expenditure function.

vii. Does this production function has decreasing, constant, or increasing return to scale? Does the firm has increasing or decreasing marginal costs? Would your answer change if its production function was $Y = K^{2/3}L^{2/3}$?

The production function has decreasing return to scale:

$$Y(tK, tL) = (tK)^{1/3}(tL)^{1/3} = t^{2/3}K^{1/3}L^{1/3} < tY(K, L)\forall t > 1 \tag{11}$$

The marginal cost for this cost function is increasing:

$$MC = \frac{dC}{dY} = 3Y^{1/2}r^{1/2}w^{1/2}; \quad \frac{dMC}{dY} = \frac{d^2C}{dY^2} = \frac{3}{2} \cdot \frac{r^{1/2}w^{1/2}}{Y^{1/2}} > 0 \tag{12}$$

You can check for yourself that $Y = K^{2/3}L^{2/3}$ has increasing return to scale. For $Y = K^{2/3}L^{2/3}$ the FOC will still imply $wL = rK$ (it is a Cobb-Douglas function with equal power coefficients for K and L). We can repeat the derivation from (vi):

$$\begin{cases} Y = K^{2/3}L^{2/3} \\ wL = rK \end{cases} \Rightarrow \begin{cases} Y = (w/r)^{2/3}L^{4/3} \\ Y = (r/w)^{2/3}K^{4/3} \end{cases} \Rightarrow \begin{cases} K = (\frac{w}{r})^{1/2}Y^{3/4} \\ L = (\frac{r}{w})^{1/2}Y^{3/4} \end{cases}$$

$$C \equiv rK + wL \Rightarrow C = 2Y^{3/4}r^{1/2}w^{1/2} \tag{13}$$

The marginal cost for this cost function is decreasing:

$$MC = \frac{dC}{dY} = \frac{3}{2} \cdot Y^{-1/4}r^{1/2}w^{1/2}; \quad \frac{dMC}{dY} = \frac{d^2C}{dY^2} = -\frac{3}{8} \cdot \frac{r^{1/2}w^{1/2}}{Y^{5/4}} < 0 \tag{14}$$

A more general conclusion is true: if a production function has decreasing returns to scale, the cost function associated with it has increasing marginal cost. And vice versa, if a production function has increasing returns to scale, the cost function associated with it has decreasing marginal cost.

3 Profit Function

Consider a firm, which produces only one output according to the production function $20x - x^2$, where x is the amount of the only input used. The firm gets \$$p$ for each unit of the output it produces and pays \$$w$ for each unit of the input it uses.

i. Set up the profit function for this firm

$$\Pi = p \cdot (20x - x^2) - w \cdot x \tag{15}$$

ii. Find the amount of the input the firm should use to get the maximum possible profit. Under what values of w will the firm produce nothing (that is, the optimal x will be equal to 0)?

The optimal amount of input is found by solving:

$$\max_x p \cdot (20x - x^2) - w \cdot x \Rightarrow MR = MC \Rightarrow p \cdot (20 - 2x) = w \Rightarrow x = 10 - \frac{w}{2p} \tag{16}$$

$$x = 10 - \frac{w}{2p} \leq 0 \Rightarrow w \geq 20p \tag{17}$$

Note that since the firm cannot use negative amount of the input (which would effectively mean producing it), all values of w ***greater than*** $20p$ ***will also make it shut the production down (employ*** $x = 0$***).***

iii. Find the factor demand function

Since in (ii) we solved the problem for arbitrary values of* p *and* w*,* $x = 10 - \frac{w}{2p}$ *is the factor demand function.

iv. Assume $w = 10$ and $p = 1$. Suppose that the government first levied the 32% profit tax. Now it is thinking about replacing the profit tax by the tax on x, which effectively means increasing the price of x from $\$w$ to $\$(w + t)$. What tax rate t will the government have to set to get the same revenue as before? How does social welfare changes after the profit tax is replaced by the tax on x?

Hint #1: The social welfare is the sum of the profit the firm makes and the tax revenue. Do not care about the consumers - they are "absent" in the problem.

Hint #2: The government gets the same revenue from both taxes if $tx_2 = 0.32\Pi_1$*, where* x_2 *is the optimal amount of input after the tax on* x *is levied, and* Π_1 *is the maximum profit under the profit tax.*

Under the profit tax, the firm solves

$$\max_x (1 - 0.32) \cdot \{p \cdot (20x - x^2) - w \cdot x\} \Rightarrow x = 10 - \frac{w}{2p} \tag{18}$$

$$x = 10 - \frac{w}{2p},\ w = 10,\ p = 1 \Rightarrow\ x = 5 \Rightarrow\ \Pi = 1 \cdot (20 \cdot 5 - 5^2) - 10 \cdot 5 = 25 \tag{19}$$

Here $\Pi = SW$***, since under the profit tax the government and the firm just divide the before-tax profit between themselves.***

Under the tax on x the firm solves

$$\max_x p \cdot (20x - x^2) - (w + t) \cdot x \Rightarrow x = 10 - \frac{w + t}{2p} \tag{20}$$

$$x = 10 - \frac{w + t}{2p},\ w = 10,\ p = 1 \Rightarrow x = 5 - \frac{t}{2} \tag{21}$$

$$tx_2 = 0.32\Pi_1 \Rightarrow t \cdot \frac{10 - t}{2} = 0.32 \cdot 25 \Rightarrow t^2 - 10t + 16 = 0 \Rightarrow t = 2 \textit{ or } t = 8 \tag{22}$$

$$t = 2 \Rightarrow x = 4 \Rightarrow SW = \Pi + tx = 20 \cdot 4 - 4^2 - 12 \cdot 4 + 2 \cdot 4 = 24 \tag{23}$$

$$t = 8 \Rightarrow x = 1 \Rightarrow SW = \Pi + tx = 20 \cdot 1 - 1^2 - 18 \cdot 1 + 8 \cdot 1 = 9 \tag{24}$$

So, the government will choose t=2, as it yields higher SW. Yet, this SW=24 is lower than SW=25 under the profit tax, hence the profit tax

is better. The reason it is better is that it is nondistorting, i.e. it does not influence the amount of the input the firm employs and the amount of the output it produces.

4 Entry and Exit in Competitive Markets

Consider a bunch of firms with the common cost function $TC = 200 + 5Q + Q^2$, which exist in a competitive environment. The price at which they can sell the product is \$25.

i. What is MC for these firms? What are VC and FC? What is AVC?

$$MC = \frac{dTC}{dQ} = 5 + 2Q;\ VC = 5Q + Q^2;\ FC = 200;\ AVC = \frac{VC}{Q} = 5 + Q$$

ii. What is the optimal (profit-maximizing) quantity for each firm?

$$TR = PQ = 25Q \Rightarrow MR = 25;\ MC = MR \Rightarrow 25 = 5 + 2Q \Rightarrow Q = 10$$

iii. What profit each firm gets at the optimal quantity? In the short run, will they stay in the business or leave it?

$$\Pi = TR - TC = 25 \cdot 10 - 200 - 5 \cdot 10 - 10^2 = -100;\ AVC = 5 + 10 < p = 25$$

They will stay in the business even though they make a loss, because leaving it (producing Q=0) is more costly – it actually brings a loss of -200.

iv. What is the quantity each firm will produce in the long run?

In the long run

$$AC = MC \Rightarrow 5 + 2Q = \frac{200}{Q} + 5 + Q \Rightarrow Q = \frac{200}{Q} \Rightarrow Q = 10\sqrt{2} \approx 14.14$$

v. What happens to the price, the size of a firm, the number of firms in the industry, and their total production in the long run, compared to the short run?

Hint: You do not have to calculate the exact numbers here, just write down the intuition based on the calculations you did so far.

In the long run $AC = MC = p \Rightarrow p = 33.28$ ***- the price increases. Q rises means that each firm which stays in the industry will be bigger (and more efficient since it reaches the minimum efficient scale). The number of firms drops, since the profit was negative in the short run and so some of them will choose to exit. The total production drops as well because the aggregate market demand curve is negatively sloped and consumers will choose to consume less at the higher prices.***

Remark: You have to understand clearly the difference between the totally flat demand curve each competitive firm faces and the negatively sloped aggregate demand curve. The flat demand for each firm only shows that under perfect competition a firm is a price takes and cannot charge more than their competitors do, since nobody will purchase from it is it does. The aggregate demand is negatively sloped because it reflects consumers' preferences and is not influenced by the market structure.

5 Birthrate and Economic Cynicism

Suppose that both men and women want to have children, and the supply side is taken by Nature, who allows any couple to have as many children as they want at a certain price, determined by the standards of living and the opportunity costs of having a baby. However, both men and women have other interests beyond having children. Discuss the impacts of the following on the birthrate:

i. The increased share of women in the work force (allegedly driven by reduced wage differential between women and men)

The increased wages for women have two effects. First, it raises the

opportunity costs of having a baby (foregone earnings of the wife). The supply curve shifts upwards and the equilibrium birthrate falls. On the other hand, when the wife's salary increases, the family will become wealthier and will want more children (again assuming that children are a normal good). So, the demand curve will move up, and the equilibrium birthrate will rise. The overall effect on the equilibrium birthrate is ambiguous.

ii. The increase in college enrollment (both for males and females)

Increase in male college enrollment will have two effects on the demand for children. More men of college age will be studying instead of working, so they will not be able to support a family. It reduces the demand for babies, so the birthrate falls. The second effect is that men are likely to be paid more after college. Assuming that children are a normal good, they will want more of them. The demand curve for children will therefore move upwards, and the birthrate rises. Overall, since we do not know, which effect is stronger, we conclude that the influence of increased college enrollment for men has an ambiguous impact on the birthrate.

Increase in female college enrollment has the same impact as increase in male college enrollment, but for the fact that the increased women's wage after college will also cause the supply curve shift upwards, because the costs of having a baby will rise. So, we have three effects now, two of which are negative (reduced demand for babies for women of college age and increased costs of having a baby after college) and one is positive (increased wealth after college). The overall effect is again ambiguous.

iii. Economy-wide recession

A recession has two impacts. First, families become less wealthy and want less children. Second, the opportunity cost of having children goes down - say, if one parent is unemployed, he or she can take care of the baby, and the family will save on kindergarten and babysitters. So, both

the demand and supply curves go down, and the effect on the equilibrium birthrate is ambiguous.

iv. The stipends for would-be parents

The stipend for would-be parents moves the supply curve downwards (the costs of having a baby are reduced). The demand curve moves upwards, since would-be parents are wealthier after the stipend. So, the birthrate will surely increase.

v. Relaxed immigration policy (assume here that immigrants have the same standards of living as natives)

Increasing immigration is likely to create some tension in the labor market, where labor supply will increase, and labor demand will stay the same, causing equilibrium wages to drop. So, the impact of increased immigration is the same as the impact of slumping economy discussed above - we cannot tell where the equilibrium birthrate goes.

Two short remarks are of order. First, notice that immigration per se does not moves the demand for children curve, since our analysis is in terms of birthrate, not the number of children born, so it does not matter that the number of couples increases as immigration increases. Second, notice the importance of our assumption that immigrants are exactly the same as the natives. In many countries, immigration increases the birthrate for two reasons. First, the immigrants have lower standards of living (the price of having a baby is lower for them) and have higher demand for children due to their cultural background. Second, the immigrants mostly supply unqualified labor, of which many economies are short, so the increased immigration produces less tension in the labor market.